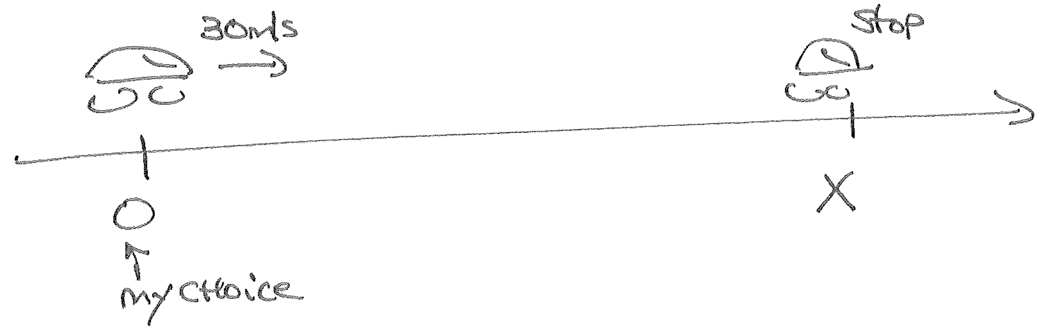
Physics 160, Constant Acceleration Examples

Example I



How long and how far while stopping.

KNOWN: $X_0 = 0$, $V_{x0} = 30\,m/s$, $a_x = -2.5\,m/s^2$, $V = 0$ ← stops

↑ opposite to velocity for decreasing speed

UNKNOWN: t, x

$V_x = V_{0x} + a_x t \Rightarrow 0 = 30\,m/s - (2.5\,m/s^2)t \Rightarrow t = \frac{30\,m/s}{2.5\,m/s^2} = 12s$

Now $X = X_0 + V_{0x}t + \frac{1}{2}a_x t^2 \Rightarrow X = 0 + 30\,m/s(12s) + \frac{1}{2}(-2.5\,m/s^2)(12s)^2$

↳ $12s \times 12s$

$\Rightarrow X = 360m + \frac{1}{2}(-2.5\,m/s^2)(144s^2) = 360m - 180m = 180m$

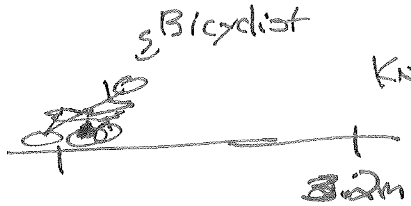
AS A VERIFICATION: $V^2 = V_{0x}^2 + 2a_x(X - X_0)$

$$\Rightarrow 0 = (30 m/s)^2 + 2(-2.5 m/s^2)(X - 0)$$

$$\Rightarrow X = \frac{-(30 m/s)^2}{2(-2.5 m/s^2)} = \frac{-900 m^2/s^2}{-5 m/s^2} = 180 m$$

Unit: $\frac{m^2}{s^2} \times \frac{s^2}{m} = m$

<u>BONUS</u>: PROBLEM-SOLVING EXERCISE



KNOWN: $V_{0x} = 0$, $a_x = 0.67 m/s^2$, $X_0 = 0$, $X = 3.2 m$

UNKNOWN: t, V_x

$$X = X_0 + V_{0x}t + \frac{1}{2}a_x t^2 \Rightarrow 3.2 m = 0 + 0 + \frac{1}{2}(0.67 m/s^2)t^2$$

$$\Rightarrow t = \sqrt{\frac{2(3.2 m)}{0.67 m/s^2}} = \sqrt{9.55225 s^2} = 3.09 s$$

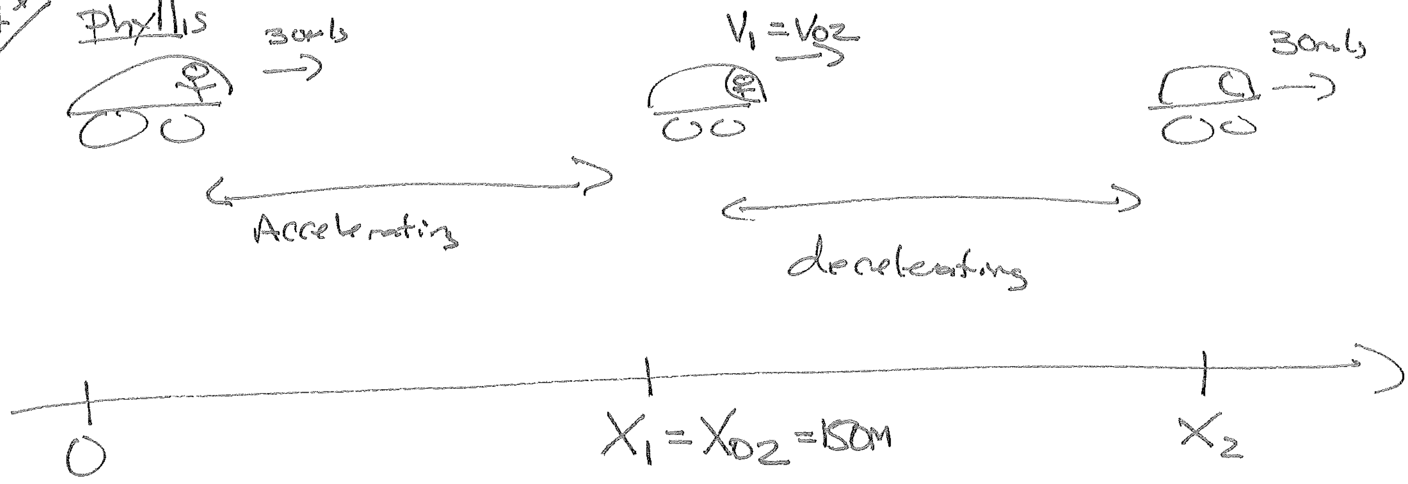
Unit: $m \times \frac{s^2}{m} = s^2$

$$V_x = V_{0x} + a_x t \Rightarrow V_x = 0 + (0.67 m/s^2)(3.09 s) = 2.07 m/s$$

OR $V_x^2 = V_{0x}^2 + 2a_x(X - X_0) \Rightarrow V_x^2 = 0^2 + 2(0.67 m/s^2)(3.2 m)$

$$\Rightarrow V_x = \sqrt{4.288 m^2/s^2} = 2.07 m/s$$

Ex II:



Two Motions, the First leading Directly into the Second

$\Rightarrow$ Final Position of First = initial position of 2^{ND}

Also, Final Velocity of First = initial Velocity of 2^{ND}

1^{st} Motion: Known: $X_{01} = 0$, $V_{01} = 30m/s$, $a_1 = 1.25m/s^2$, $X_1 = 150m$

Unknown: V_1, t_1

2^{ND} Motion: Known: $X_{02} = 150m$, $V_2 = 30m/s$, $t_2 = 5s$

Unknown: V_{02}, a_2, X_2

From 1^{st} Motion: $V_1^2 = V_{01}^2 + 2a_1(X_1 - X_{01}) \Rightarrow V_1^2 = (30m/s)^2 + 2(1.25m/s^2)(150m)$

$$\Rightarrow V_1^2 = 900m^2/s^2 + 375m^2/s^2 = 1275m^2/s^2$$

$$\Rightarrow V_1 = \sqrt{1275m^2/s^2} = 35.7m/s$$

Now: $V_1 = V_{01} + a_1 t_1 \Rightarrow 35.7\,m/s = 30\,m/s + (1.25\,m/s^2)t_1$

$$\Rightarrow t_1 = \frac{35.7\,m/s - 30\,m/s}{1.25\,m/s^2} = \frac{5.7\,m/s}{1.25\,m/s^2} = 4.56\,s$$

Now look at second motion, with new knowns: $V_{02} = 35.7\,m/s$

All equations have acceleration in them, so

$$V_2 = V_{02} + a_2 t_2 \Rightarrow 30\,m/s = 35.7\,m/s + a_2(5s)$$

$$\Rightarrow a_2 = \frac{30\,m/s - 35.7\,m/s}{5s} = \frac{-5.7\,m/s}{5s} = -1.14\,m/s^2$$

$$X_2 = X_{02} + V_{02}t_2 + \tfrac{1}{2}a_2 t_2^2 \Rightarrow X_2 = 150m + (35.7\,m/s)(5s) + \tfrac{1}{2}(-1.14\,m/s^2)(5s)^2$$

$$\Rightarrow X_2 = 150m + 178.5m - 14.25m$$

$$= 314.25m$$

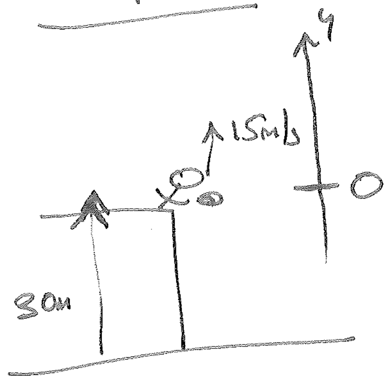
Finally: Total Elapsed time?

Remember t_1 = elapsed time for 1st motion

t_2 = elapsed time for 2nd

$$\Rightarrow t_{total} = t_1 + t_2 = 4.56s + 5s = 9.56s$$

Example III



KNOWN: $V_{0y} = +15\,m/s$ ← up is positive

gravity is DOWN $\Rightarrow a_y = -g = -9.8\,m/s^2$

- How fast after 3s? $t = 3s$

$V_y = V_{0y} + a_y t \Rightarrow V_y = 15\,m/s - 9.8\,m/s^2(3s) = 15\,m/s - 29.4\,m/s$

$= -14.4\,m/s$

$= 14.4\,m/s$ DOWN

- How High?

Now KNOWN: $V_{0y} = 15\,m/s$, $a_y = -9.8\,m/s^2$, $V_y = 0$ at MAX height, $y_0 = 0$

UNKNOWN: y, t

$V_y^2 = V_{0y}^2 + 2a_y(y - y_0) \Rightarrow 0^2 = (15\,m/s)^2 + 2(-9.8\,m/s^2)(y - 0)$

$\Rightarrow y = \frac{-(15\,m/s)^2}{2(-9.8\,m/s^2)} = \frac{-225\,m^2/s^2}{-19.6\,m/s^2} = +11.4796\,m = 11.5\,m$

- How long to hit GROUND?

IF we Keep $y_0 = 0$, $y = -30\,m$, $V_{0y} = 15\,m/s$, $a_y = -9.8\,m/s^2$

UNKNOWN: V_y, t

Note: $V_y \neq 0$ since this is the instant Before it Hits the GROUND.

$$y = y_0 + V_{0y}t + \frac{1}{2}a_y t^2$$

$$\Rightarrow -30m = 0 + (15m/s)t + \frac{1}{2}(-9.8m/s^2)t^2$$

$$\Rightarrow -30m = (15m/s)t - 4.9m/s^2 t^2$$ ← QUADRATIC EQUATION

$$\Rightarrow +4.9m/s^2 t^2 - (15m/s)t - 30m = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-15m/s) \pm \sqrt{(15m/s)^2 - 4(4.9m/s^2)(-30m)}}{2(4.9m/s^2)}$$

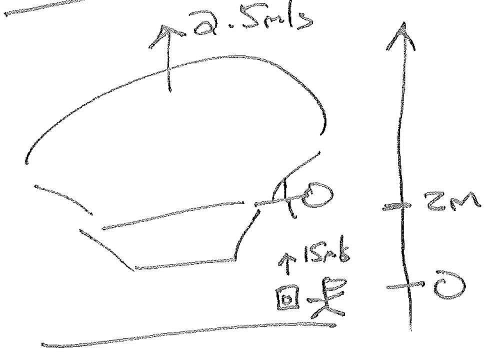
$$= \frac{+15m/s \pm \sqrt{225m^2/s^2 + 588m^2/s^2}}{9.8m/s^2} = \frac{+15m/s \pm \sqrt{813m^2/s^2}}{9.8m/s^2}$$

$$\Rightarrow t = \frac{15m/s + \sqrt{813m^2/s^2}}{9.8m/s^2} = \frac{43.513m/s}{9.8m/s^2} = 4.44s$$

$$\text{OR } t = \frac{15m/s - \sqrt{813m^2/s^2}}{9.8m/s^2} = \frac{-13.513m/s}{9.8m/s^2} = -1.38s$$

Not physical, so Reject

Example IV



Two moving objects ⇒ separate lists

FOR MAN in BALLOON: $y_{0M} = 2m$, $V_{0M} = 2.5 m/s$

MAN

$a_M = 0$

For Camer: $y_{0C} = 0$, $V_{0C} = 15 m/s$

Camera

$a_C = -9.8 m/s^2$

FOR MAN to Catch Camera, They Have to be at the SAME PLACE: $y_M = y_C$

$$y = y_0 + V_{0y}t + \tfrac{1}{2}a_y t^2 \Rightarrow y_M = 2m + (2.5m/s)t + 0$$

$$y_C = 0 + (15m/s)t + \tfrac{1}{2}(-9.8m/s^2)t^2$$

$y_M = y_C$ gives EQN. for t: $2m + (2.5m/s)t = (15m/s)t - 4.9m/s^2\,t^2$

$$\Rightarrow +4.9m/s^2\,t^2 + (2.5m/s)t - (15m/s)t + 2m = 0$$

$$\Rightarrow +4.9m/s^2\,t^2 + [(2.5m/s) - (15m/s)]t + 2m = 0$$

$$\Rightarrow 4.9m/s^2\,t^2 - (12.5m/s)t + 2m = 0$$

QUADRATIC EQN: $t = \frac{-(-12.5 m/s) \pm \sqrt{(12.5 m/s)^2 - 4(4.9 m/s^2)(2m)}}{2(4.9 m/s^2)}$

$$\Rightarrow t = \frac{+12.5 m/s \pm \sqrt{117.05 m^2/s^2}}{9.8 m/s^2}$$

$$\Rightarrow t = \frac{12.5 m/s + \sqrt{117.05 m^2/s^2}}{9.8 m/s^2} \approx \frac{23.319 m/s}{9.8 m/s^2} = 2.38 s$$

$$\text{or } t = \frac{12.5 m/s - \sqrt{117.05 m^2/s^2}}{9.8 m/s^2} = \frac{1.681 m/s}{9.8 m/s^2} = 0.172 s$$

Two ~~e~~physical times because MAN gets two chances! First time at $t = 0.172 s$, if he misses BALLOON will catch CAMERA again because it is slowing down while BALLOON is rising. 2ND time $t = 2.38 s$

Looking at 1st time (MAN paying attention)

$$y_M = 2m + (2.5 m/s)(0.172 s) = 2.43 m$$

$$y_C = (15 m/s)(0.172 s) - 4.9 m/s^2 (0.172 s)^2 = 2.43 m$$