April 24, Week 14

Today: Chapter 13, Kepler's Laws

Exam #4, Next Friday, April 26

Practice Exam on Website.

Review Sessions: Thursday, April 25, 5:15PM, 114 Regener Hall

After the exam, we will start chapter 14.

Kepler's Laws

Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

Kepler's Laws

Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

Kepler's Laws:

1: Each planet's orbit traces out the shape of an ellipse with the sun located at one focus.

2: The imaginary line from the sun to a planet sweeps out equal areas in equal times.

3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.

Ellipses - The Geometric Approach

Ellipse - ovals.

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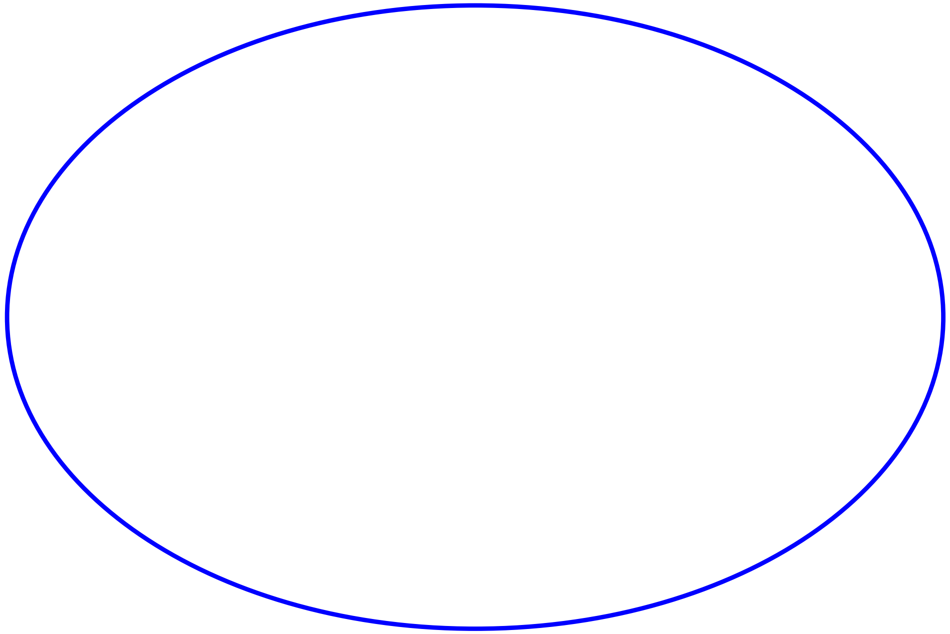
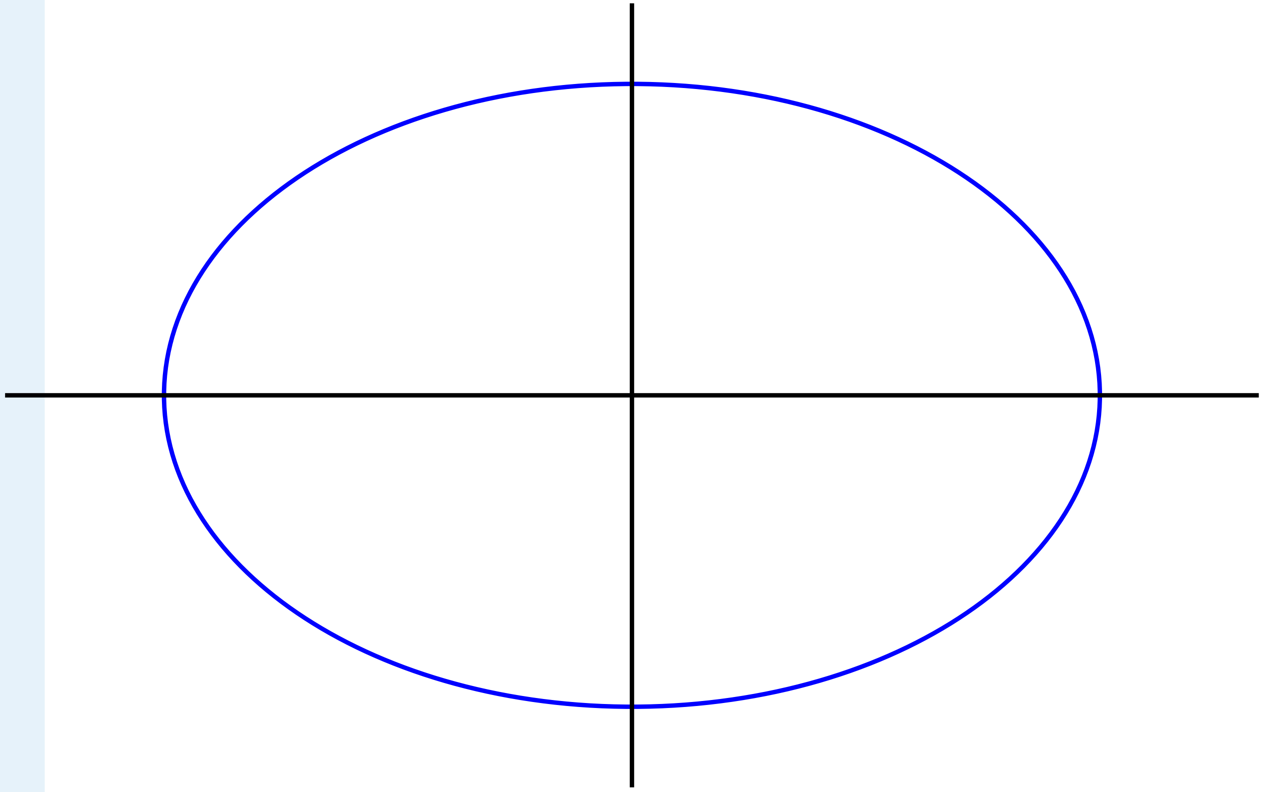
Ellipse - ovals.



Ellipses - The Geometric Approach

Ellipse - ovals.



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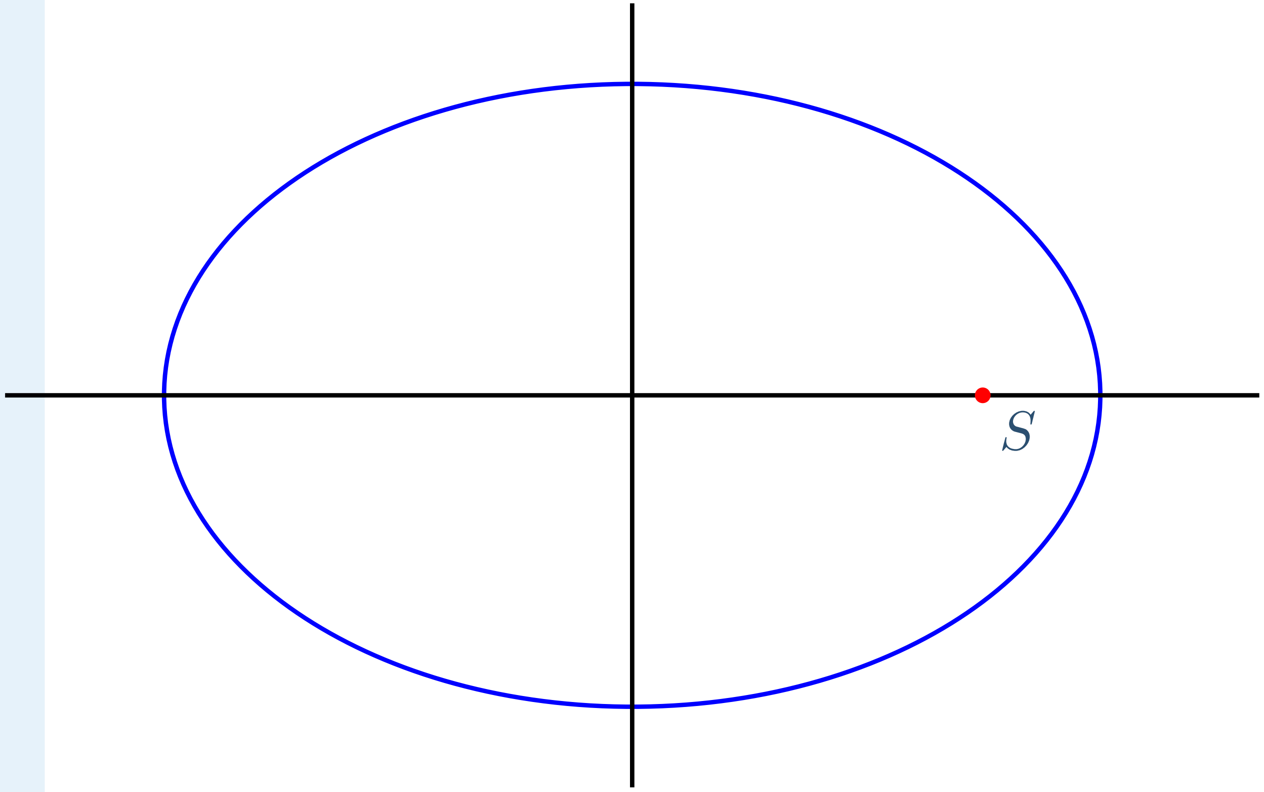

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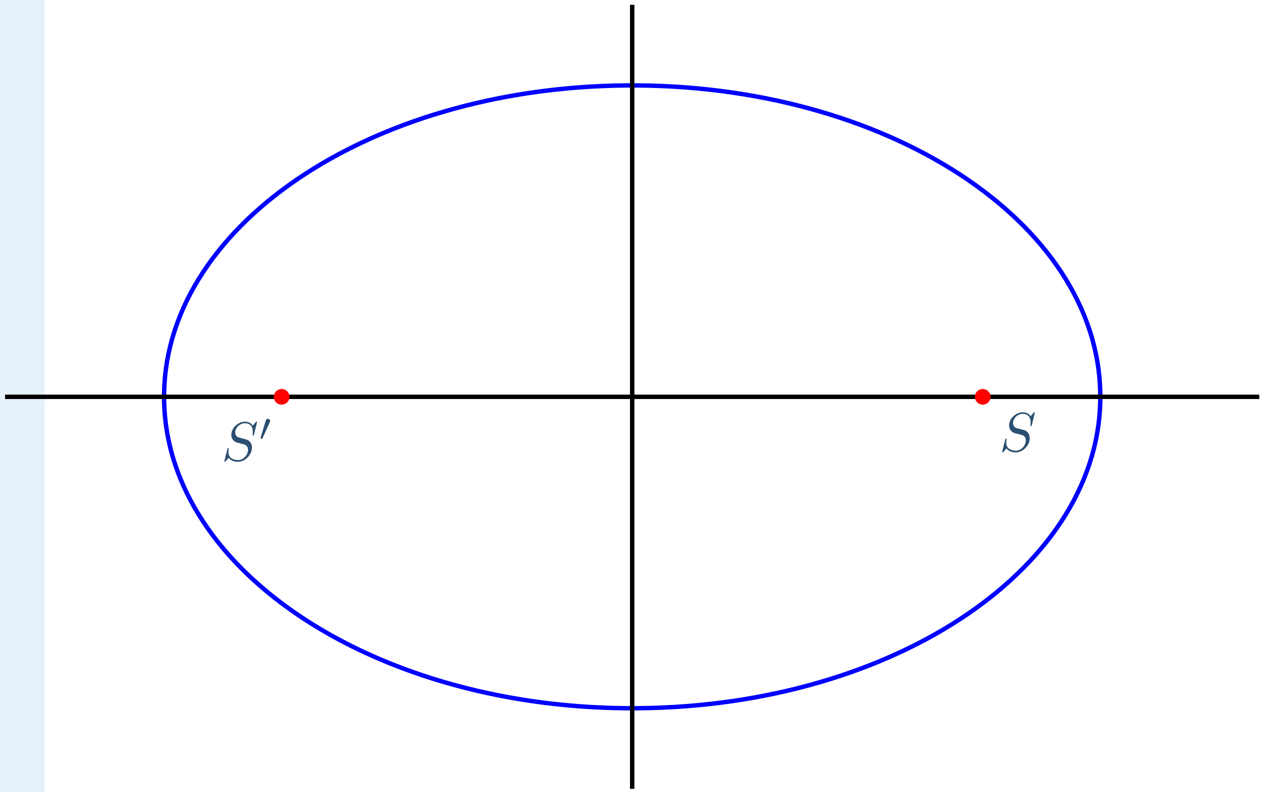

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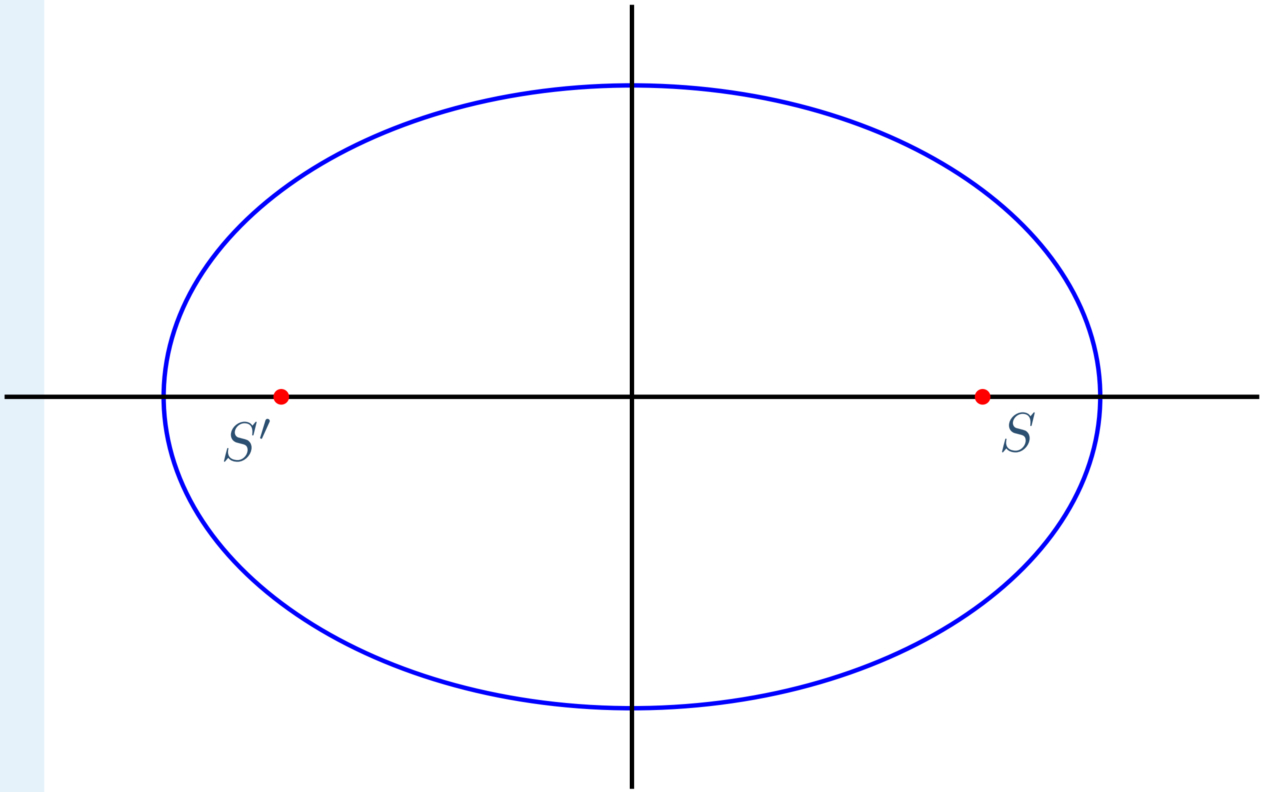

S, S' - foci

Ellipses - The Geometric Approach

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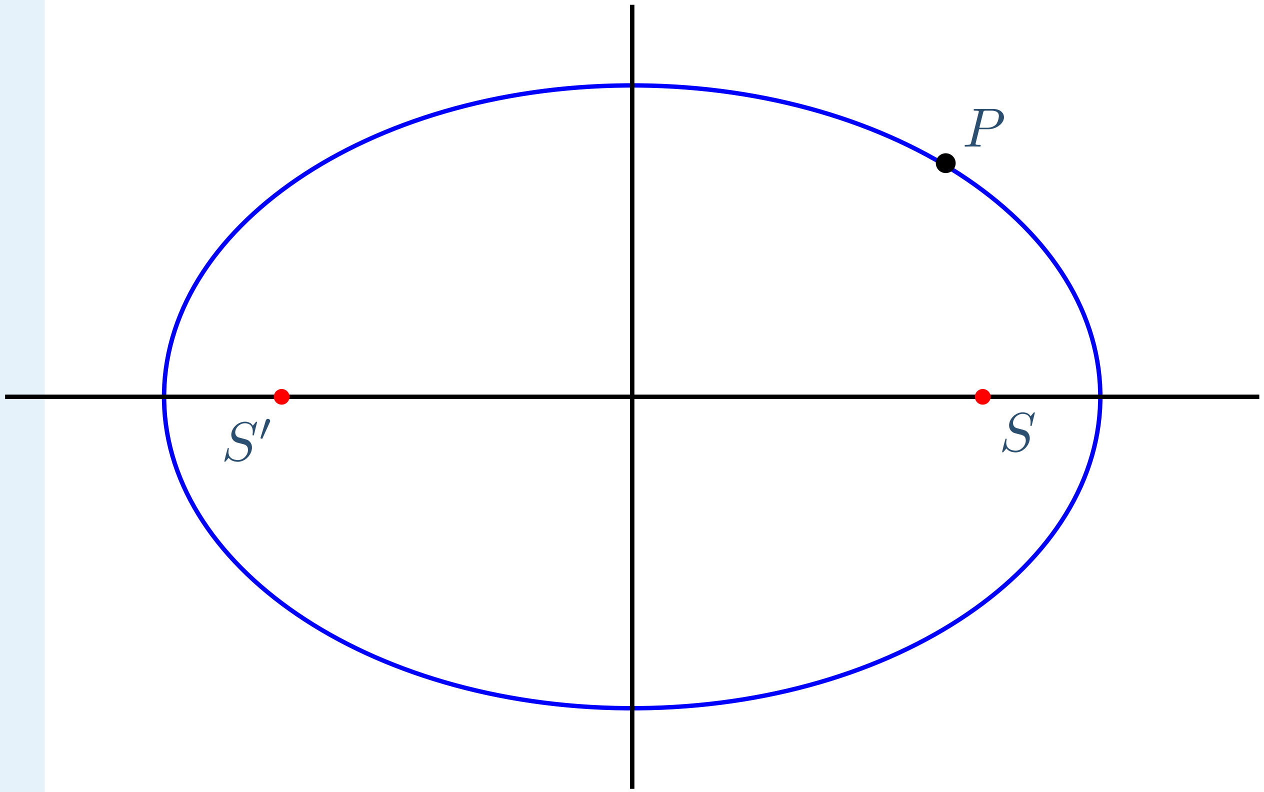

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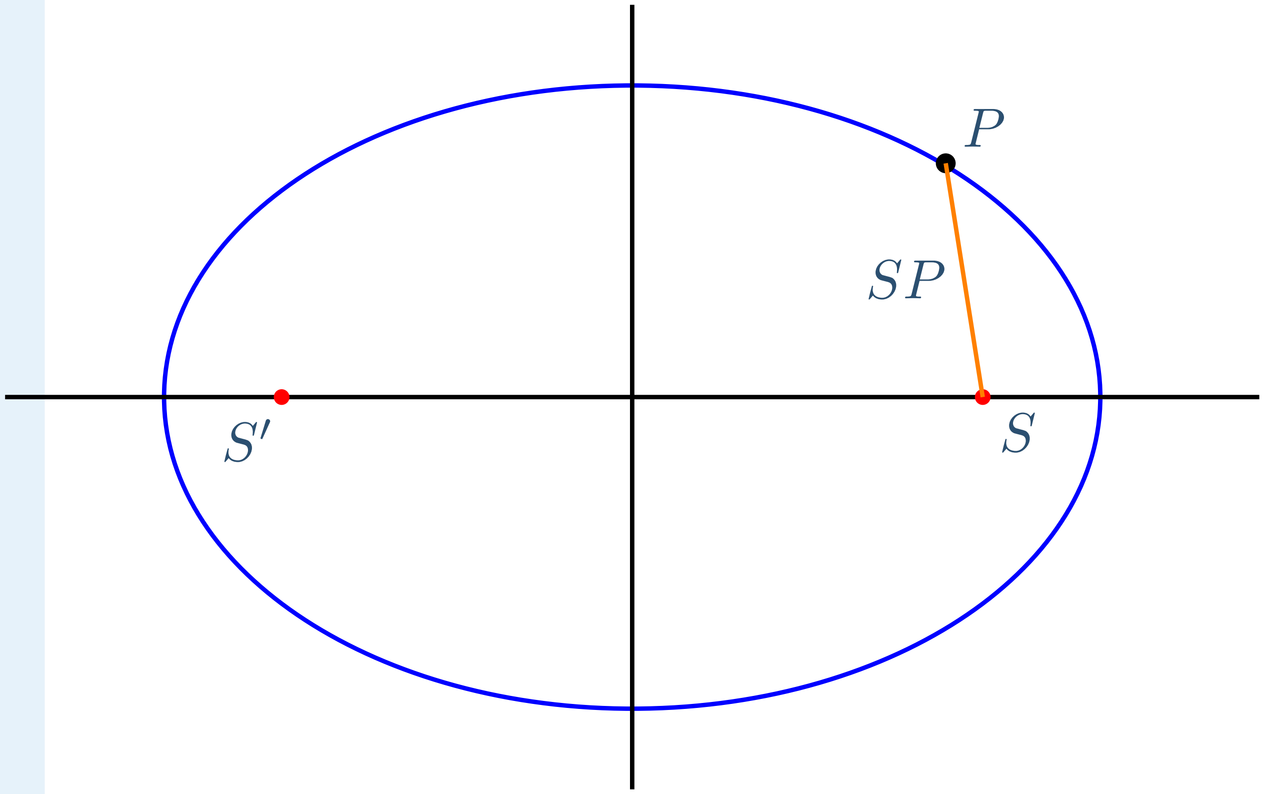

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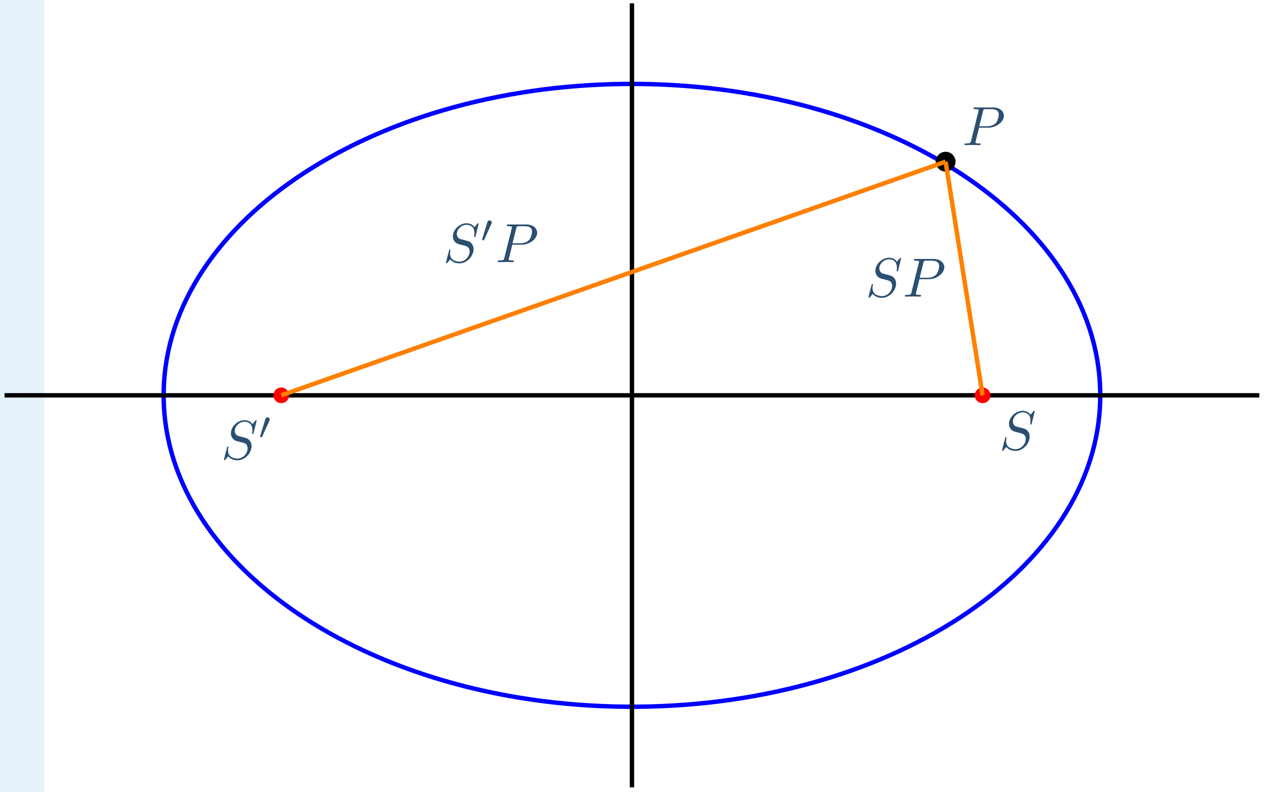

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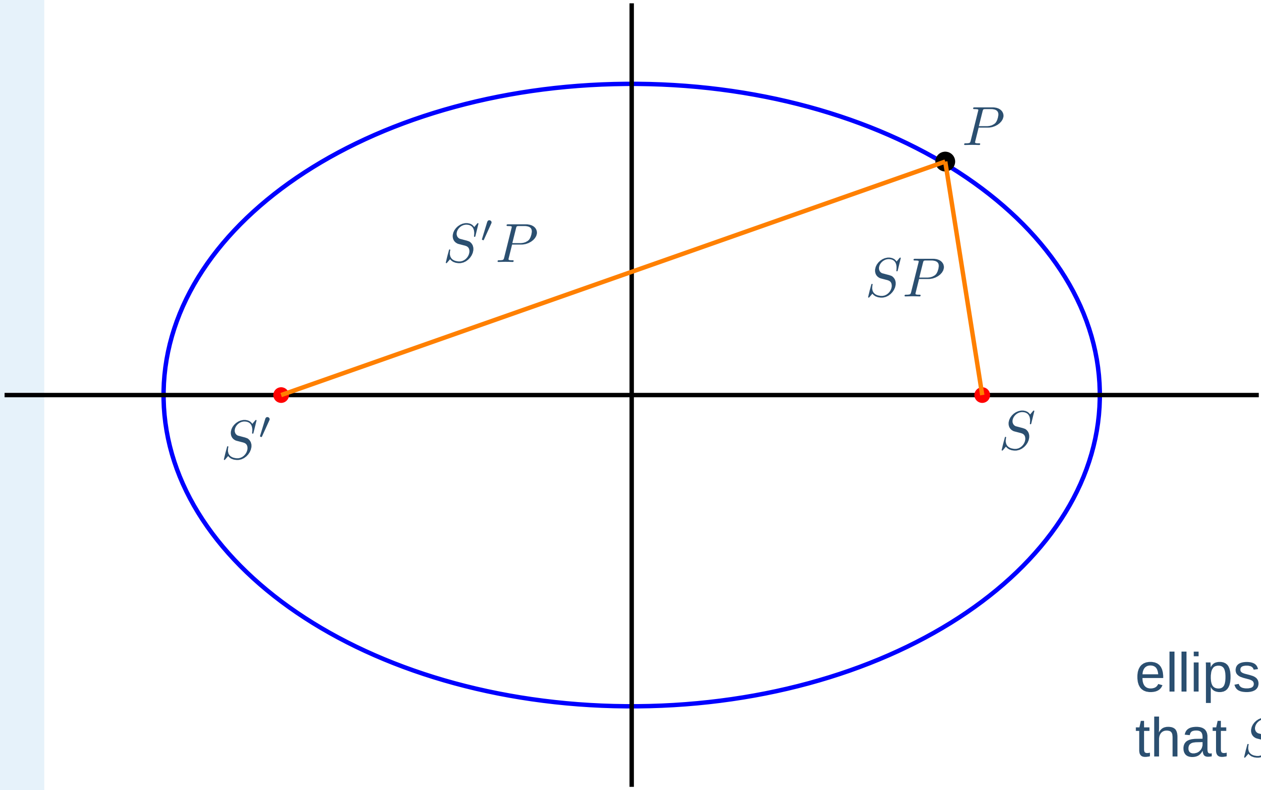
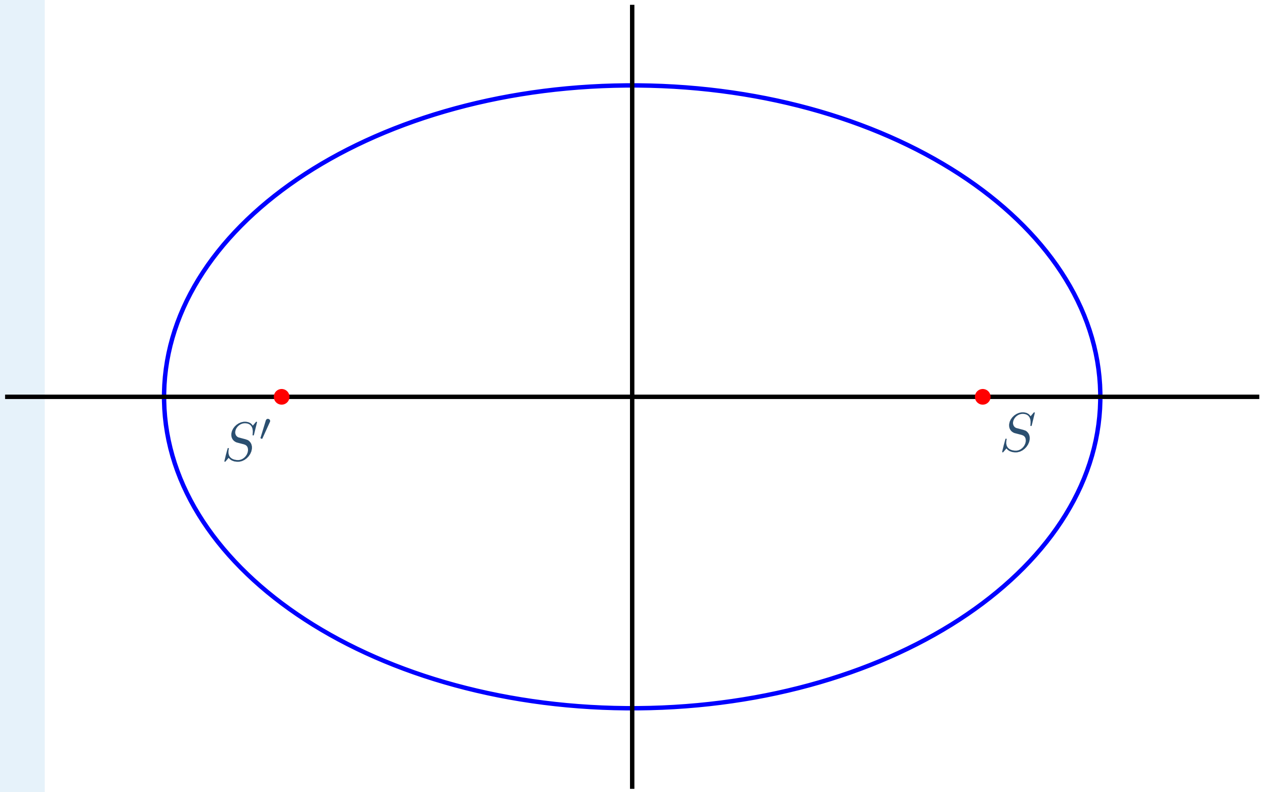


S, S' - foci

ellipse = all points P such that $SP + S'P = constant$

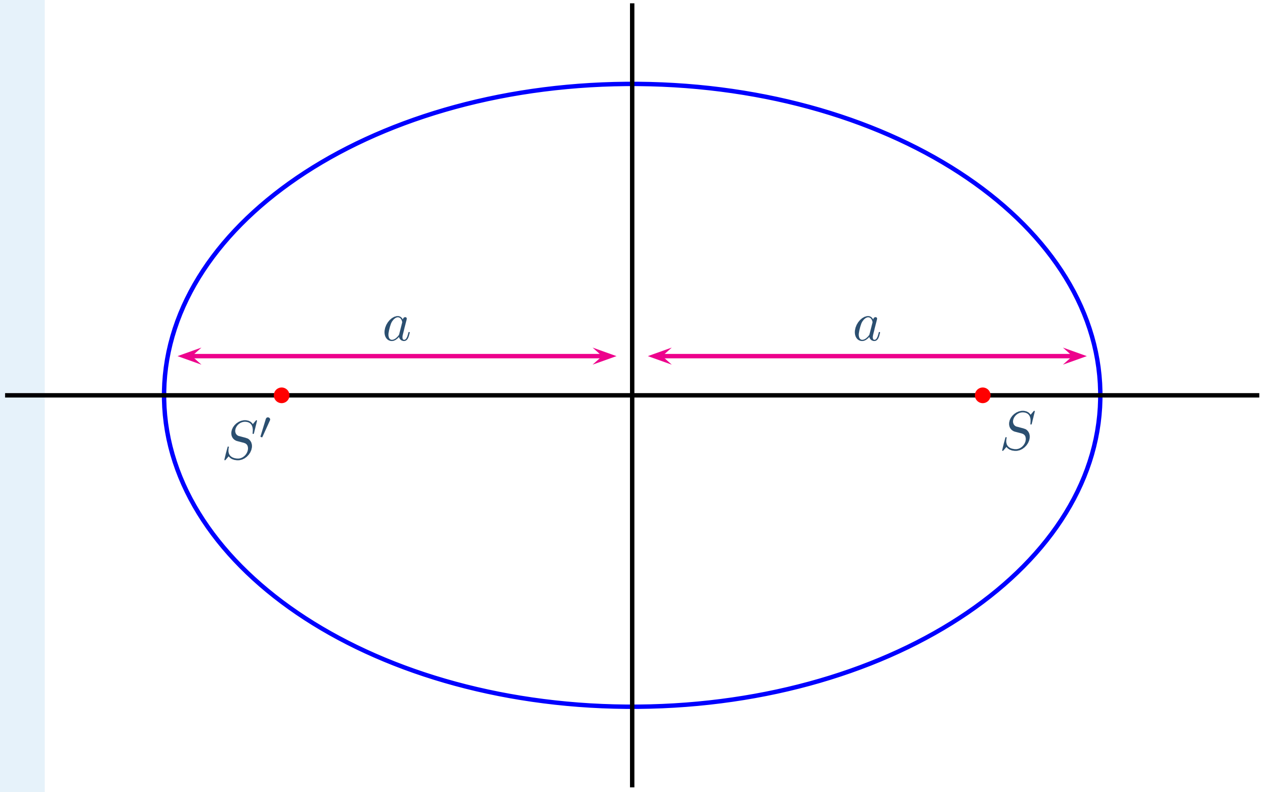
Ellipses - The Algebraic Approach

Ellipse - ovals.



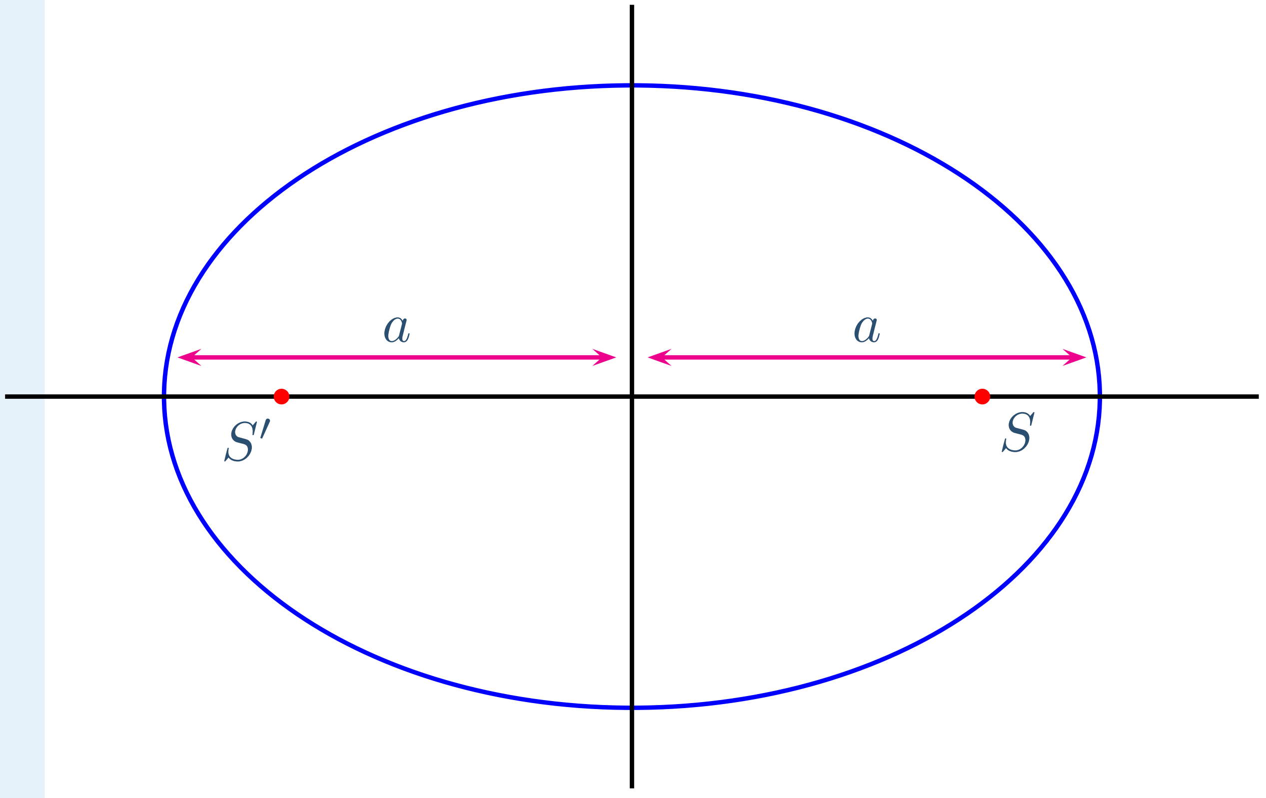
Ellipses - The Algebraic Approach

Ellipse - ovals.



Ellipses - The Algebraic Approach

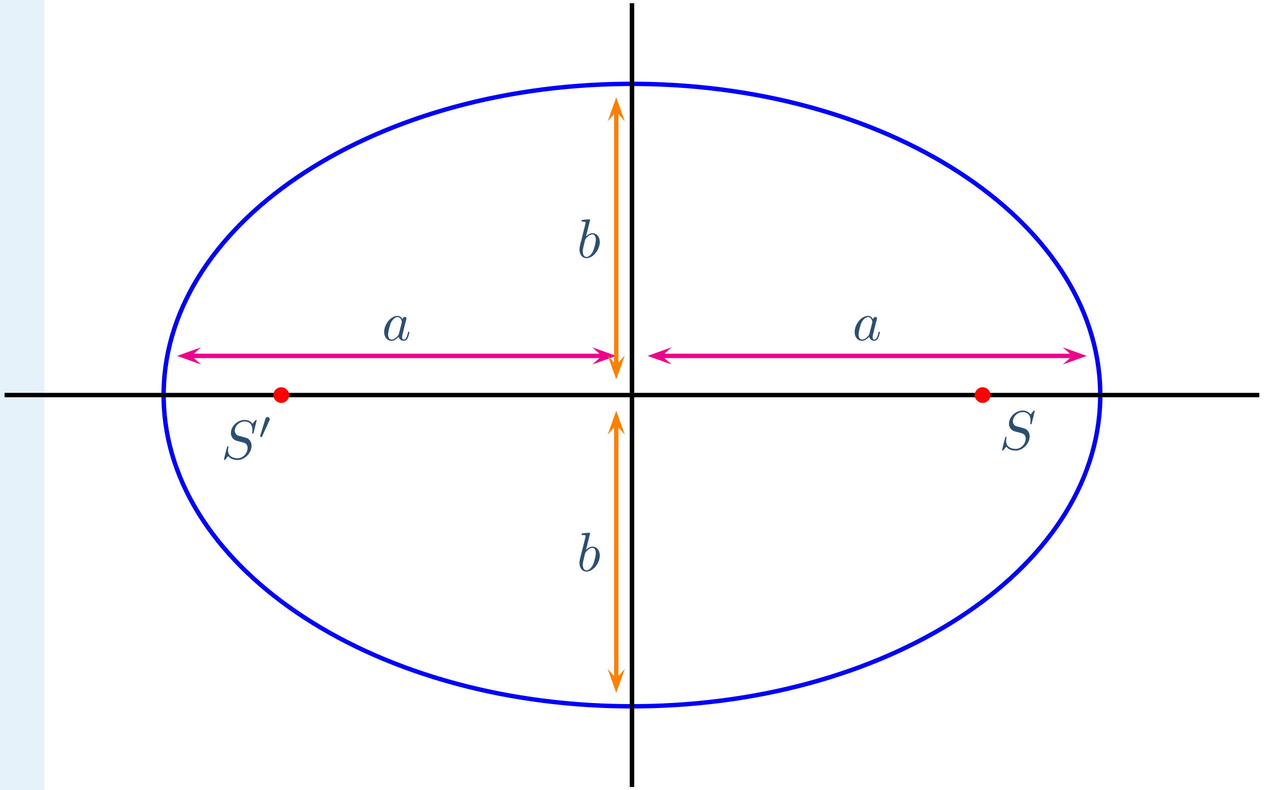
Ellipse - ovals.



a: semi-major axis

Ellipses - The Algebraic Approach

Ellipse - ovals.



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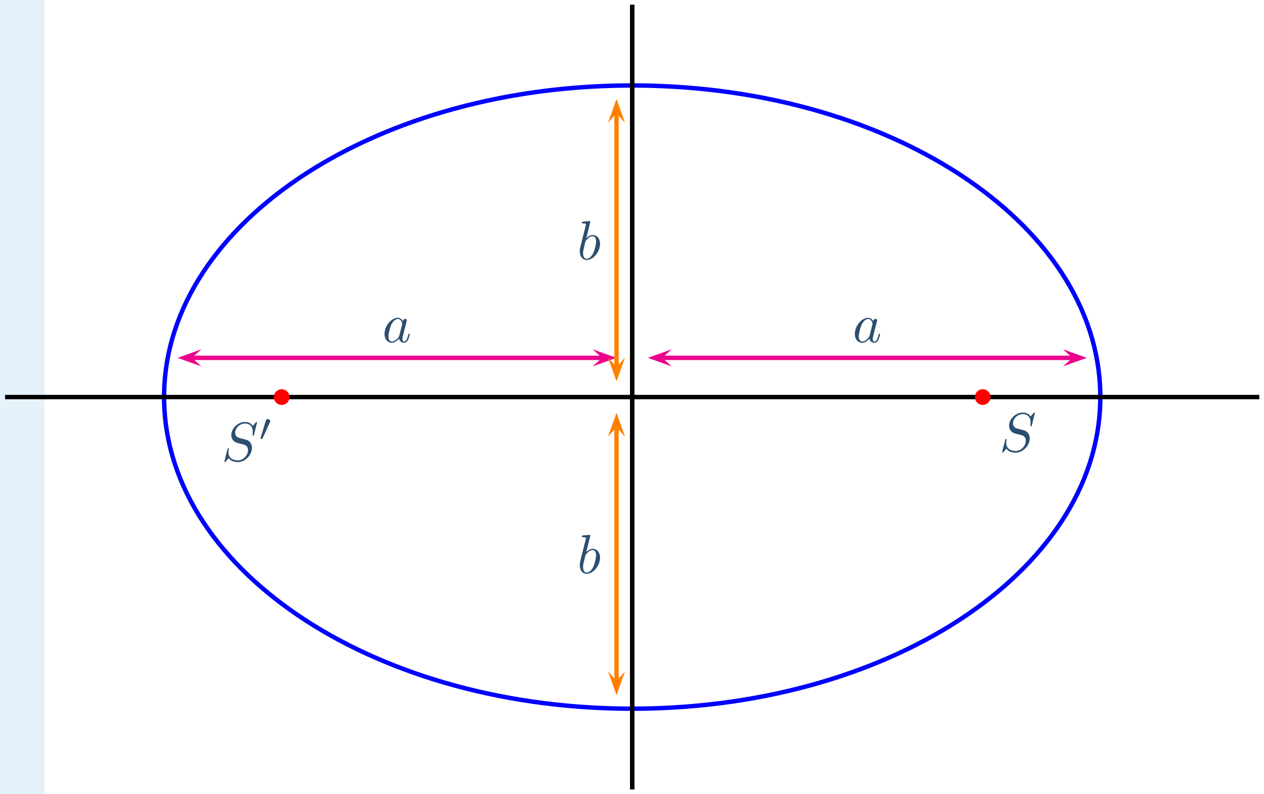

a: semi-major axis

b: semi-minor axis

Ellipses - The Algebraic Approach

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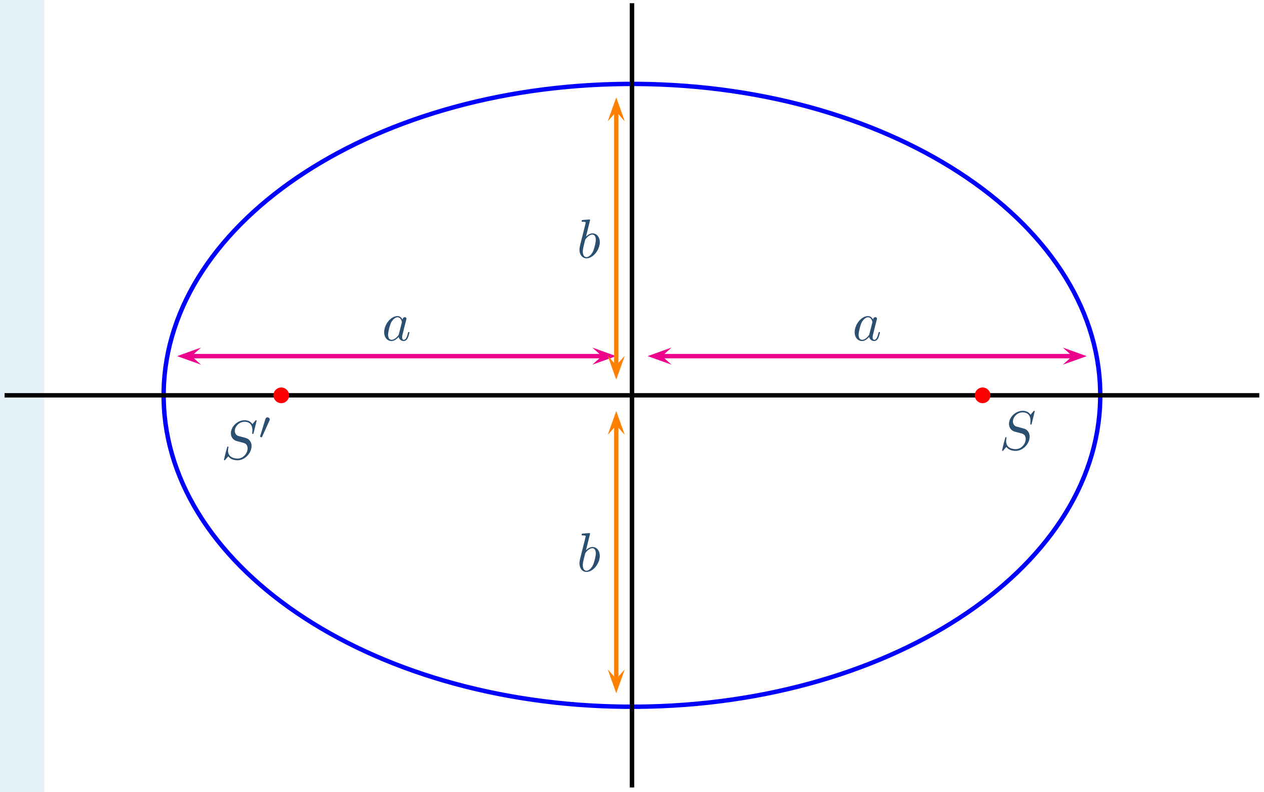


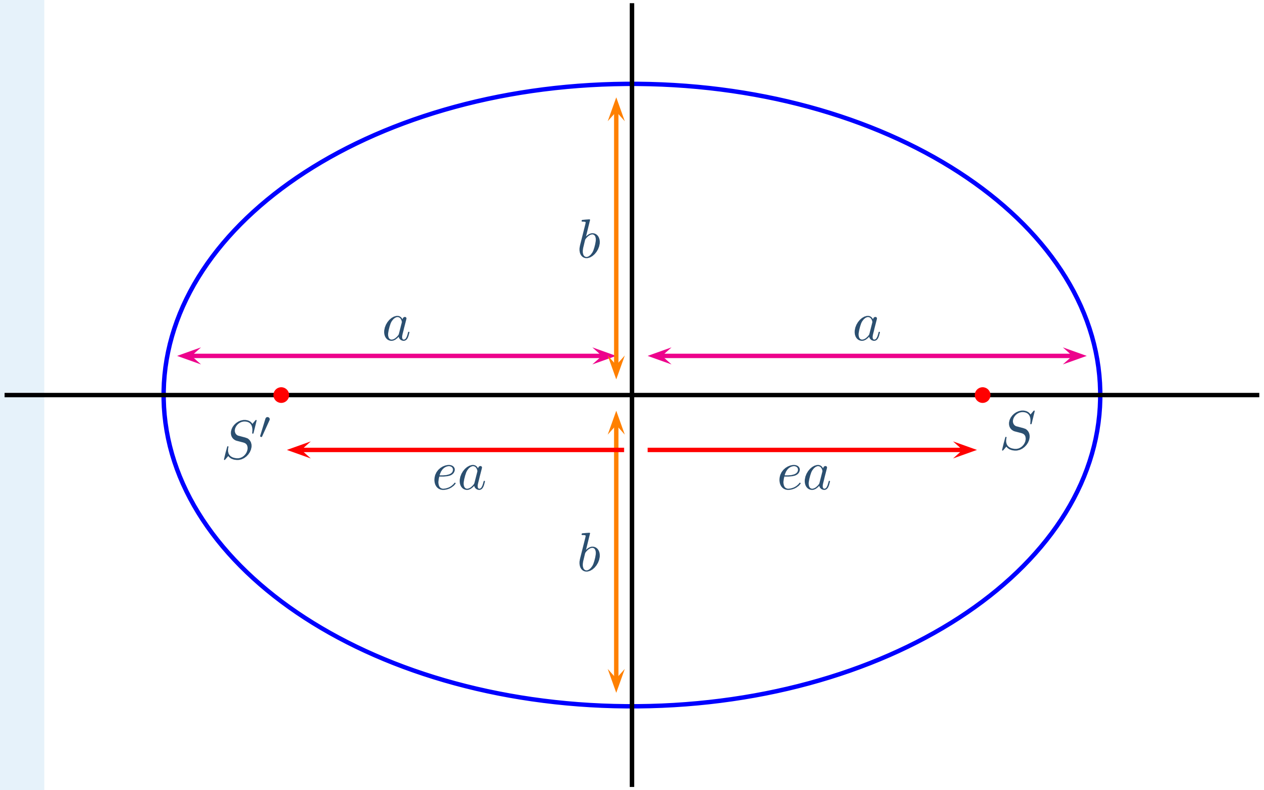
a: semi-major axis

b: semi-minor axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

Ellipses - The Algebraic Approach

Ellipse - ovals.



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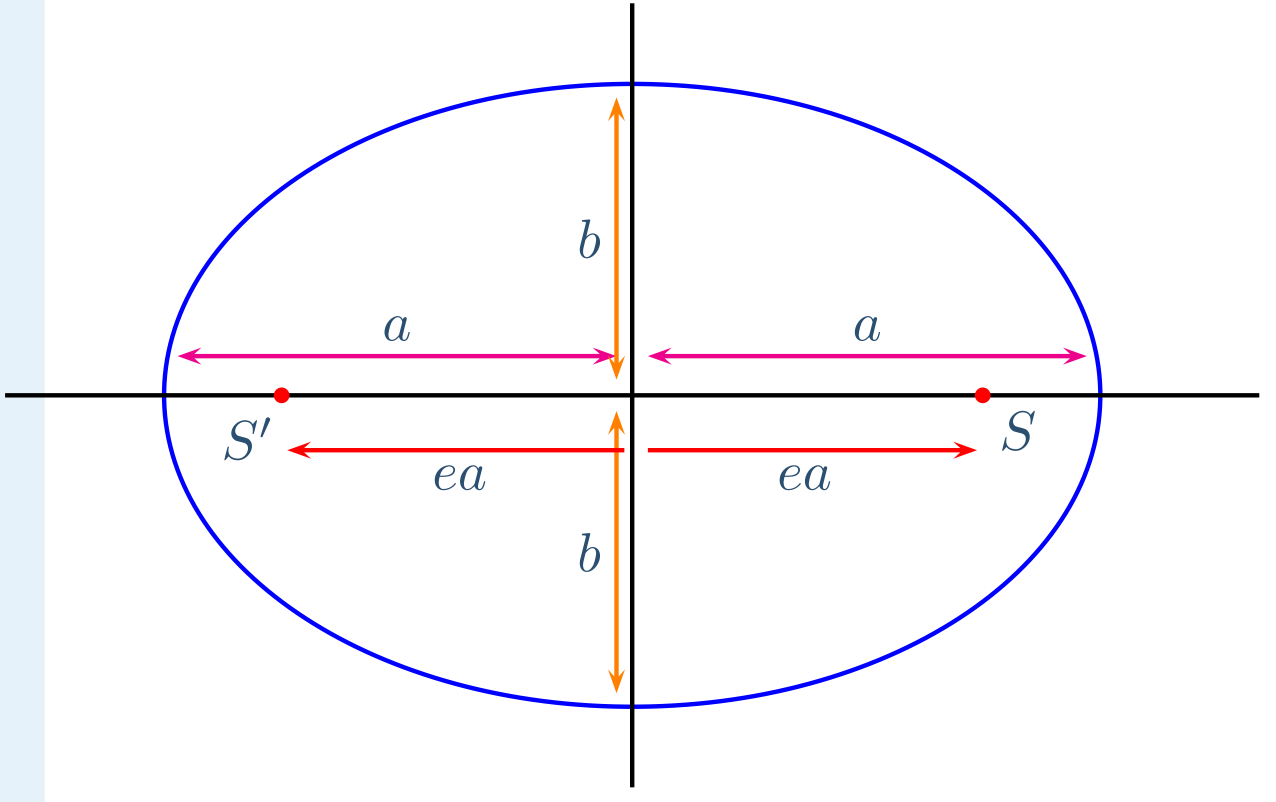


a: semi-major axis

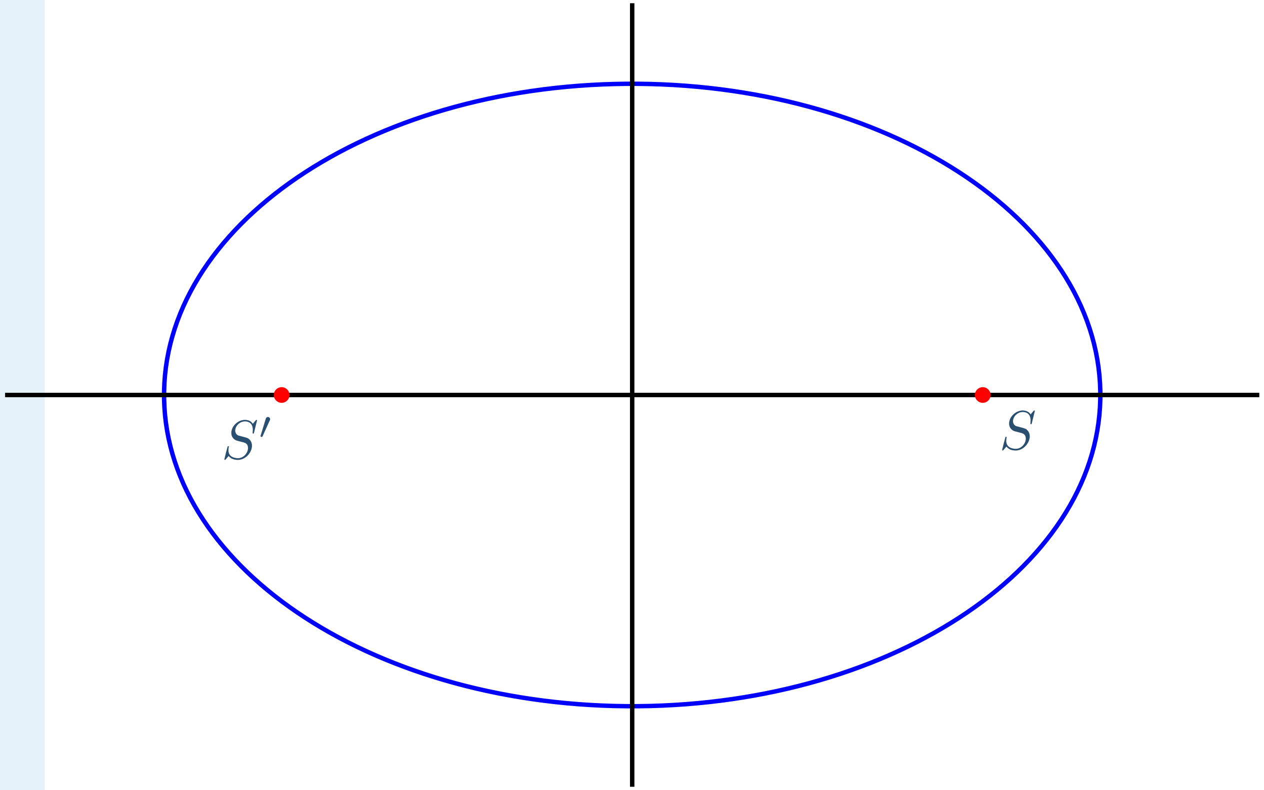
b: semi-minor axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

e: eccentricity

Eccentricity

The eccentricity gives the amount of “oval-ness" of the ellipse.



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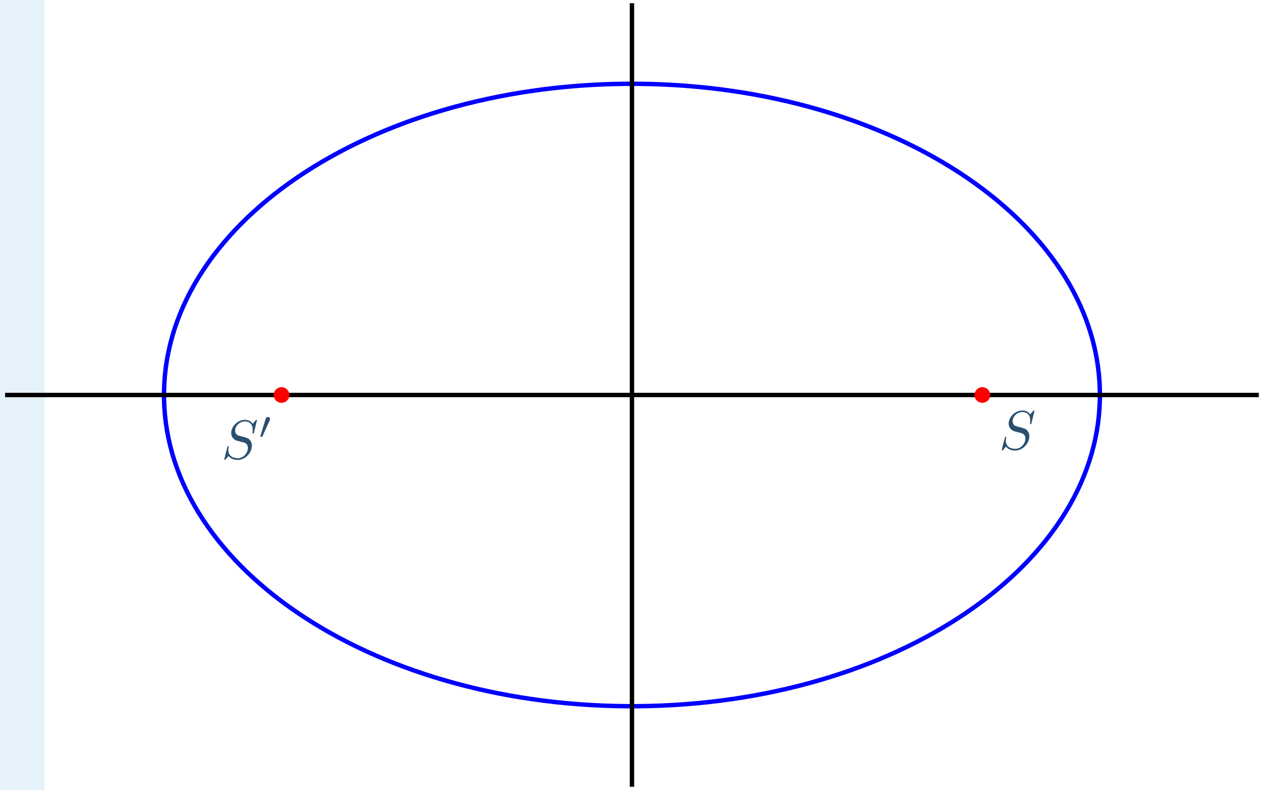

$a = 3$

$b = 2$

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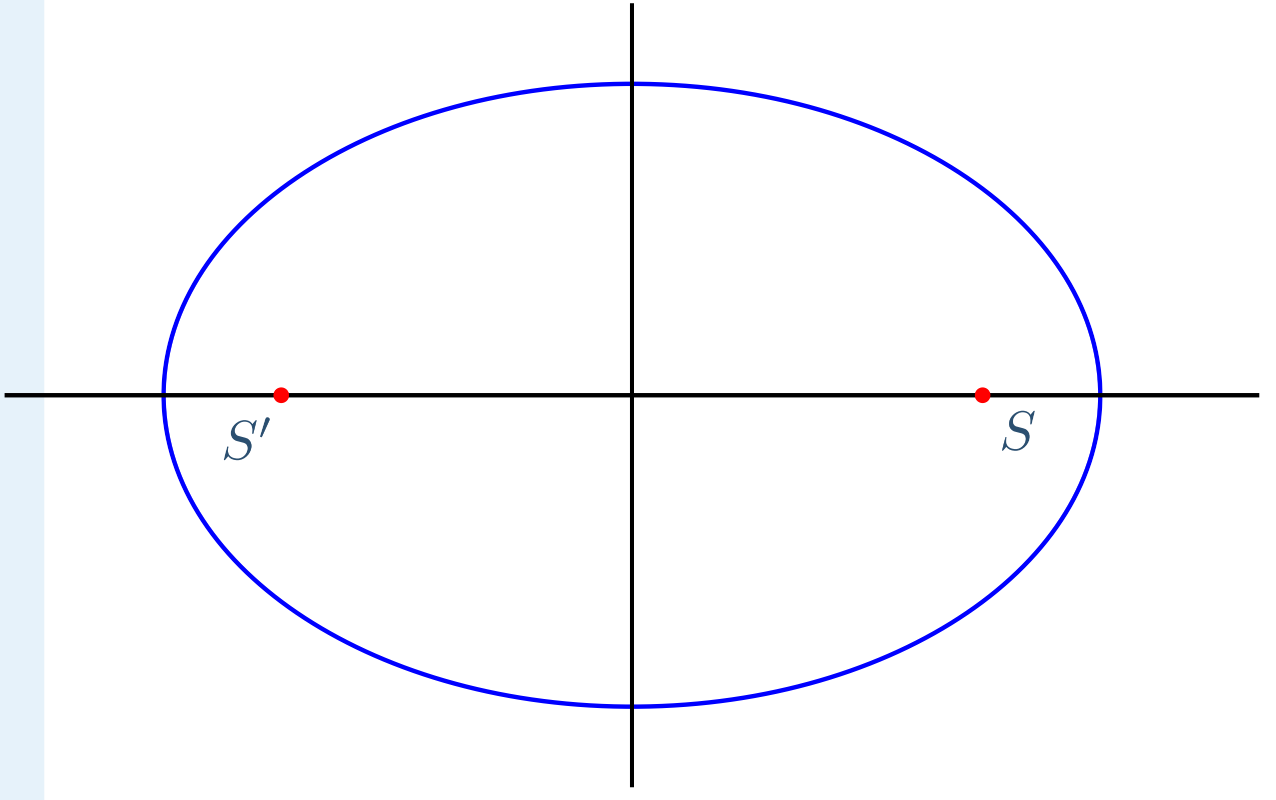

$$a = 3$$

$$b = 2$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

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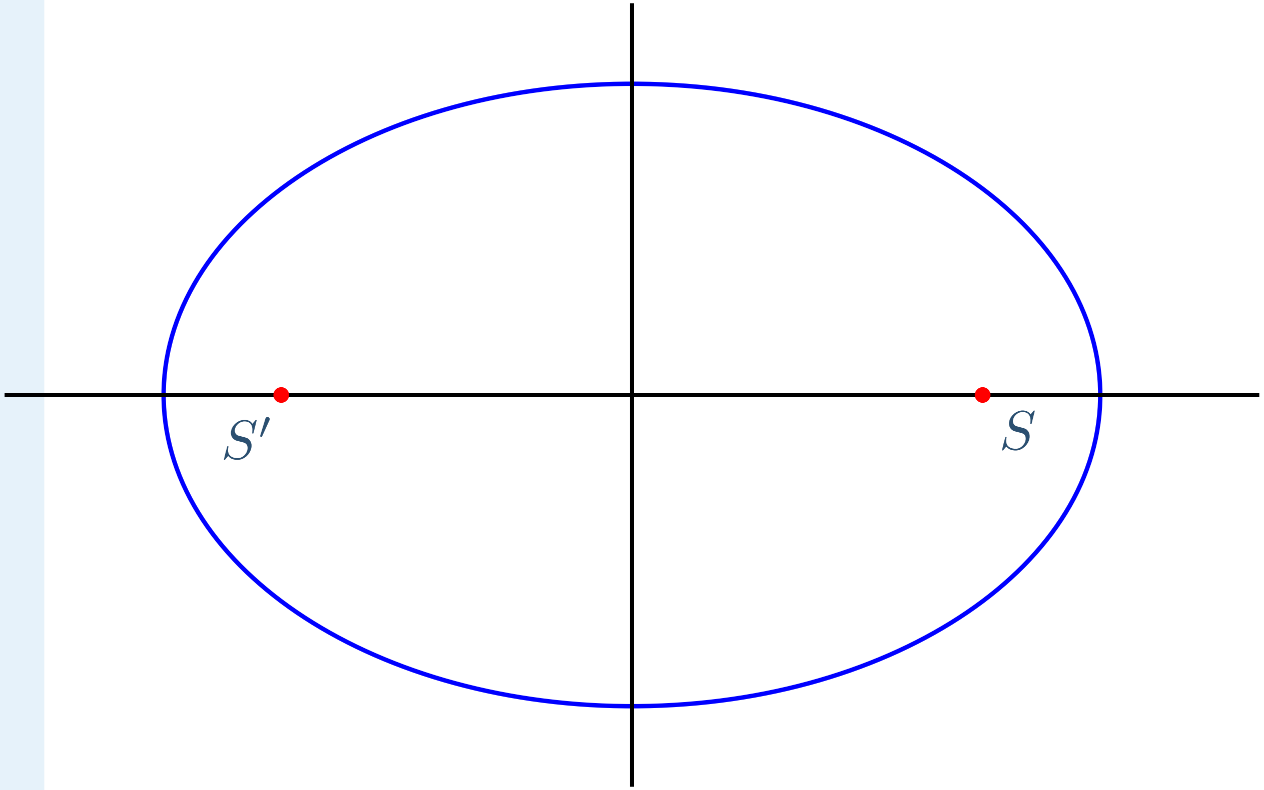

$$a = 3$$

$$b = 2$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.745$$

Eccentricity

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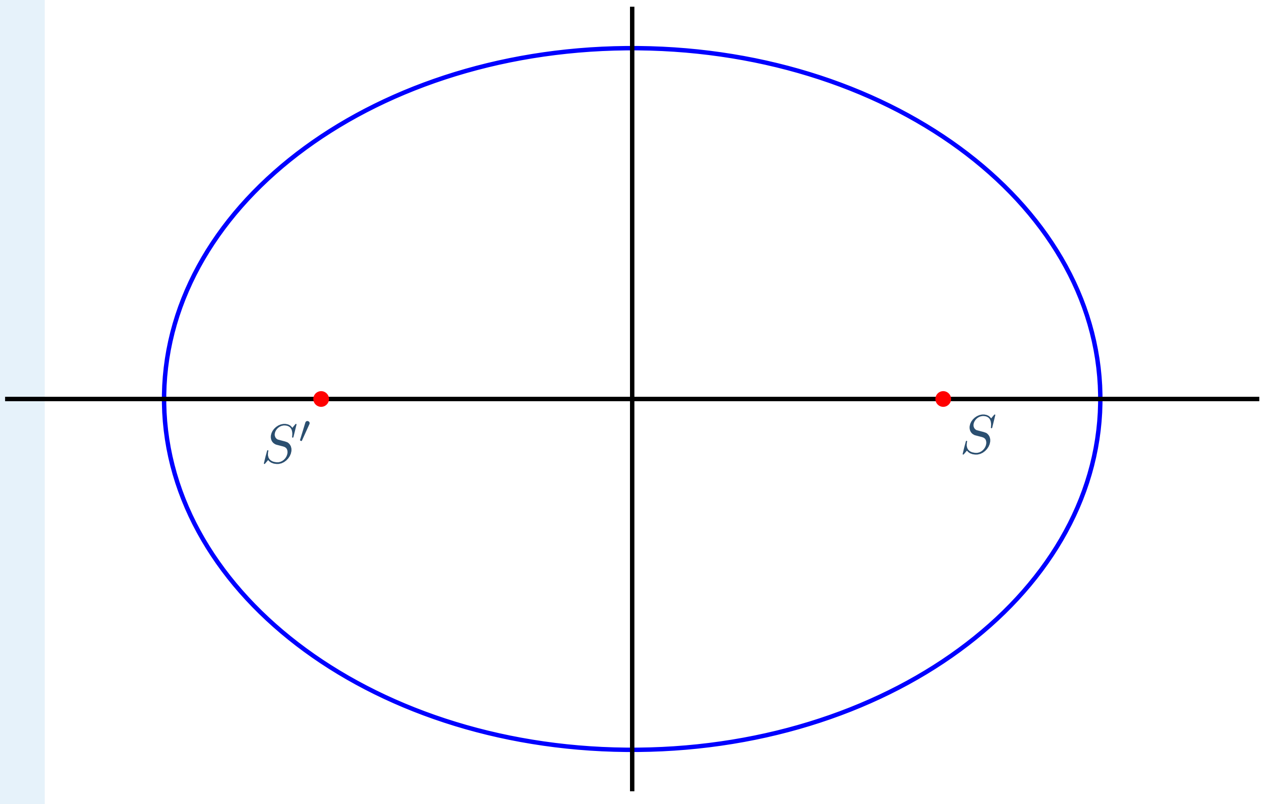

$$a = 3$$

$$b = 2.25$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.661$$

Eccentricity

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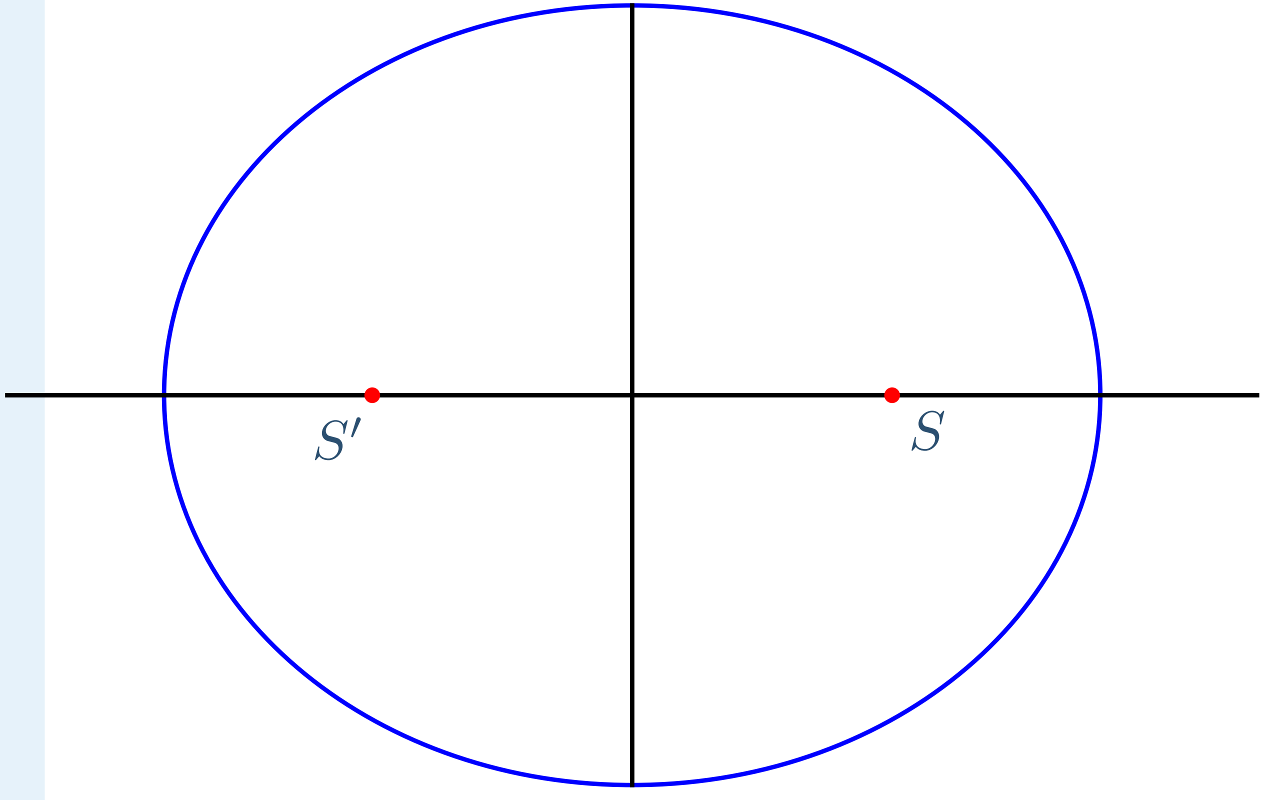

$a = 3$

$b = 2.5$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$e = 0.553$

Eccentricity

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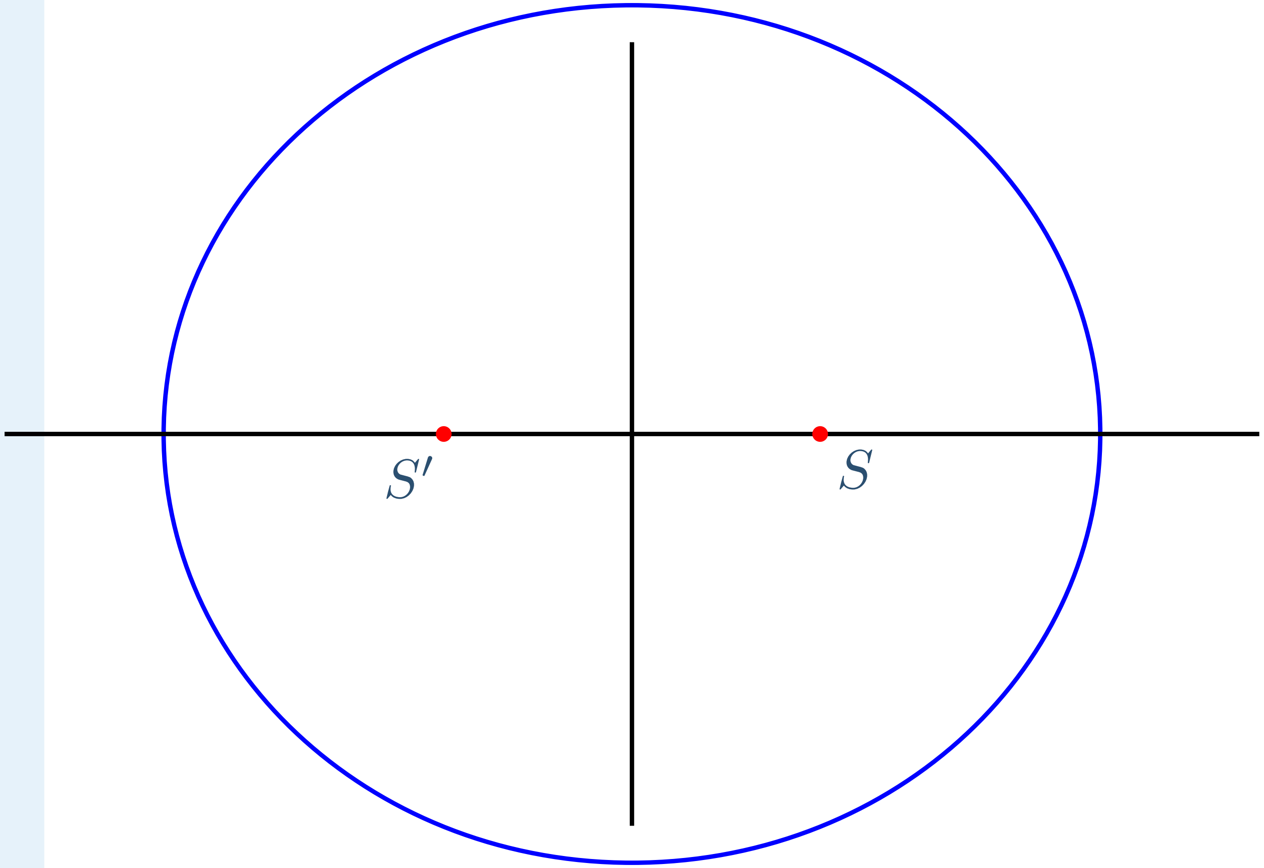

$$a = 3$$

$$b = 2.75$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.400$$

Eccentricity

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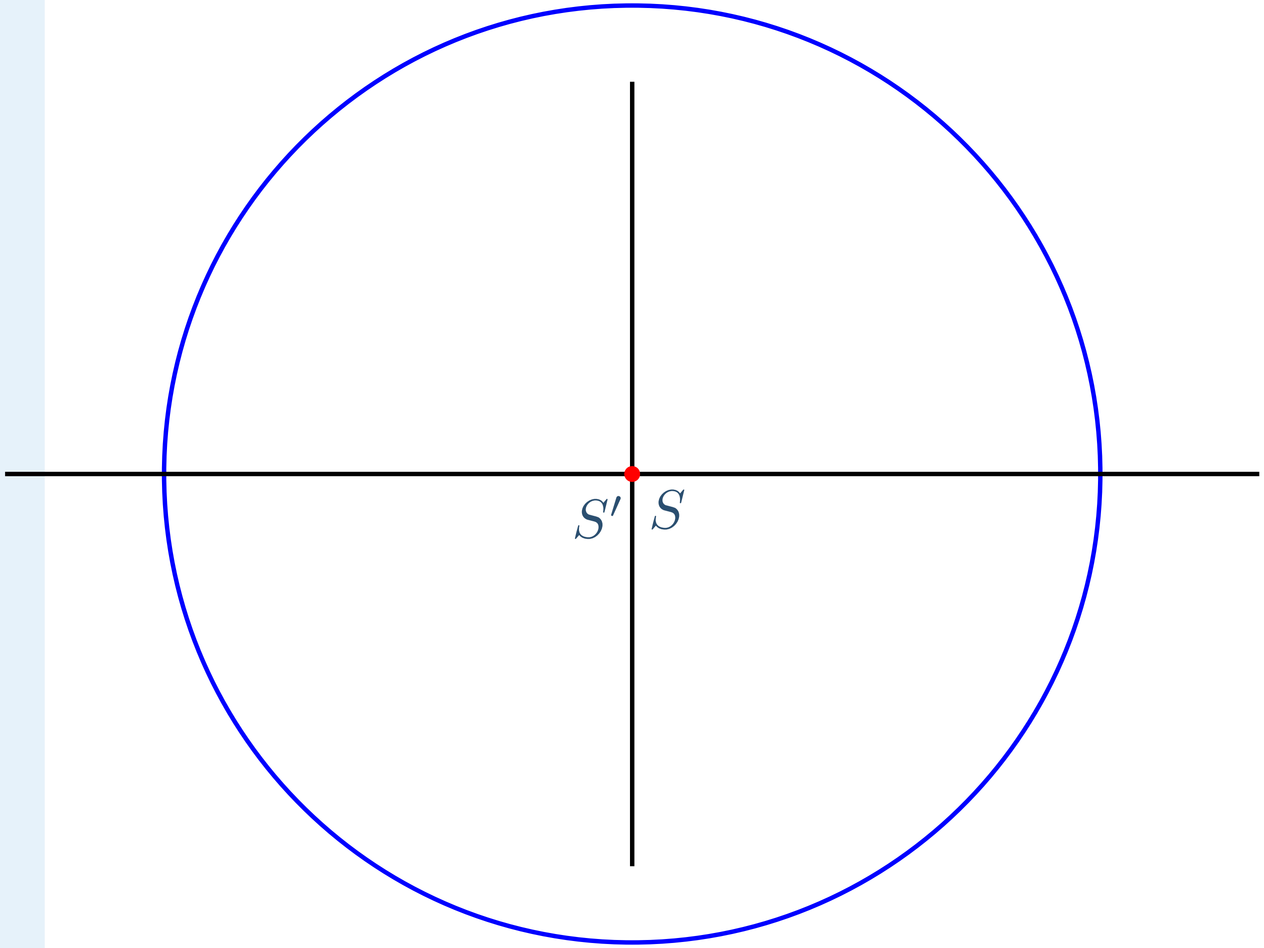

$$a = 3$$

$$b = 3$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0$$

Eccentricity

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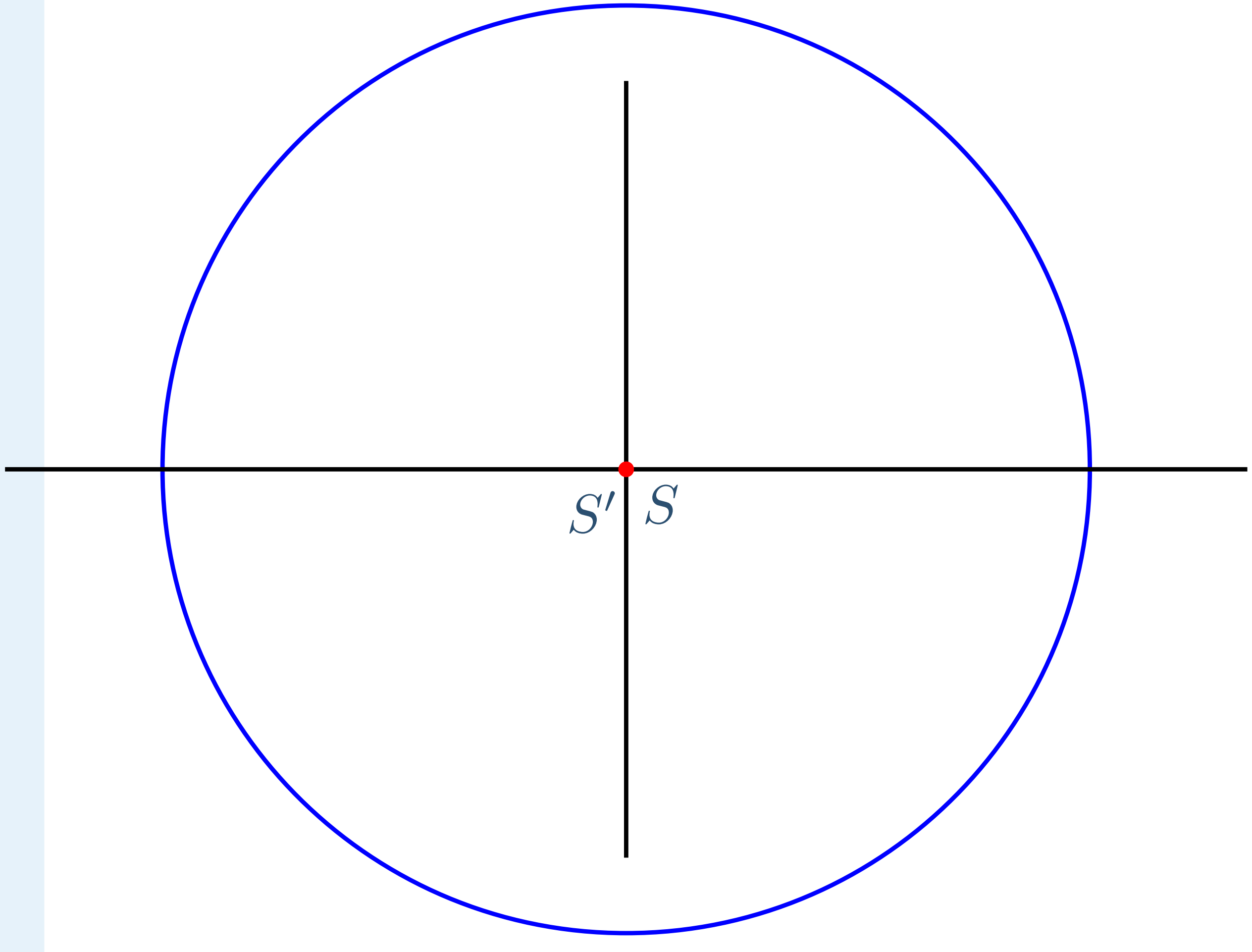

$a = 3$

$b = 3$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$e = 0$

Circle

Kepler’s First Law

Kepler’s Laws:

1: Each planet’s orbit traces out the shape of an ellipse with the sun located at one focus.

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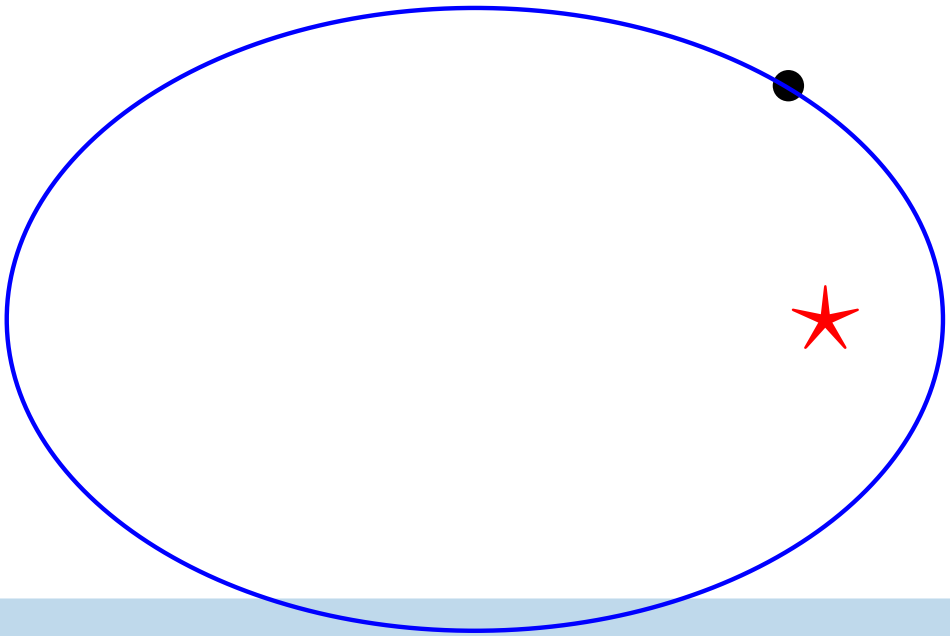

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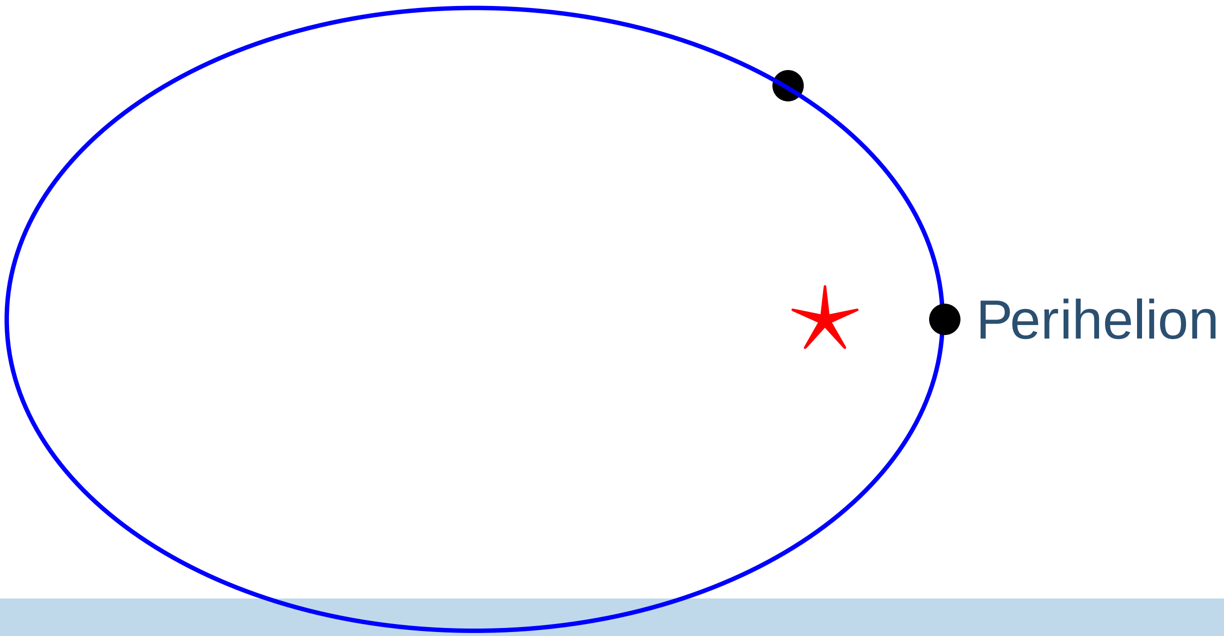

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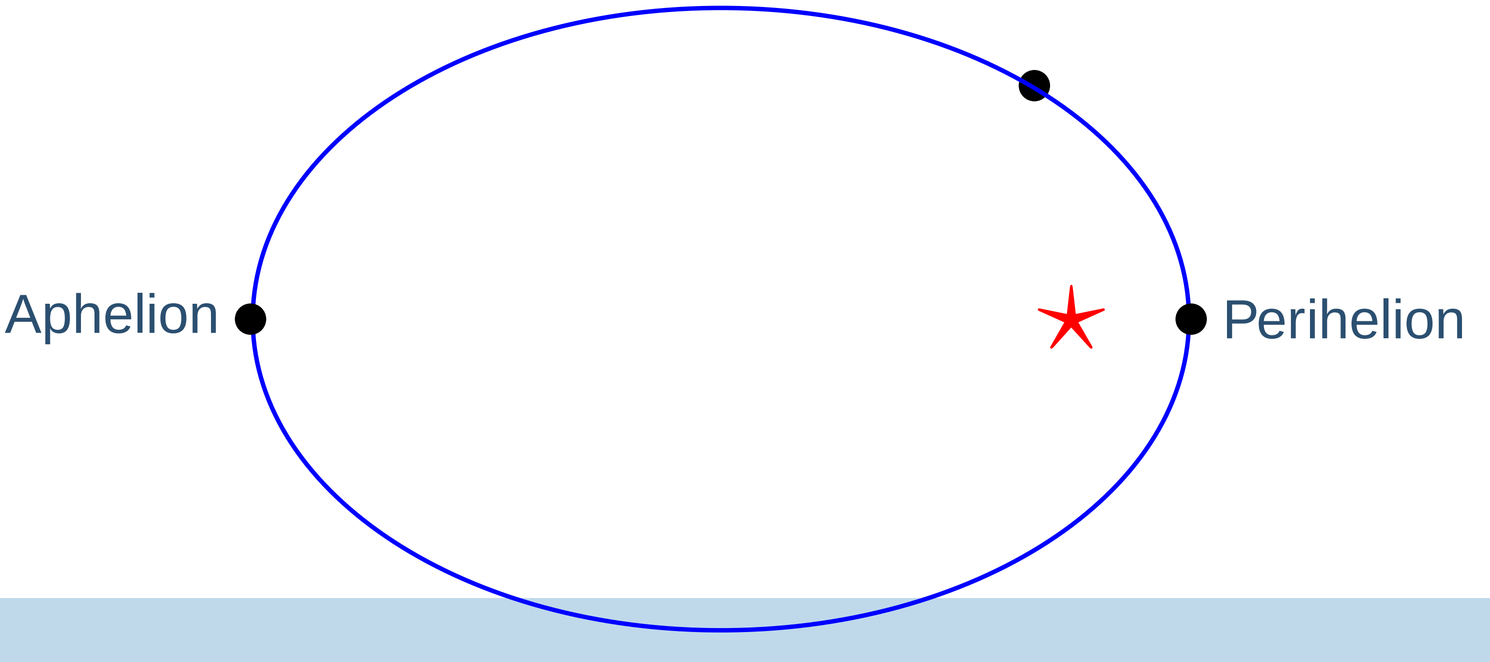

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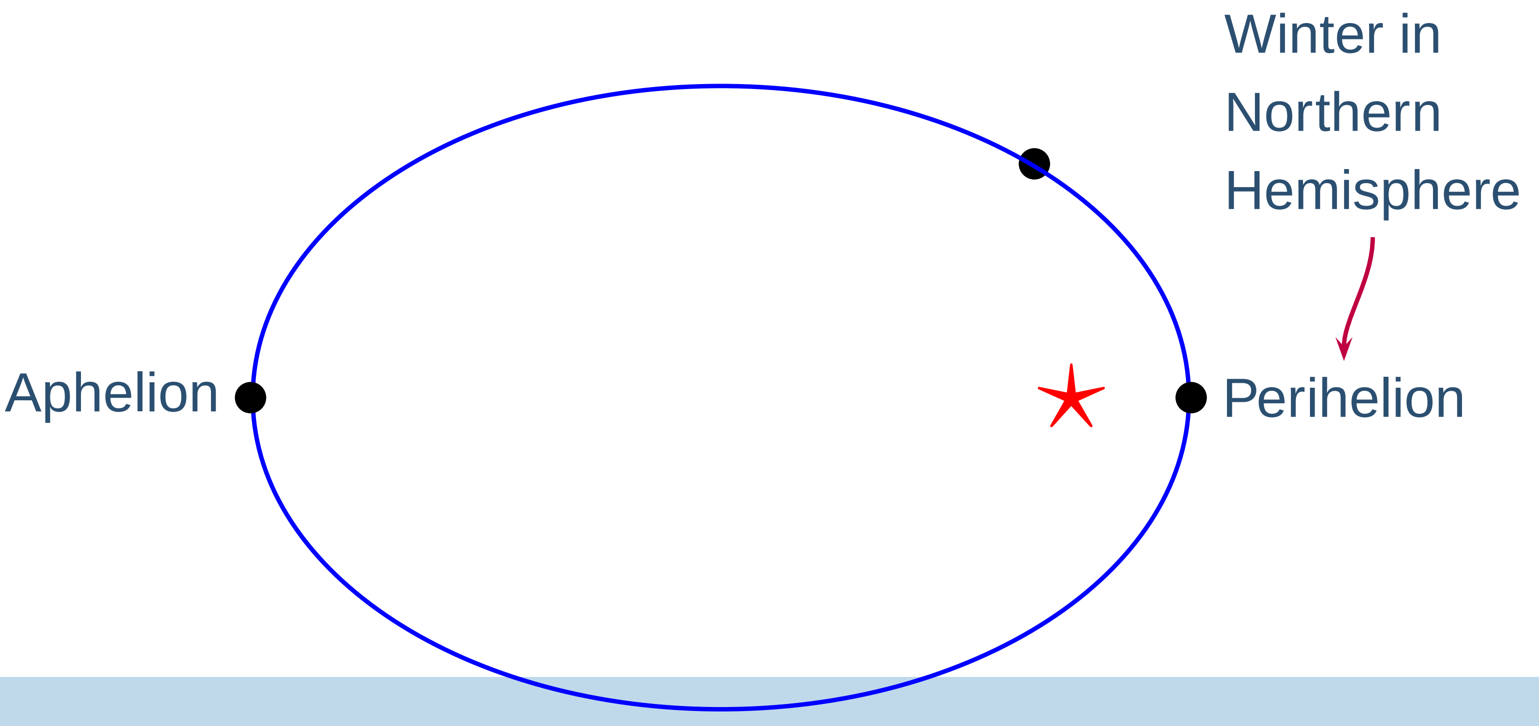

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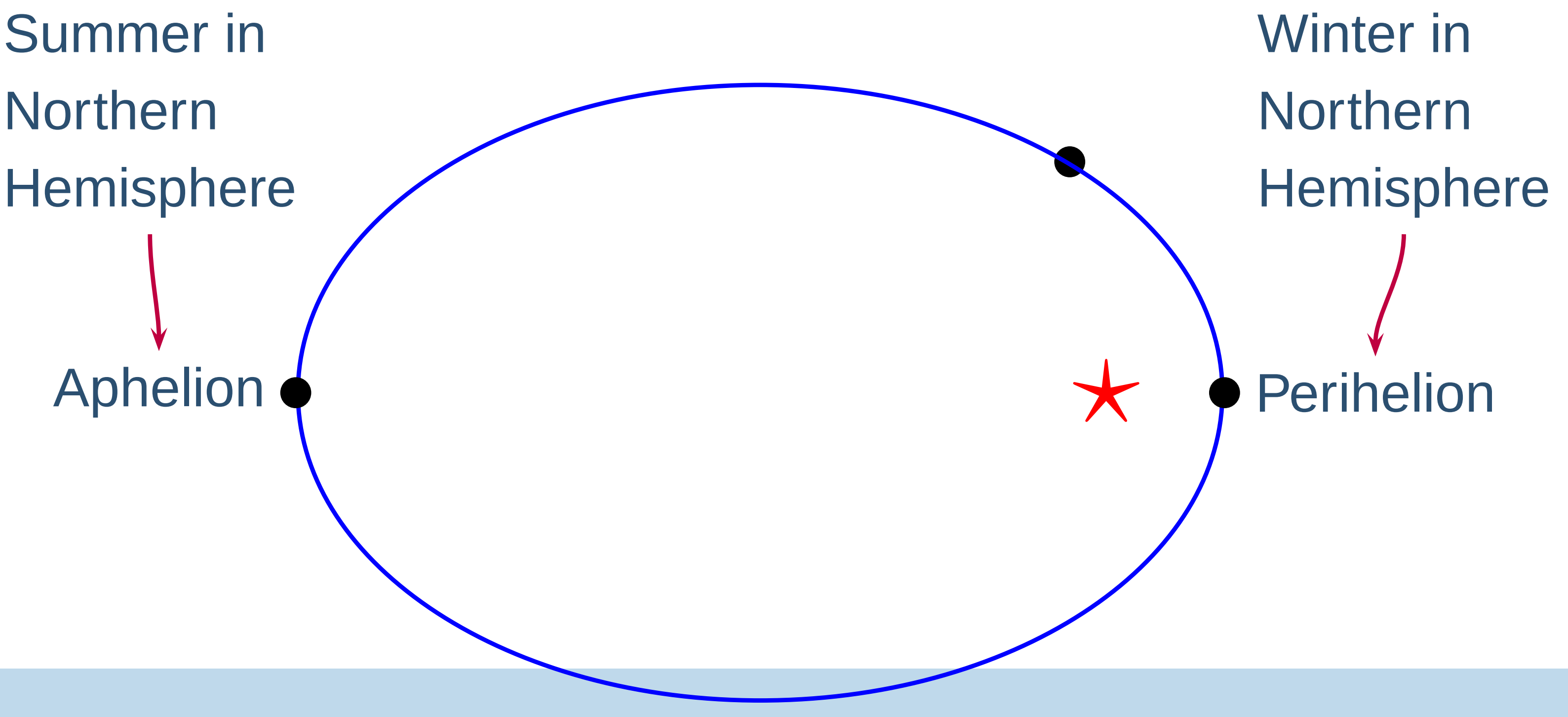
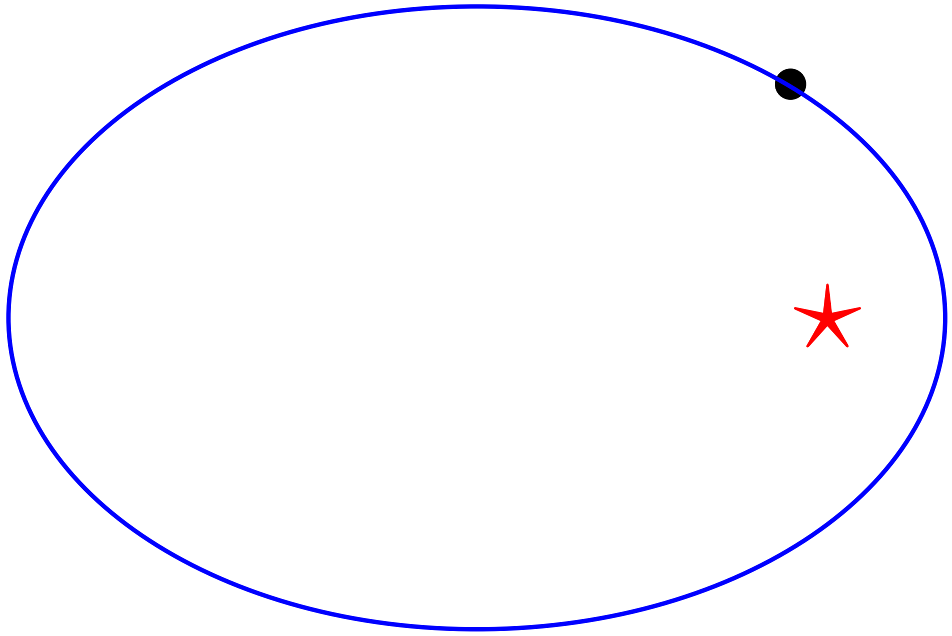


Kepler's First Law II



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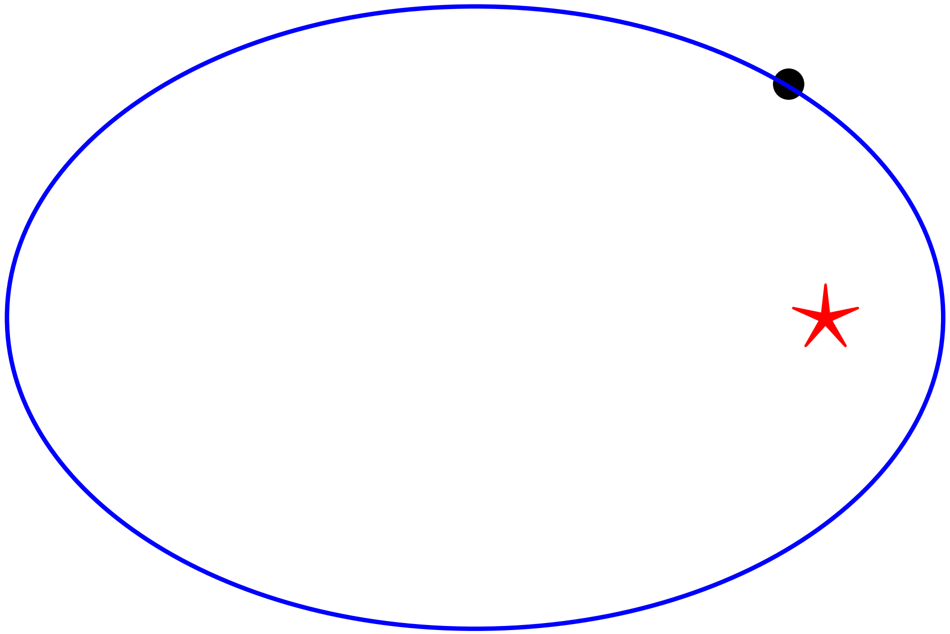
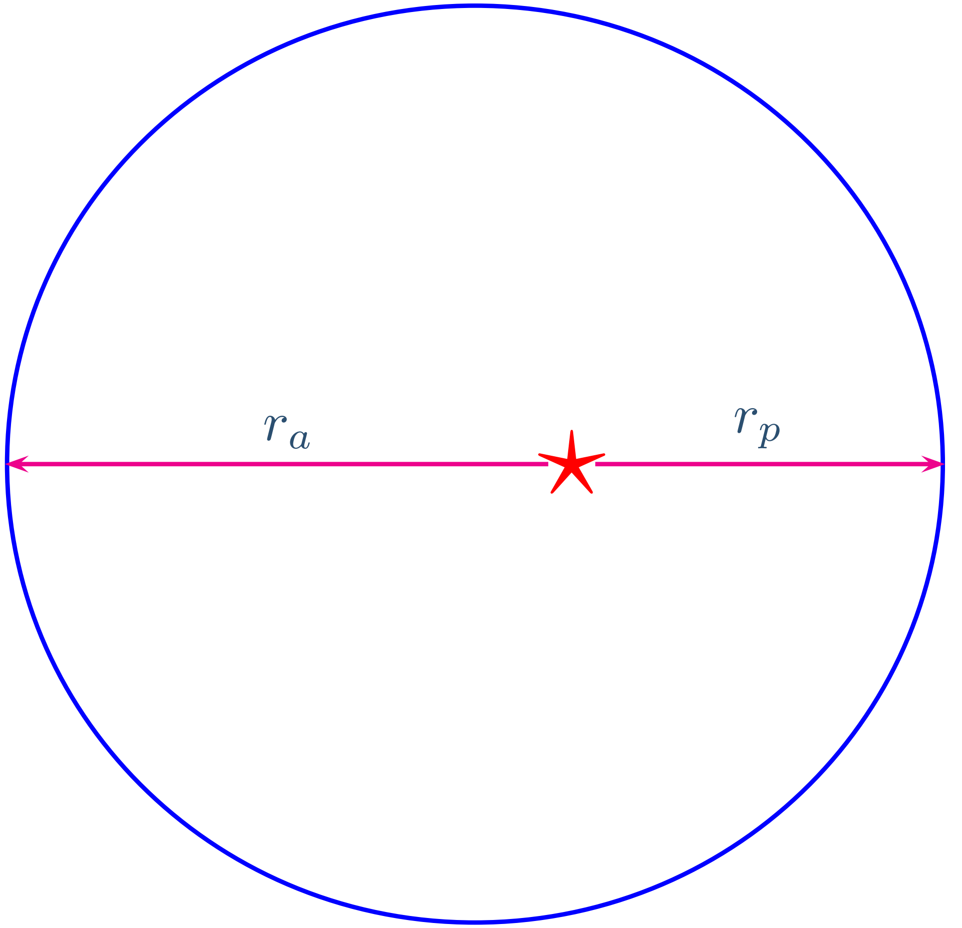


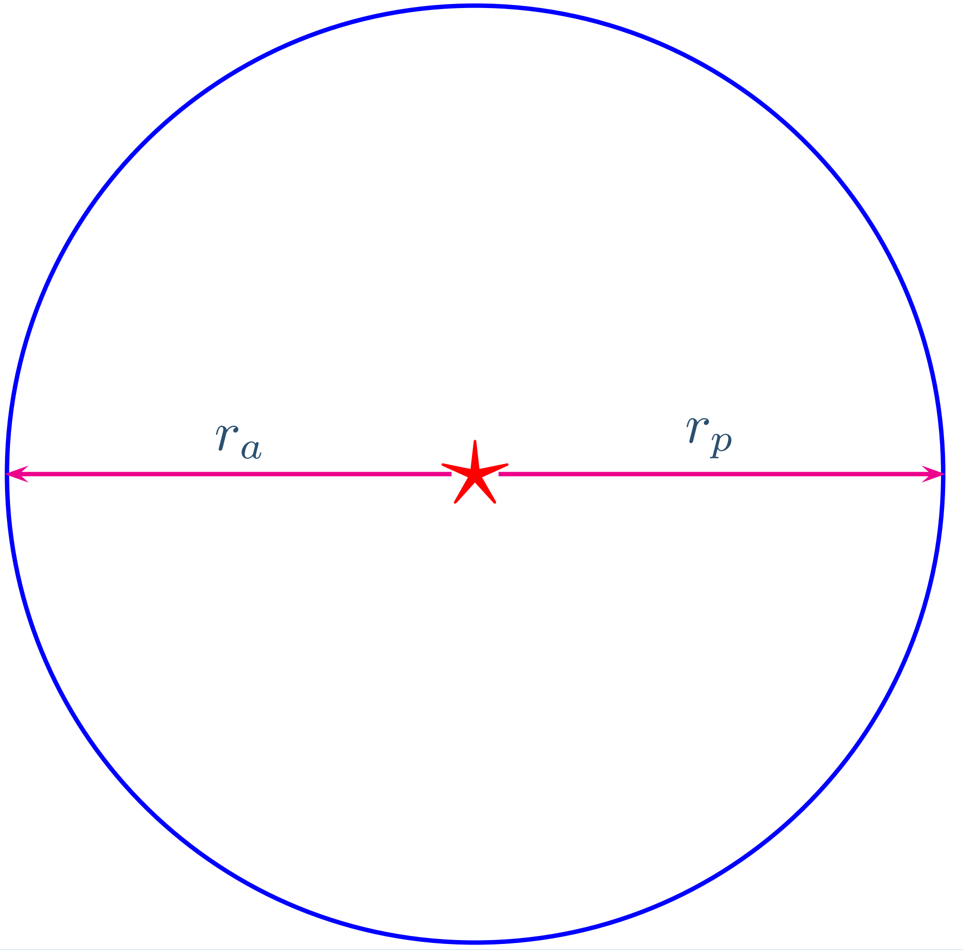
Planet	e

Kepler’s First Law II



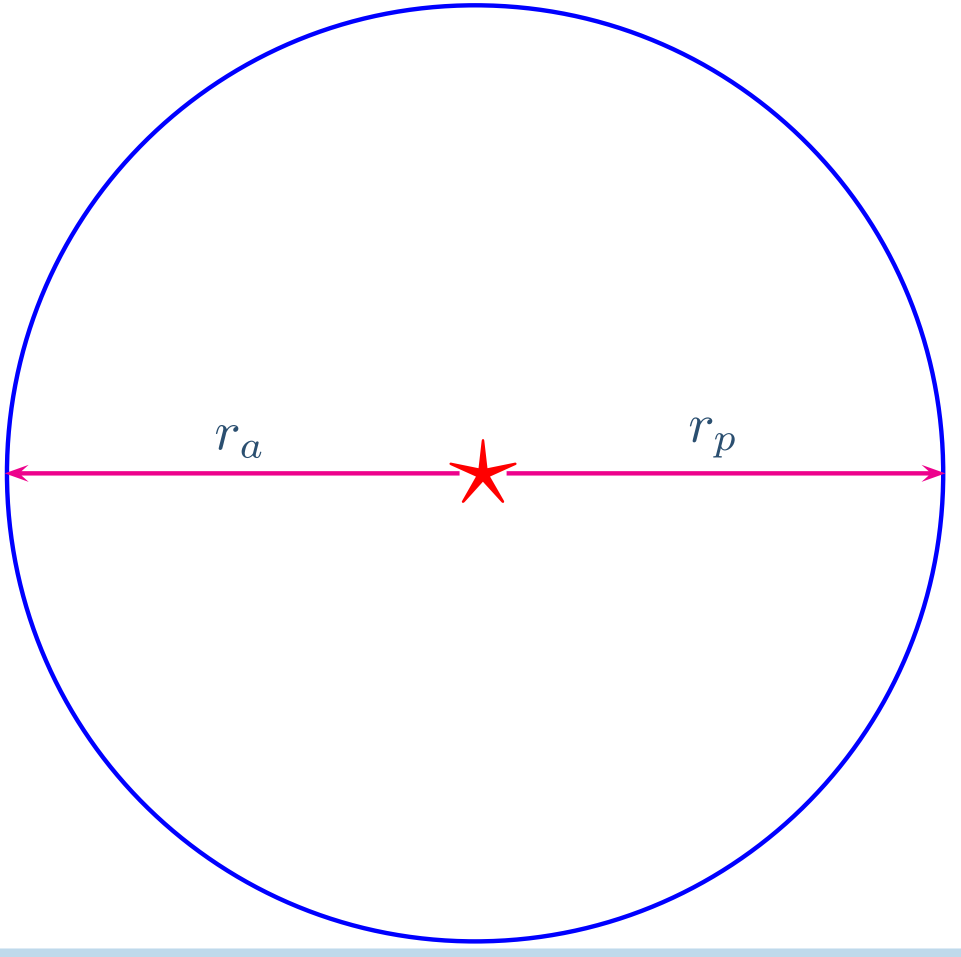
Planet	e
Mercury	0.206

Kepler's First Law II



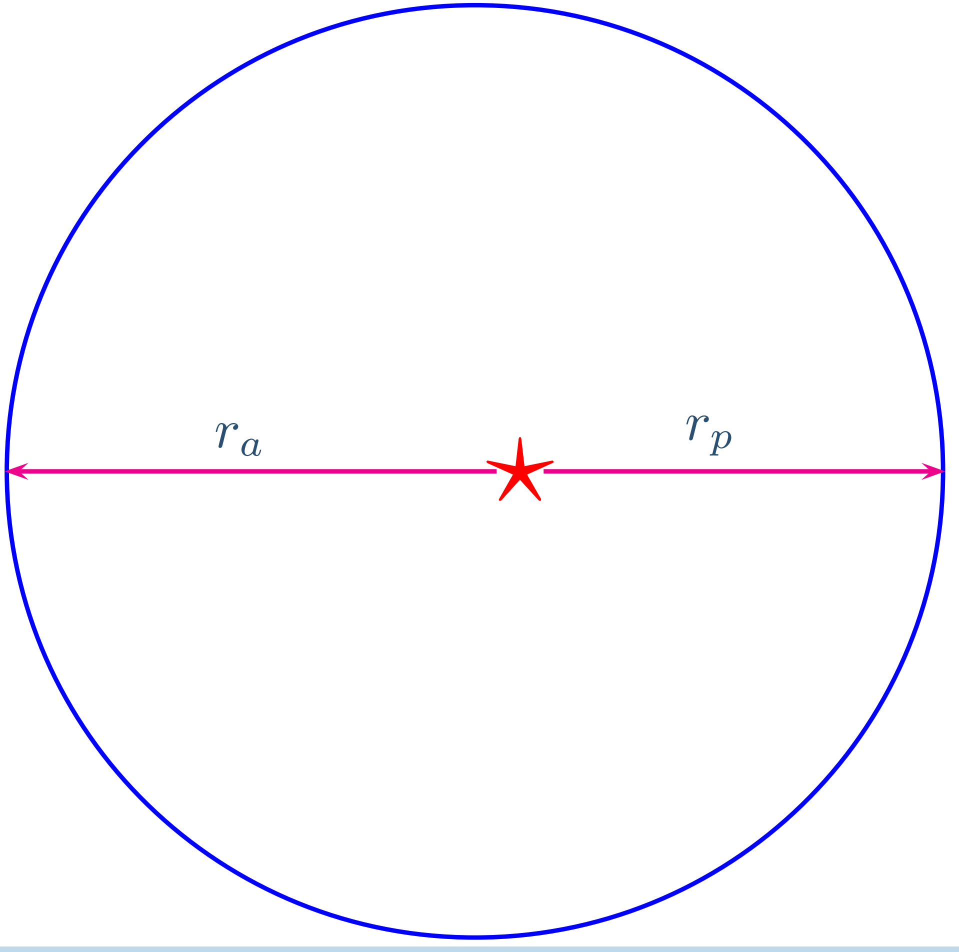
Planet	e
Mercury	0.206
Venus	0.007

Kepler's First Law II



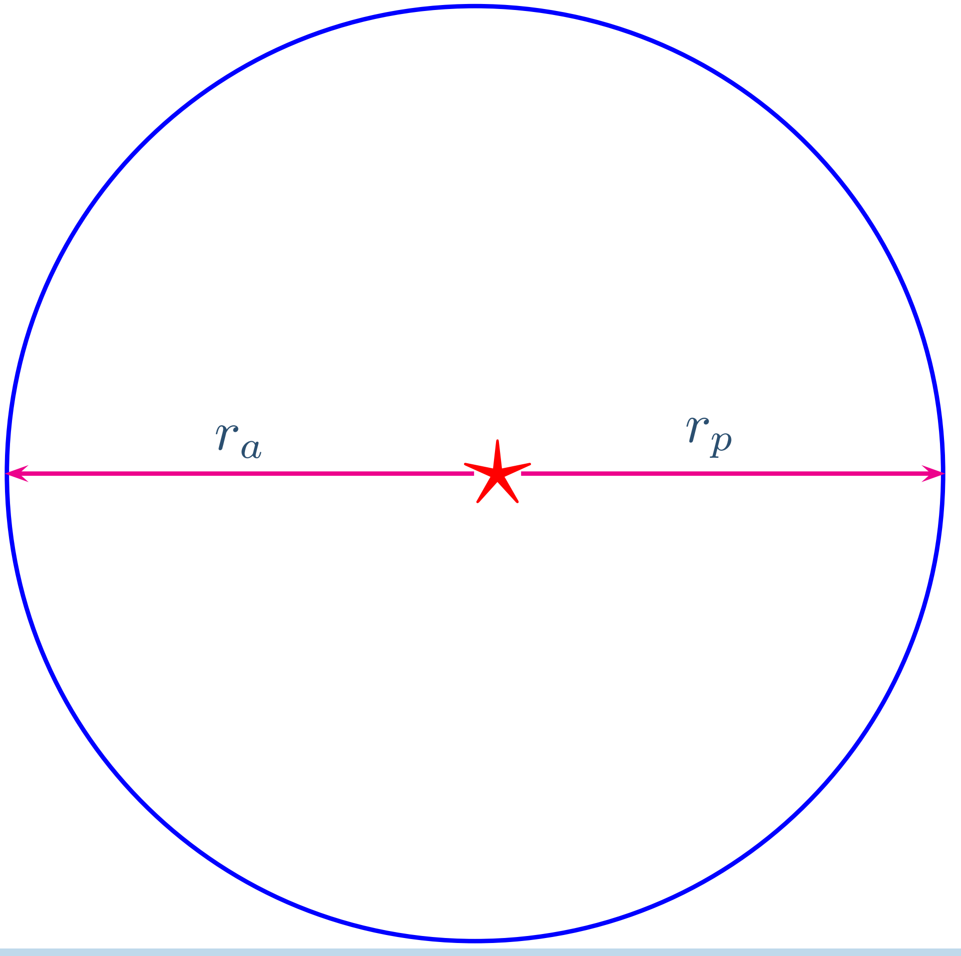
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017

Kepler's First Law II



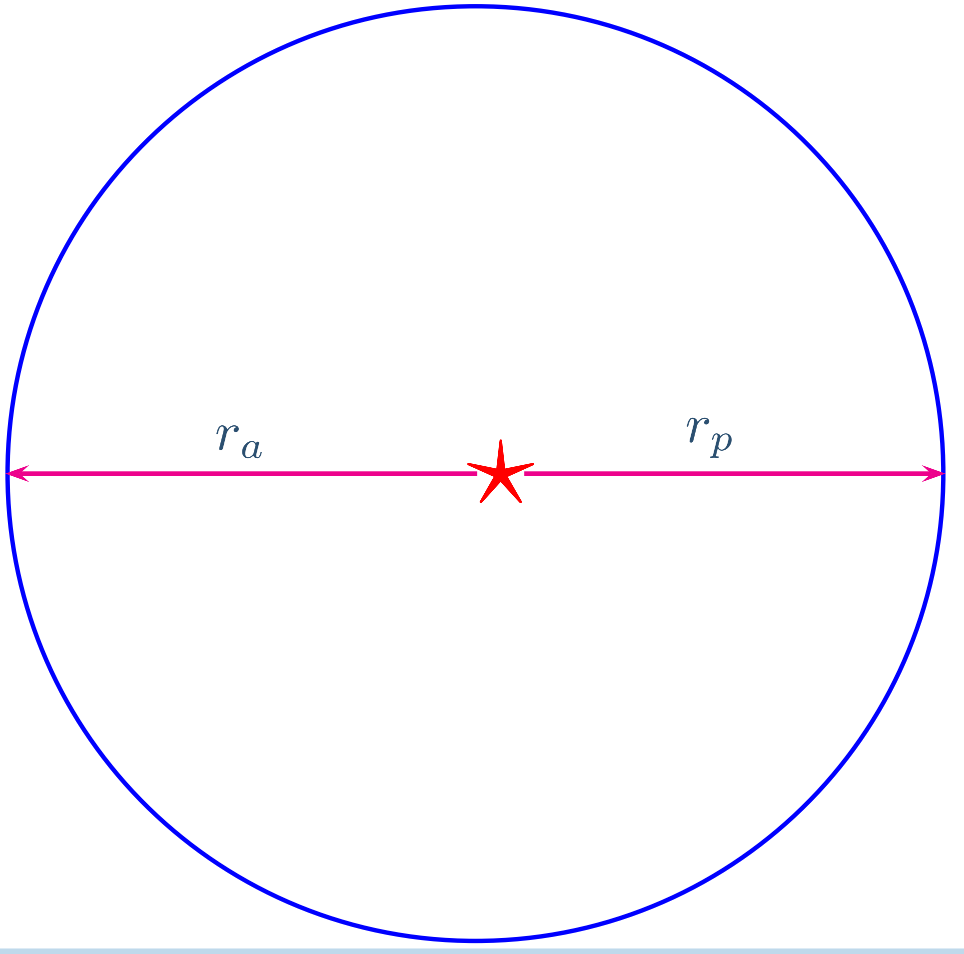
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093

Kepler's First Law II



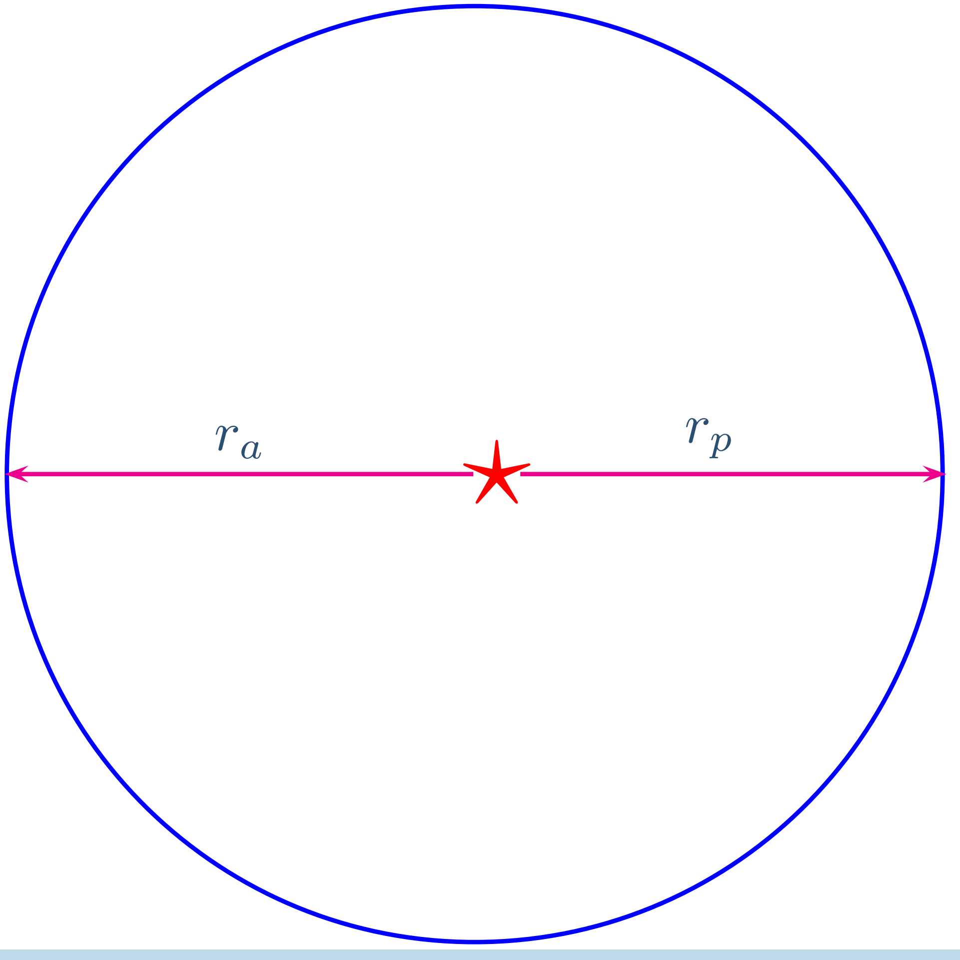
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048

Kepler's First Law II



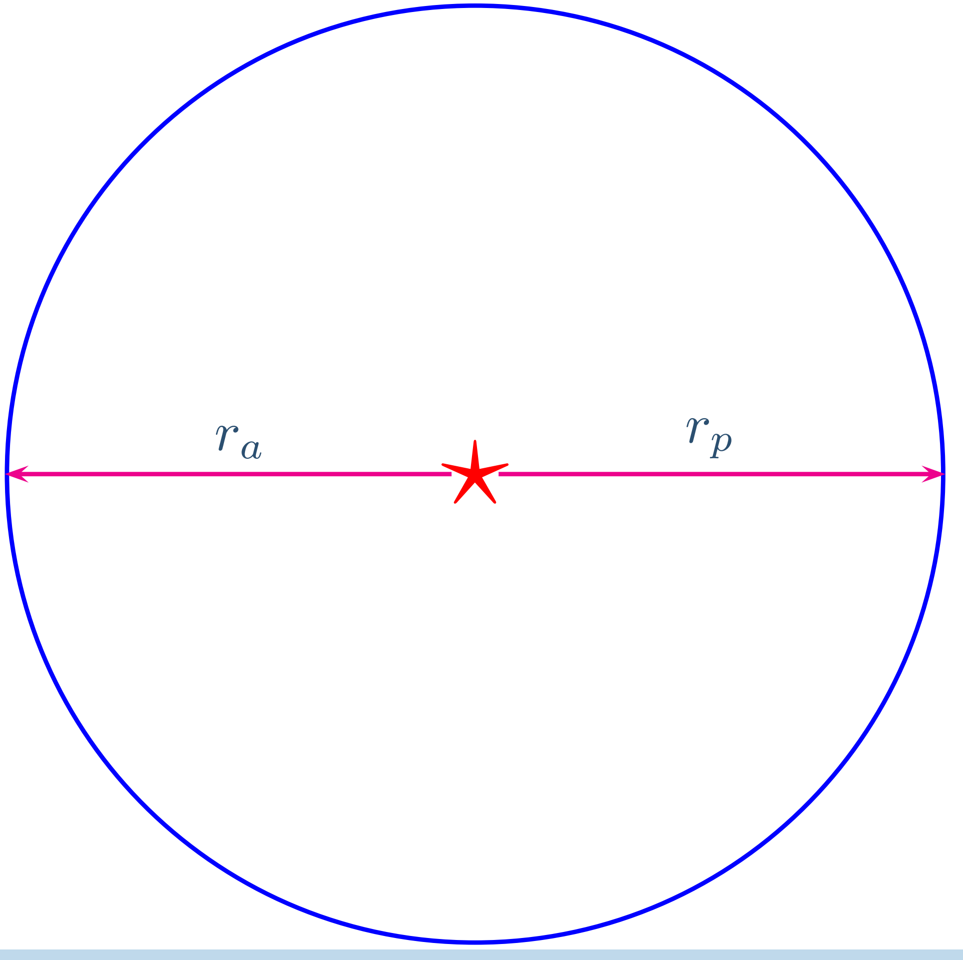
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054

Kepler's First Law II



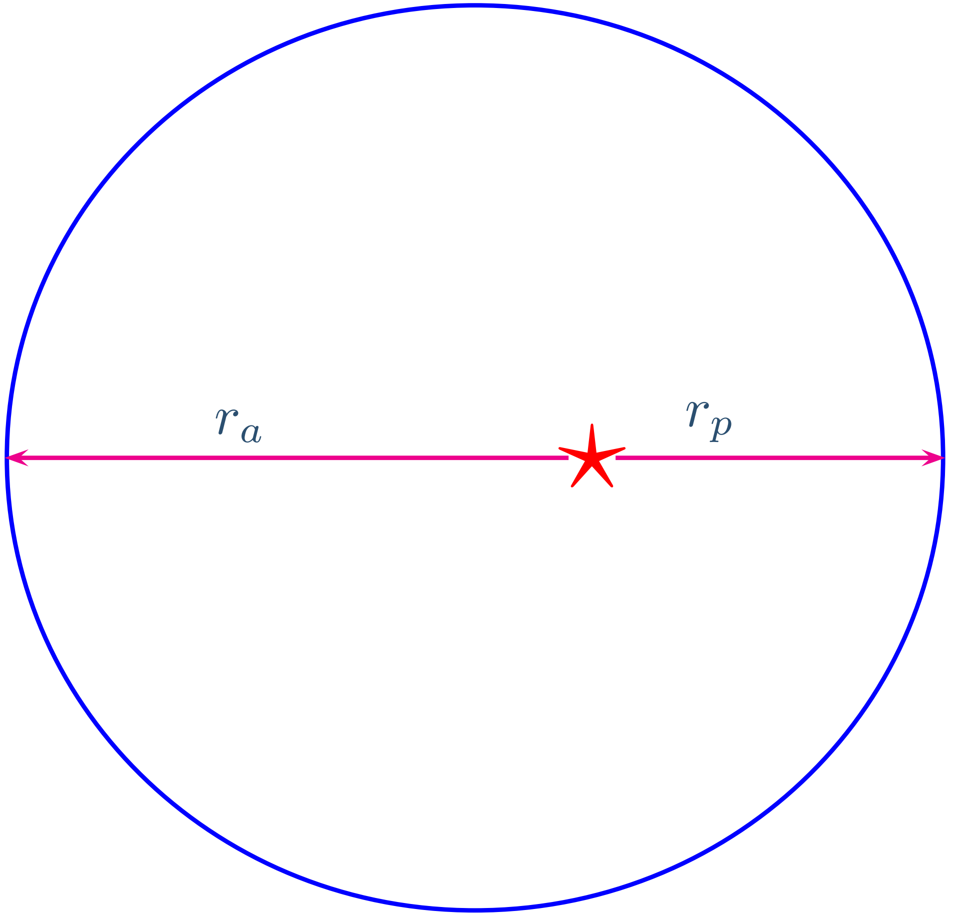
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047

Kepler's First Law II



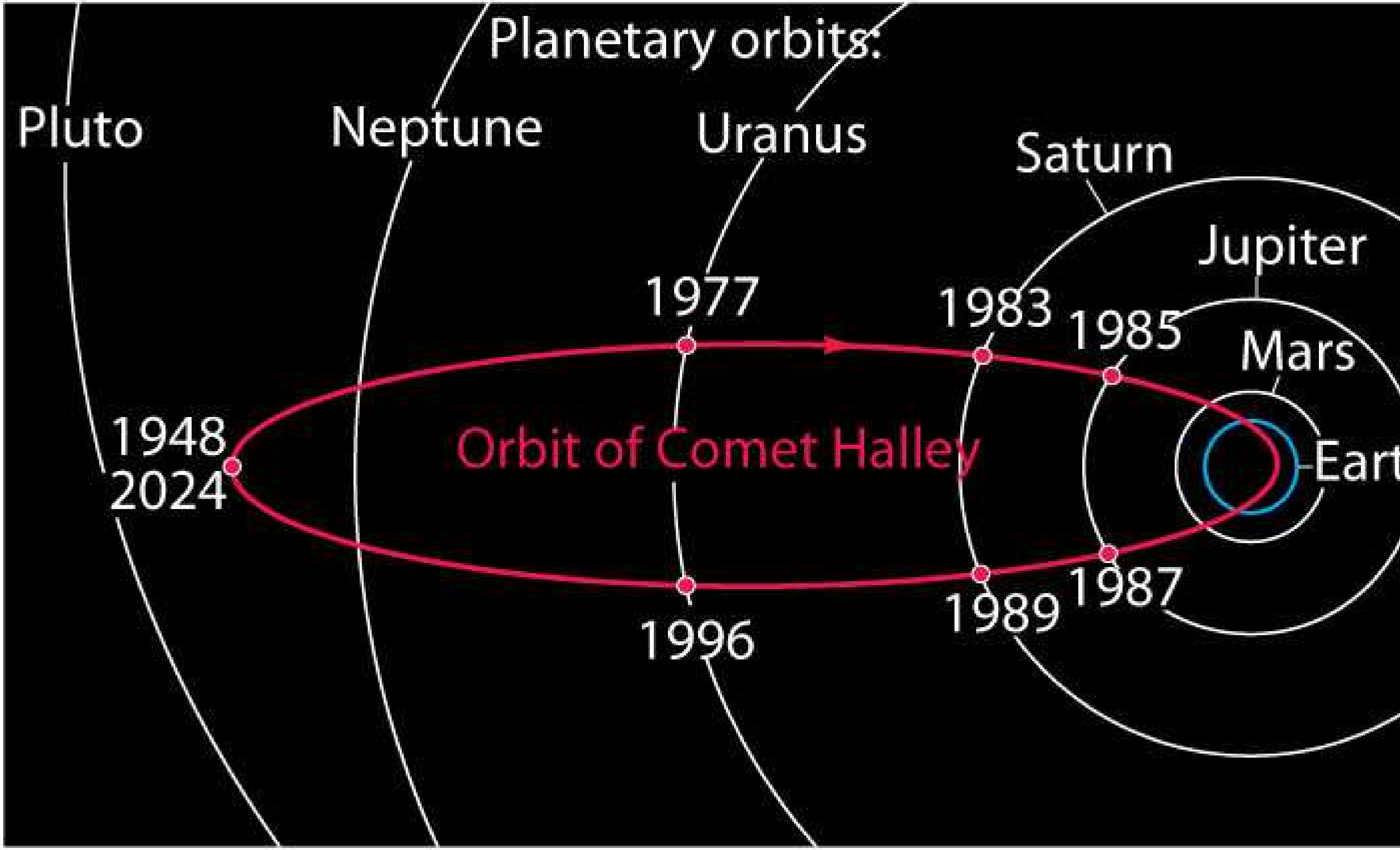
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009

Kepler's First Law II



Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009
Pluto	0.249

Kepler’s First Law III



Kepler’s Second Law

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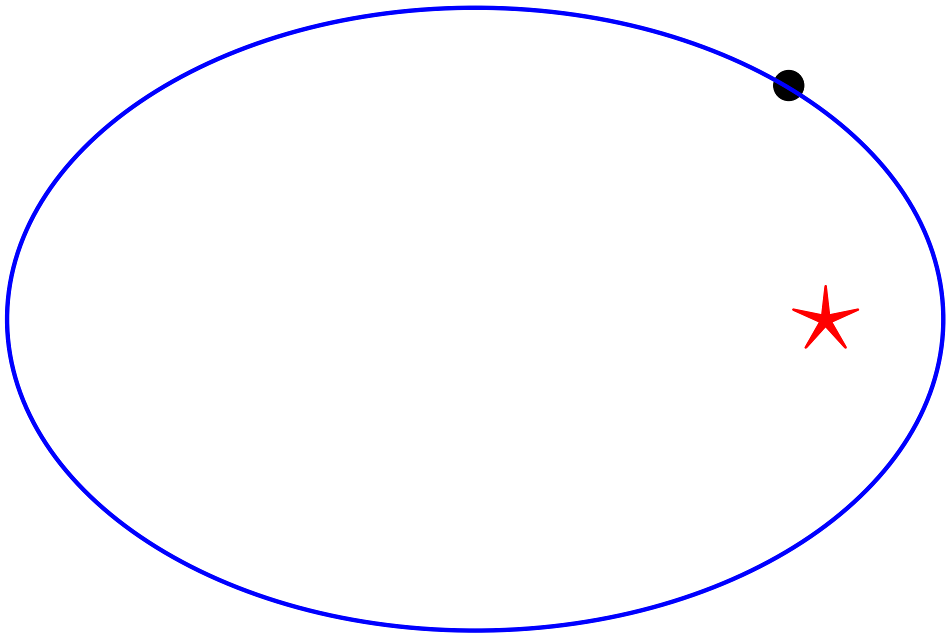

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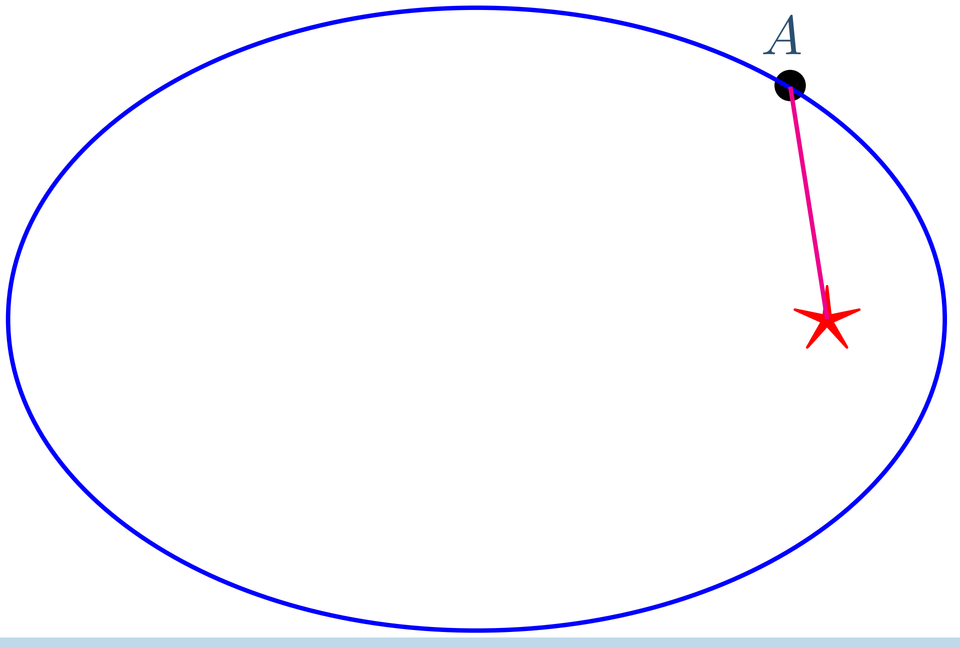

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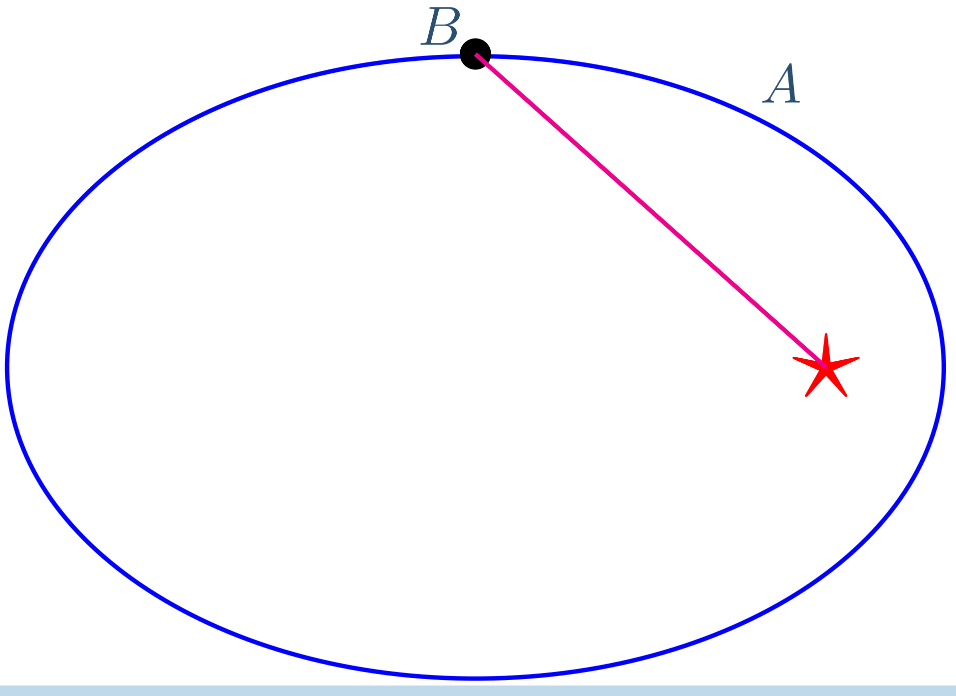

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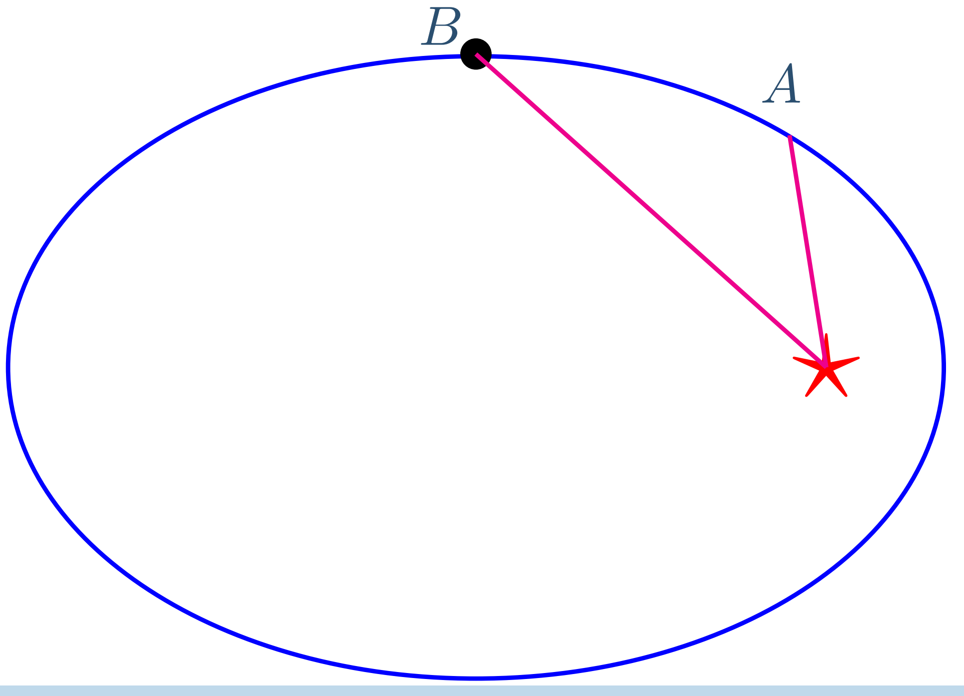

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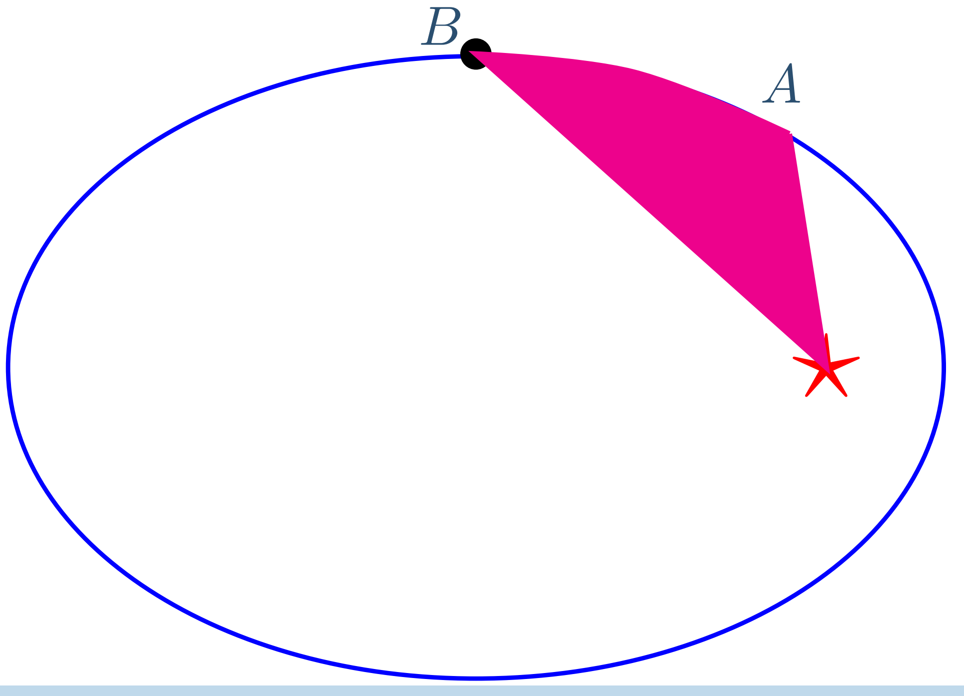

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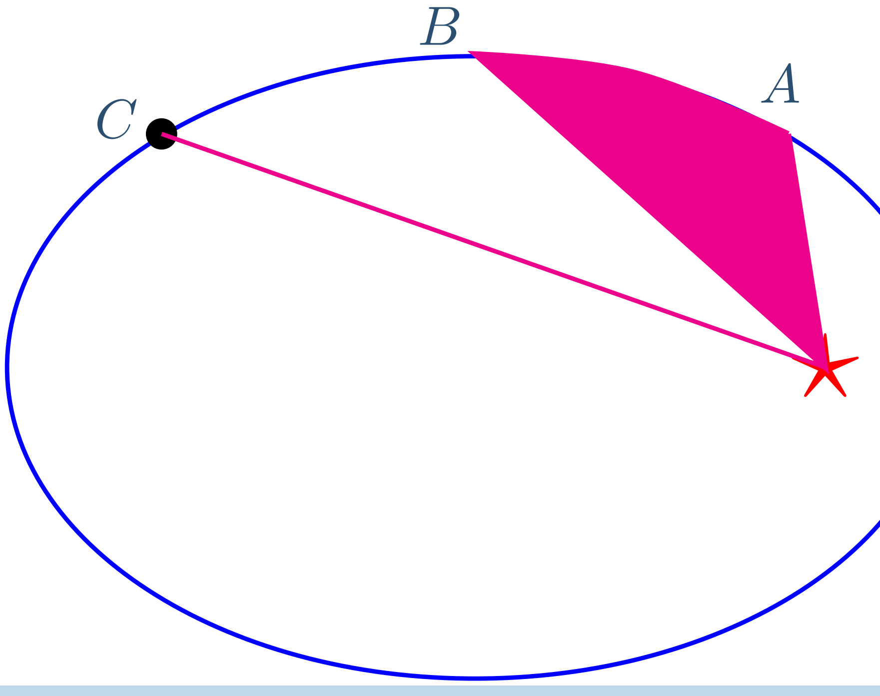

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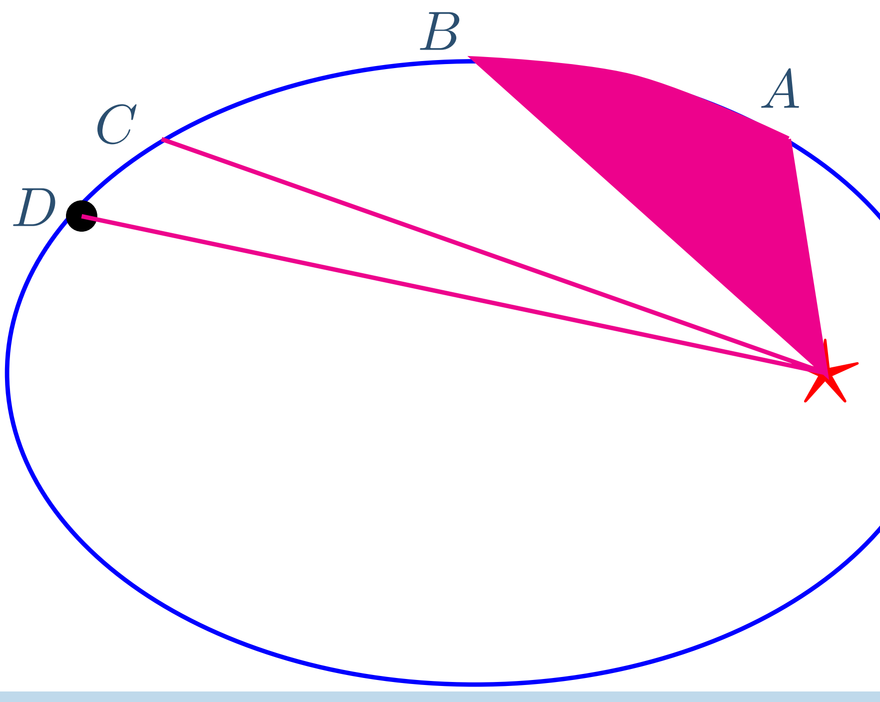

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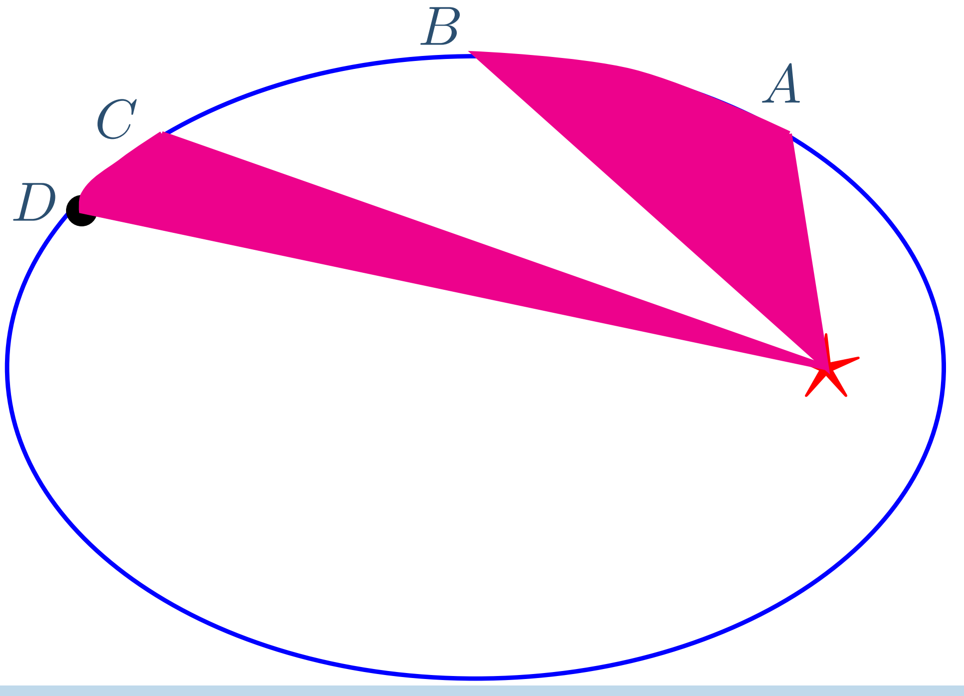

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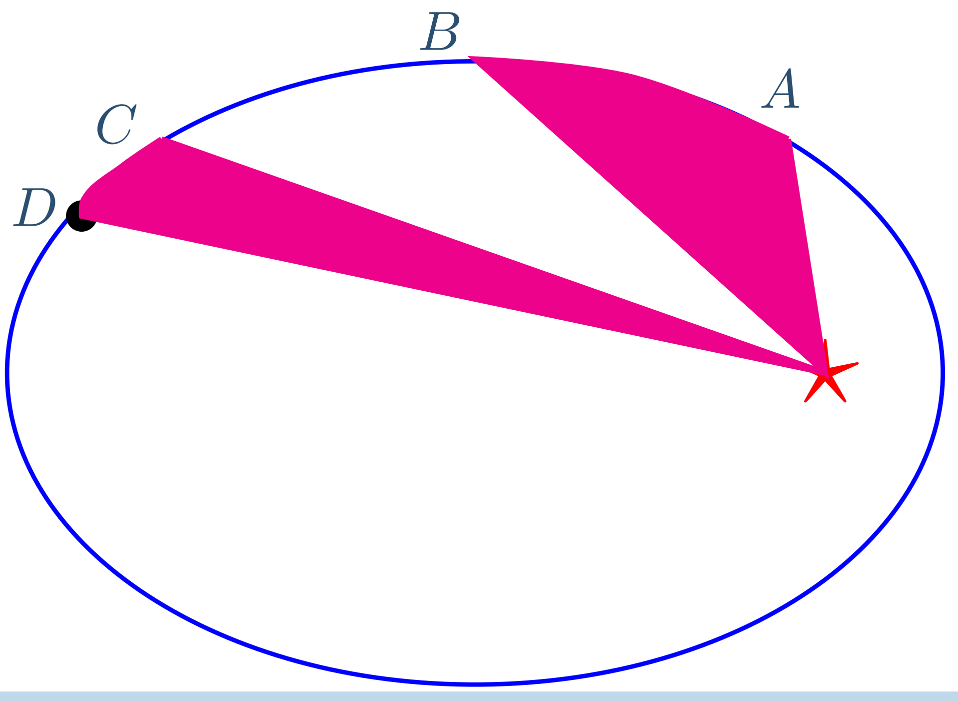

If these areas are equal then the planet takes the same amount of time going from A to B as it does going from C to D

Kepler's Second Law

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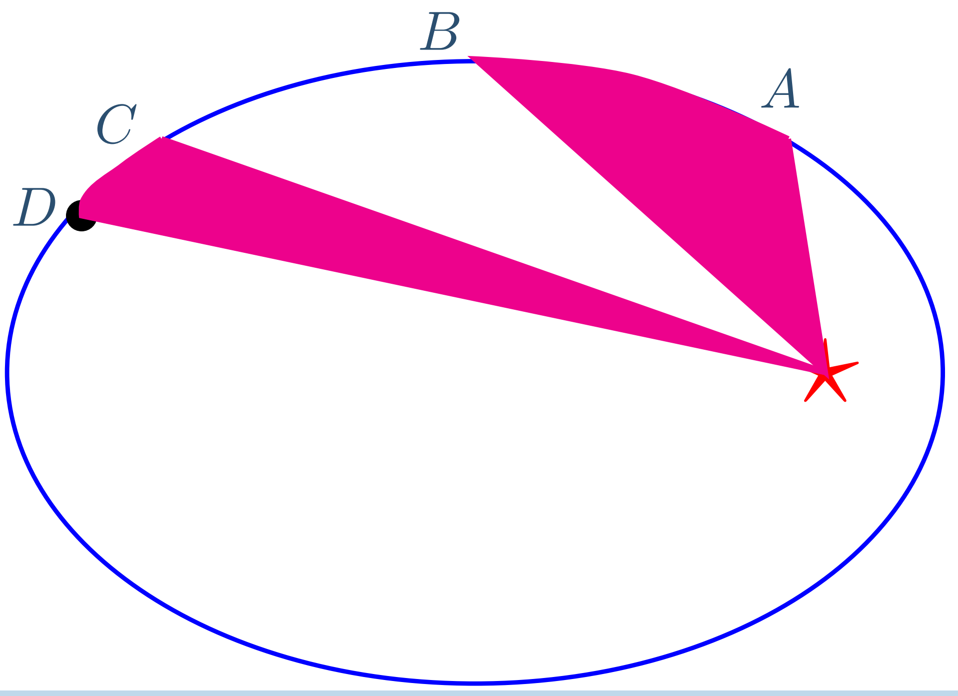

If these areas are equal then the planet takes the same amount of time going from A to B as it does going from C to D

The speed of *any* object in an elliptical orbit is NOT constant

Kepler’s Second Law II

Proof:

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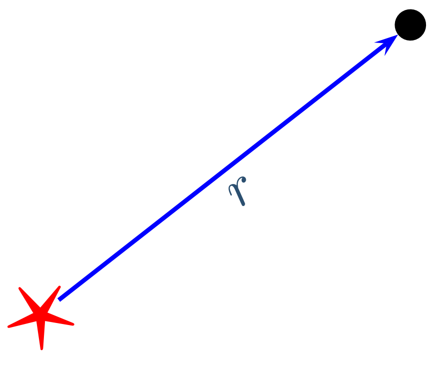

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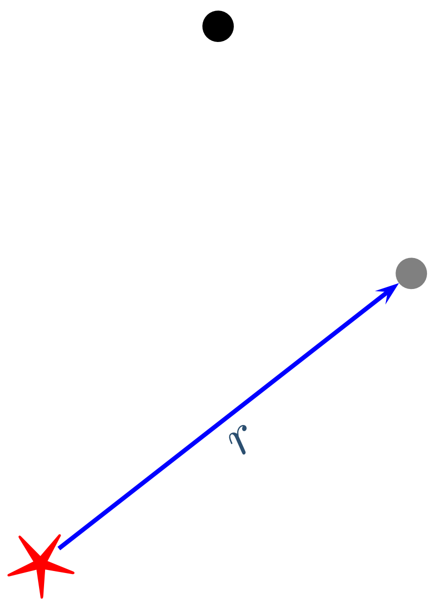

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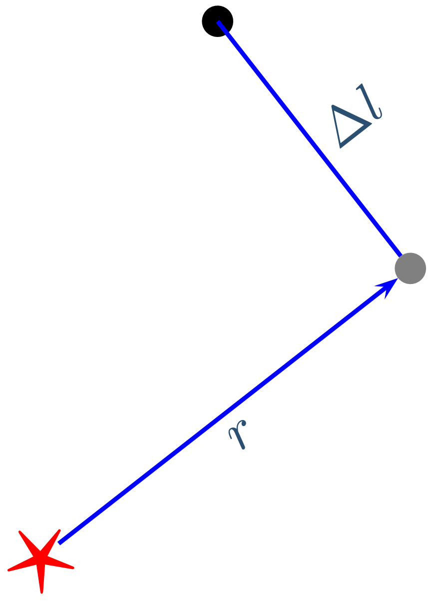

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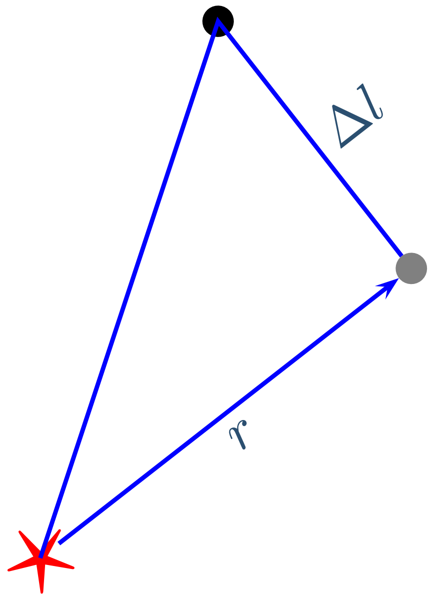

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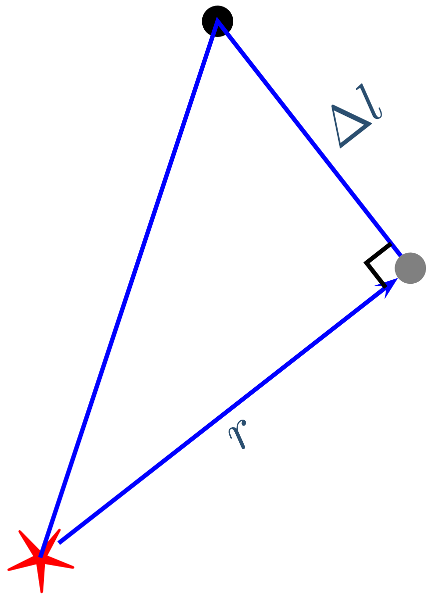

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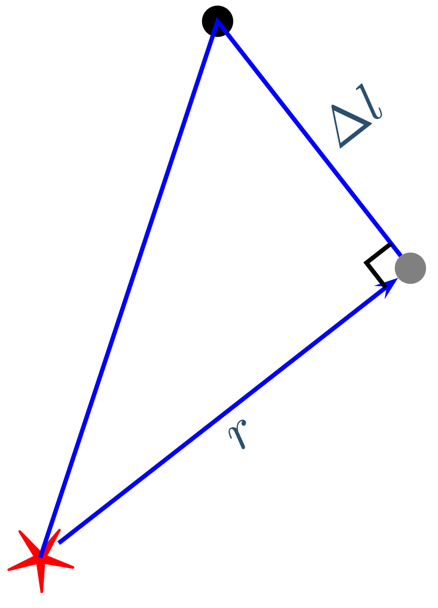

$$A \approx \frac{1}{2}(r)(\Delta l)$$

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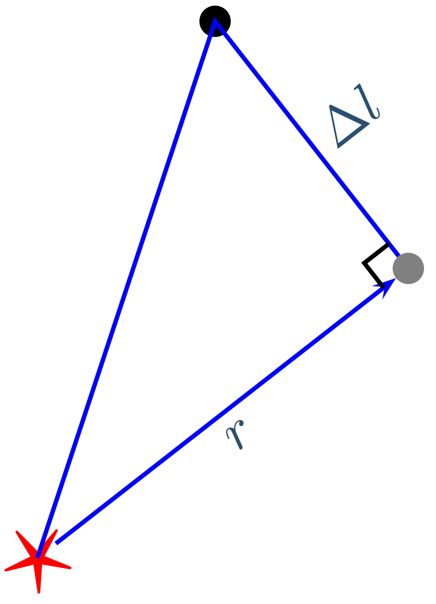

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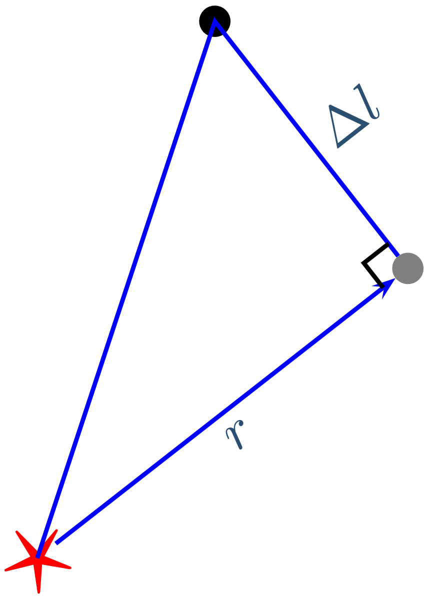

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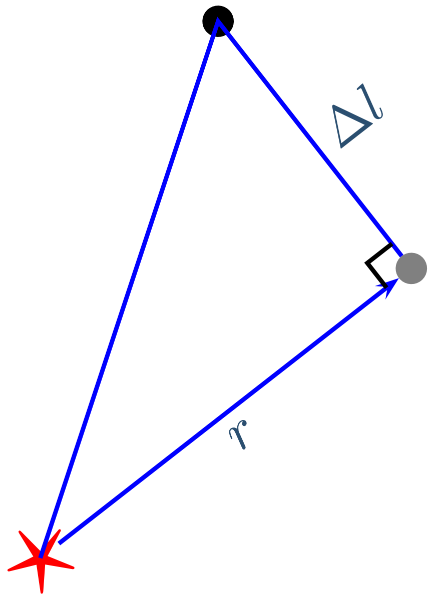

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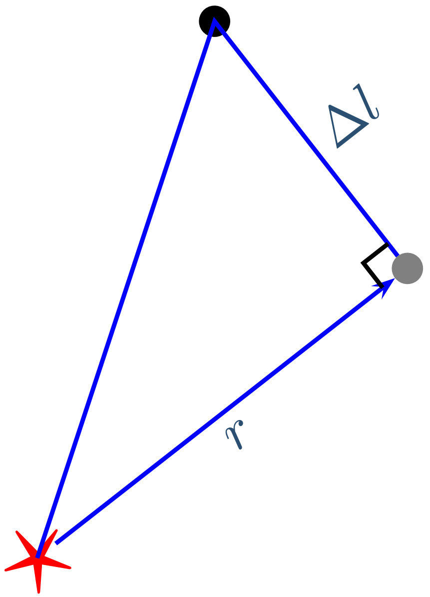

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← Tangential Velocity

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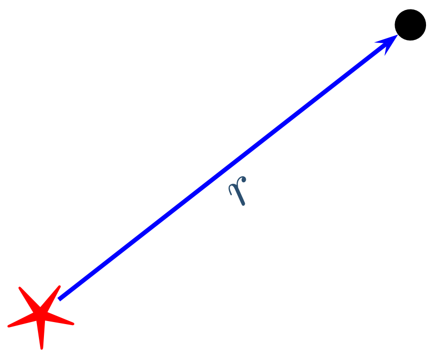

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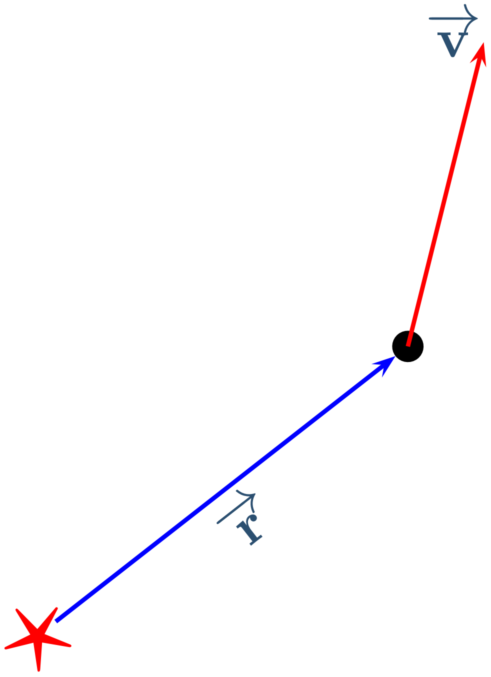

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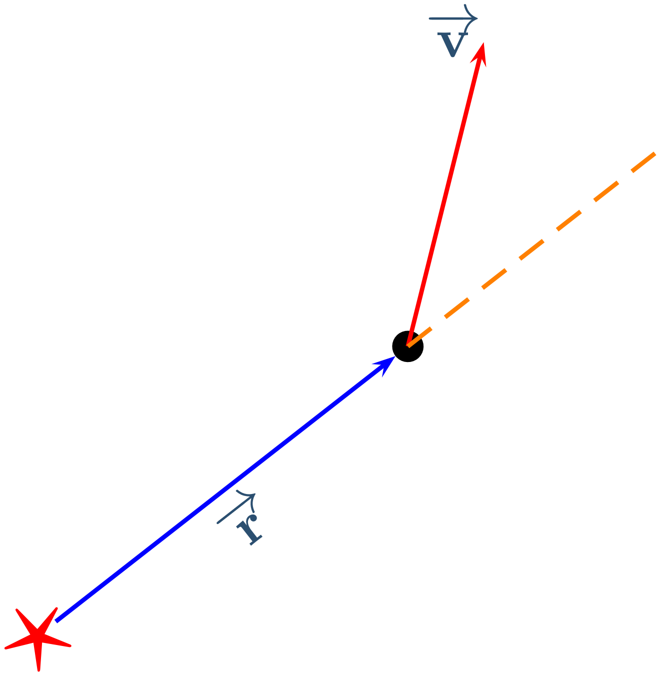

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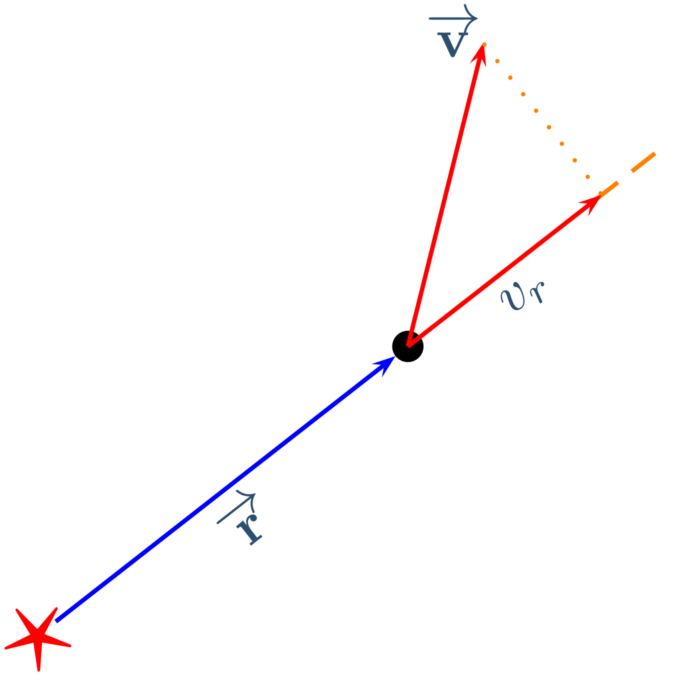

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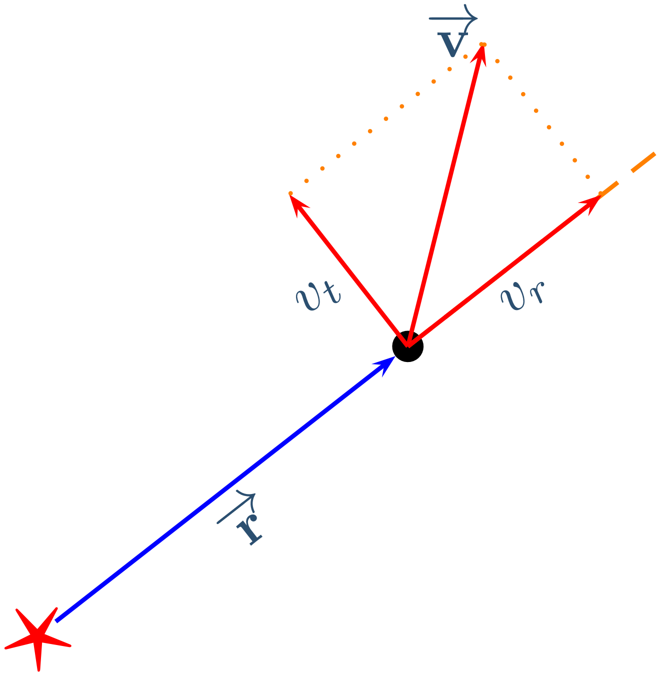

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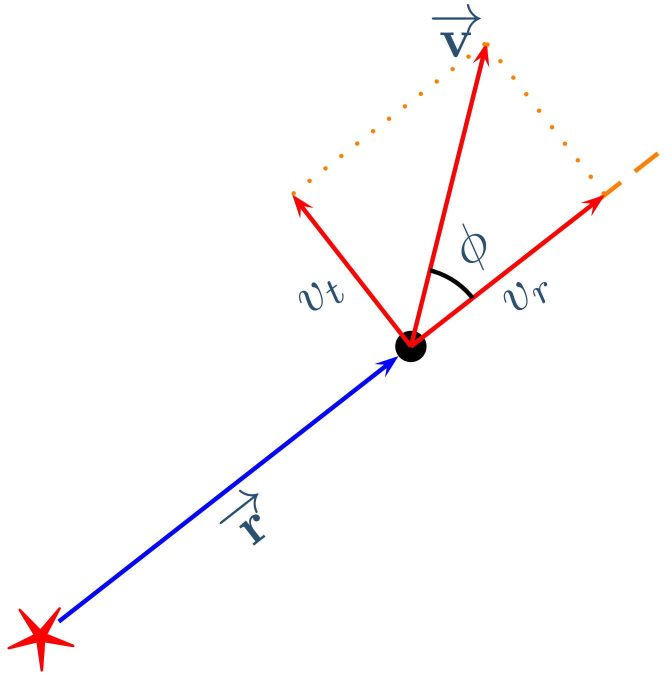

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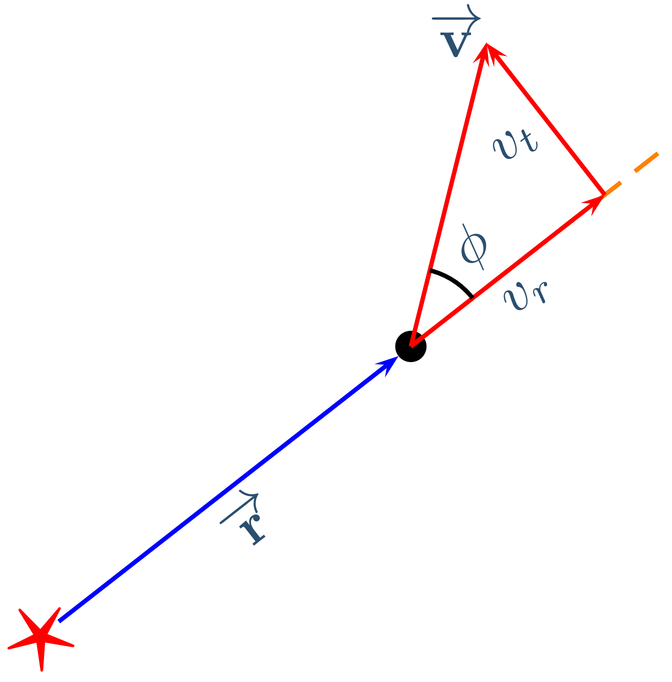

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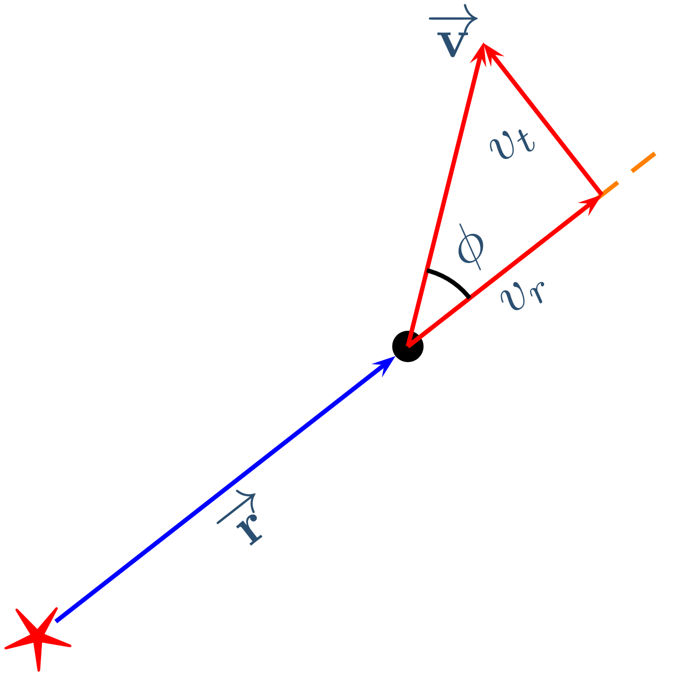

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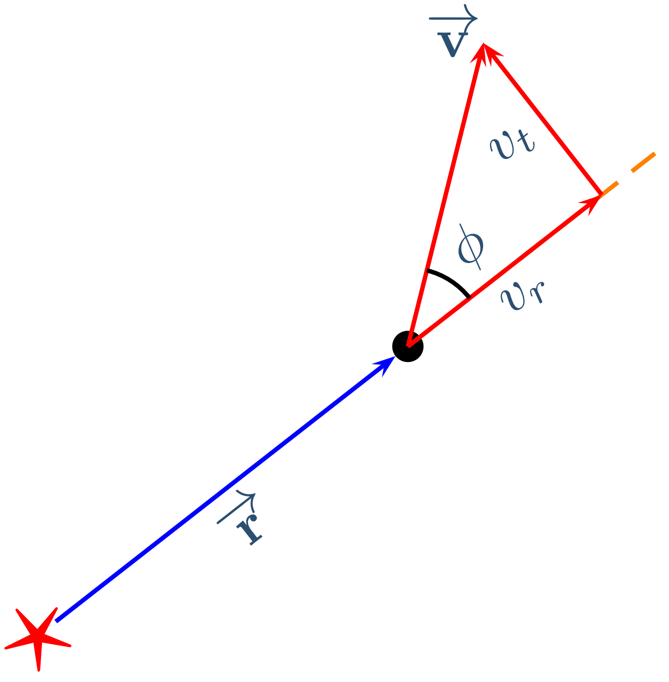

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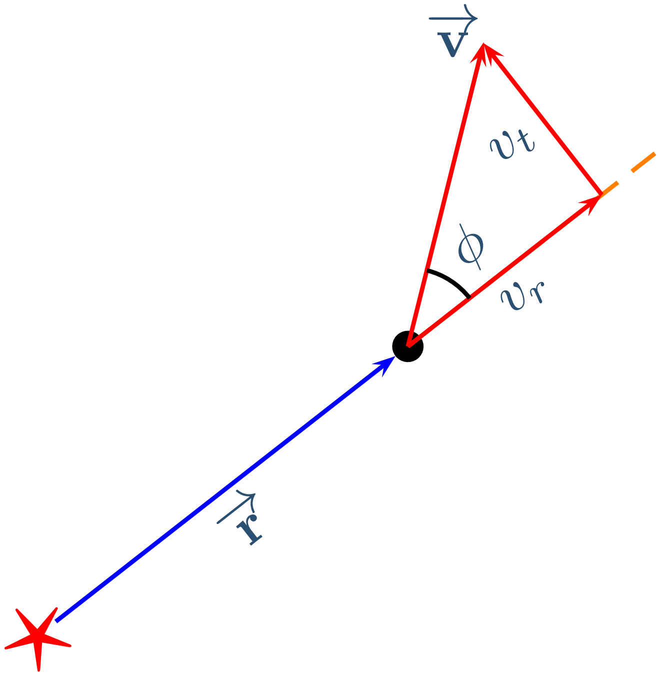

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Angular Momentum of a Satellite: $\boxed{L = Mvr\sin\phi}$

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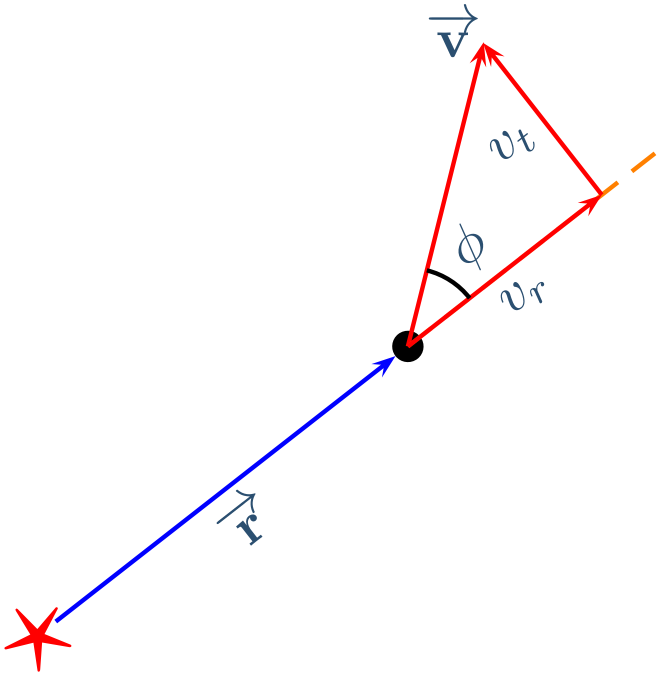

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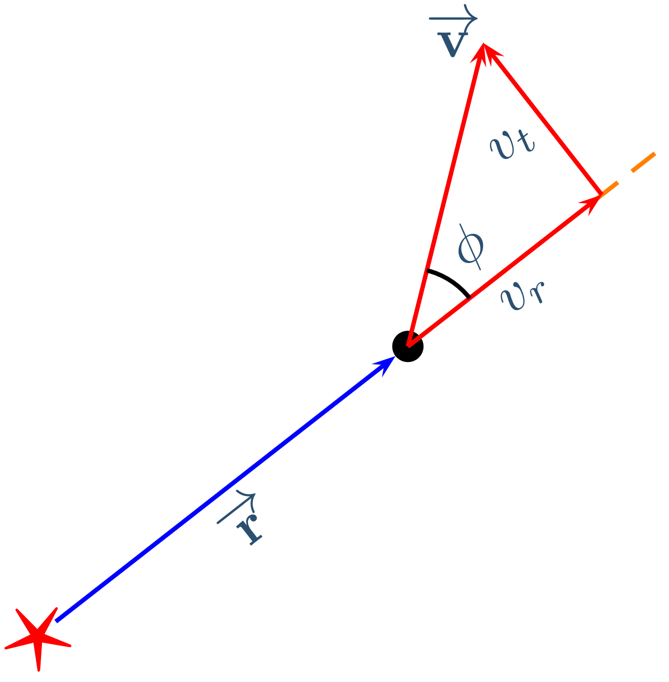

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Kepler's Second Law IV

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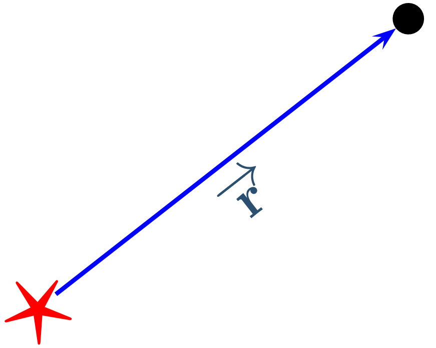

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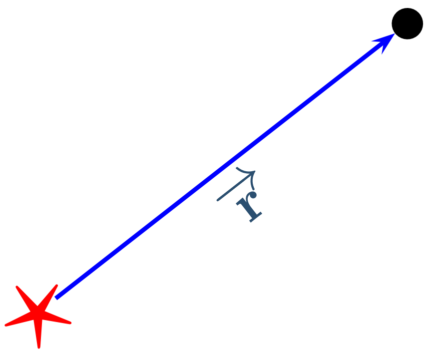

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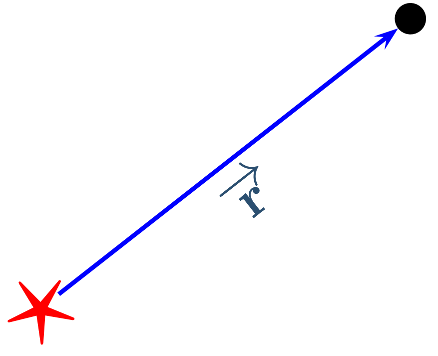

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On satellites, gravity causes no torque

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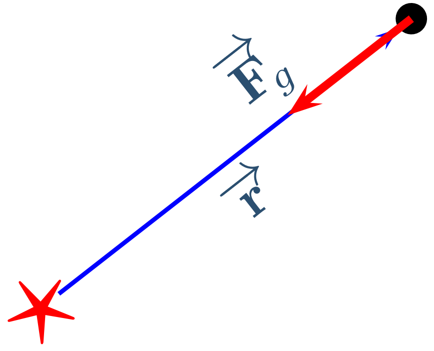

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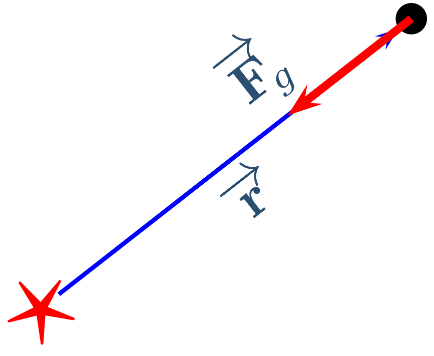

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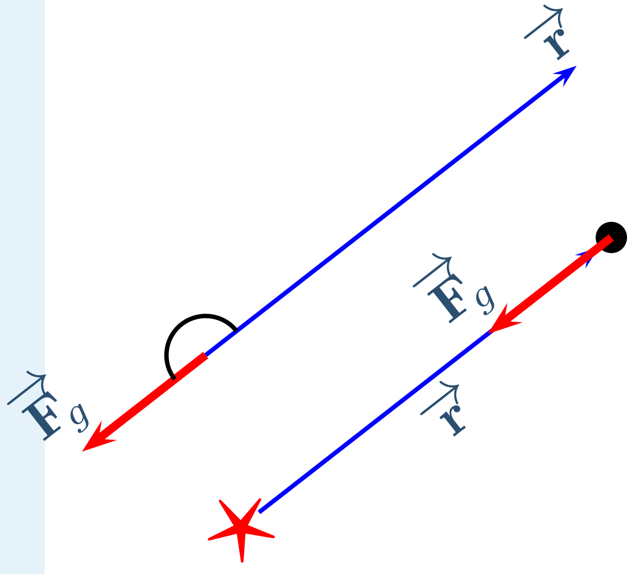

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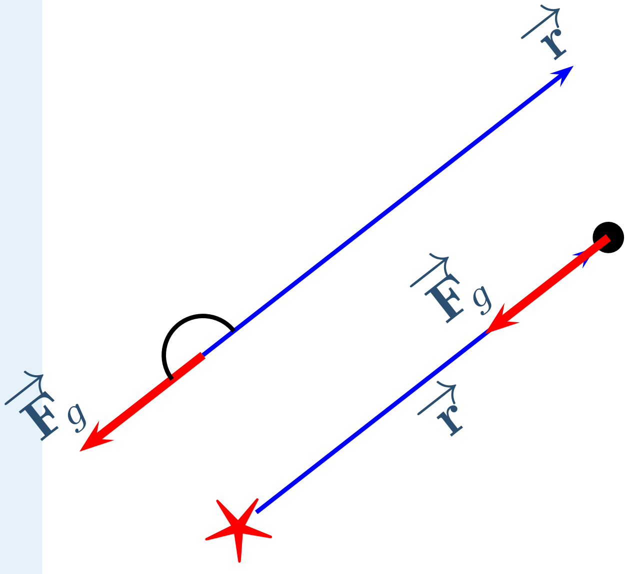

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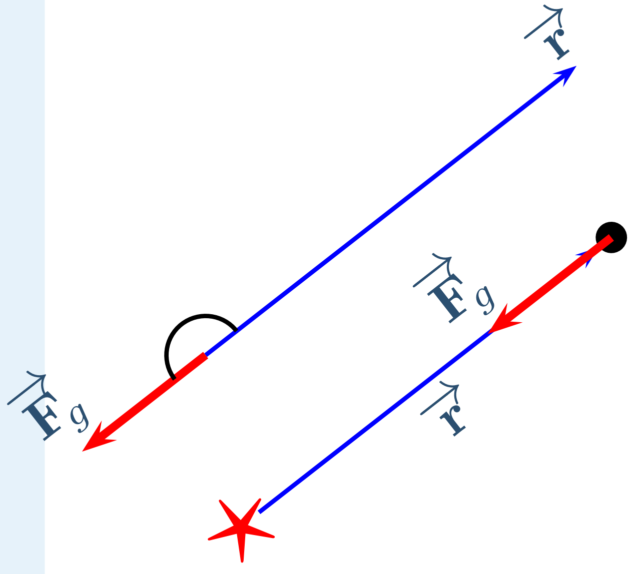

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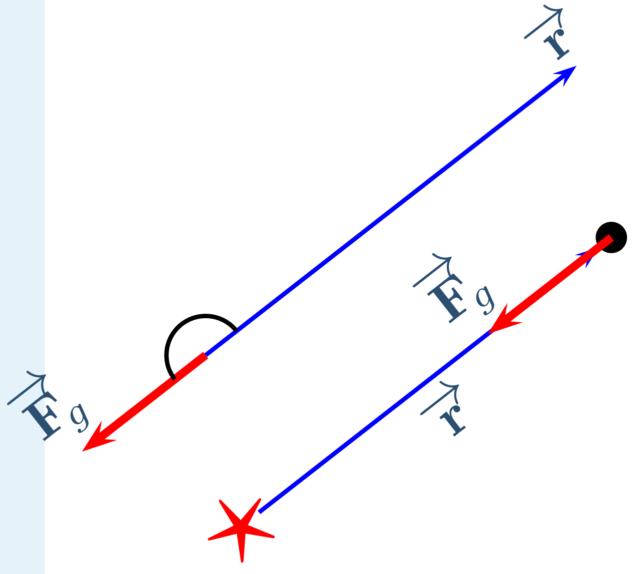

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The angular momentum of a satellite is constant

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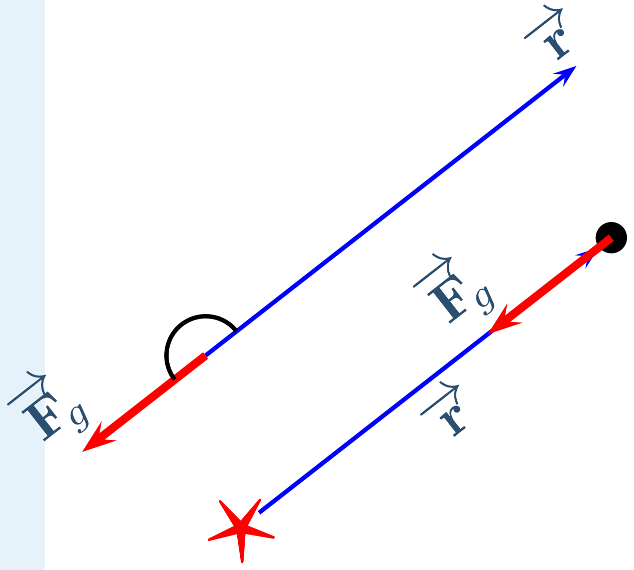

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$$\frac{dA}{dt} = constant \Rightarrow$$ equal ΔA for equal Δt

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At perihelion and aphelion the angle ϕ for satellite motion is $90°$.

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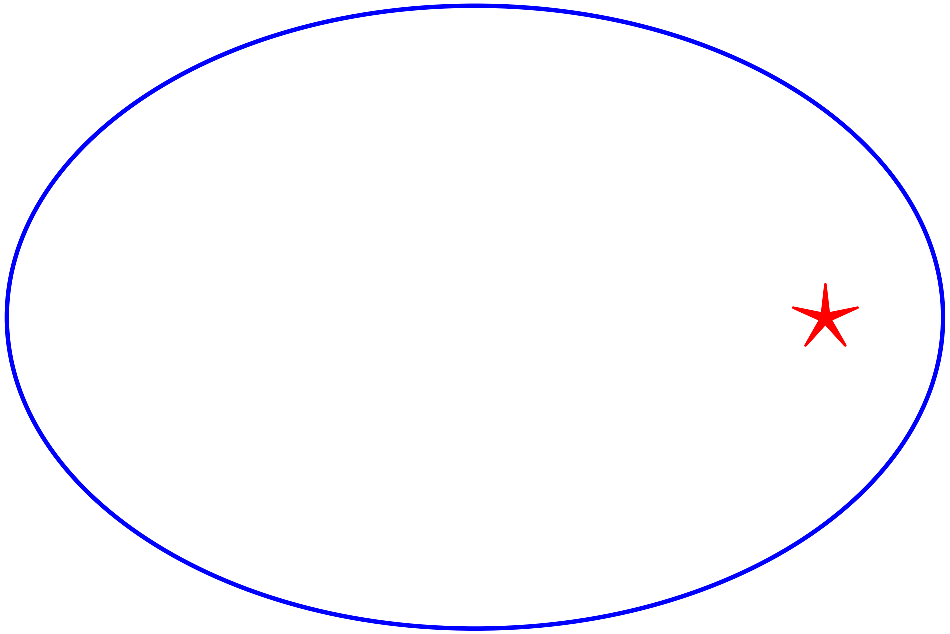

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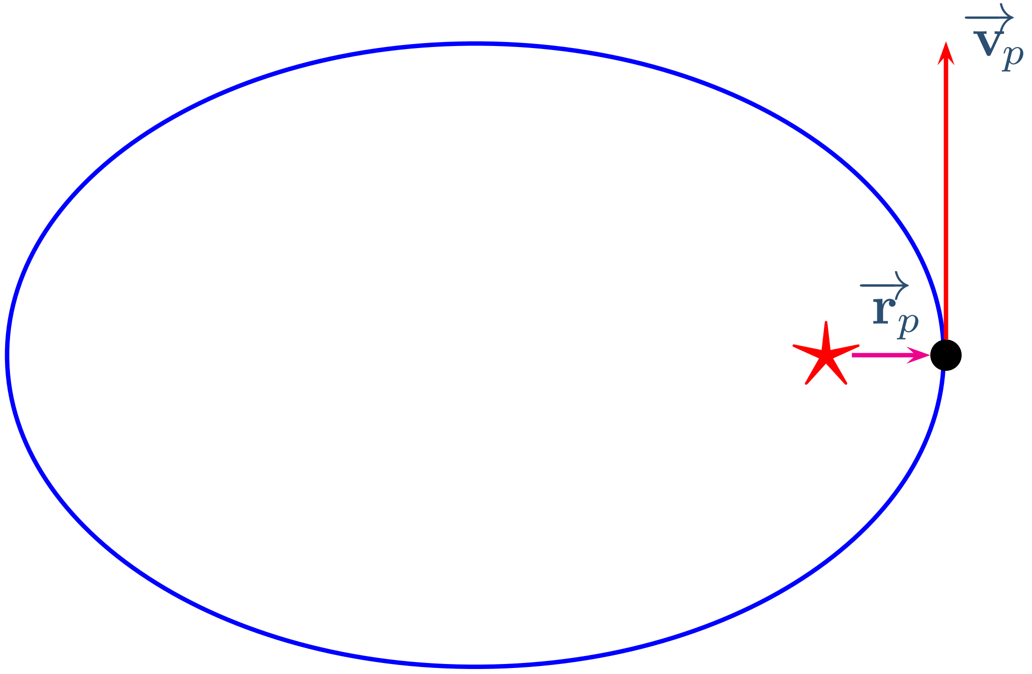

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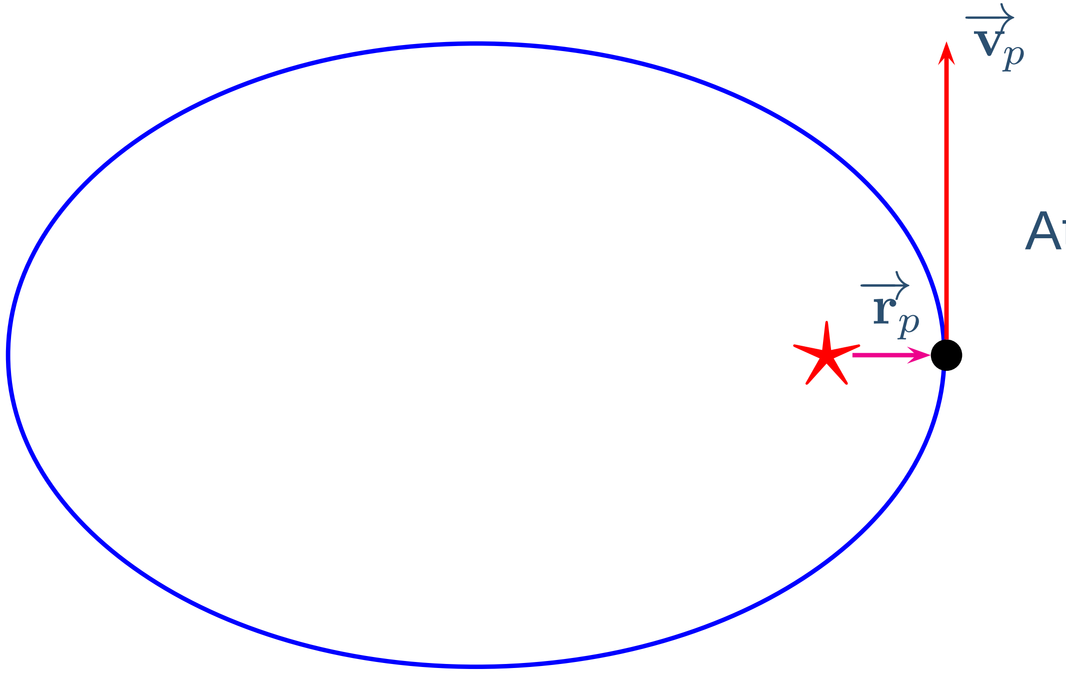

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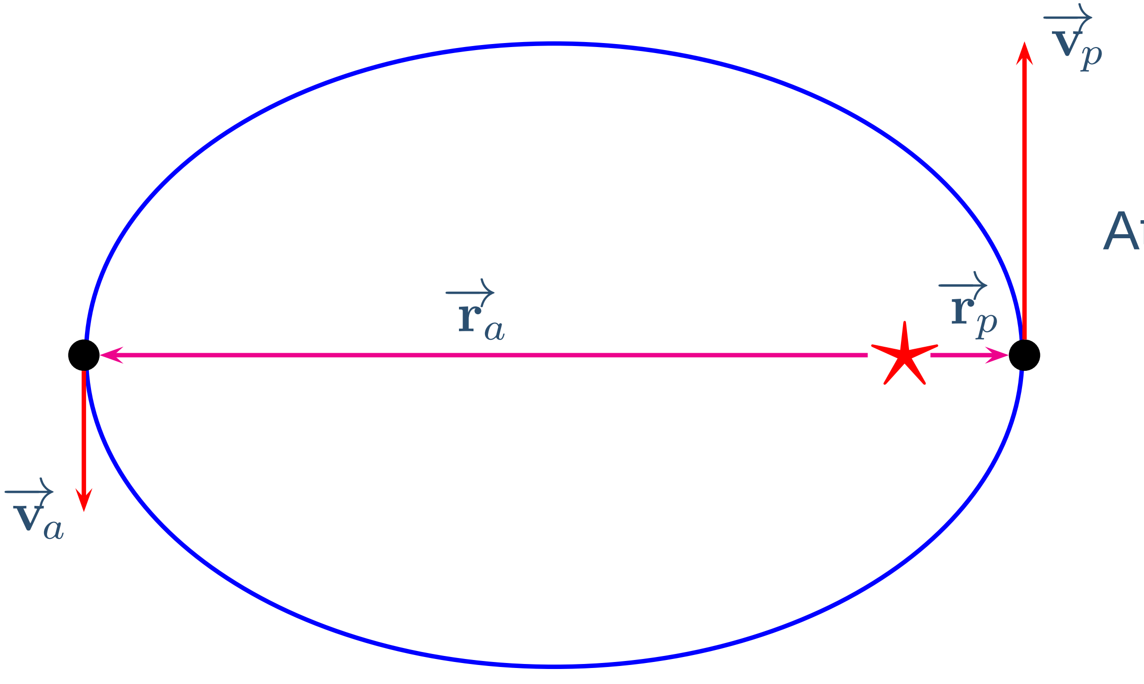

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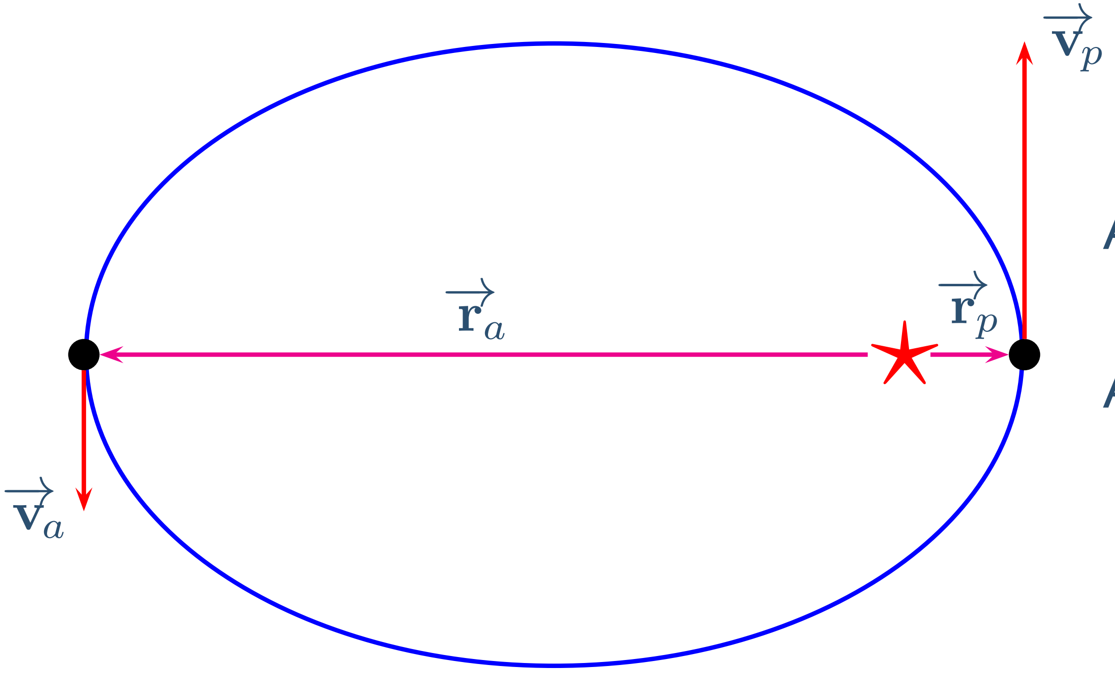

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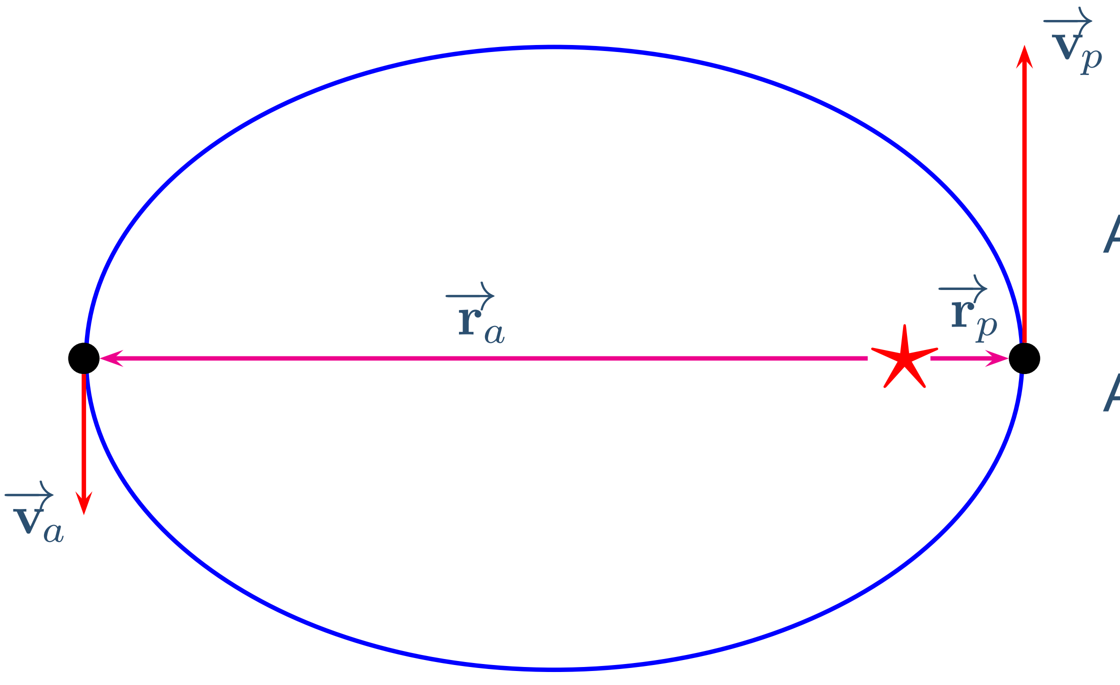

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$\Rightarrow Mv_p r_p = Mv_a r_a$

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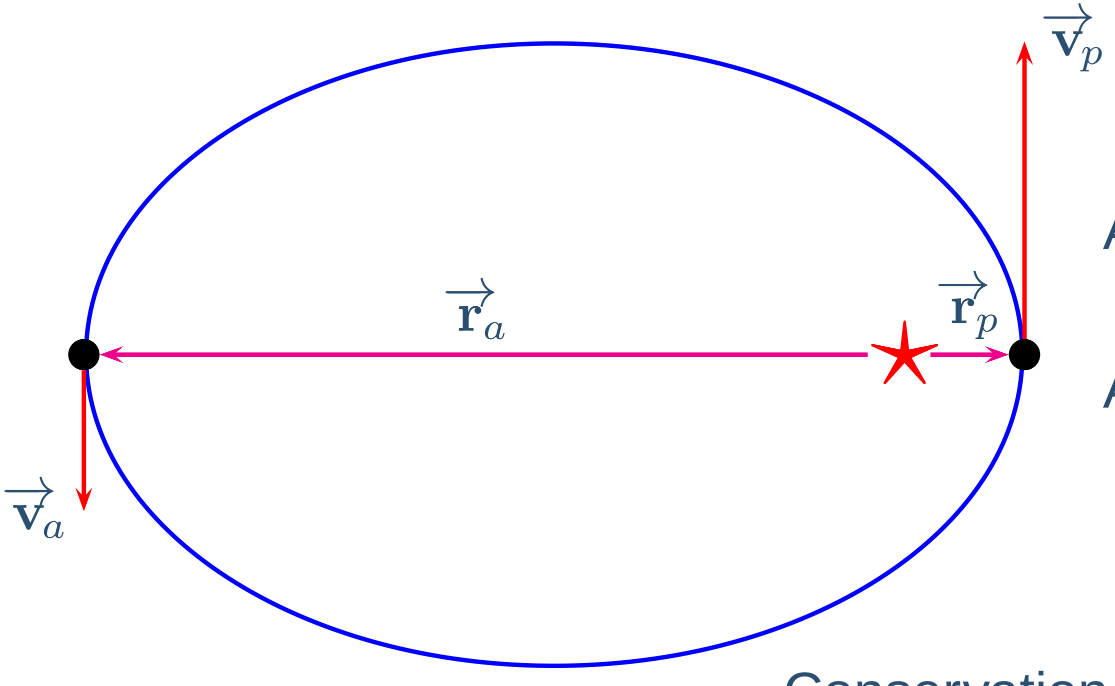

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$\Rightarrow \boxed{v_p r_p = v_a r_a}$

Kepler's Third Law

Kepler's Laws:

3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.

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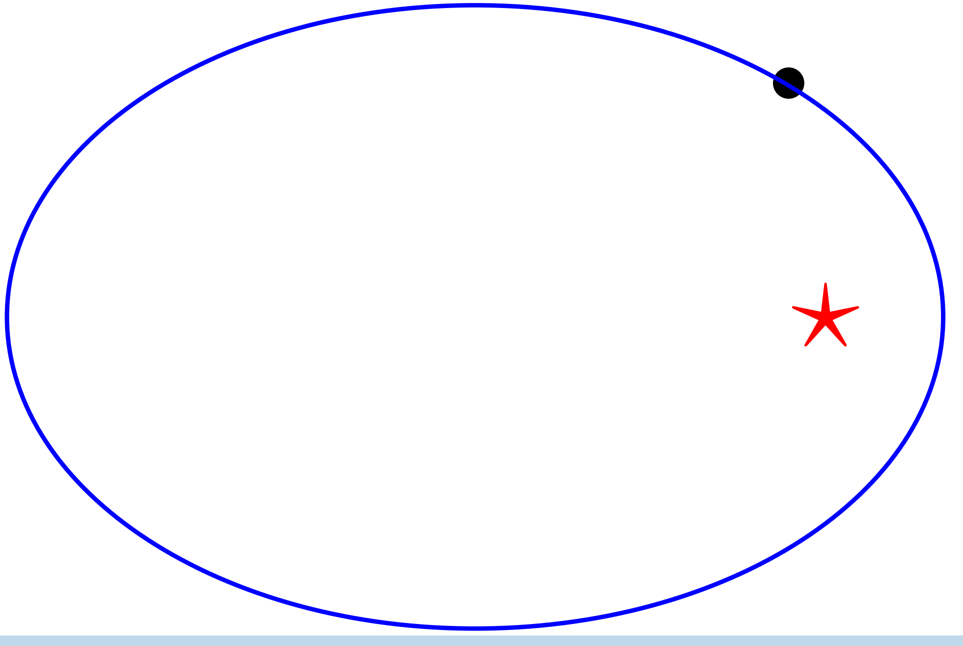

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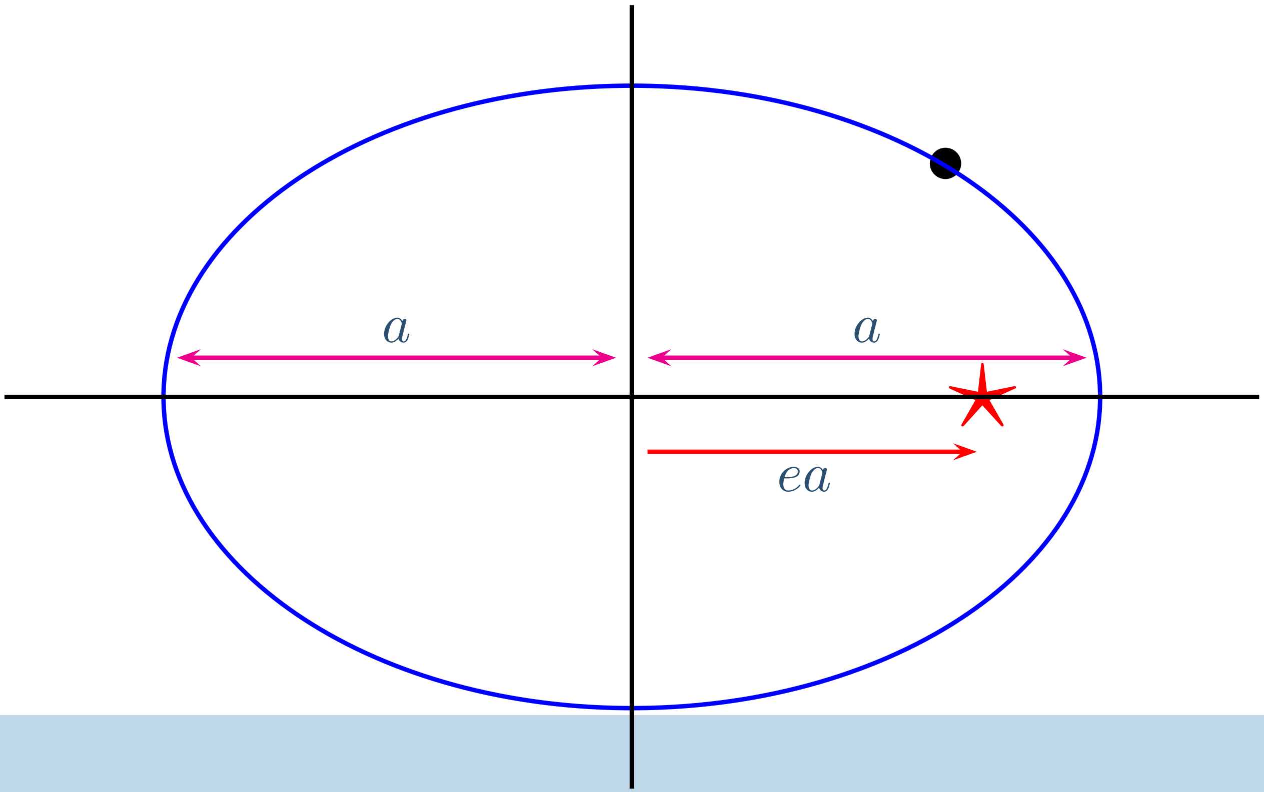

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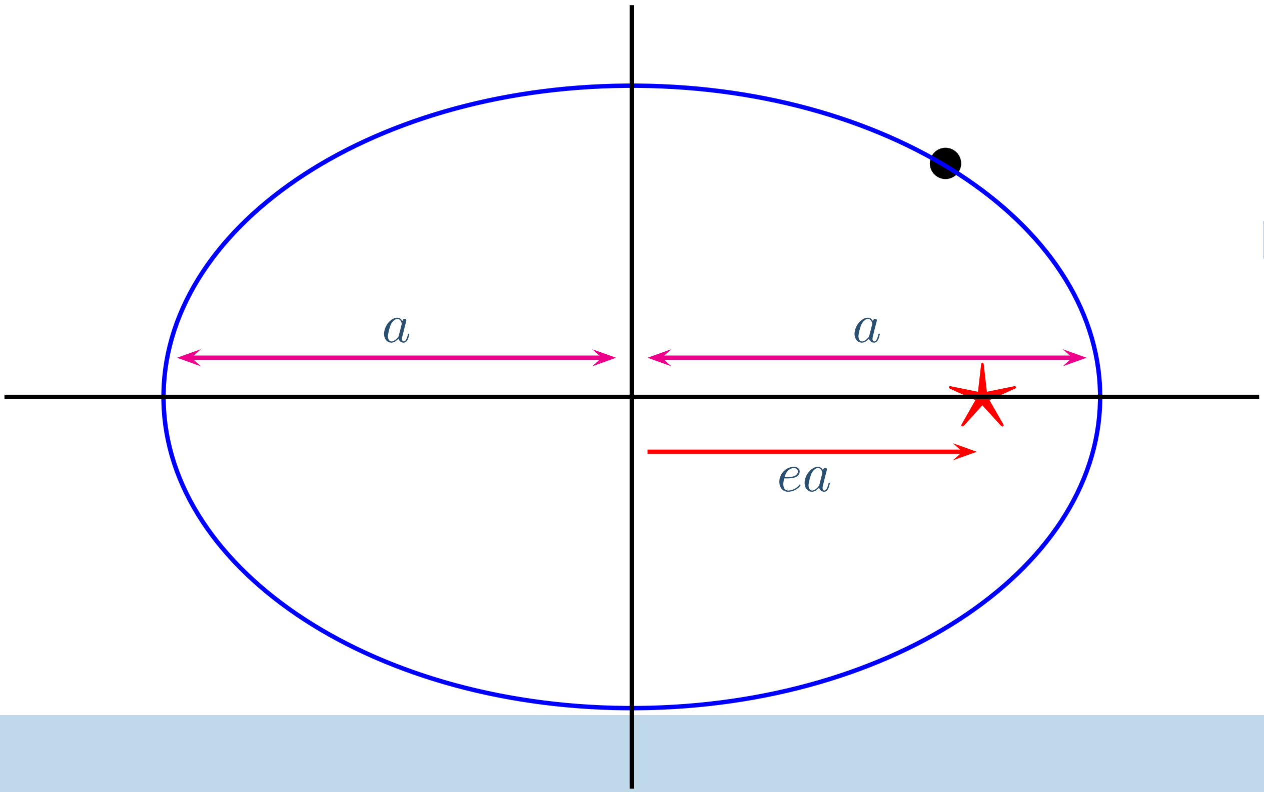

Kepler: $T \propto a^{3/2}$

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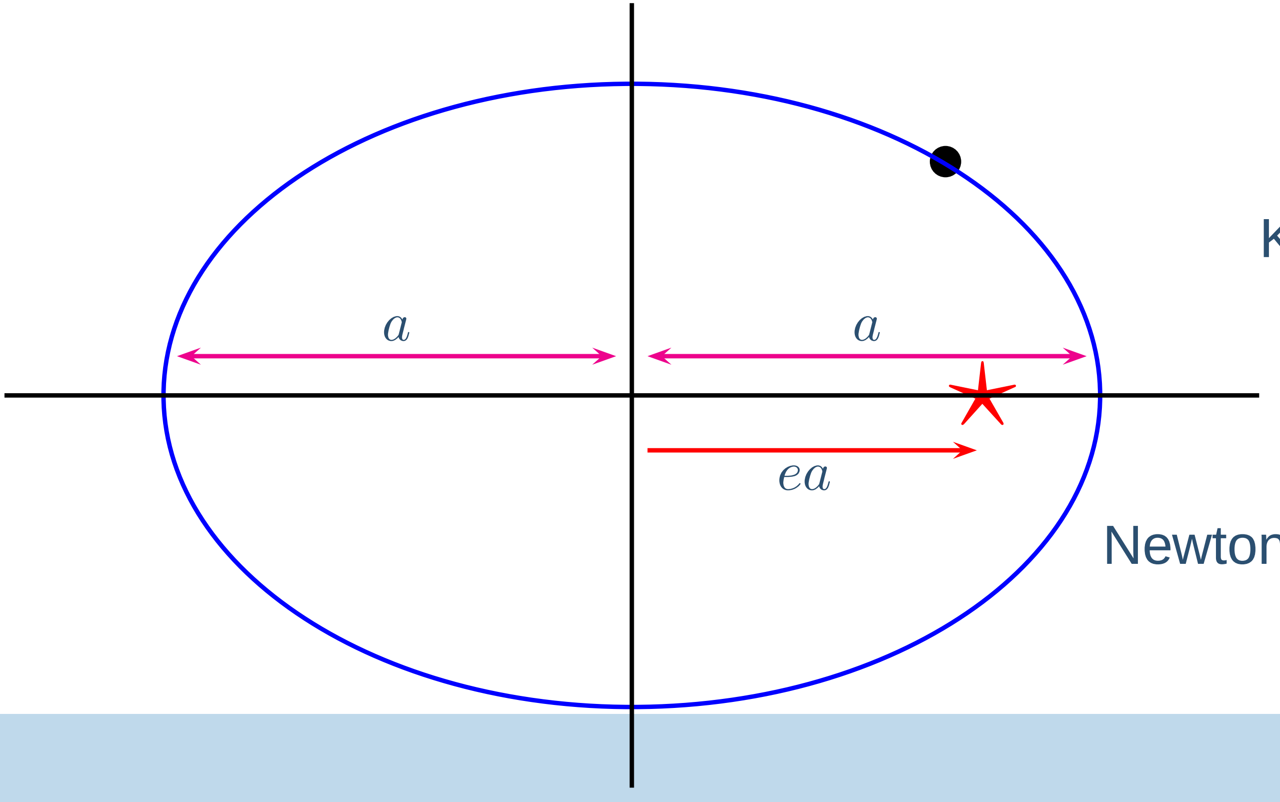

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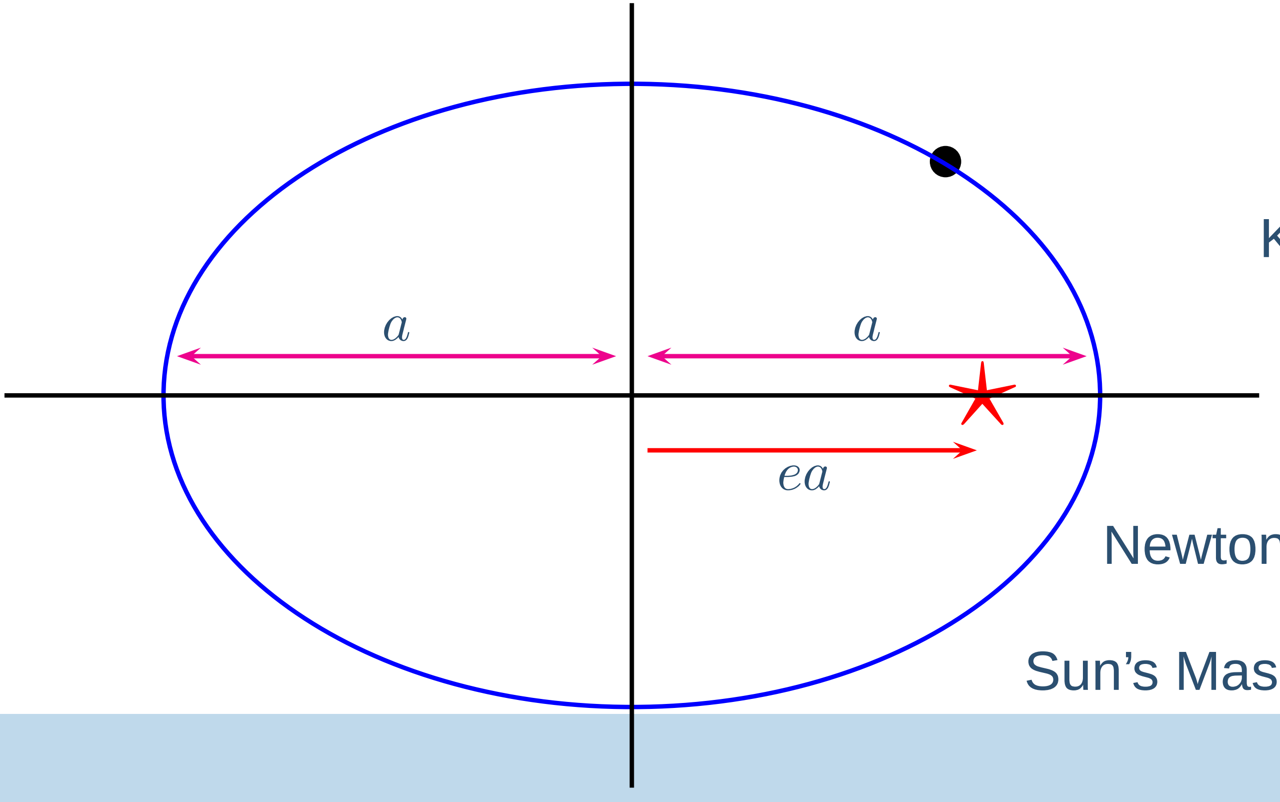

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Sun's Mass