April 22, Week 14

Today: Chapter 13, Gravity

Exam #4, Next Friday, April 26

Practice Exam on Website.

Review Sessions: Thursday, April 25, 5:15PM, 114 Regener Hall

Newton’s Law of Gravitation

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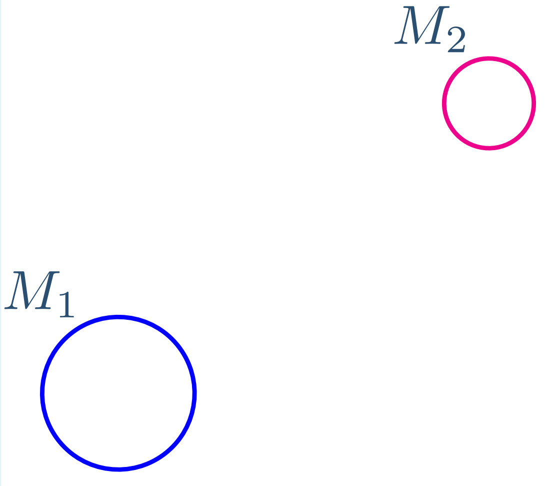

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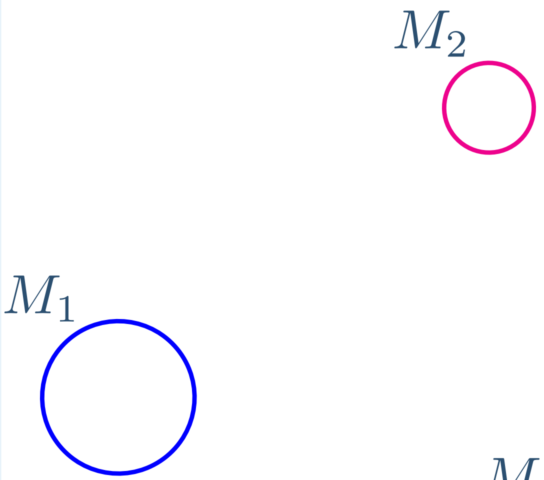

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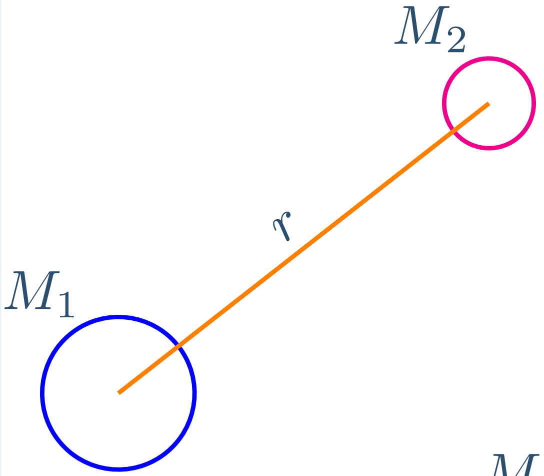

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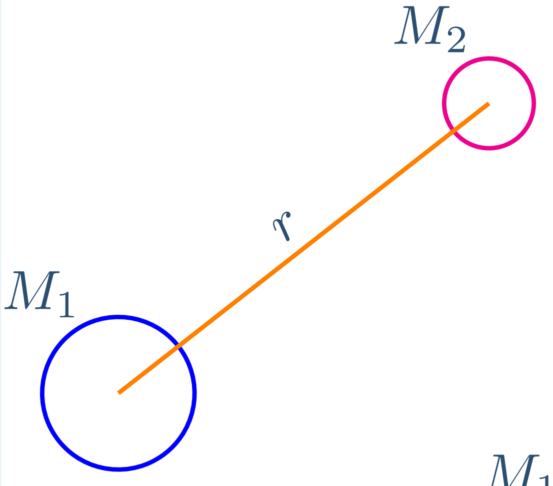

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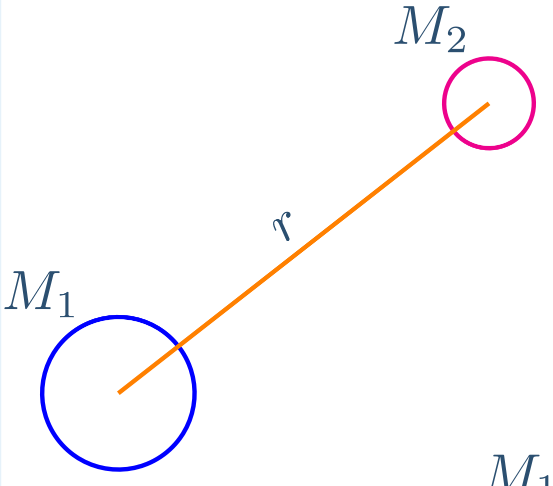

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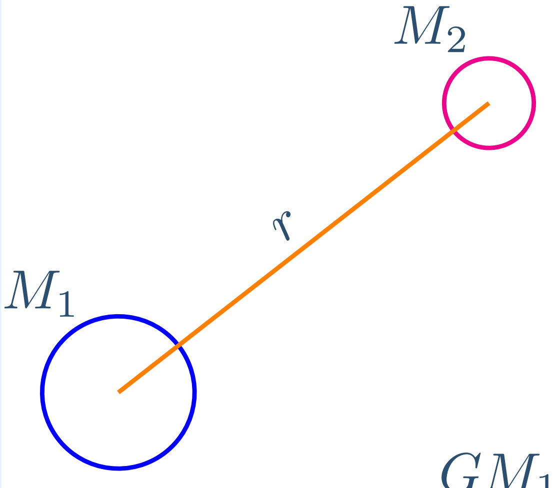

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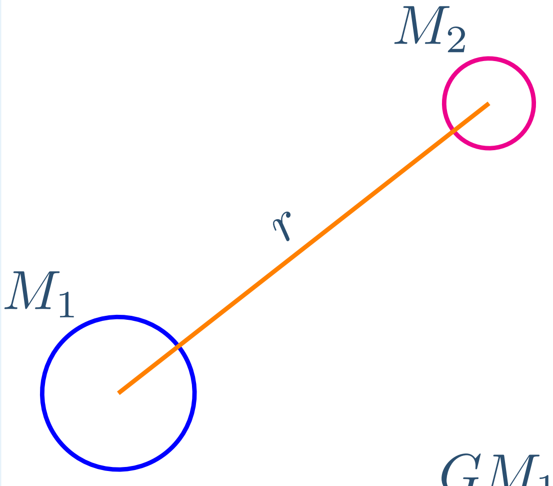

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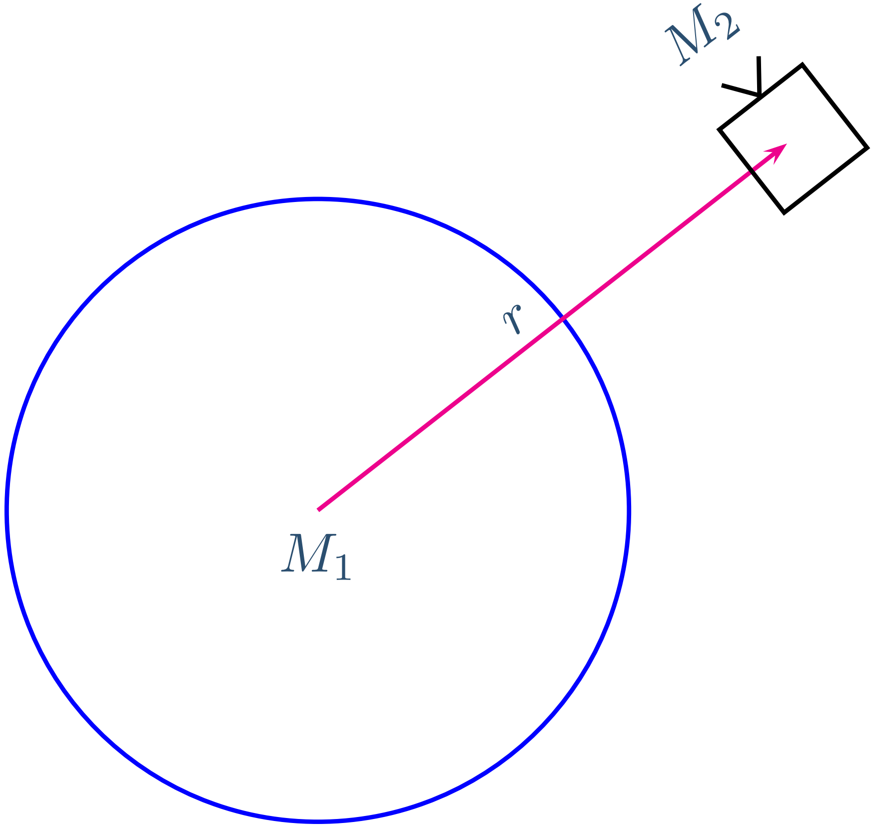
Universal Gravitational Constant:
$G = 6.67 \times 10^{-11}\, N \cdot m^2/kg^2$

The Negative Sign

For distances larger than a planet's radius:



$$U_g = -\frac{GM_1M_2}{r}$$

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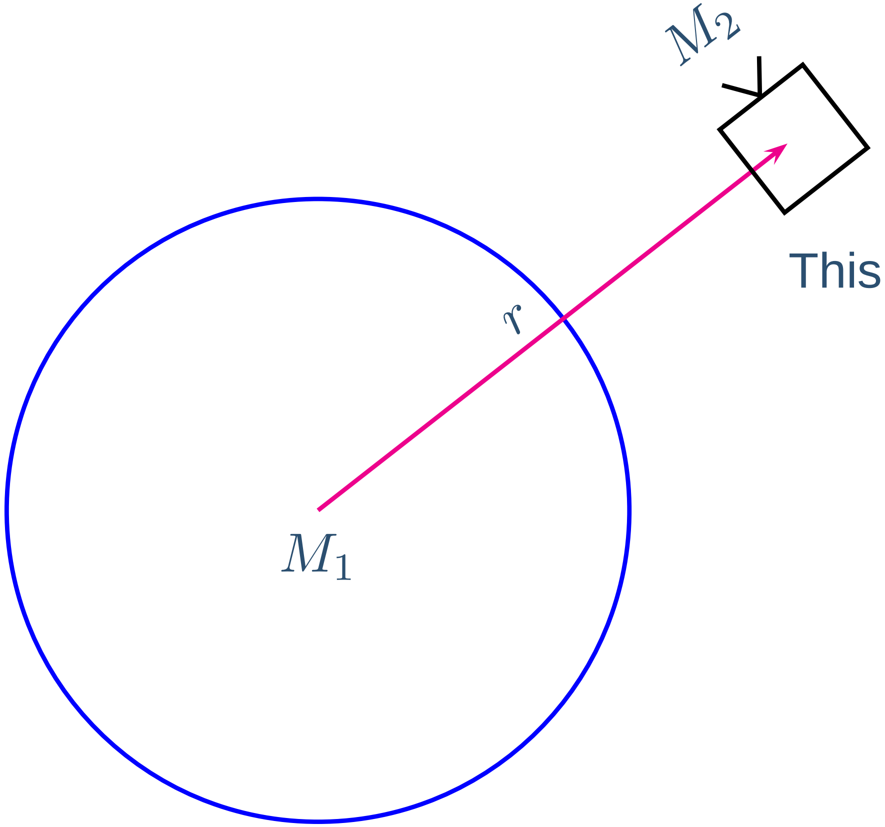

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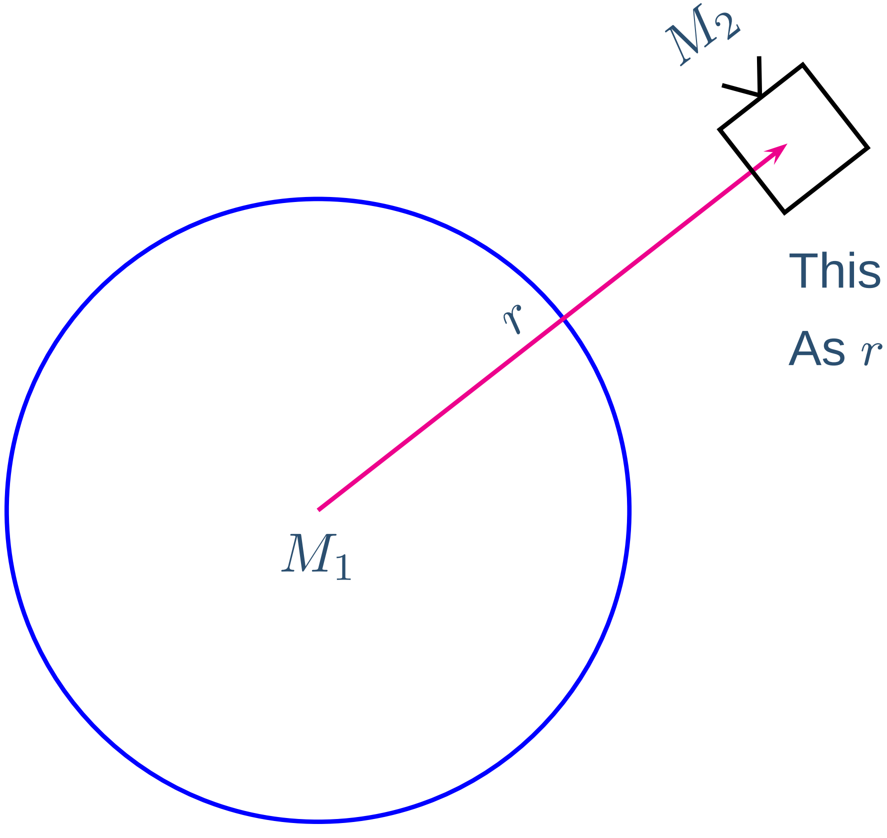

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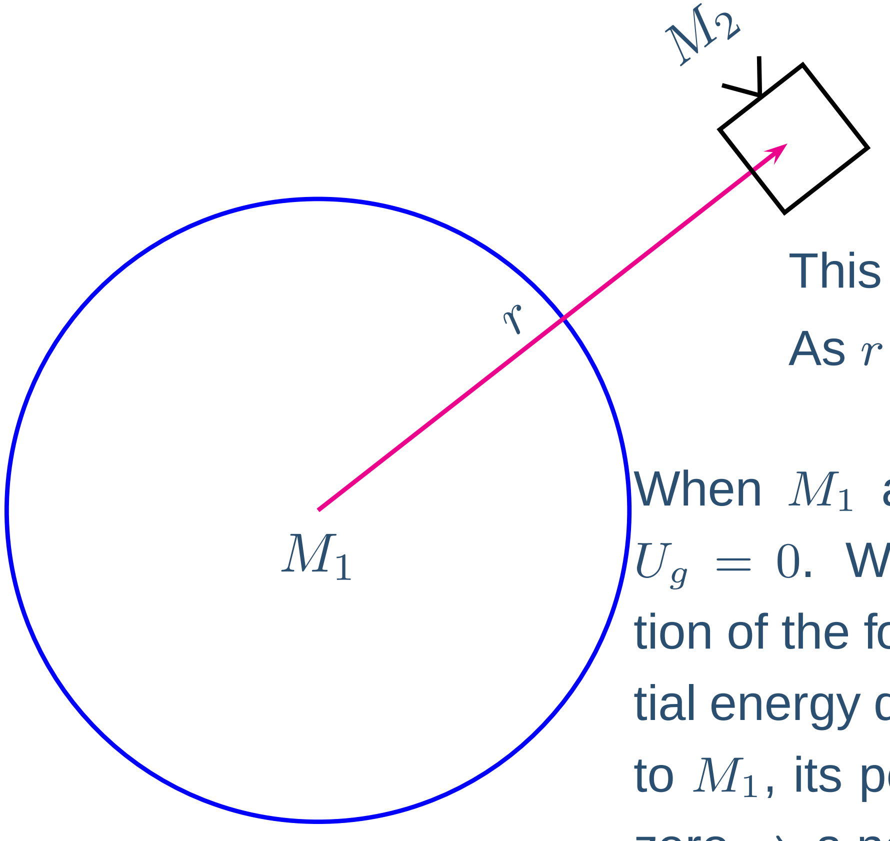

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When M_1 and M_2 are infinitely far apart $U_g = 0$. When objects move in the direction of the force acting on them their potential energy decreases $\Rightarrow$ as M_2 gets closer to M_1, its potential energy decreases from zero $\Rightarrow$ a negative amount.

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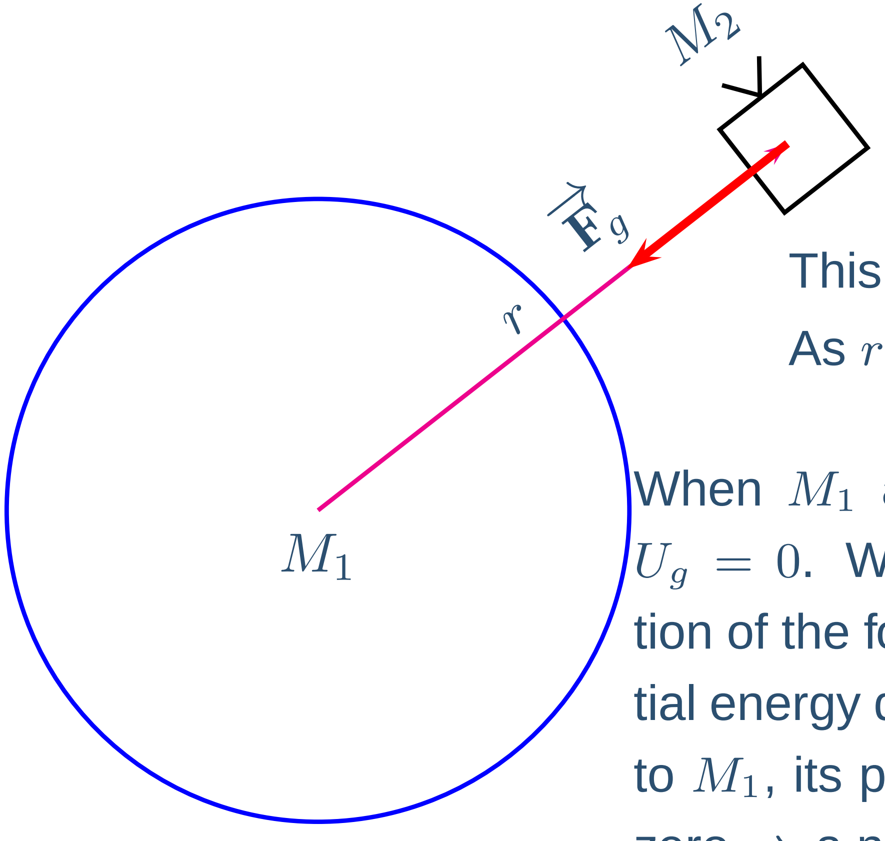

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Escape Speed

When gravity is the only force doing work on a rocket with mass M near a planet, M_P:

$$\boxed{\frac{1}{2}Mv_1^2 - \frac{GM_PM}{r_1} = \frac{1}{2}Mv_2^2 - \frac{GM_PM}{r_2}}$$

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$$v_{es} = \sqrt{\frac{2GM_P}{R}}$$

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Projectile: $y = y_0 + v_{oy}t - \frac{1}{2}gt^2$

$$\Rightarrow \Delta y = 0 - \frac{1}{2}(10\,m/s^2)(1\,s)^2$$

A New Question

The earth is not flat! It has a curvature of roughly $8000\,m$ to $5\,m$ (horizontal to vertical). A projectile is fired horizontally at $8000\,m/s$, $20\,m$ above the ground on earth. How long does it take it to hit the ground?

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Projectile: $x = x_0 + v_{ox}t \Rightarrow x = (8000\,m/s)(1\,s) = 8000\,m$

The projectile follows the curvature of the earth! Every second, the projectile goes $8000\,m$ horizontal and drops $5\,m$ vertical. It has become a satellite!

<u>Satellite</u> - Any projectile with sufficient horizontal velocity to “miss" the ground.

(e) The projectile never hits the ground.

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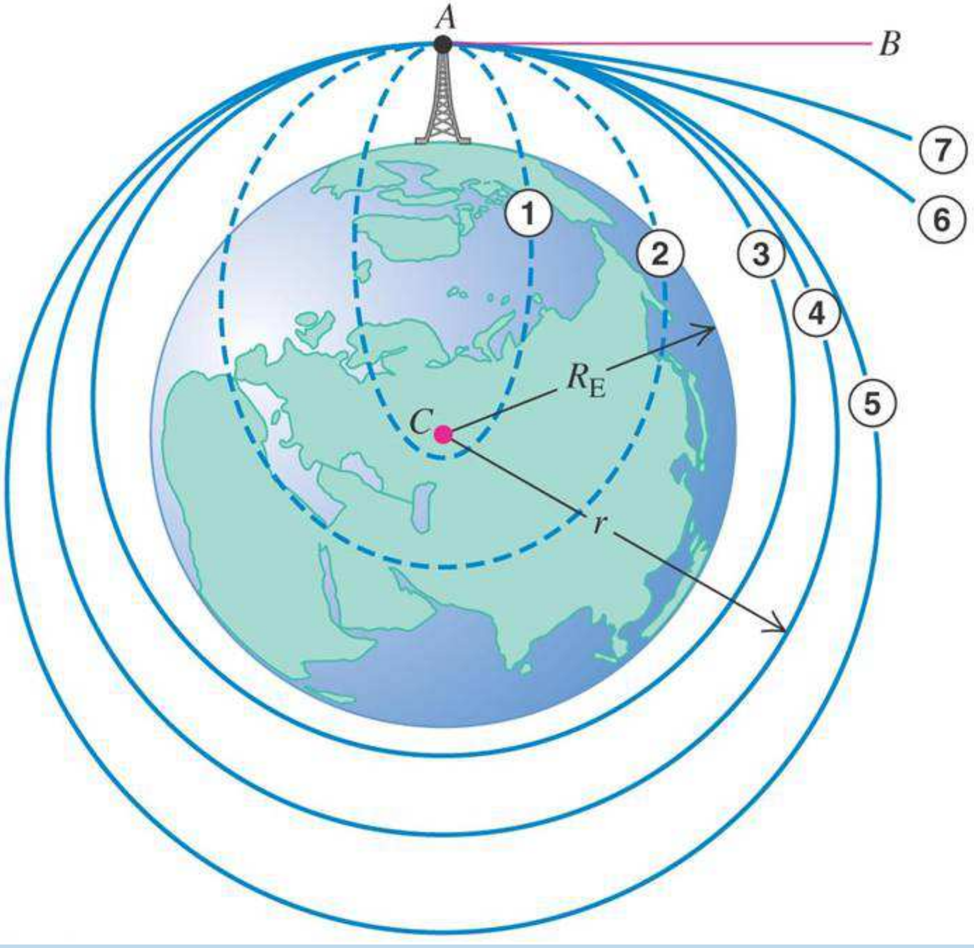
Newton showed that when gravity is the only force doing work, the only allowed closed orbits are circular or elliptical in shape. While the only open orbits are parabolic or hyperbolic.

Orbits II

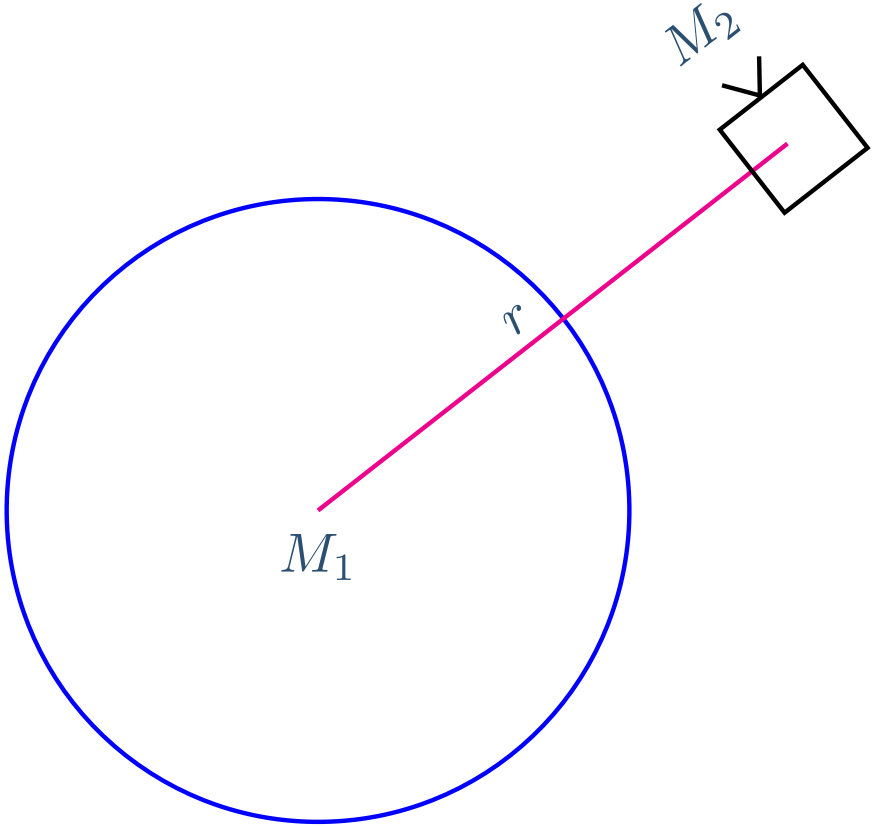
The initial velocity of the satellite determines whether the orbit is open or closed.



A projectile is launched from *A* toward *B*. Trajectories (1) through (7) show the effect of increasing initial speed.

Circular Orbits

In circular orbit, gravity creates the centripetal acceleration.



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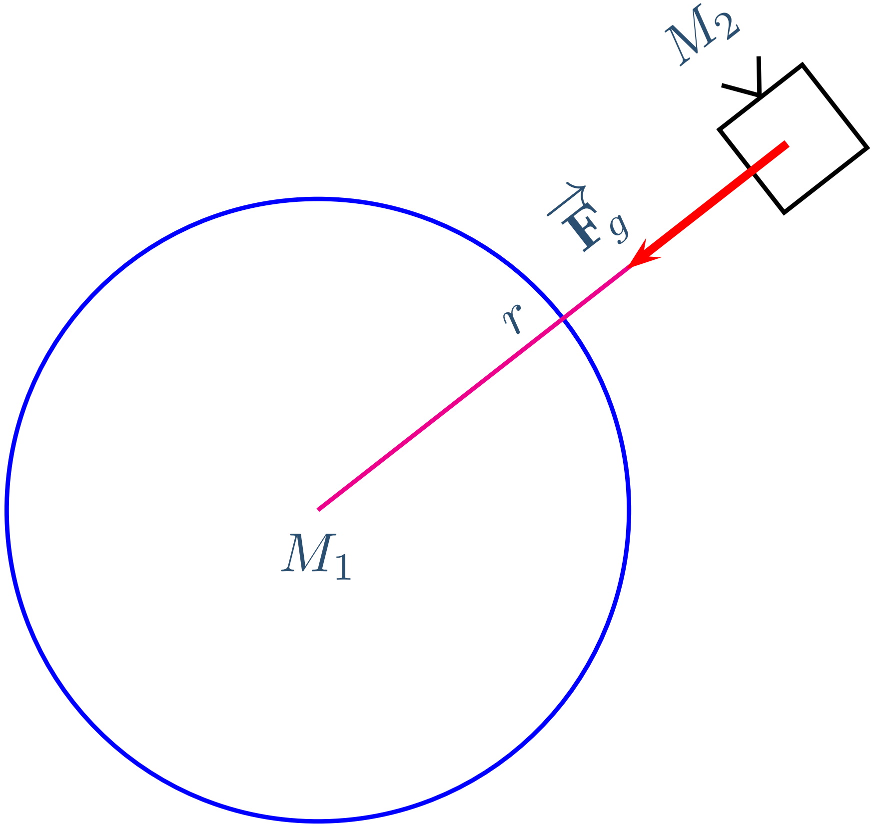

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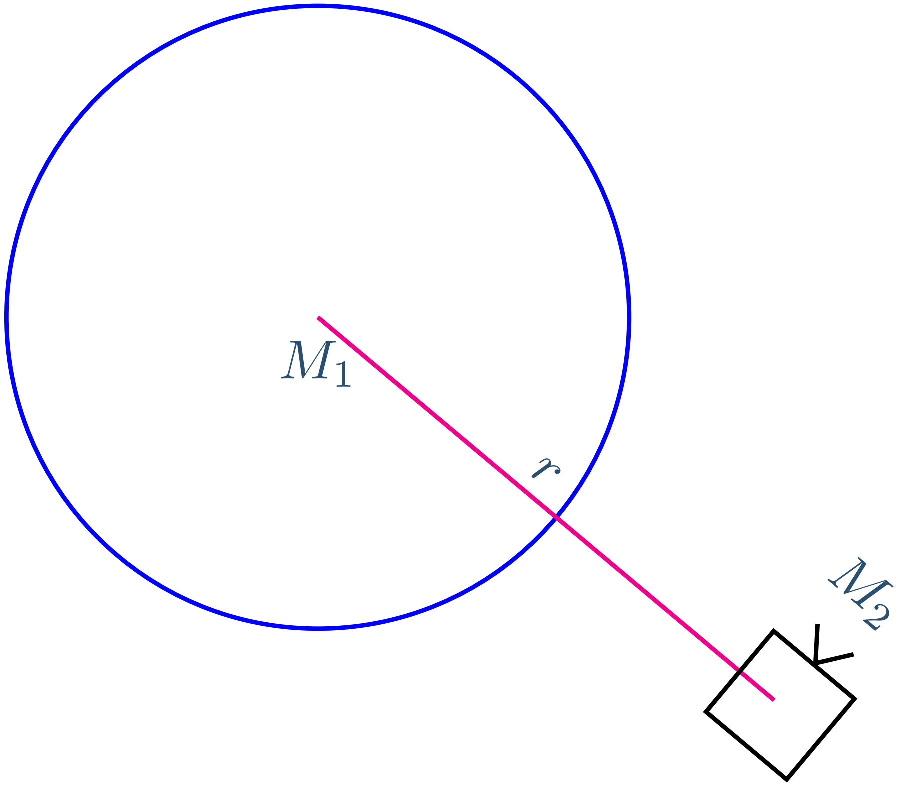

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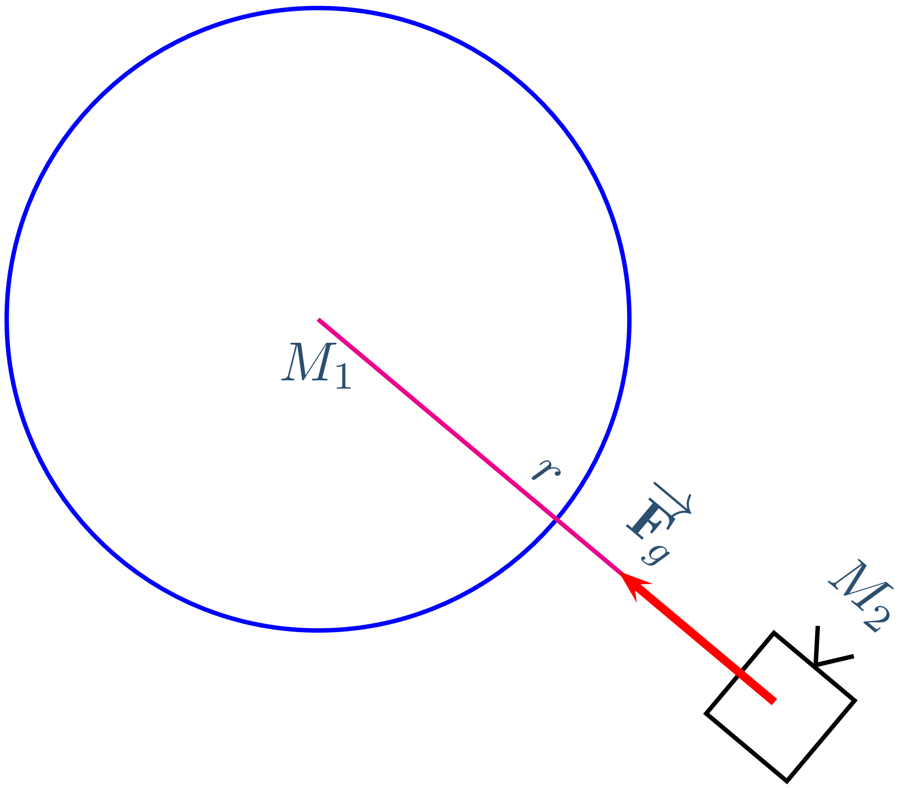

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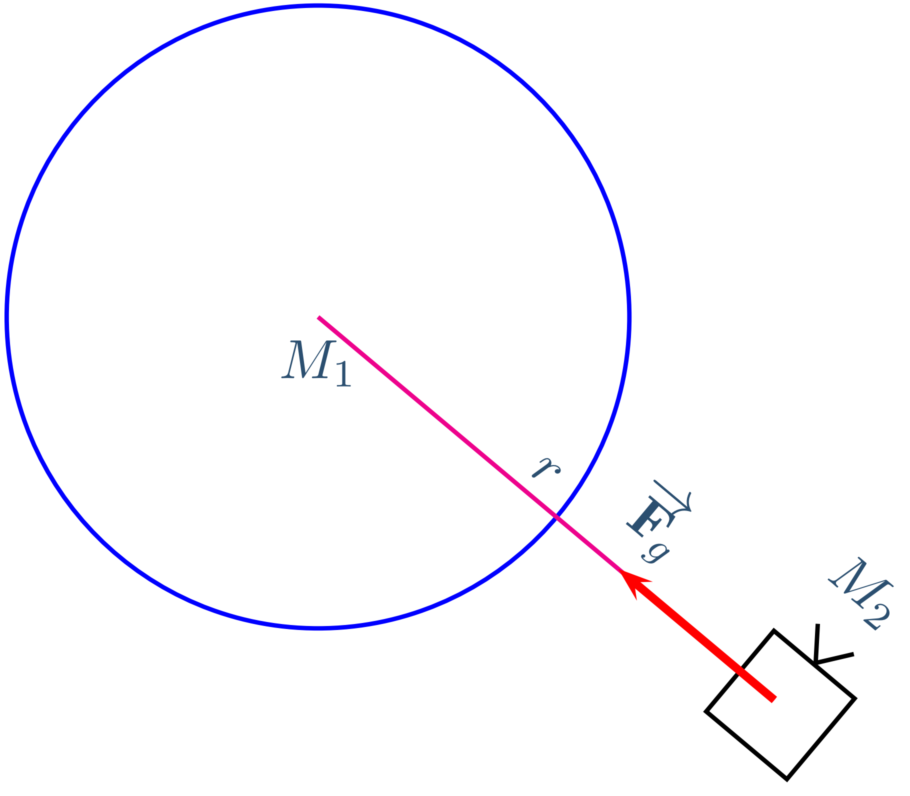

$$\sum \overrightarrow{\mathbf{F}}_2 = M_2 \overrightarrow{\mathbf{a}} \Rightarrow F_g = M_2 a_{rad}$$

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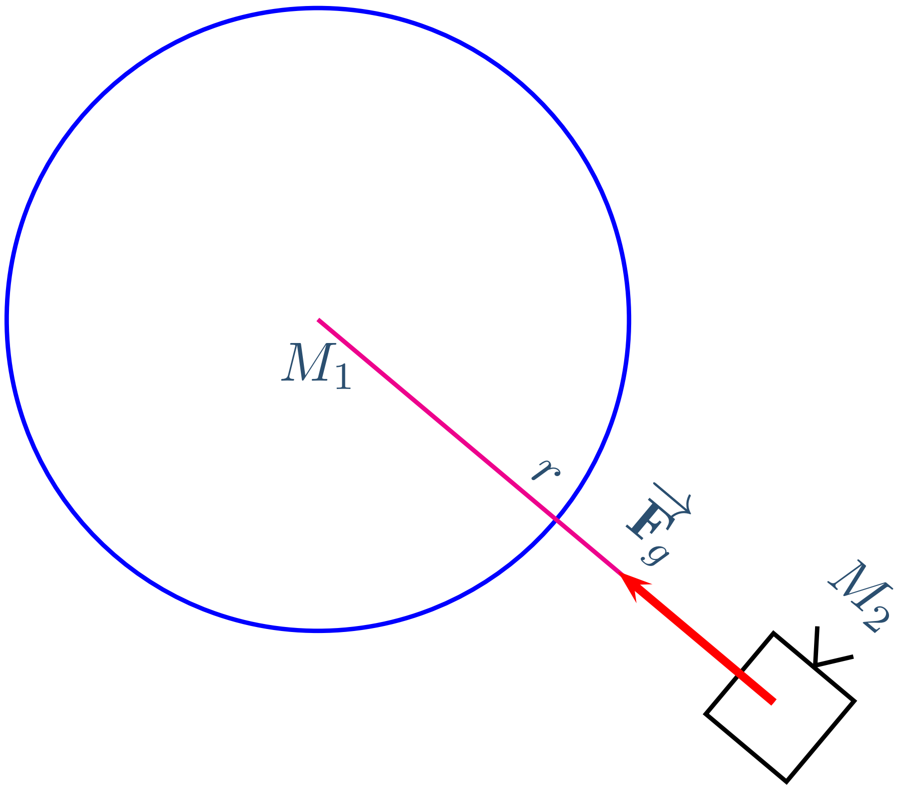

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$$\Rightarrow F_g = M_2 \frac{v^2}{r}$$

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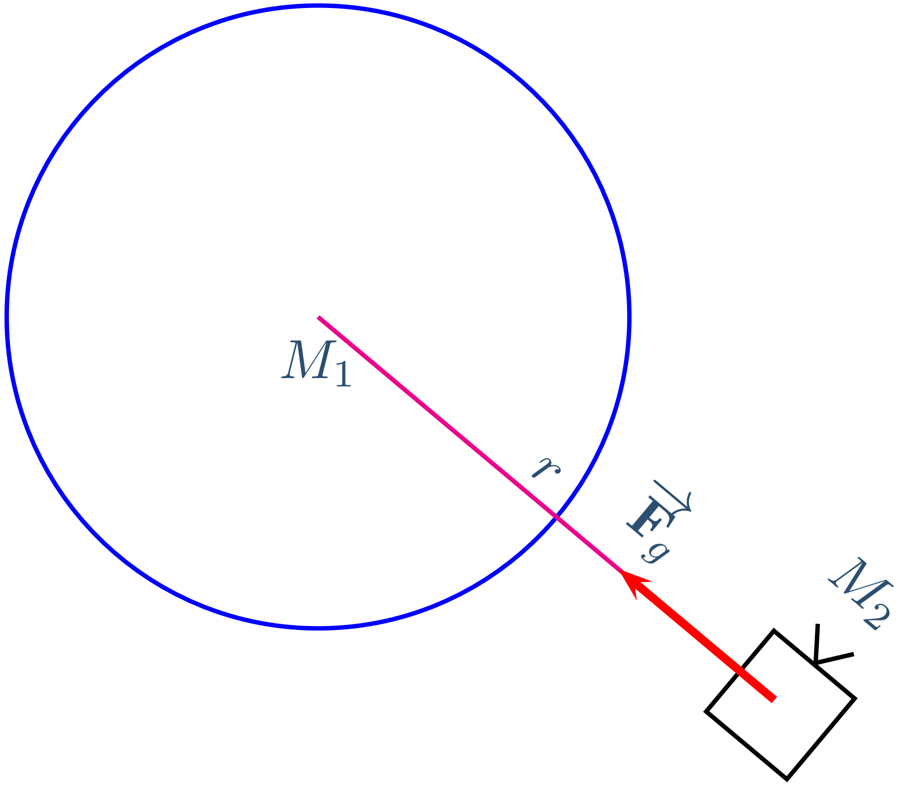

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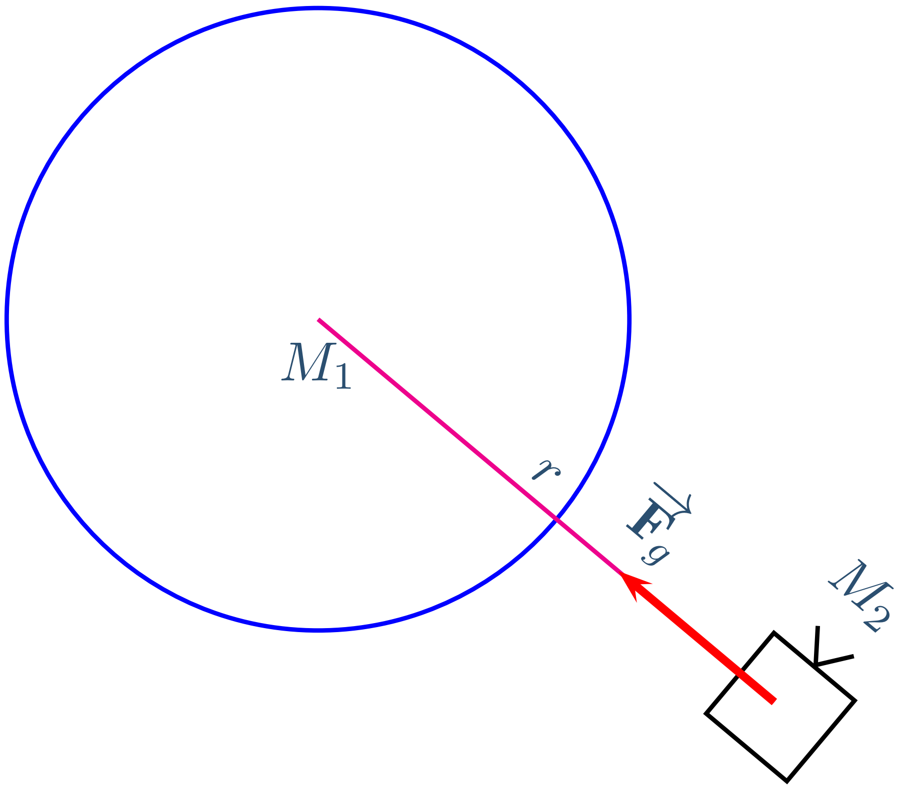

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$$v = \sqrt{\frac{GM_1}{r}}$$

Circular Orbits II

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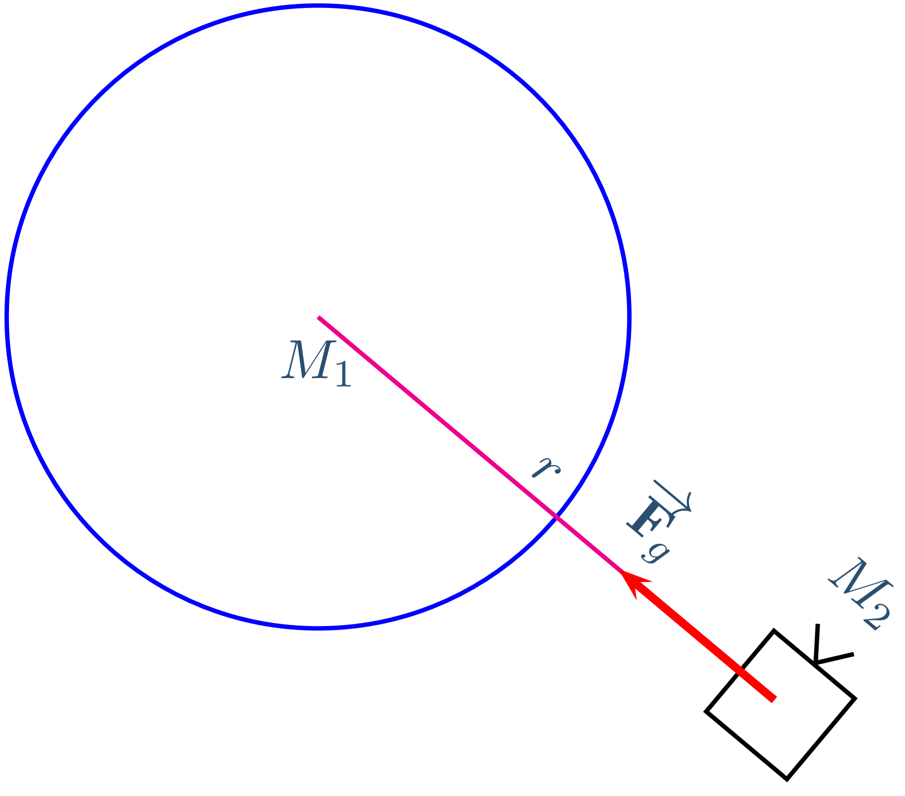

Speed: $$v = \sqrt{\frac{GM_1}{r}}$$

Circular Orbits II

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Period, T - Time for one revolution

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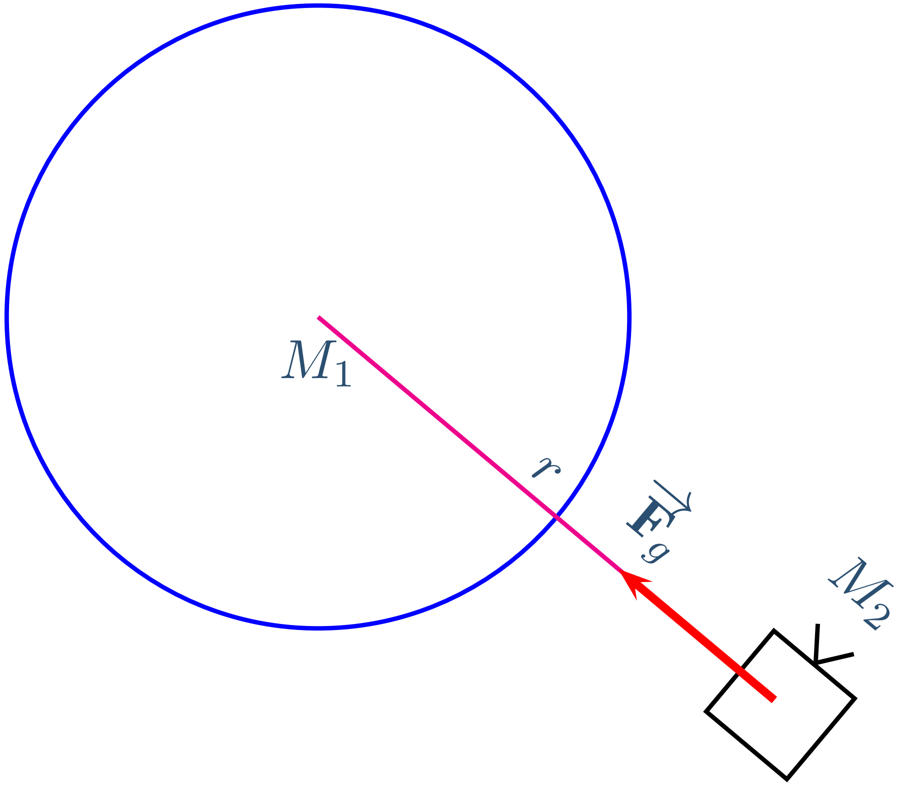

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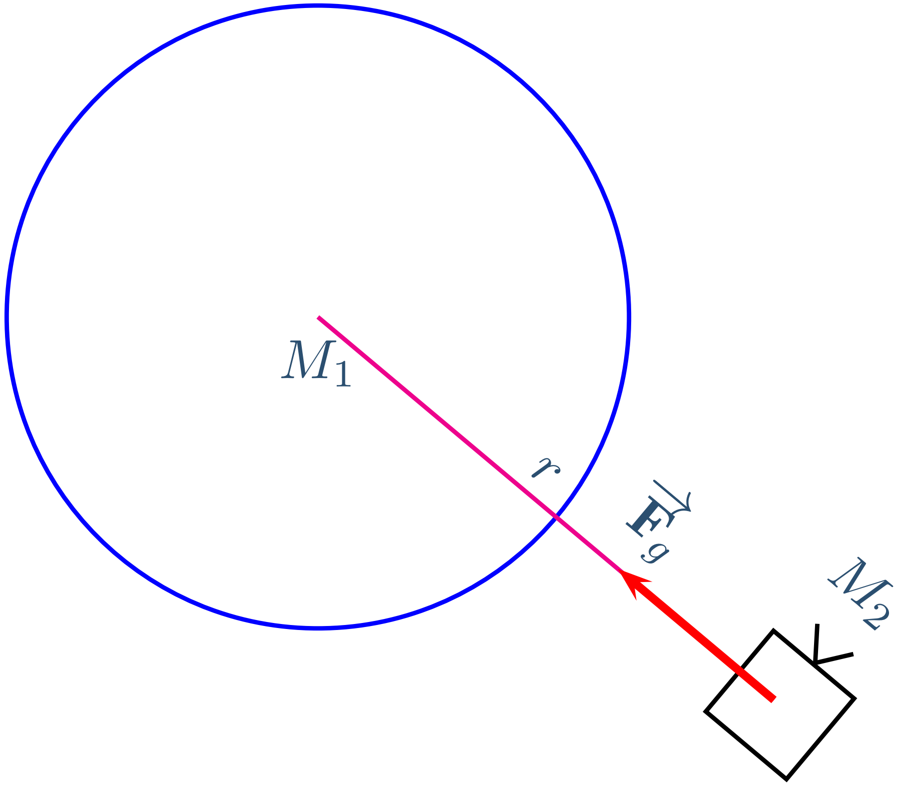

Speed: $$\boxed{v = \sqrt{\frac{GM_1}{r}}}$$

Constant Speed $\Rightarrow v = \dfrac{2\pi r}{T}$

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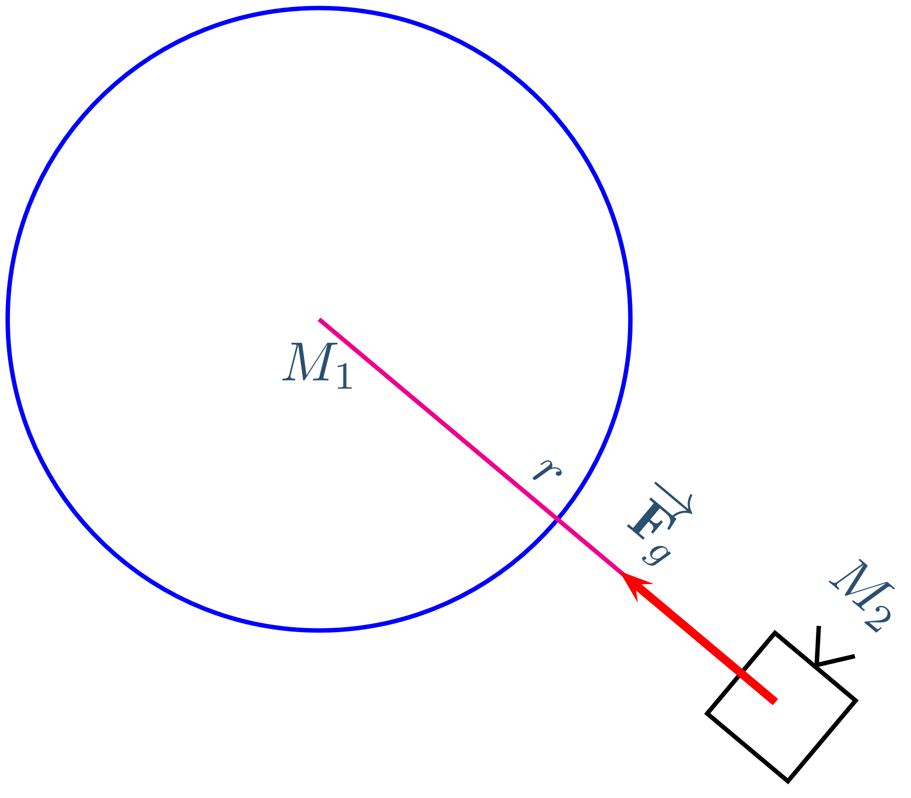

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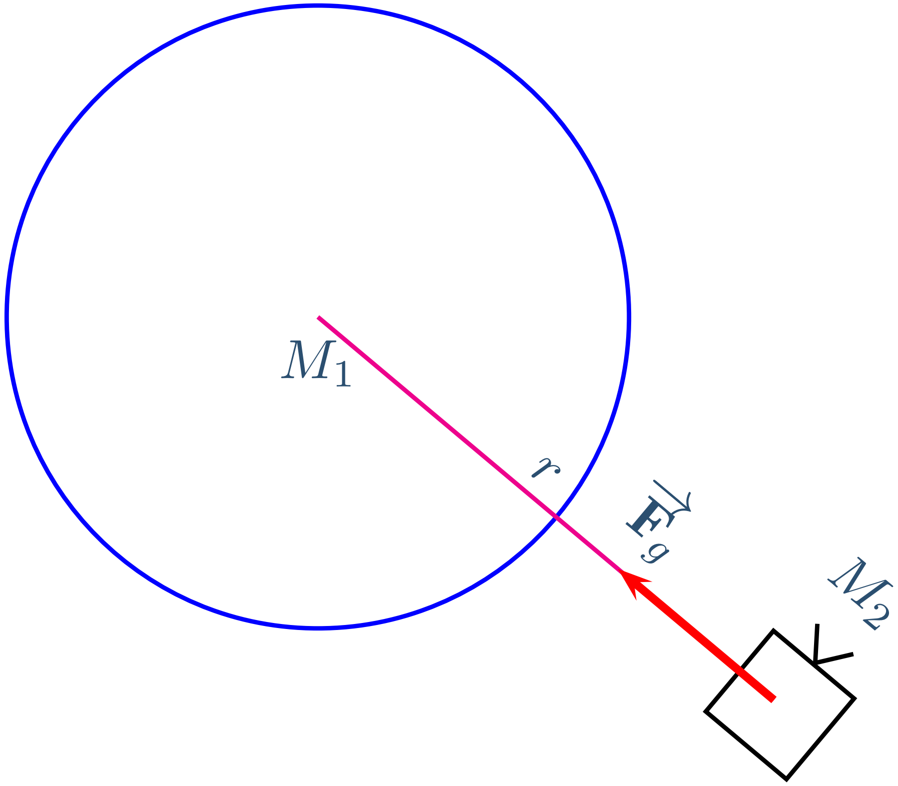

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Period: $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_1}}$$

Energy: $$E = \frac{-GM_1 M_2}{2r}$$

Kepler’s Laws

Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

Kepler's Laws

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Kepler's Laws:

1: Each planet's orbit traces out the shape of an ellipse with the sun located at one focus.

2: The imaginary line from the sun to a planet sweeps out equal areas in equal times.

3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.