April 17, Week 13

Today: Chapter 10, Angular Momentum

Homework Assignment #10 - Due April 19.

Mastering Physics: 7 problems from chapter 9

Written Question: 10.86

On Friday, we will begin chapter 13.

> From now on, Thursday office hours will be held in room 109 of Regener Hall

Newton's Second Law for Rotation

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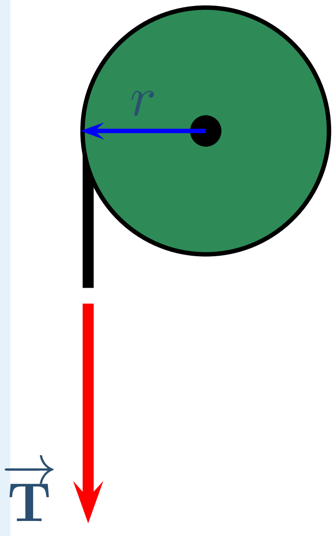
(Only true for spinning motion with the origin of your coordinates at the axis of rotation.)

Rotational Dynamics Exercise

A solid cylinder with moment of inertia $25\,kg \cdot m^2$ and radius $0.5\,m$ has a rope wrapped around it. The rope is pulled and the cylinder spins about its center with angular acceleration $1\,rev/s^2$. What is the tension in the rope? Ignore friction.



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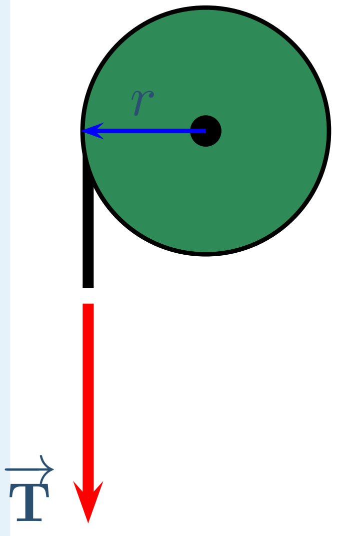

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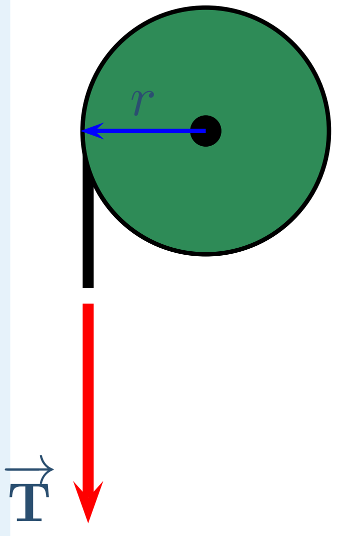

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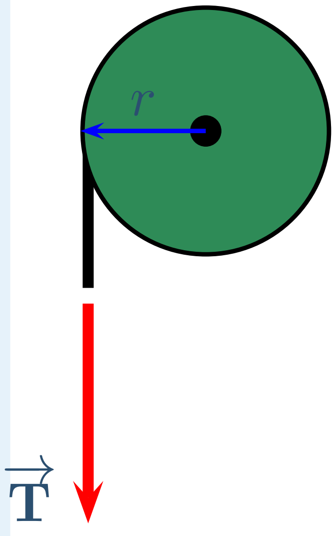

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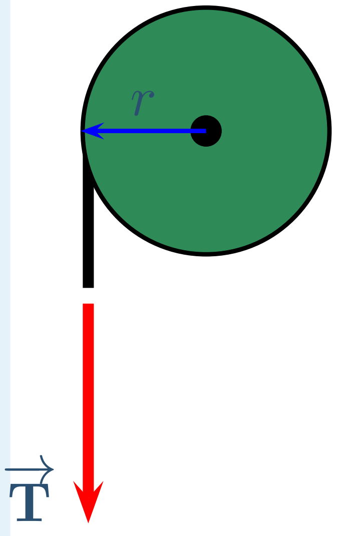

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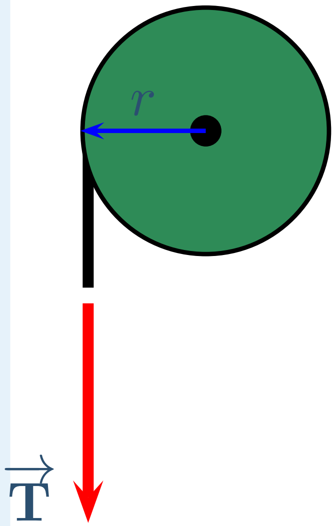

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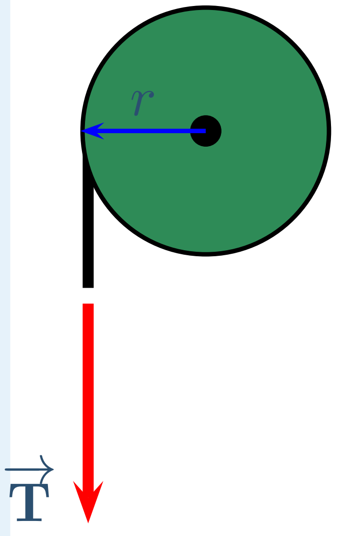

Tension only force exerting torque $\Rightarrow$ $\tau_T = I\alpha$

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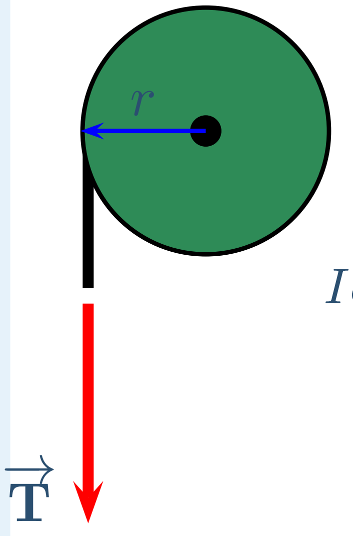

Tension only force exerting torque $\Rightarrow$ $\tau_T = I\alpha$

$$I\alpha = (25\,kg \cdot m^2)(2\pi\,rad/s^2)$$
$$= 50\pi\,N \cdot m$$

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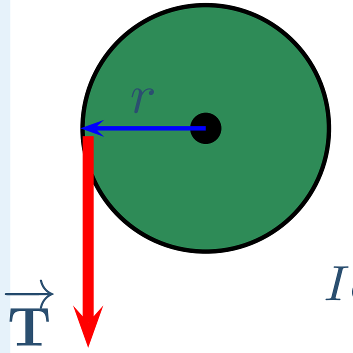

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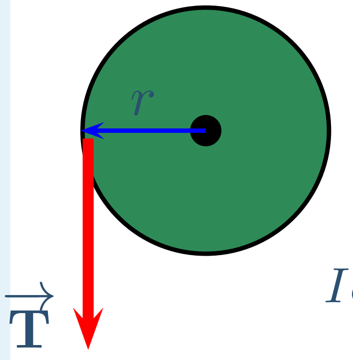

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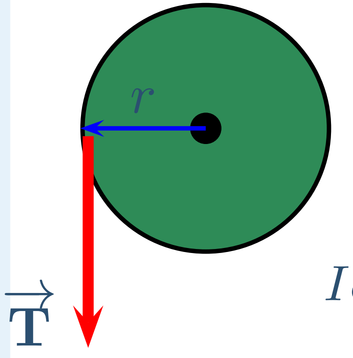

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Note: For a point particle (an object with a single value of v) going around a circle of radius r, $\boxed{L = mvr}$. (Comes from $I = mr^2$ for a particle and $v = \omega r$.)

Conservation of Angular Momentum

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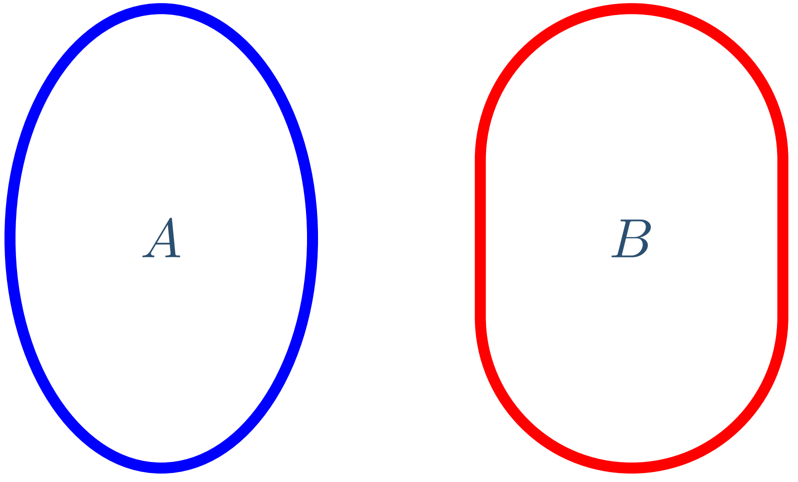

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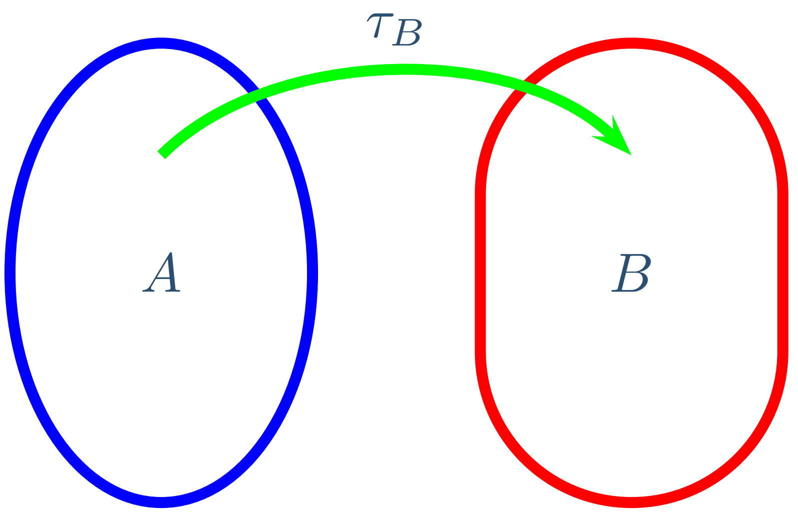

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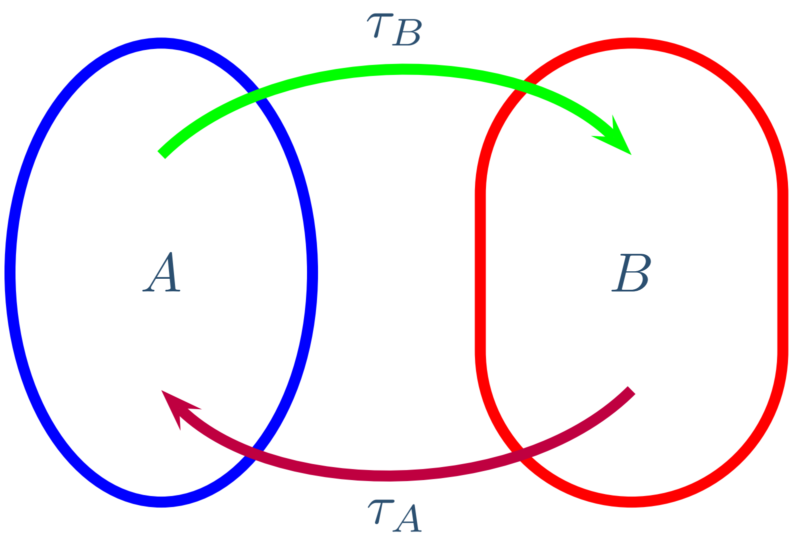

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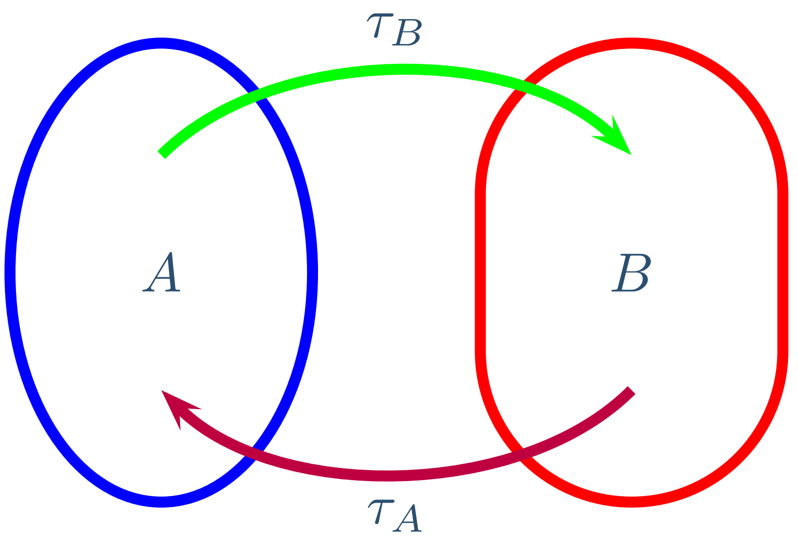

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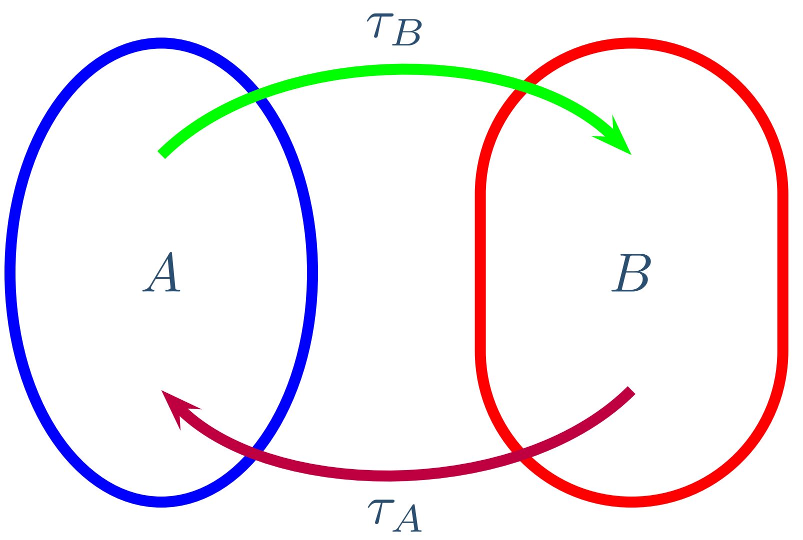

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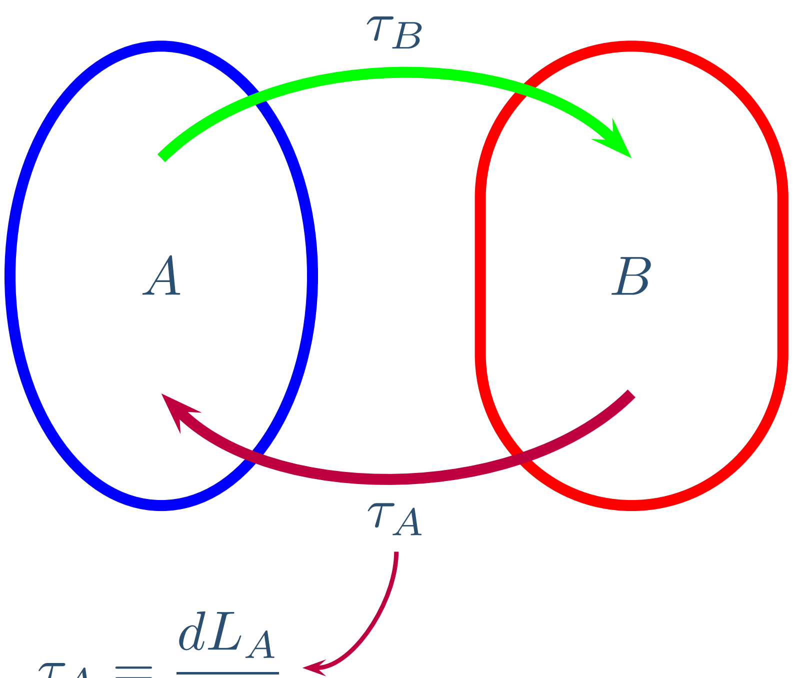

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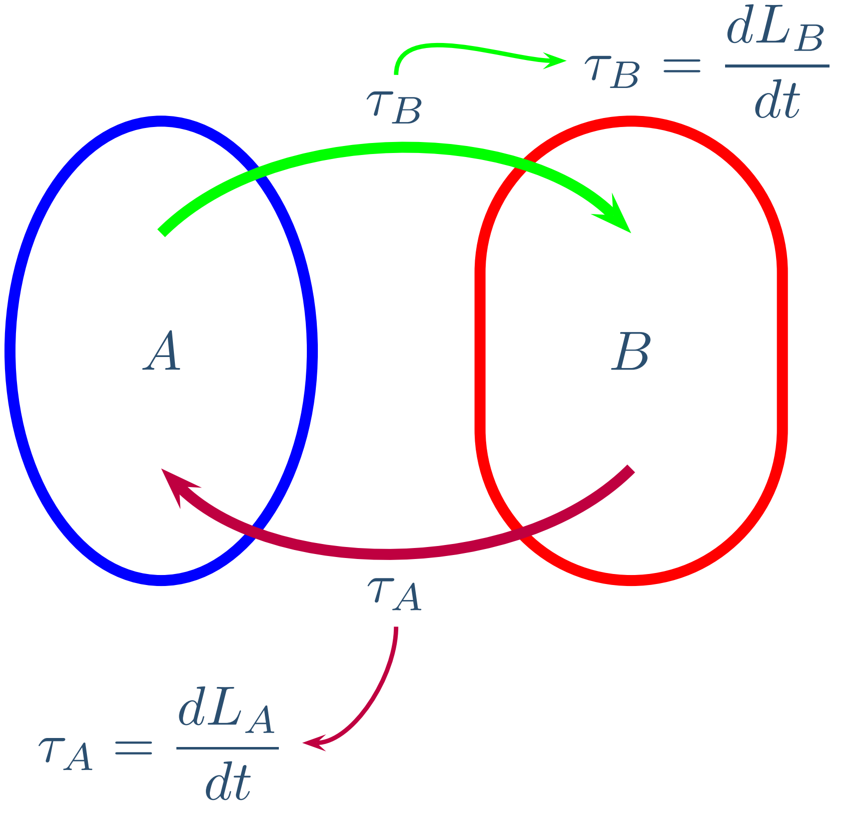

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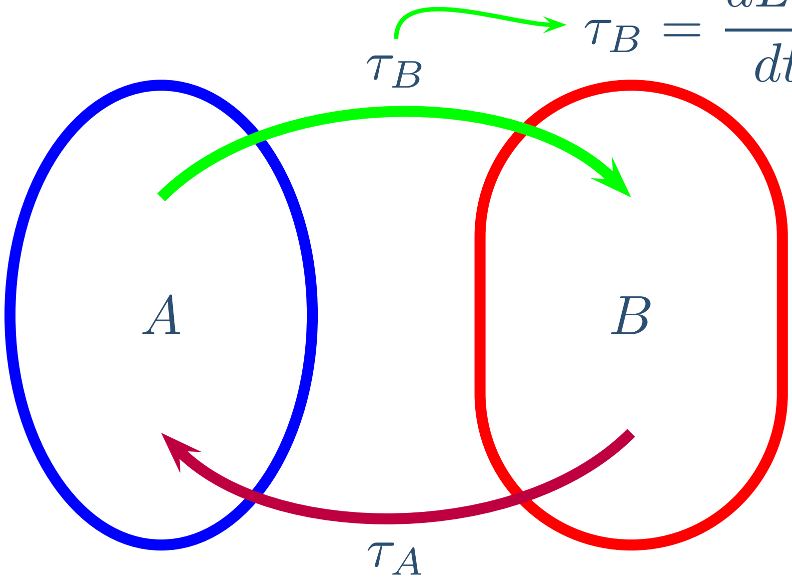

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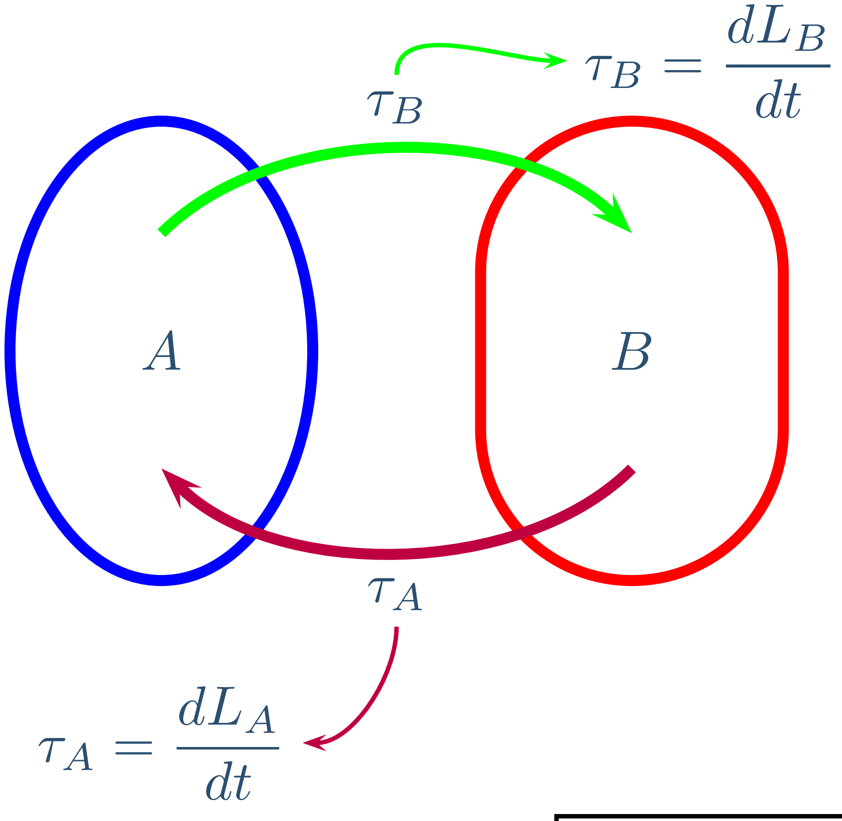

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$$\boxed{\Delta (L_A + L_B) = 0}$$

Single Object Conservation

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$$L = I\omega \Rightarrow \boxed{I_{Ai}\,\omega_{Ai} + I_{Bi}\,\omega_{Bi} = I_{Af}\,\omega_{Af} + I_{Bf}\,\omega_{Bf}}$$

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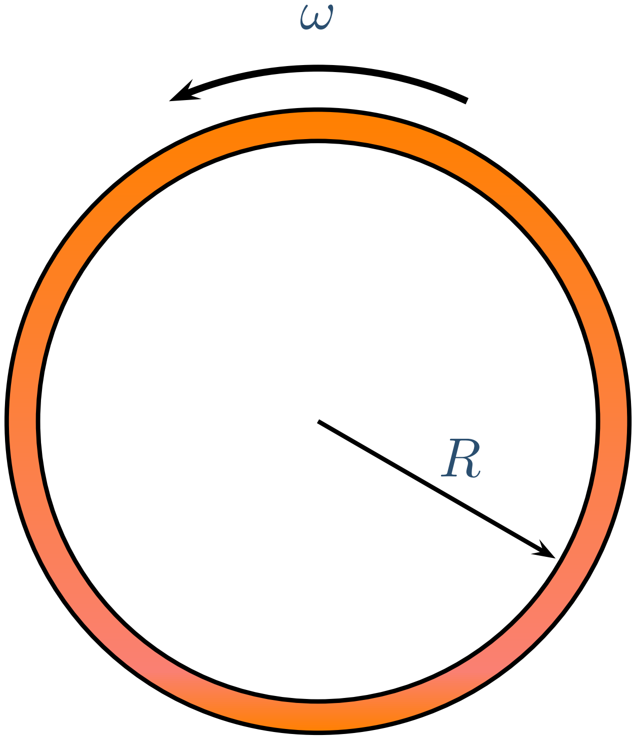
$$\boxed{I_i\omega_i = I_f\omega_f}$$

Angular Momentum Exercise

An isolated hoop of mass M and radius R is rotating about its center with angular speed $100\,RPM$. What would the hoop's angular speed become if its radius suddenly doubled without changing its mass? **Hint:** The moment of inertia for a hoop is $I = MR^2$.



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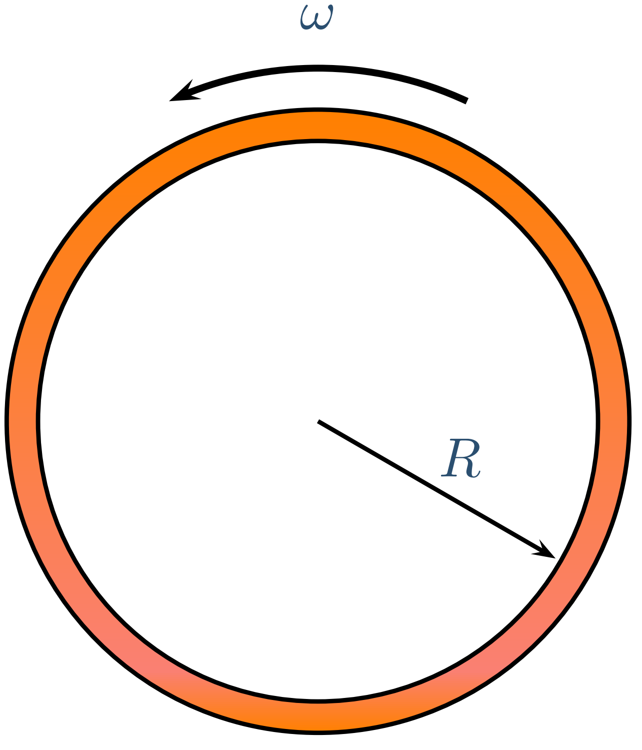

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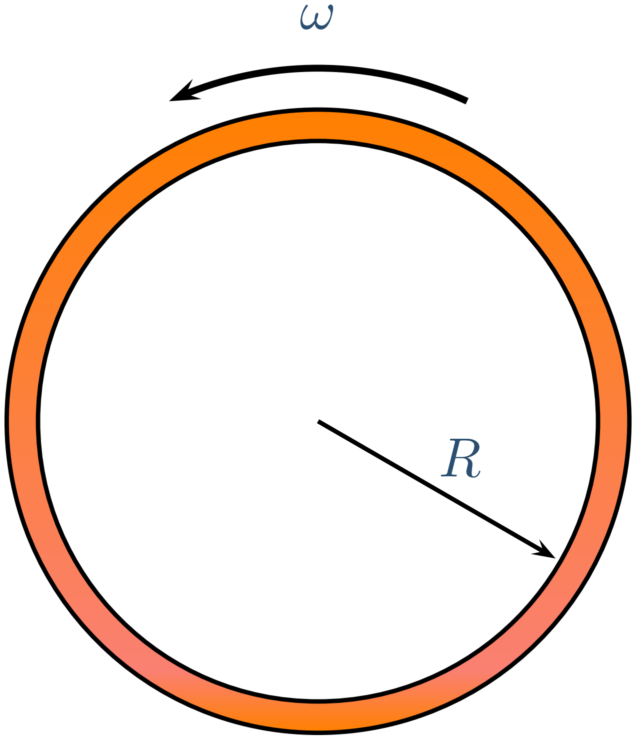

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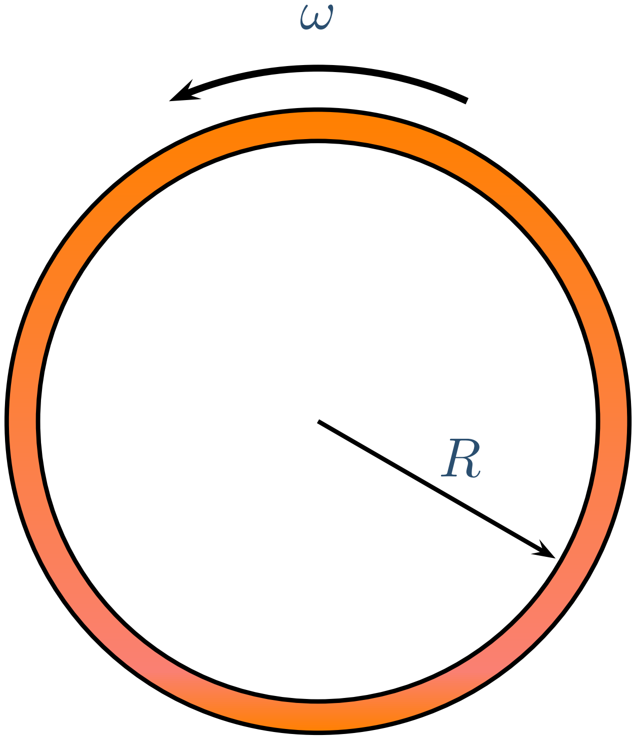

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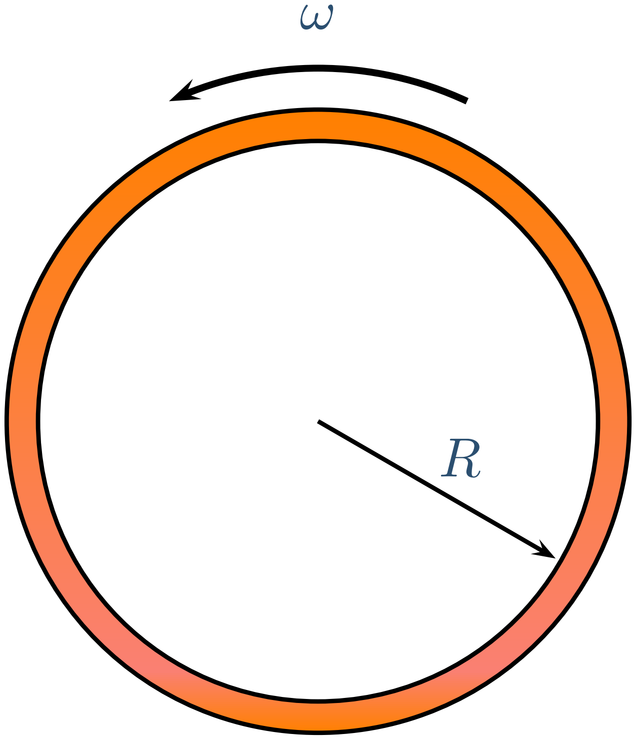

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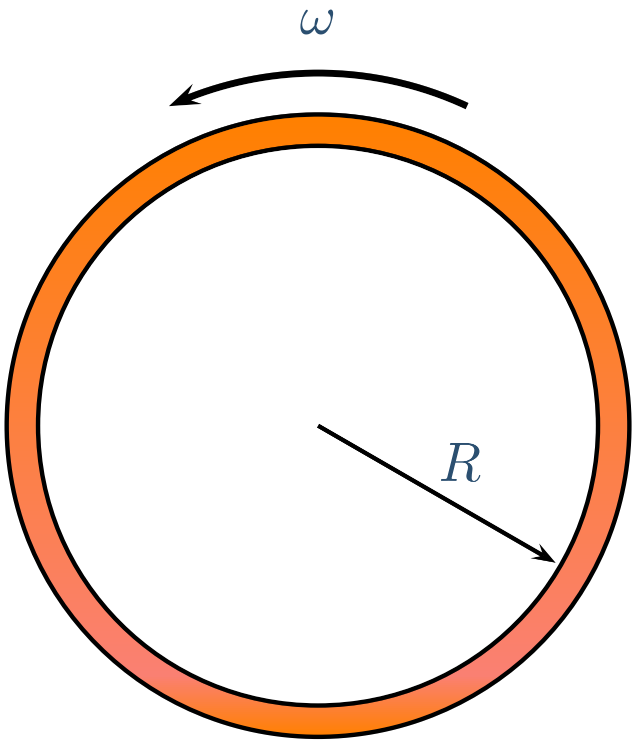

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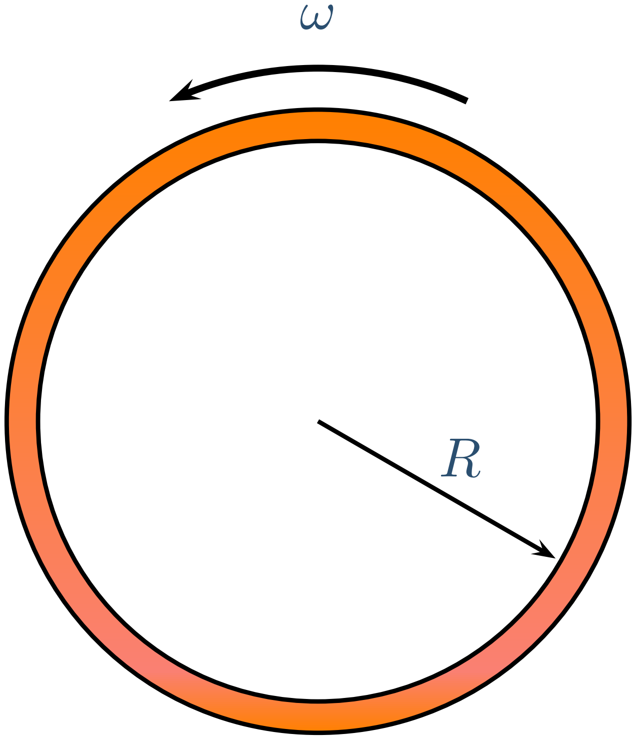

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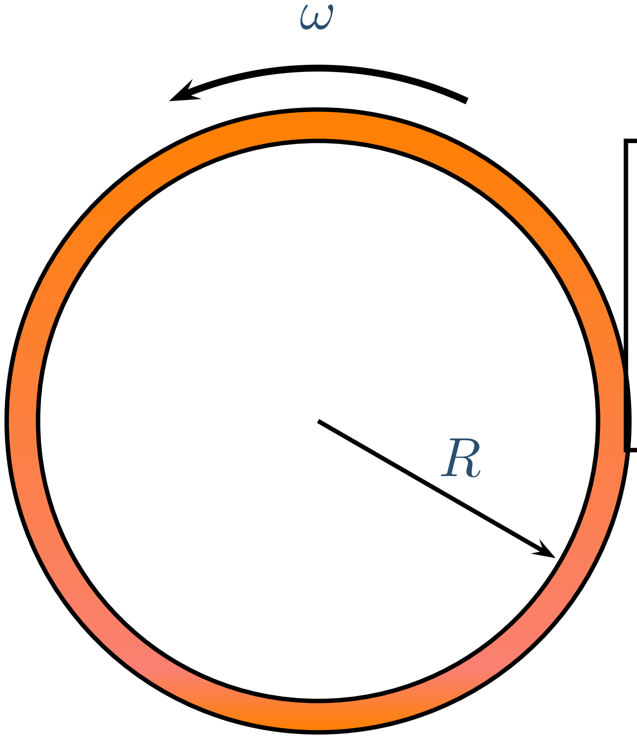
(e) $400\,RPM$

Angular Momentum Exercise

An isolated hoop of mass M and radius R is rotating about its center with angular speed $100\,RPM$. What would the hoop's angular speed become if its radius suddenly doubled without changing its mass? **Hint:** The moment of inertia for a hoop is $I = MR^2$.



Doubling radius
$\Rightarrow I_f = 4I_i$
$\Rightarrow \omega_f = \frac{1}{4}\omega_i$

(a) $25\,RPM$

(b) $50\,RPM$

(c) $100\,RPM$

(d) $200\,RPM$

(e) $400\,RPM$