April 8, Week 12

Today: Chapter 9, Linear and Angular Motion

Homework Assignment #8 - Due Today

Mastering Physics: 8 problems from chapter 8

Written Question: 8.101

Homework Assignment #9 - Due Friday, April 12.

Mastering Physics: 7 problems from chapter 9

Written Question: 10.80

Office Hours Today: 1:00-3:30 in Regener Hall 109

Relating Linear and Angular Velocity

Use the relationship $s = r\theta$ to relate linear and angular speeds.

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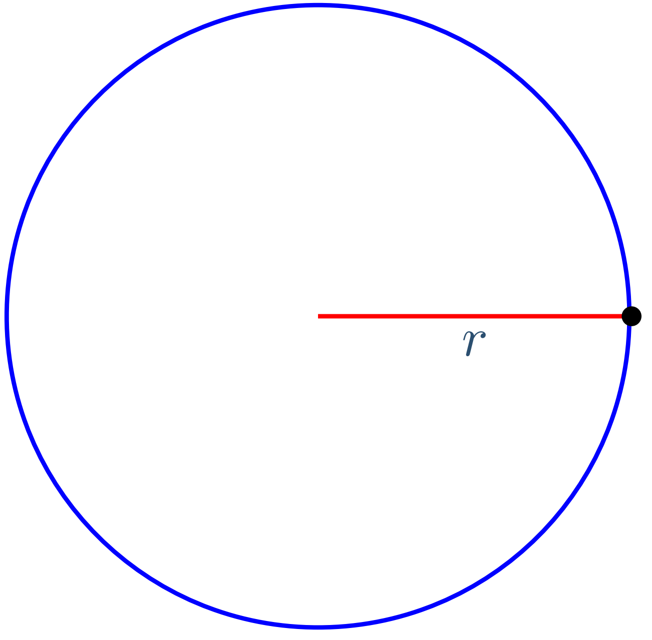

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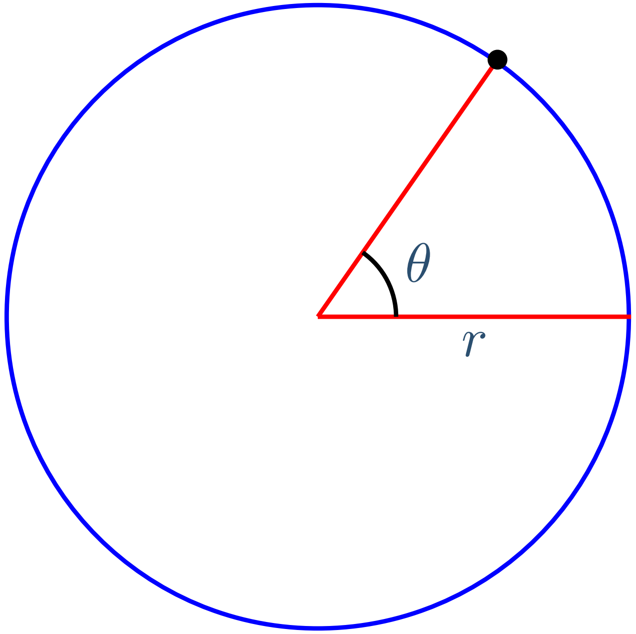

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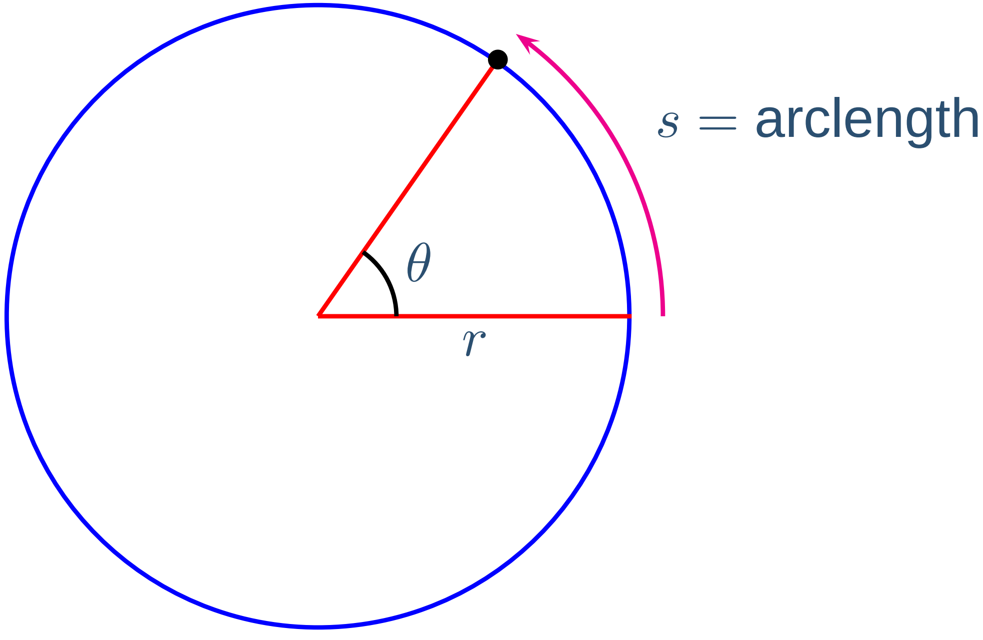

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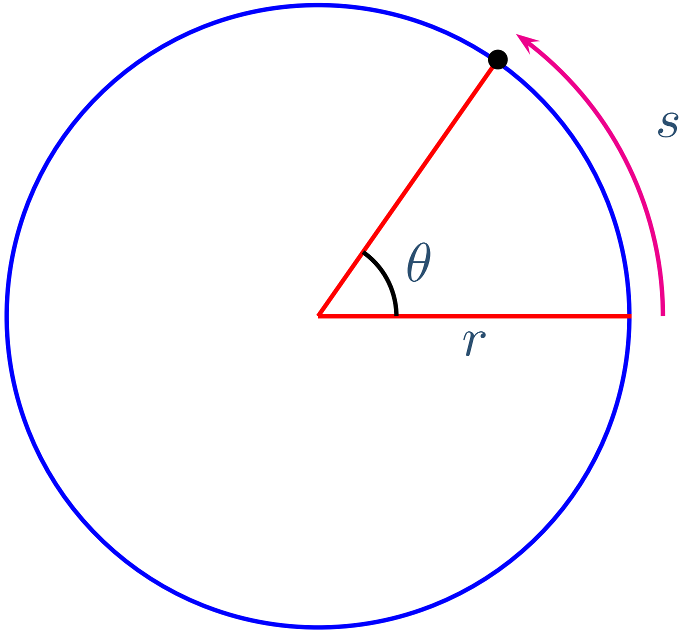

$s =$ arclength

s is the linear distance traveled

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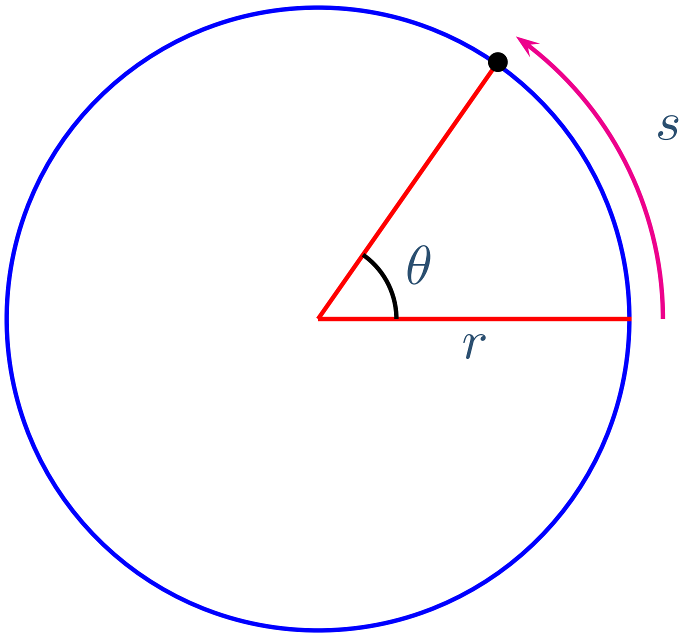

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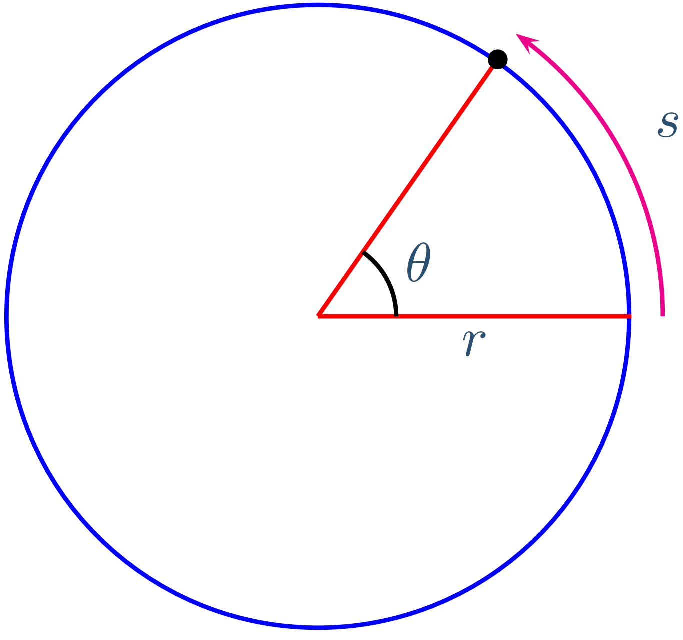

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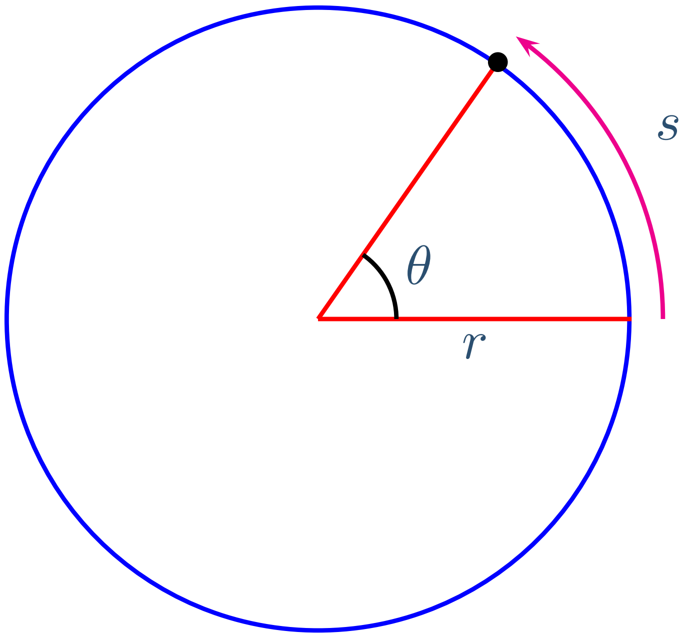

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$\frac{d}{dt}\left(s = r\theta\right)$

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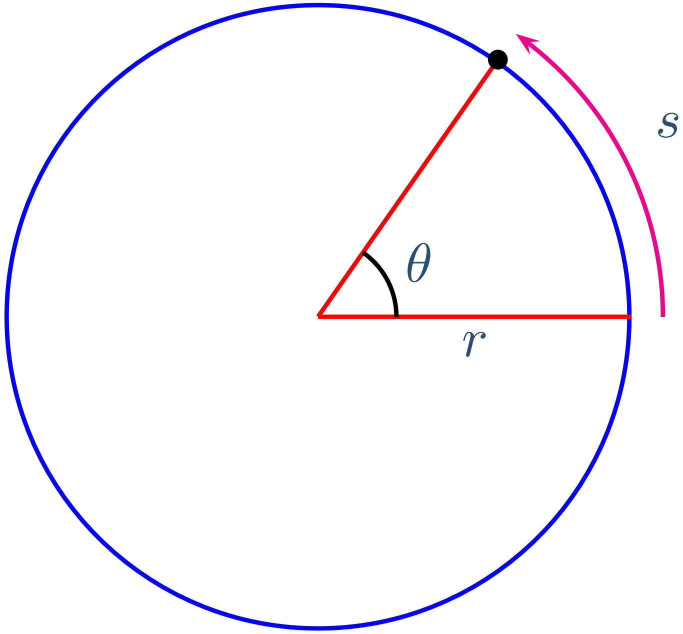

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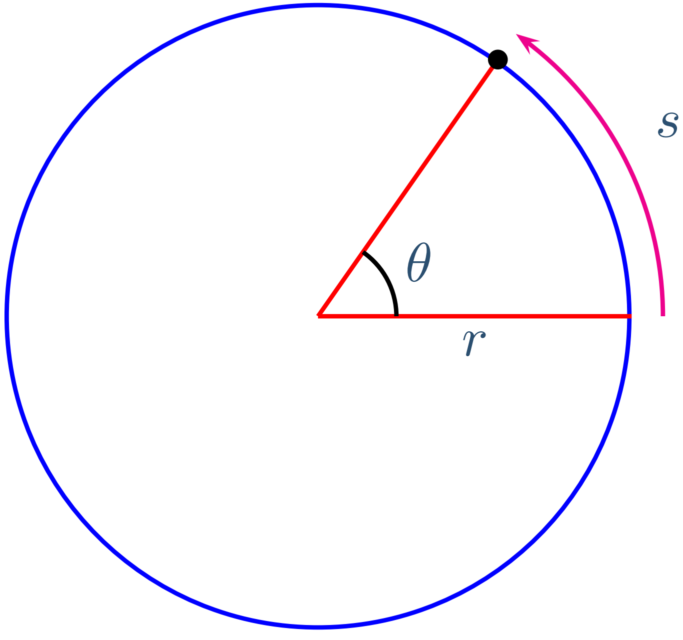

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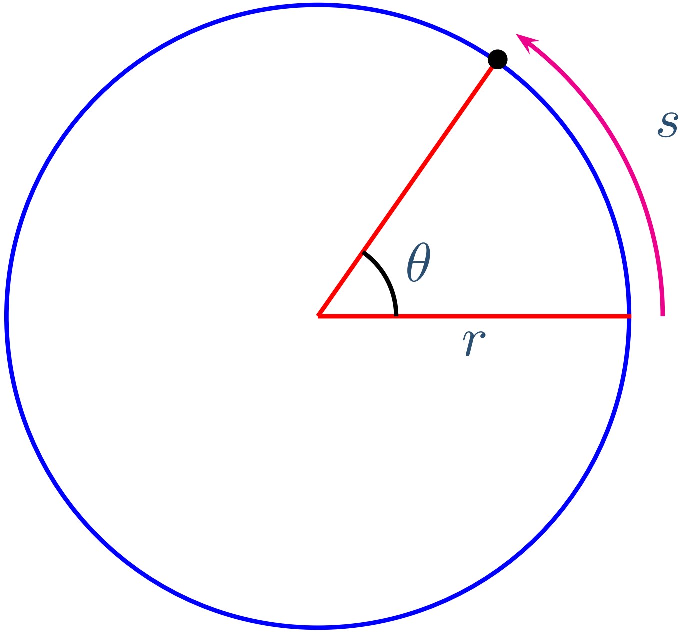

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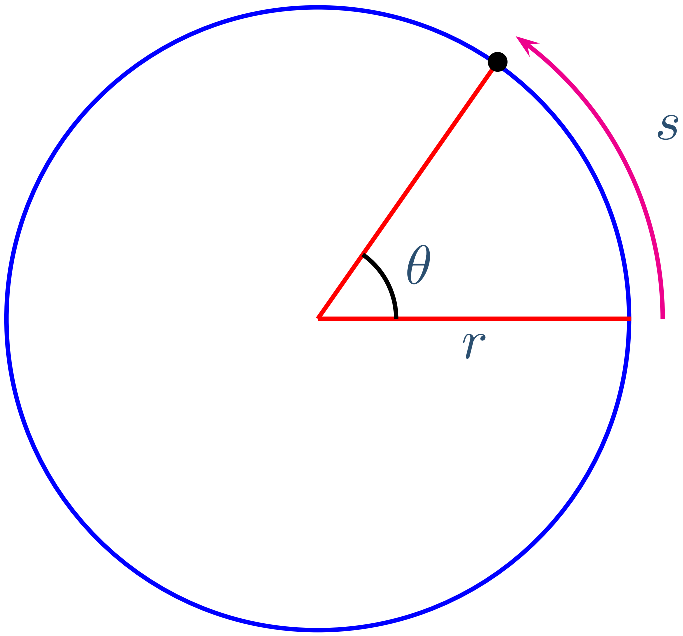

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Speeds Exercise

A wheel is spinning at $45\,RPM$. What is the linear speed of the point which is $0.25\,m$ away from the center?

$$\boxed{v = \omega r}$$

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When relating linear and angular quantities you *MUST* use radians!

The Cross Product

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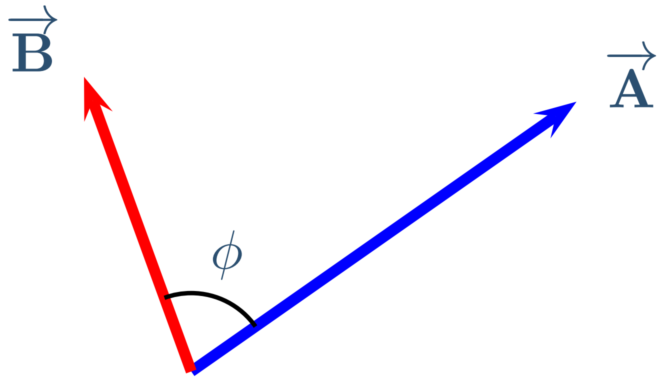

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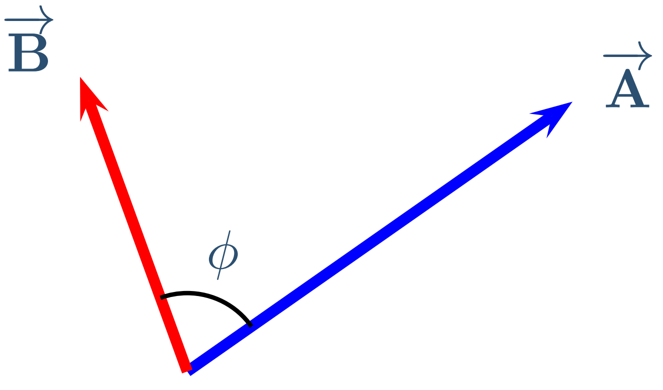

$$\left|\overrightarrow{\mathbf{A}} \times \overrightarrow{\mathbf{B}}\right| = AB\sin\phi$$

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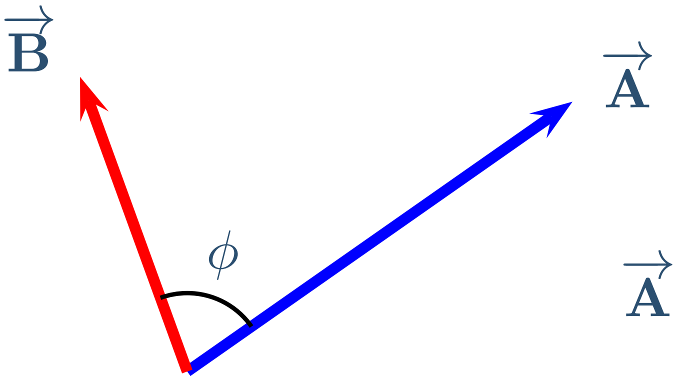

$$\left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right| = AB\sin\phi$$

$\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is perpendicular to both $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$

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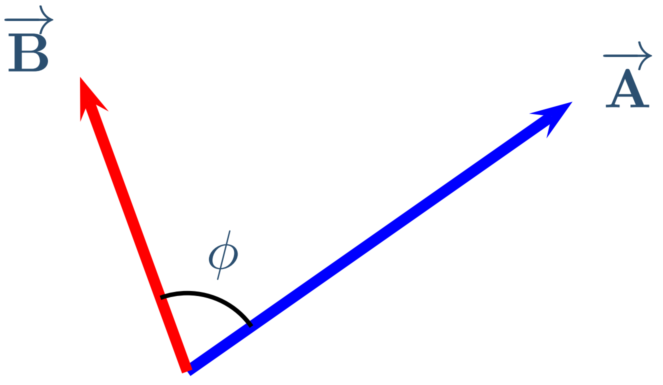
Another Right-Hand-Rule (RHR) gives direction:



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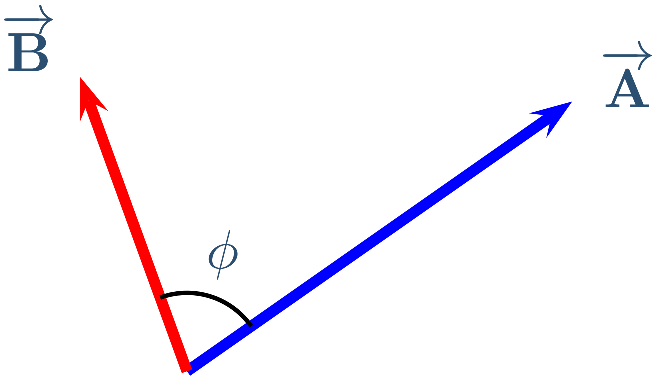

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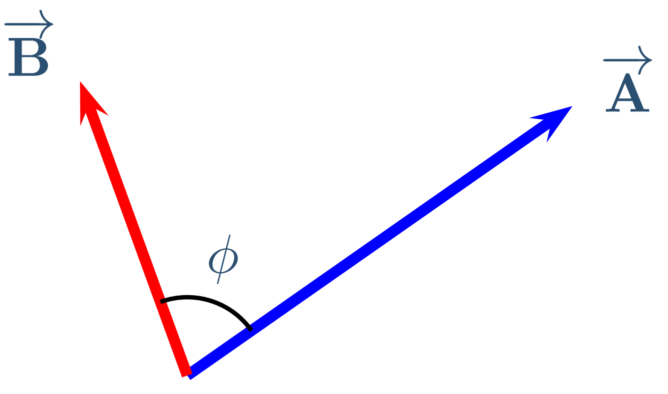

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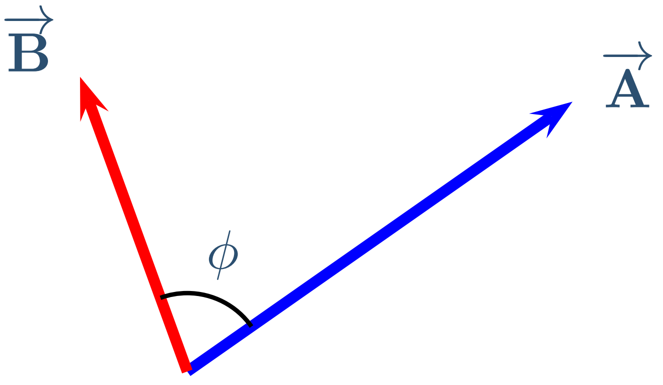

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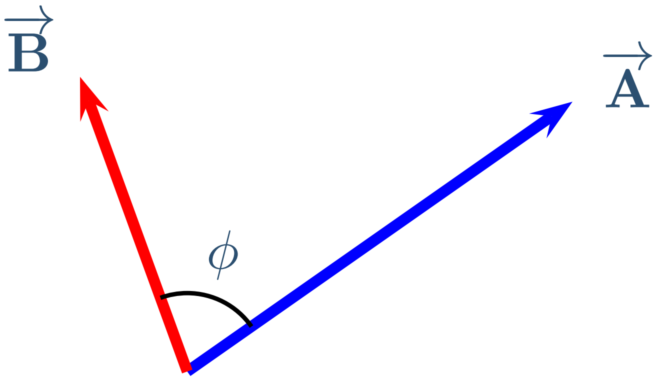

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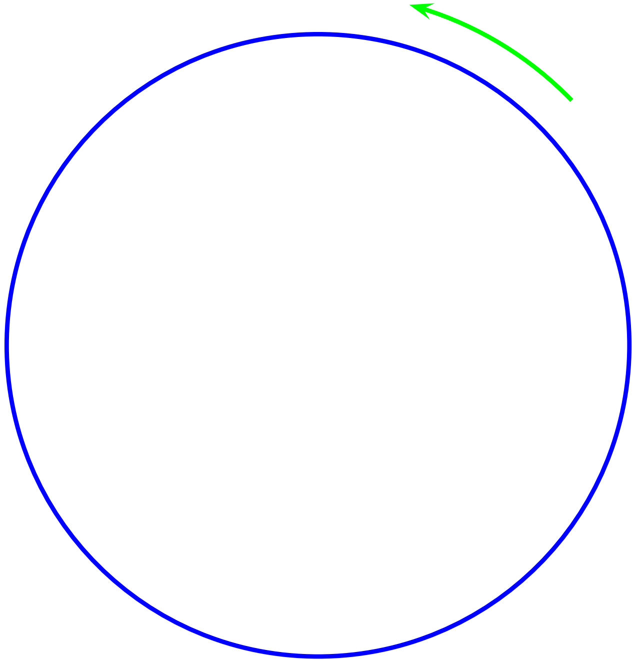
Linear and Angular Velocities



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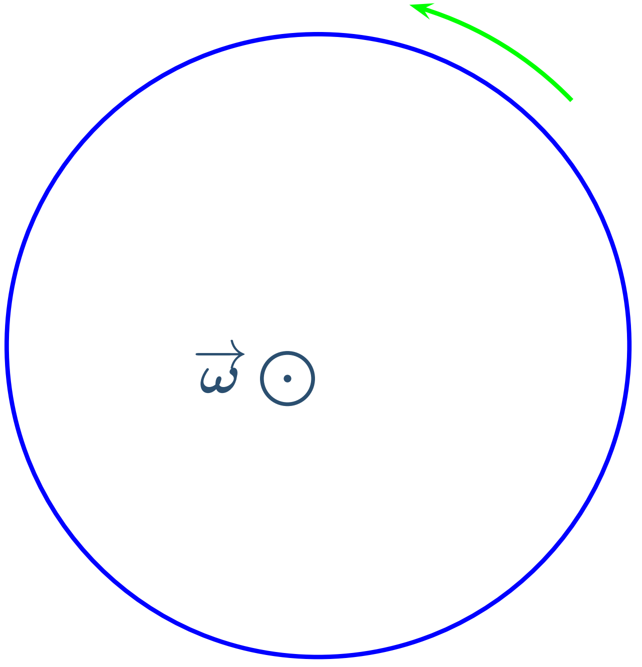

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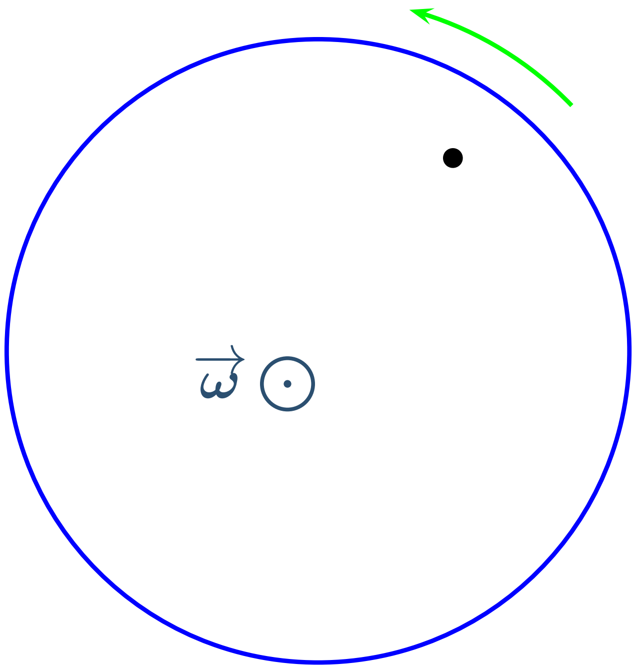

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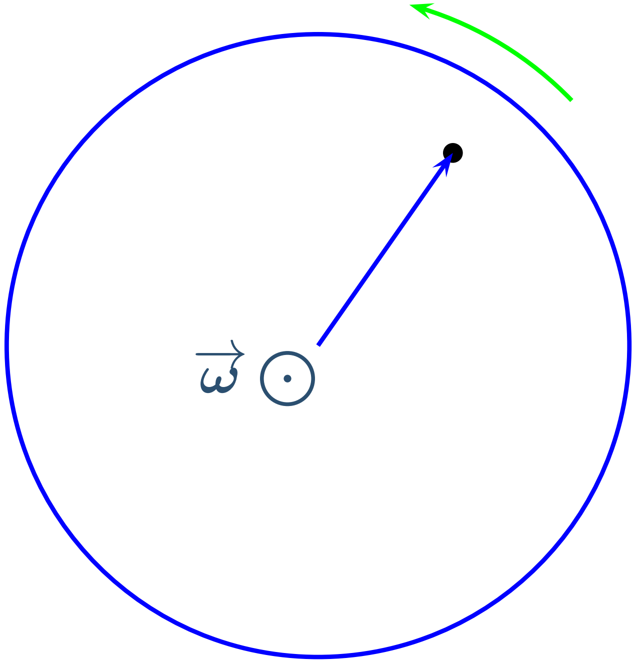

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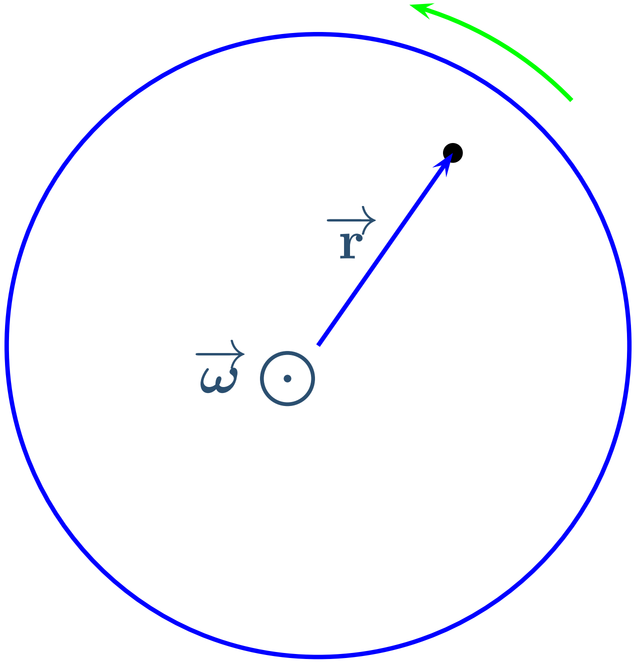

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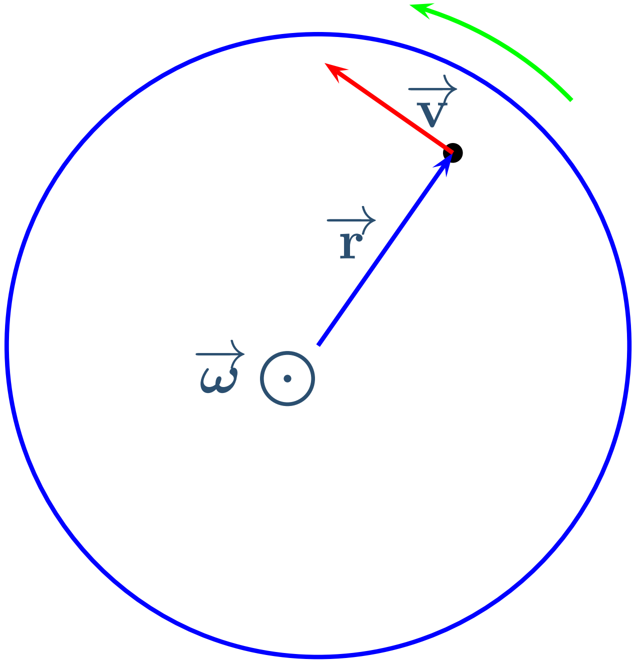

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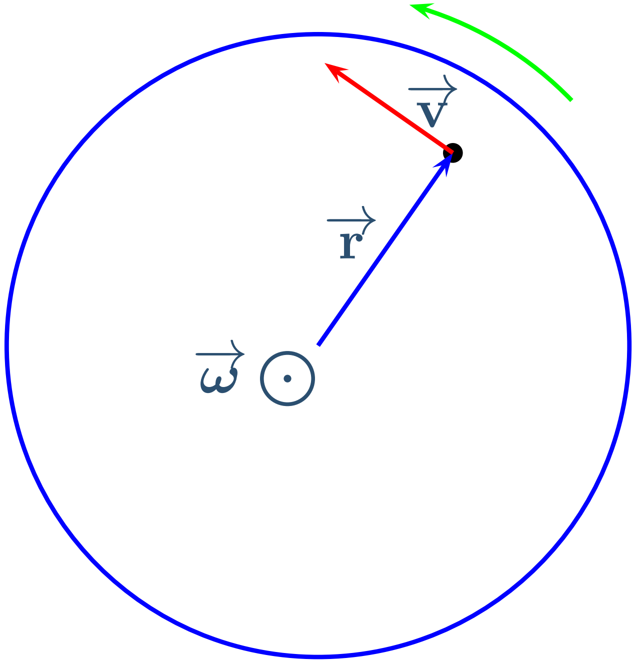
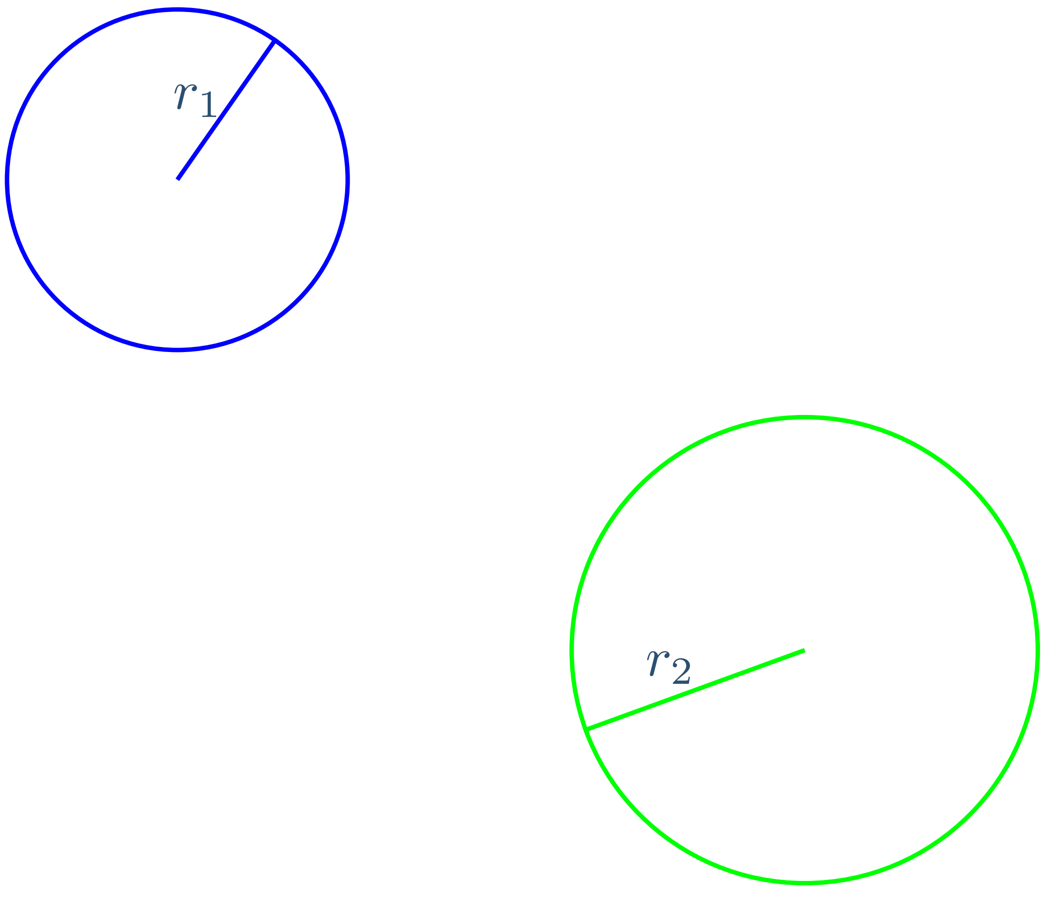


$\overrightarrow{\mathbf{r}}$ - The vector from the axis of rotation to the point of interest

$$\overrightarrow{\mathbf{v}} = \overrightarrow{\omega} \times \overrightarrow{\mathbf{r}}$$

Connected Rotating Objects



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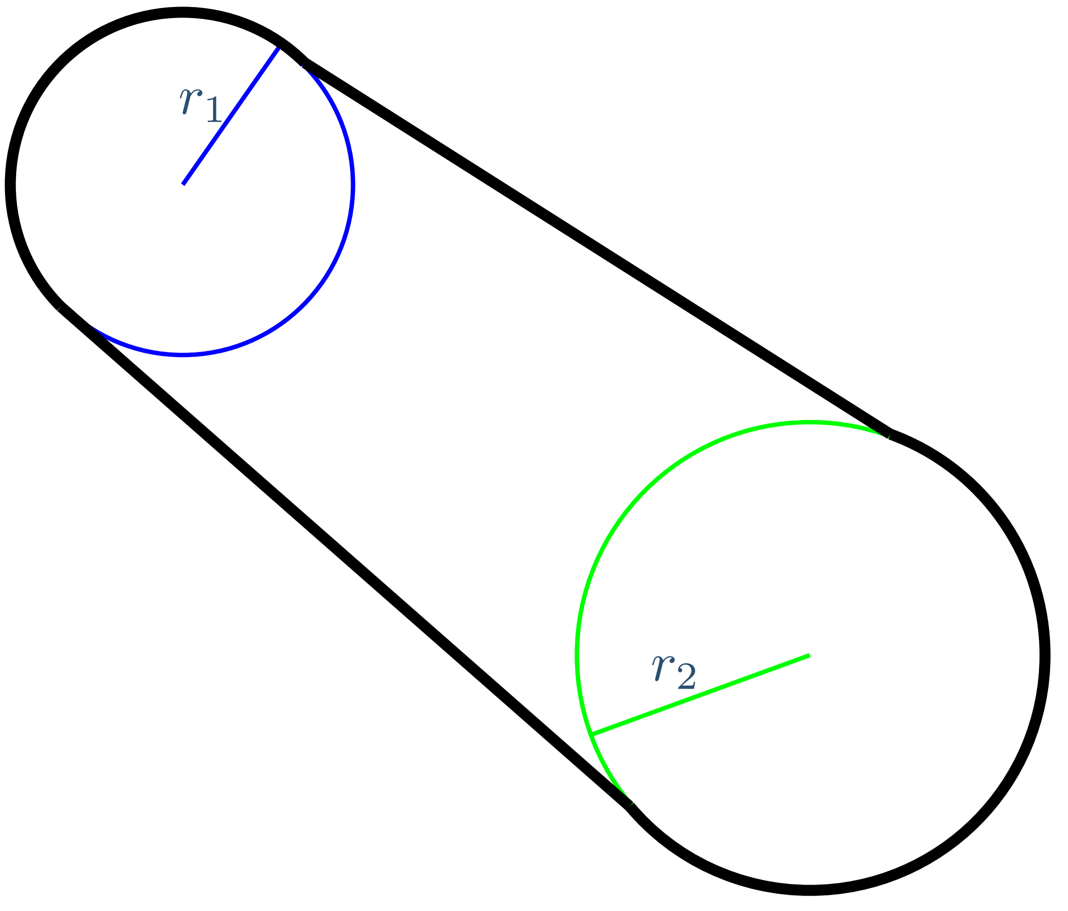

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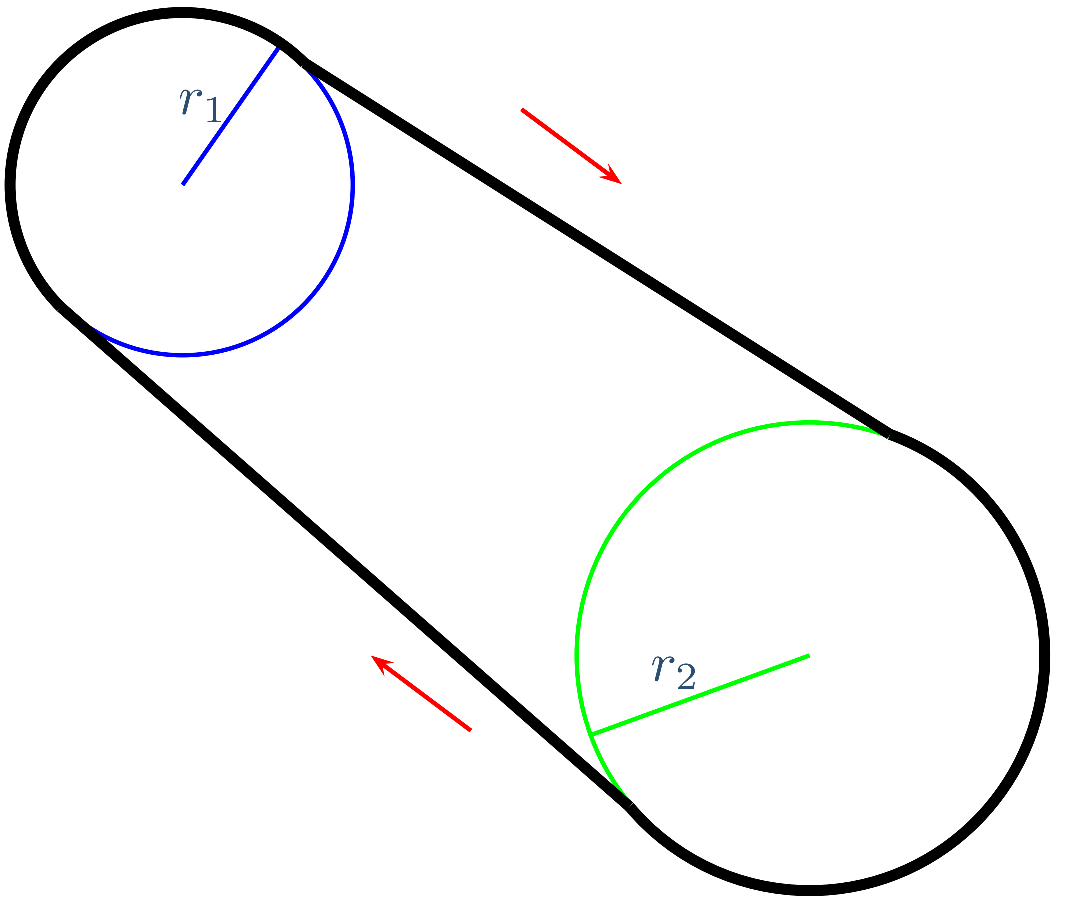

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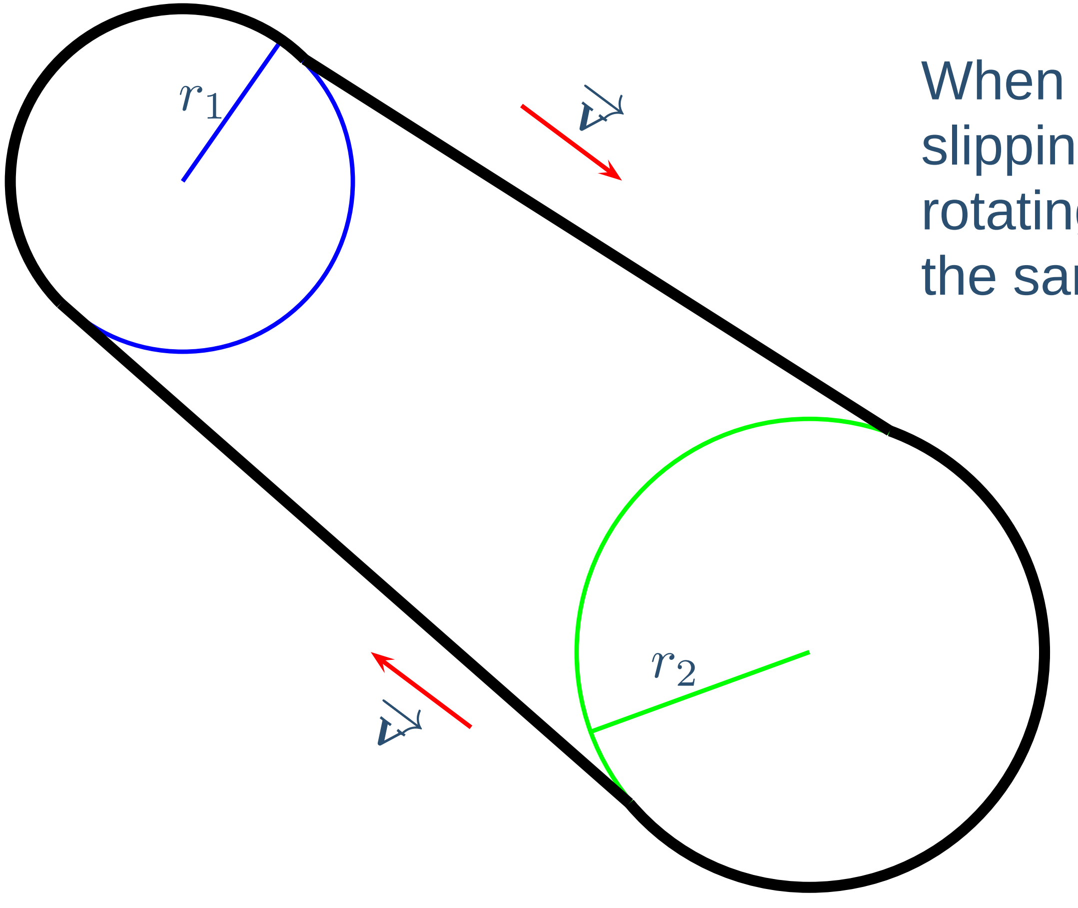

When connected by a non-slipping chain or belt, the two rotating objects must have the same linear velocity

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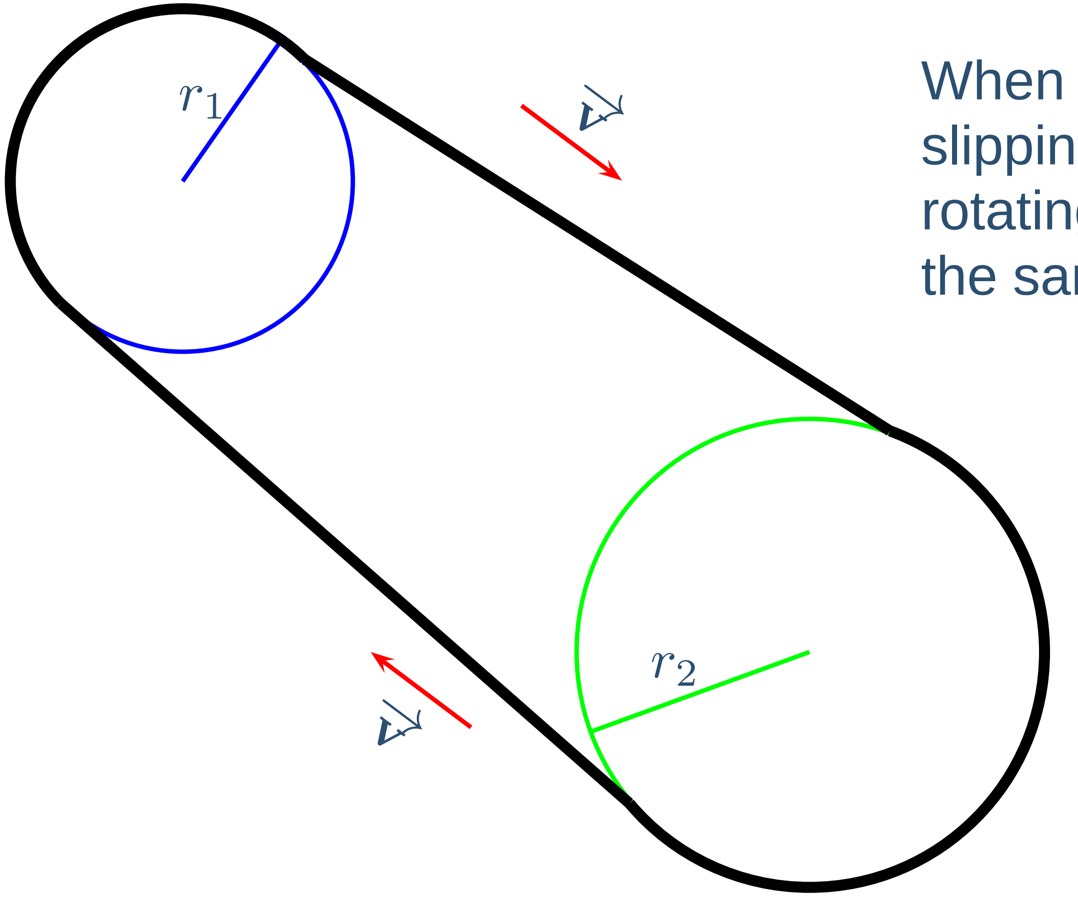

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$$v_1 = v_2$$

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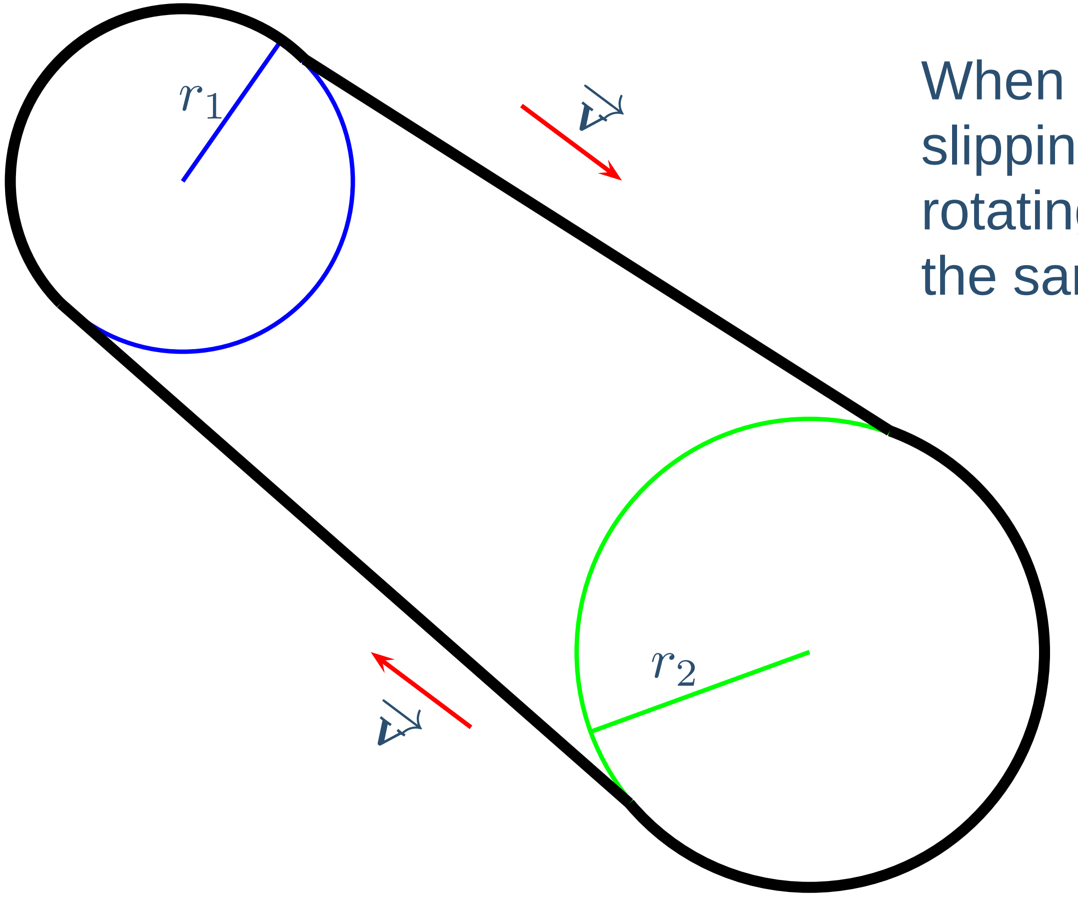

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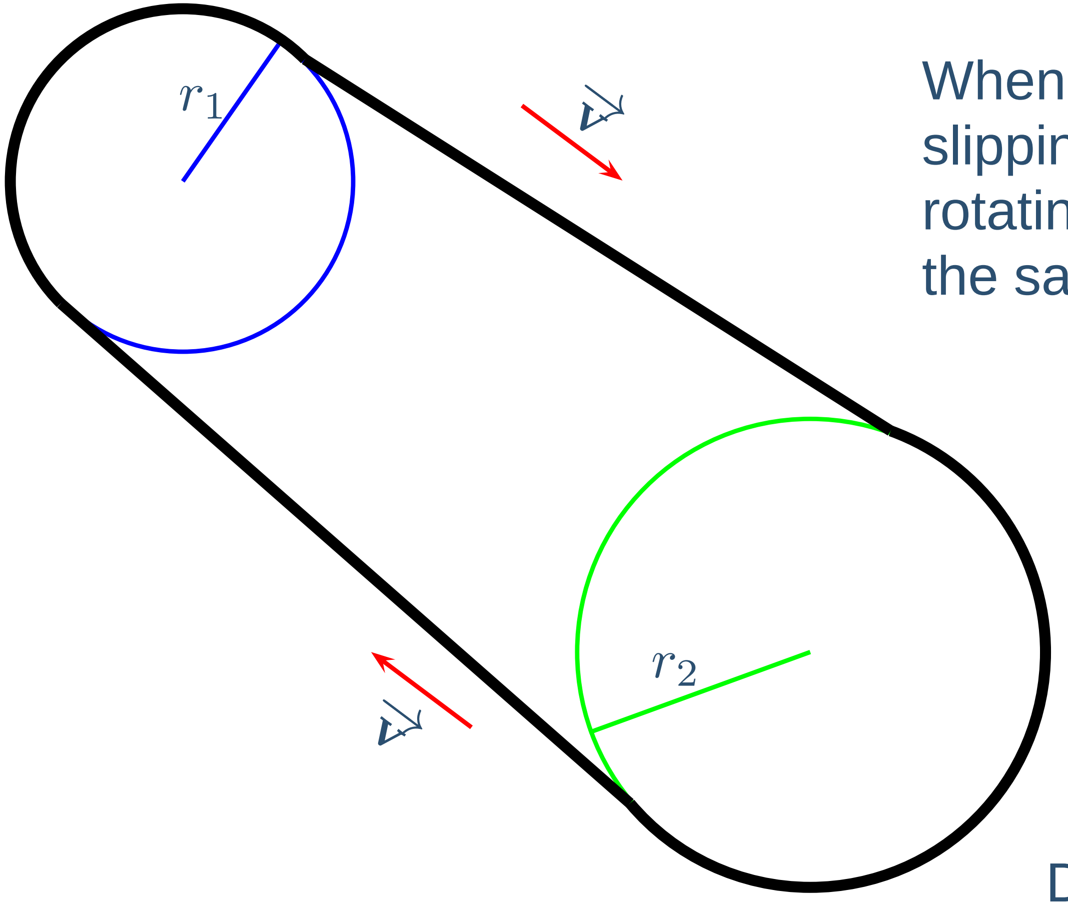

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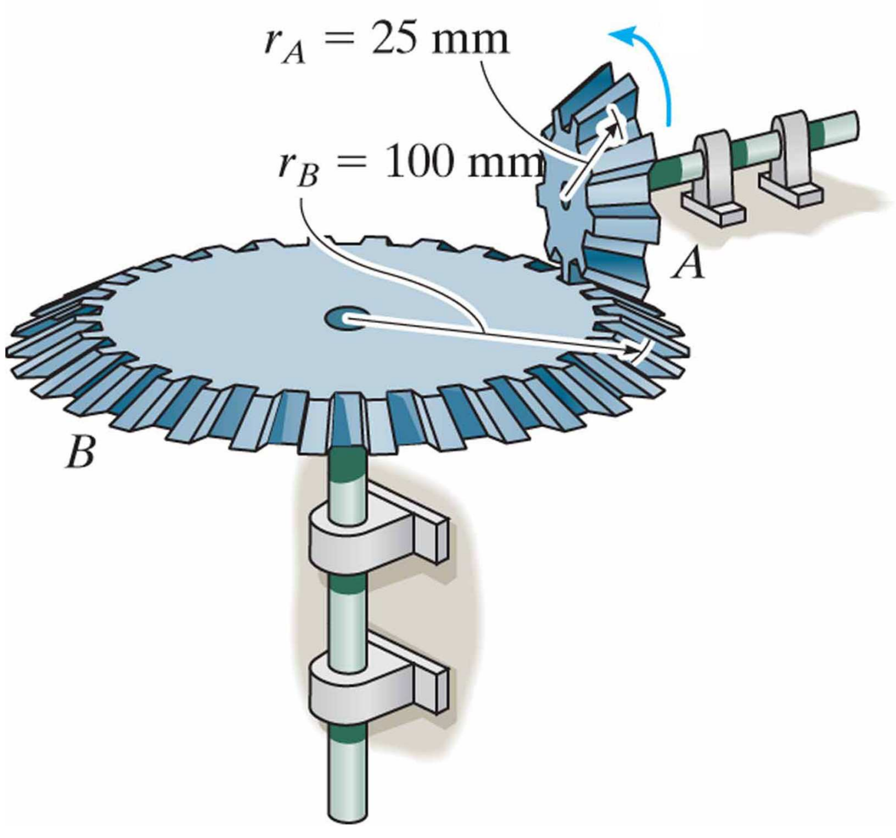
Different Angular Velocities

Connected Objects Example

With what angular velocity must gear A rotate if we wish gear B to rotate at $100\,RPM$?



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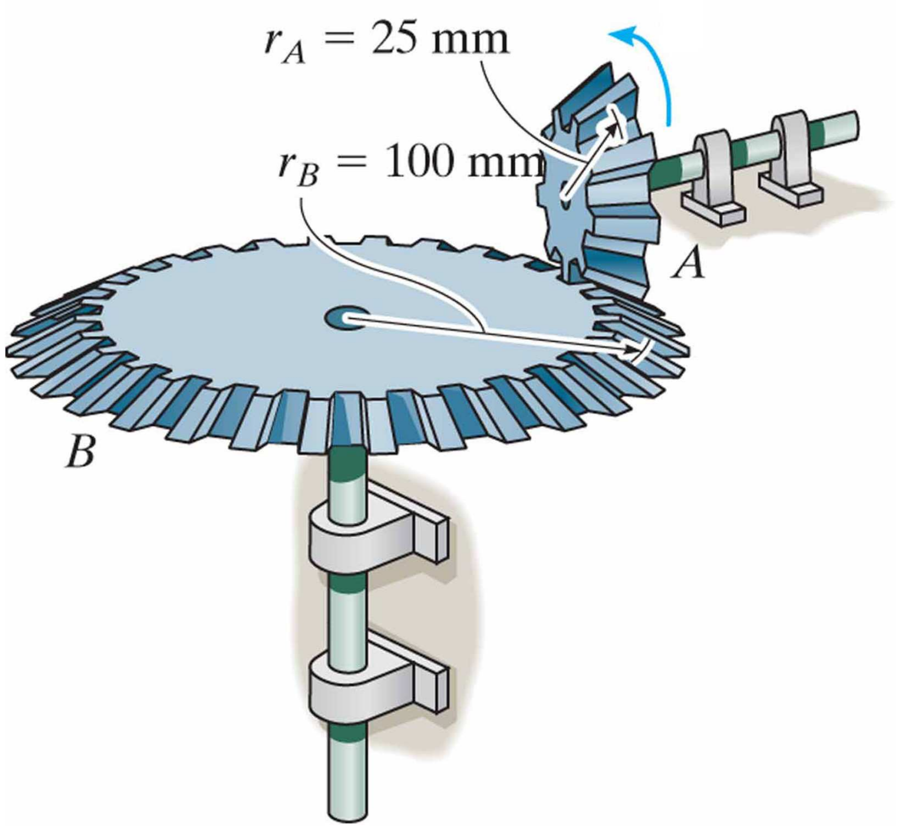

(a) $25\,RPM$

Connected Objects Example

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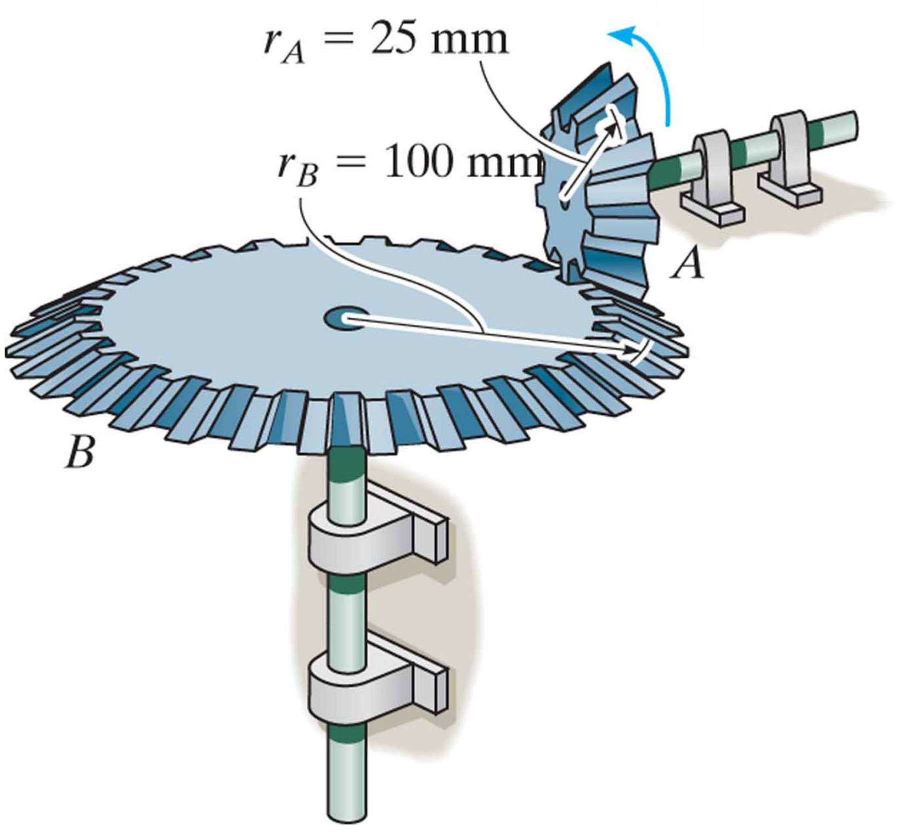

(a) $25\,RPM$

(b) $50\,RPM$

Connected Objects Example

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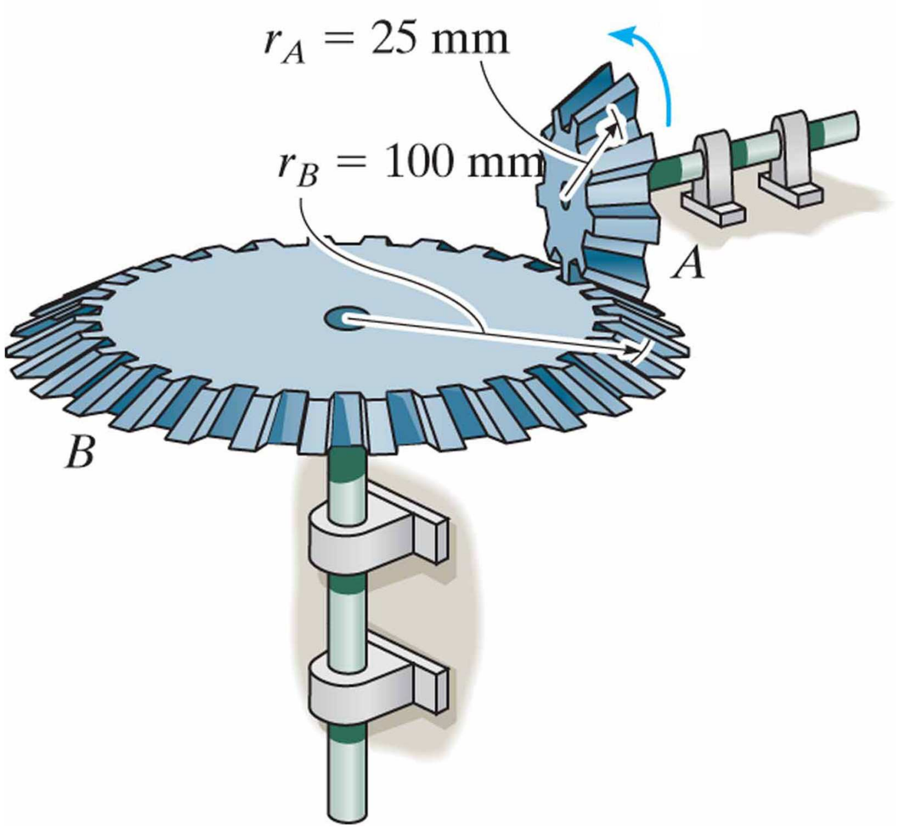

(a) $25\,RPM$

(b) $50\,RPM$

(c) $100\,RPM$

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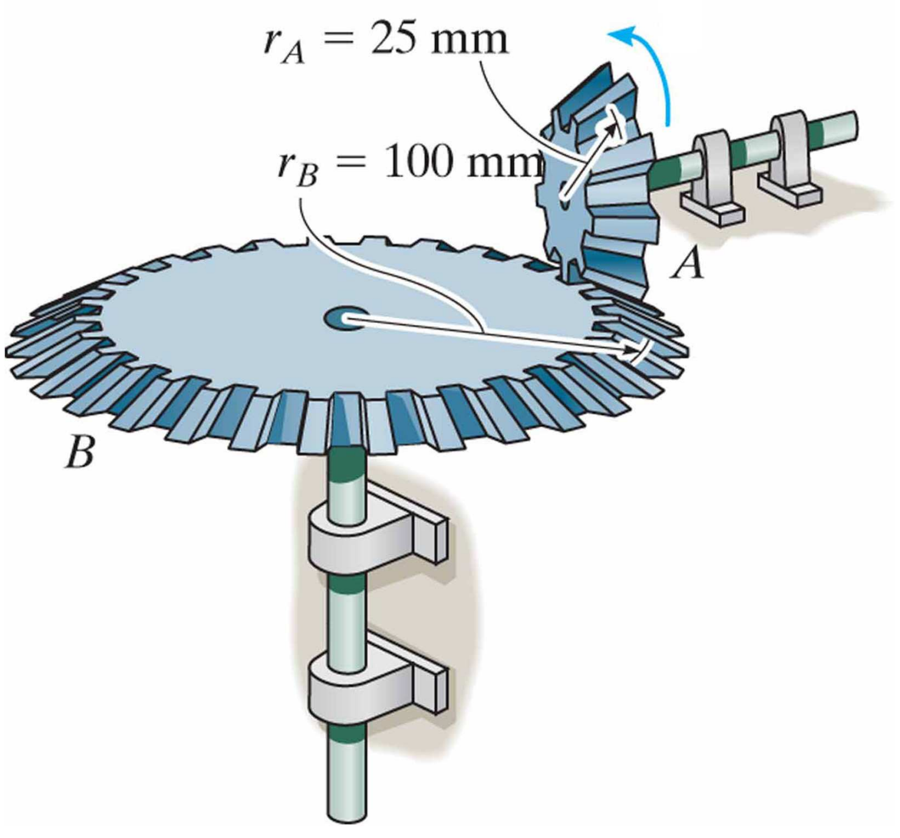


(a) $25\,RPM$

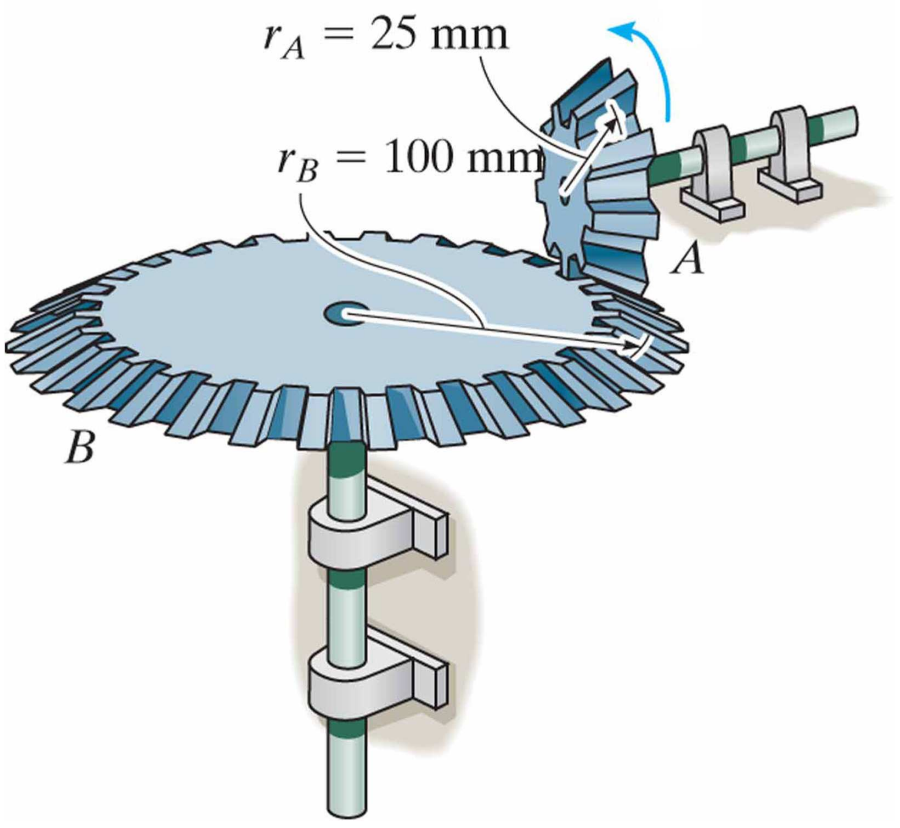
(b) $50\,RPM$

(c) $100\,RPM$

(d) $200\,RPM$

Connected Objects Example

With what angular velocity must gear A rotate if we wish gear B to rotate at $100\,RPM$?



(a) $25\,RPM$

(b) $50\,RPM$

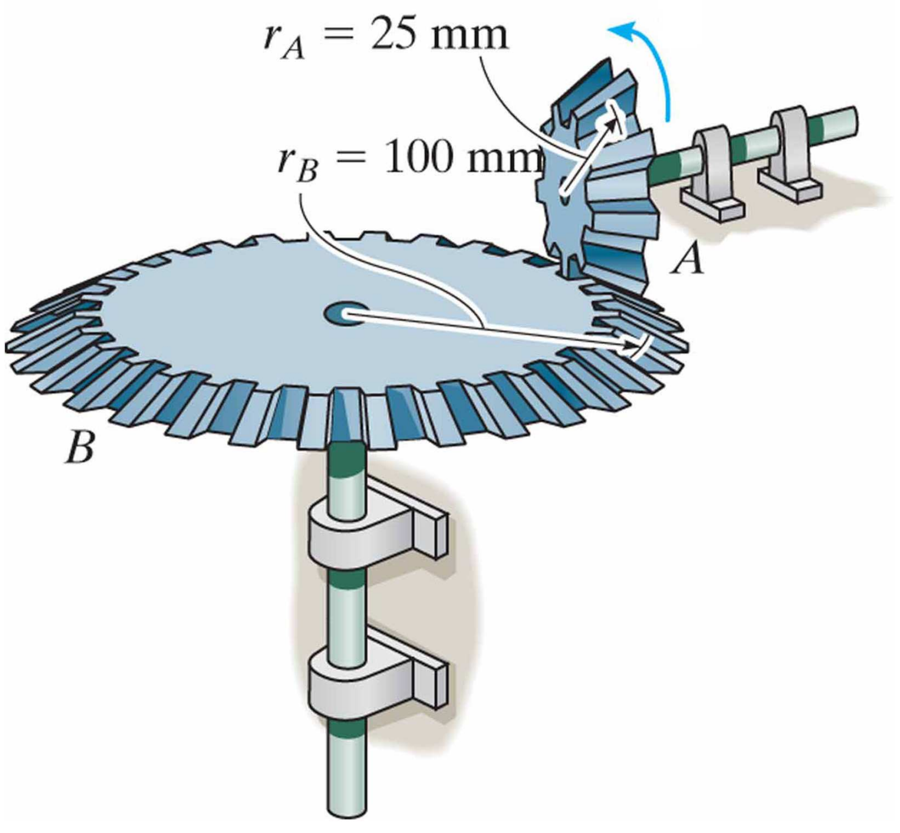
(c) $100\,RPM$

(d) $200\,RPM$

(e) $400\,RPM$

Connected Objects Example

With what angular velocity must gear A rotate if we wish gear B to rotate at $100\,RPM$?



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Connected Objects Example

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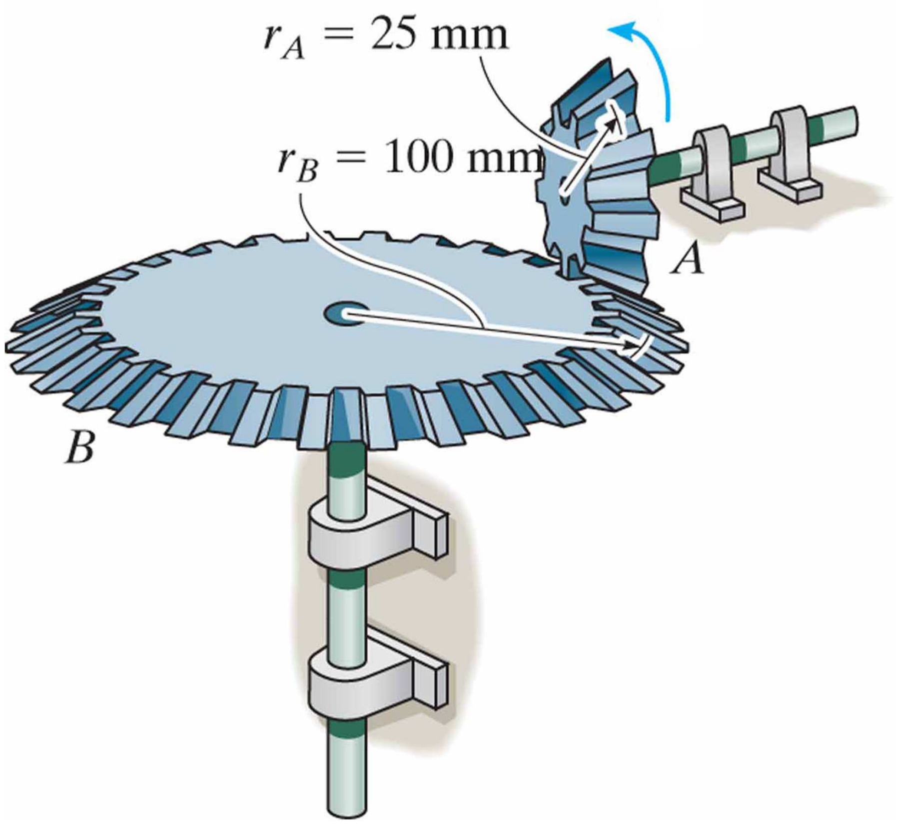

We can use any units in this equation as long as they are the same

$$r_1\omega_1 = r_2\omega_2 \Rightarrow$$

$$\omega_1 = \frac{(100\,mm)(100\,RPM)}{25\,mm}$$

(e) $400\,RPM$

Linear Accelerations

Every point on a rotating object has two acceleration components.

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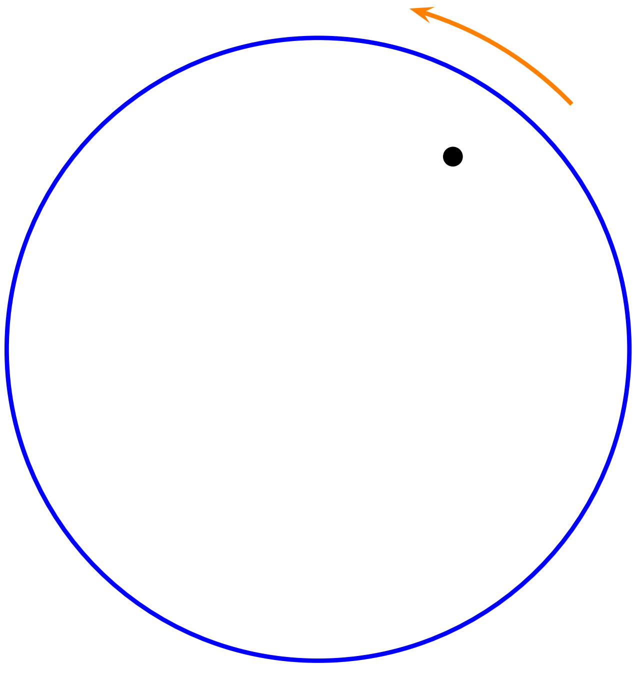

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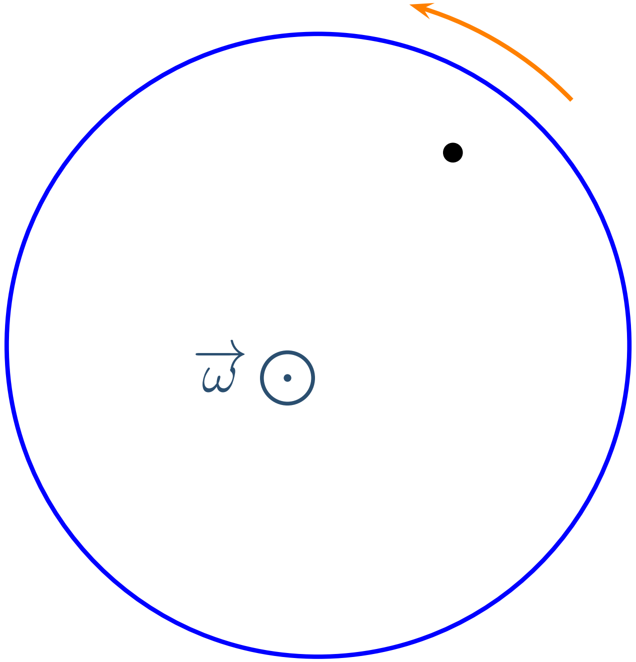

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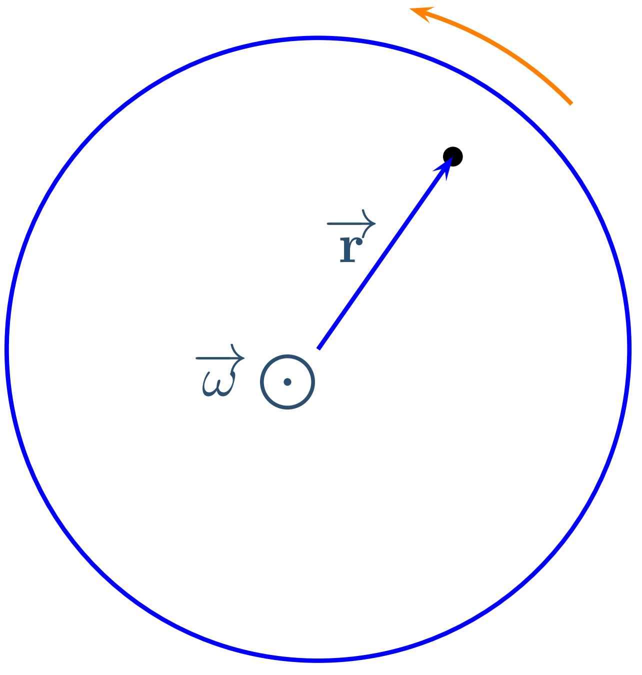

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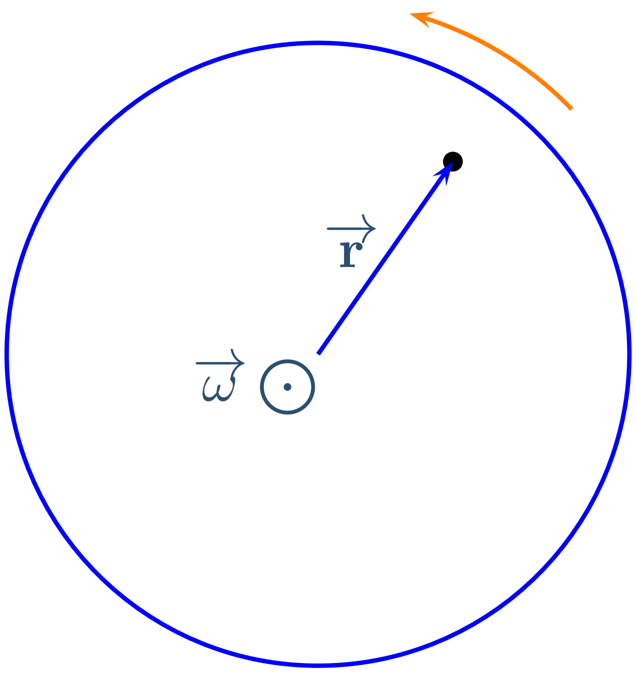

$\vec{\mathbf{a}}_{rad}$ - changes in direction

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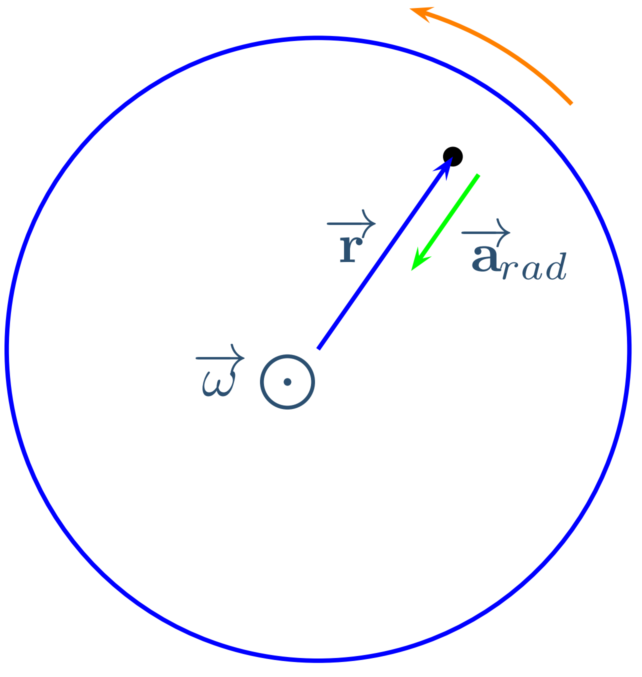

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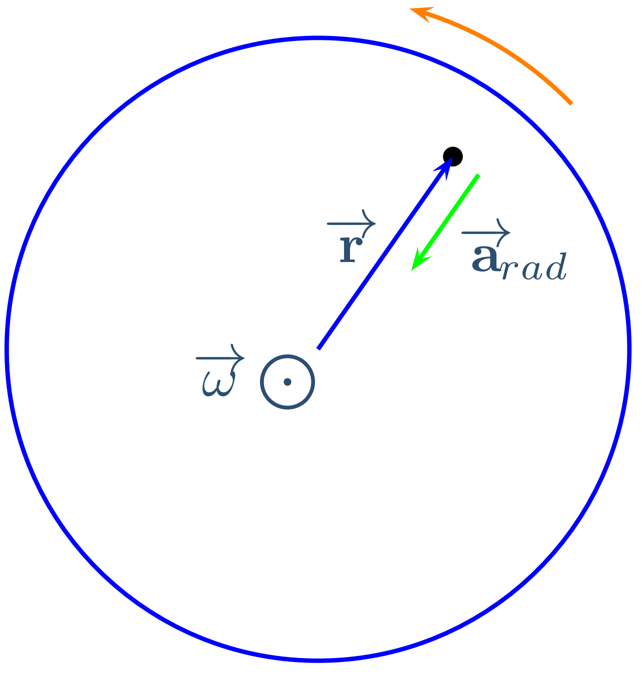

$\vec{\mathbf{a}}_{rad}$ - changes in direction

$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$

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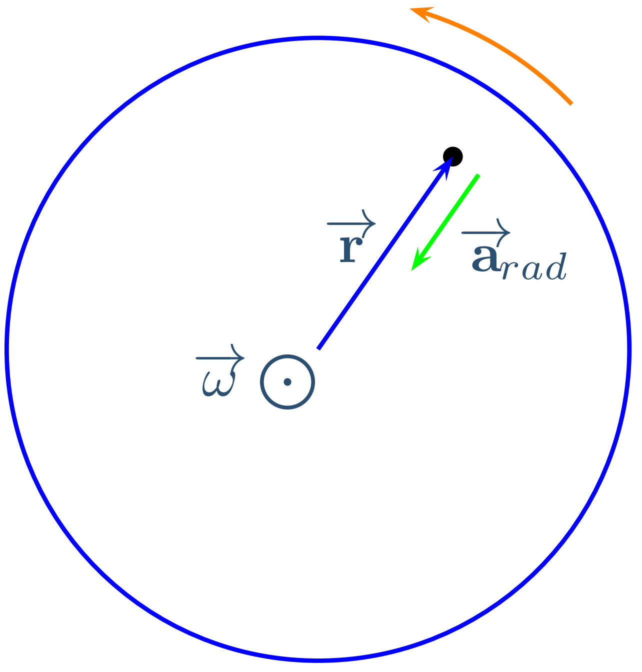

$\overrightarrow{\mathbf{a}}_{rad}$ - changes in direction

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$$\boxed{\overrightarrow{\mathbf{a}}_{rad} = -\omega^2 \overrightarrow{\mathbf{r}}}$$

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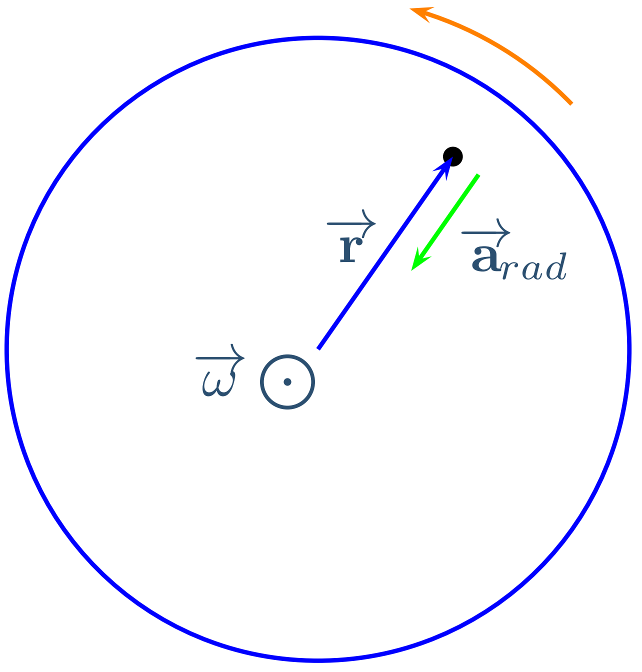

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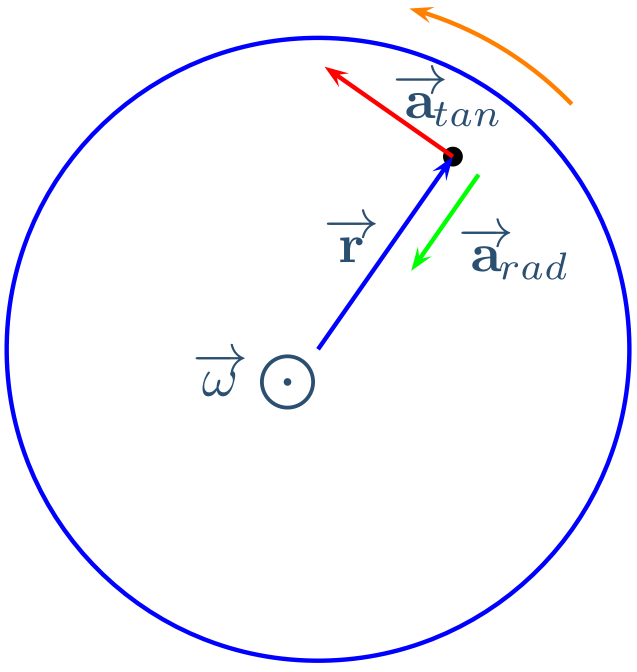

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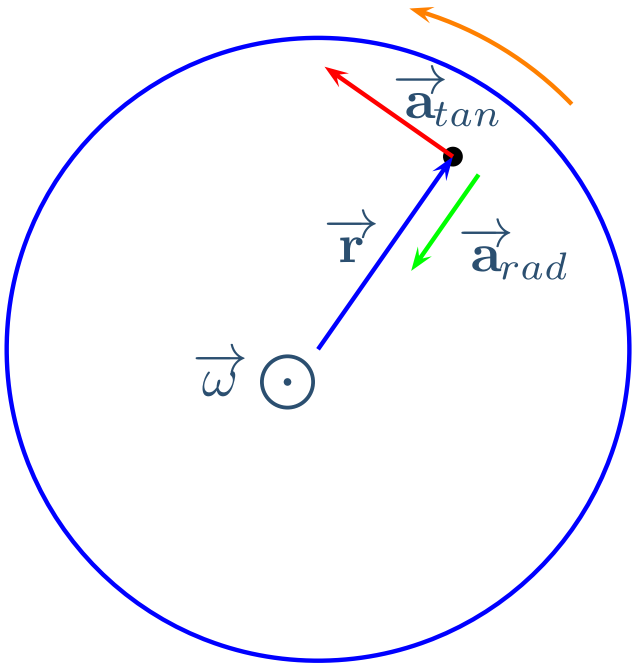

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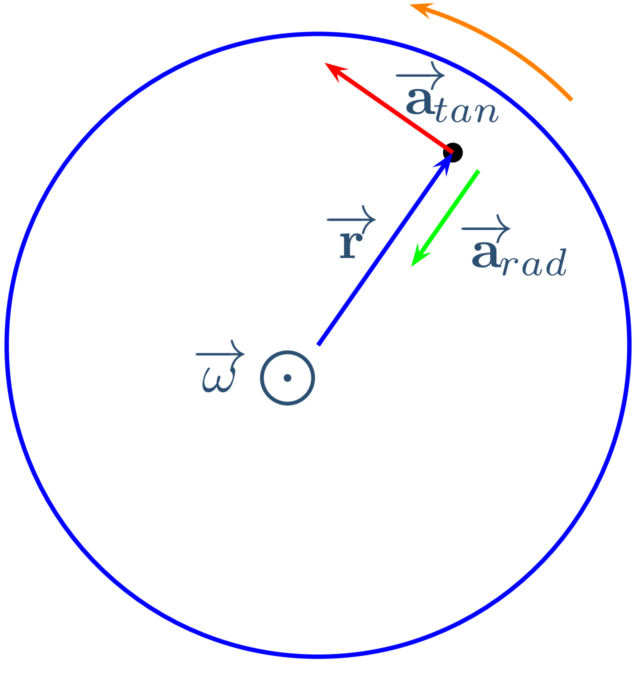

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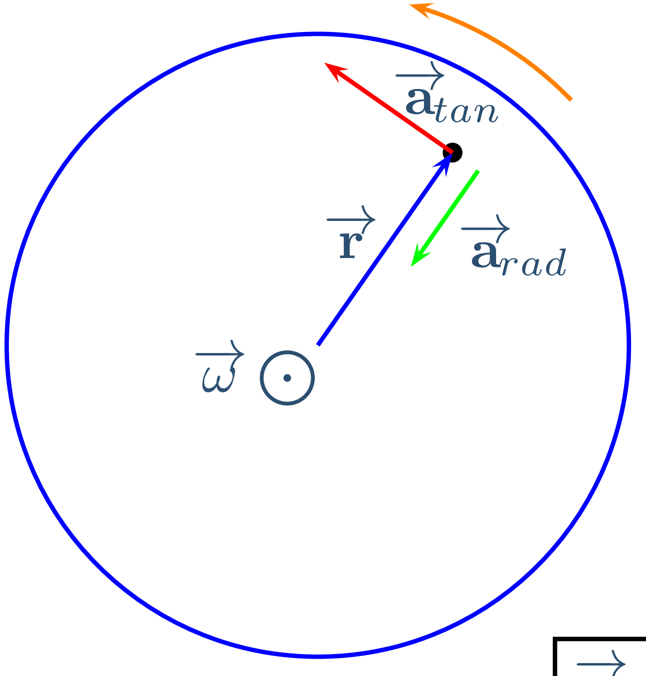

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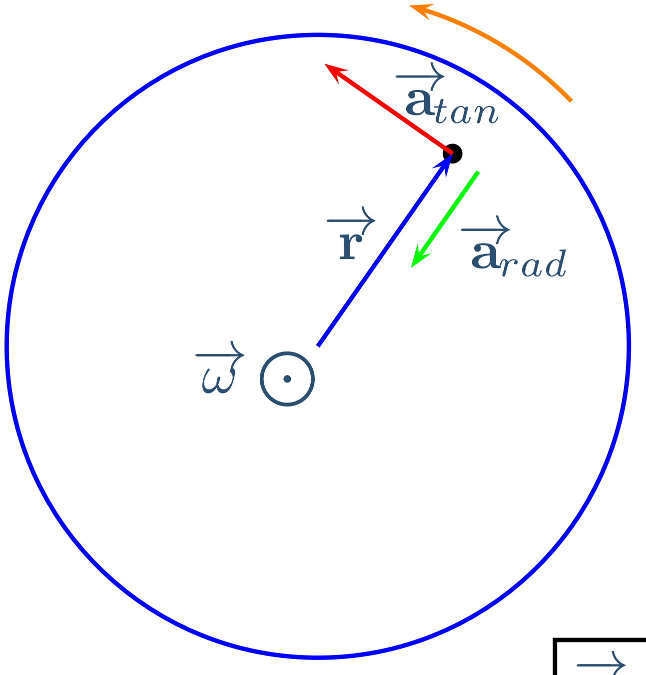

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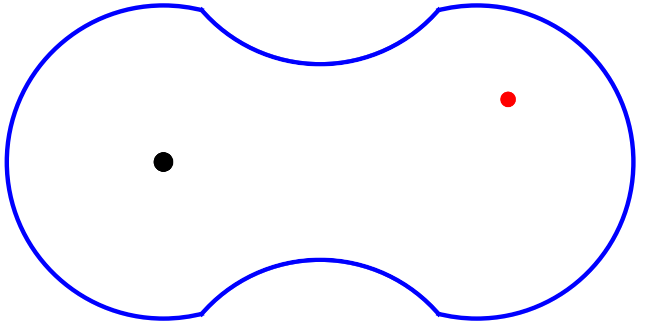
Non-Circular Objects

Putting the origin of the coordinate system at the axis of rotation allows us to use all of the equations for circular objects.



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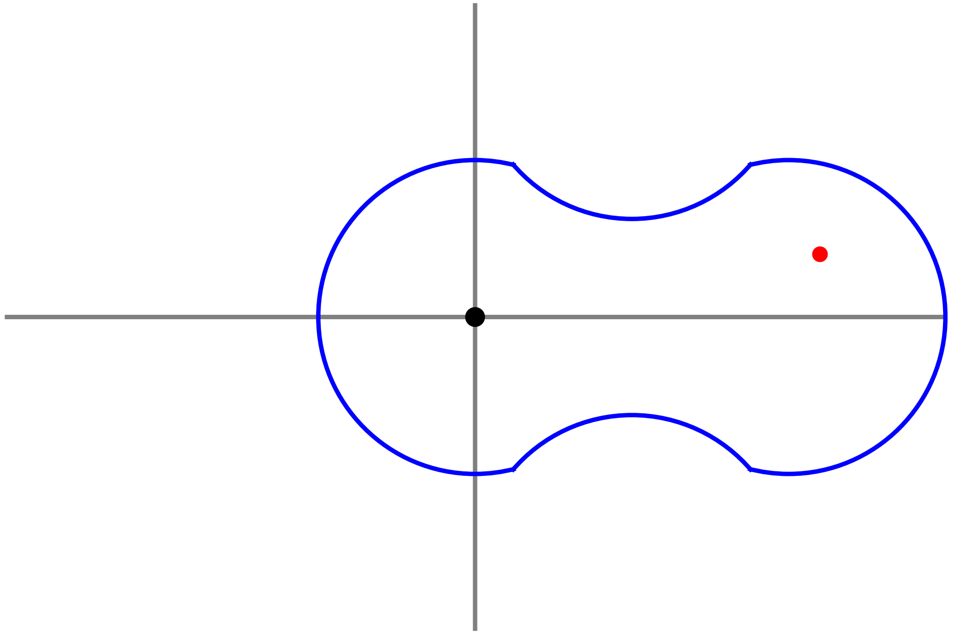

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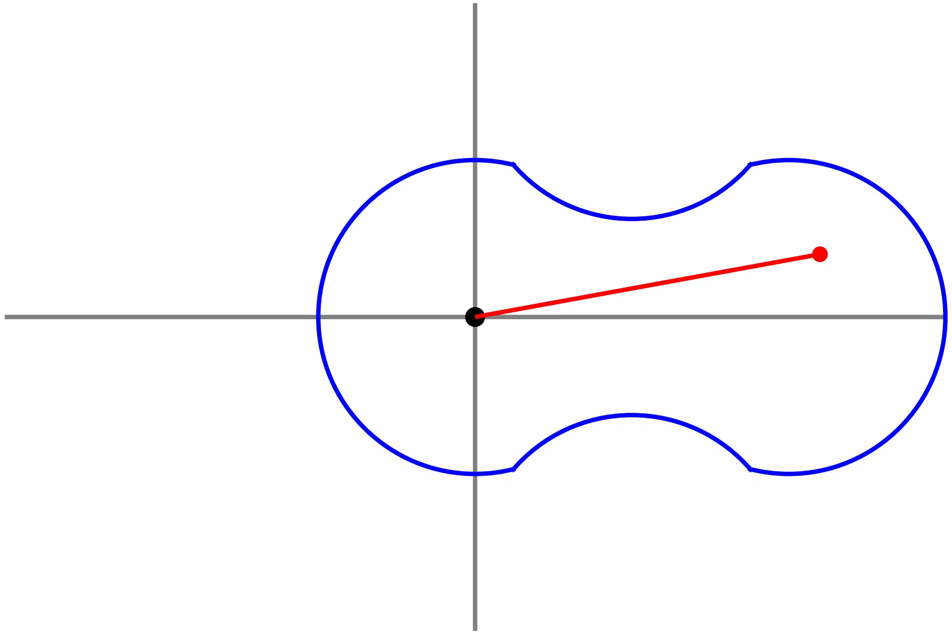

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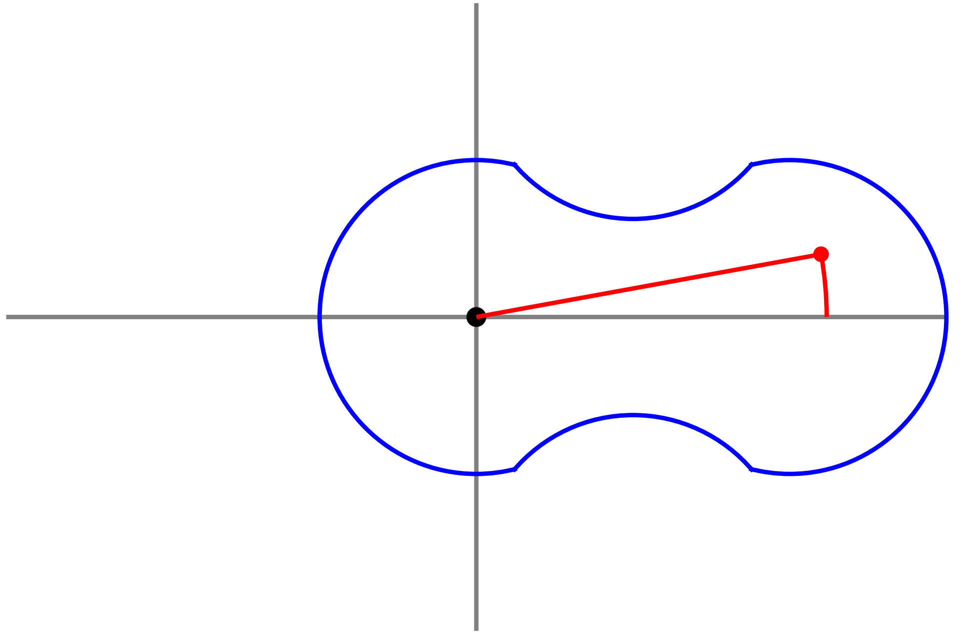

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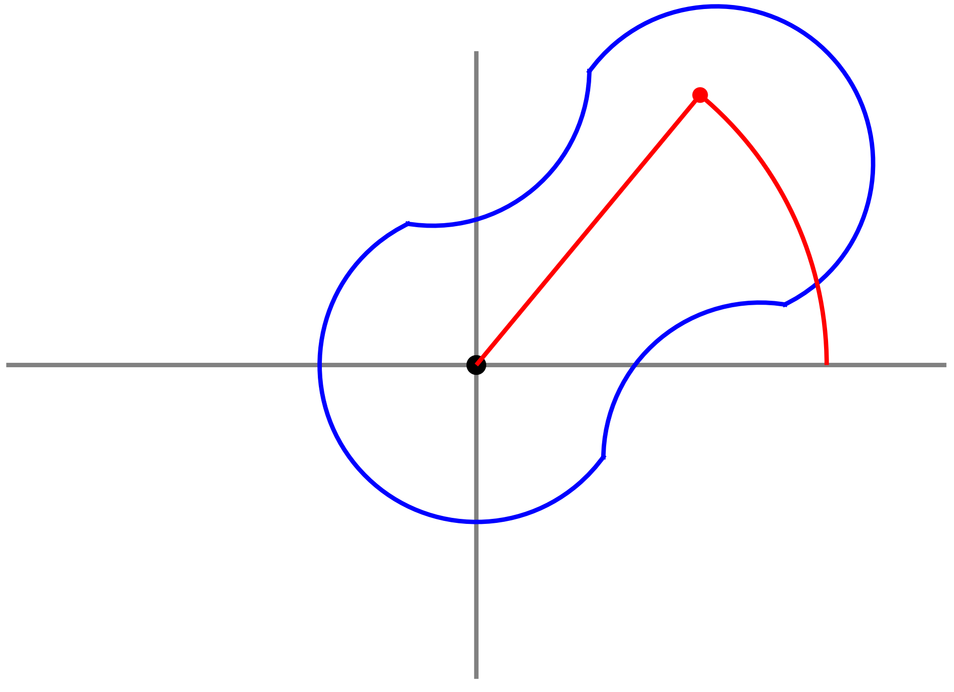

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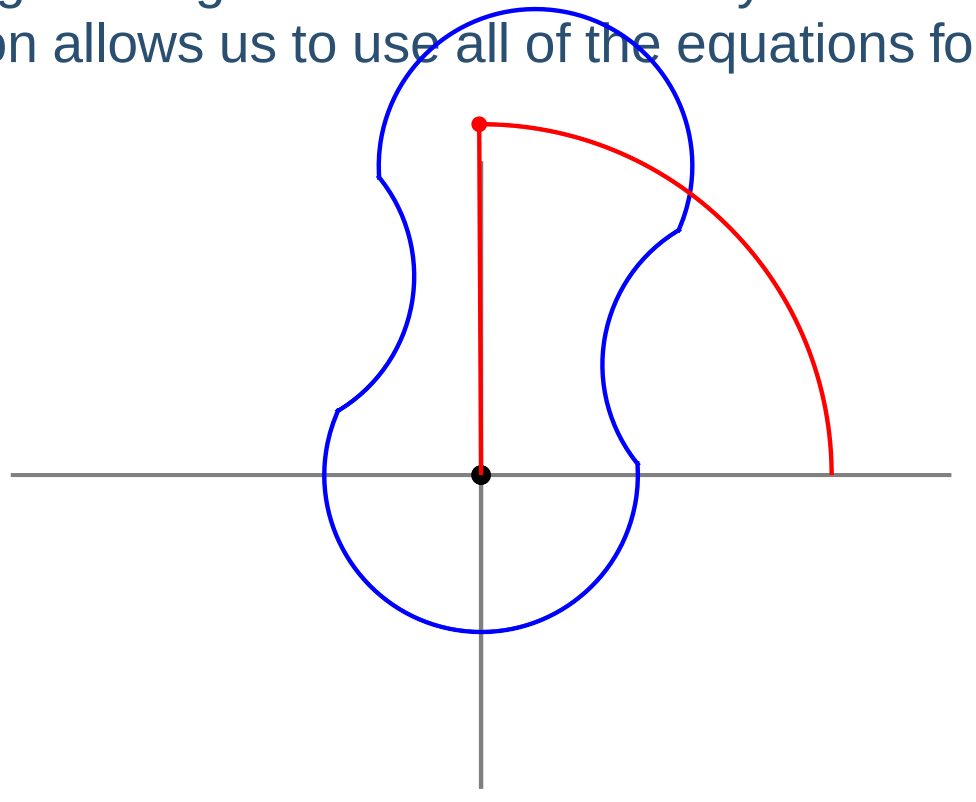

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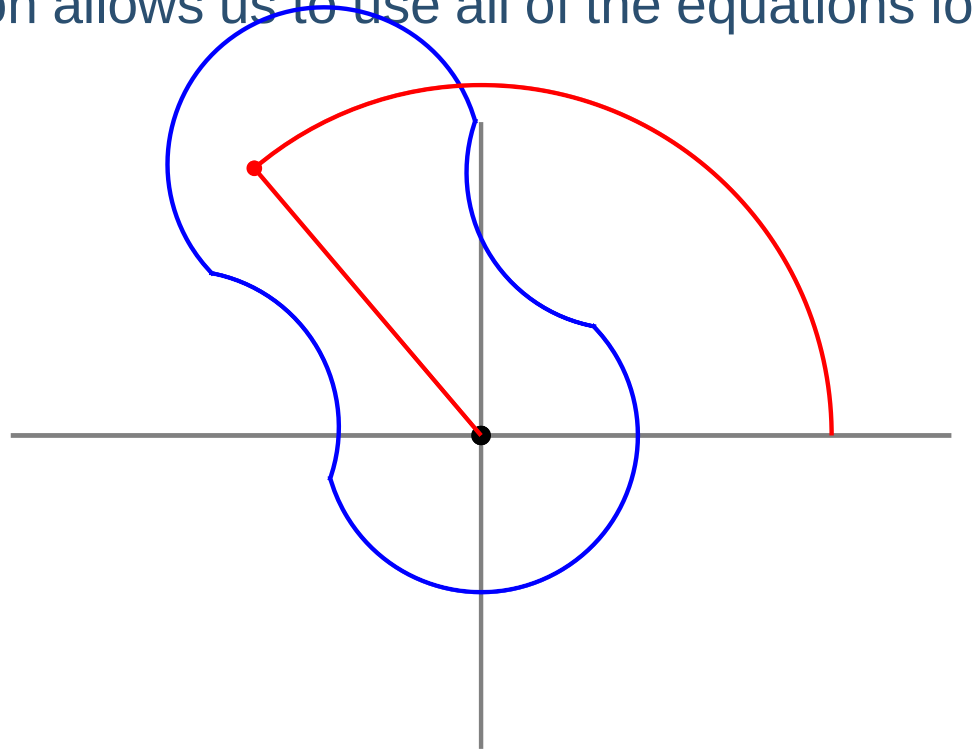

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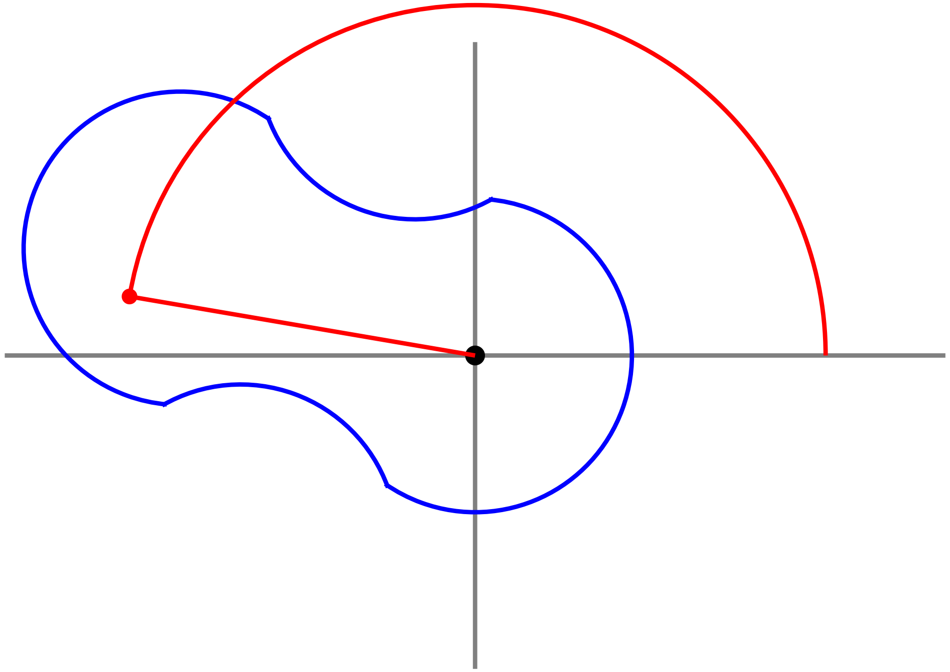

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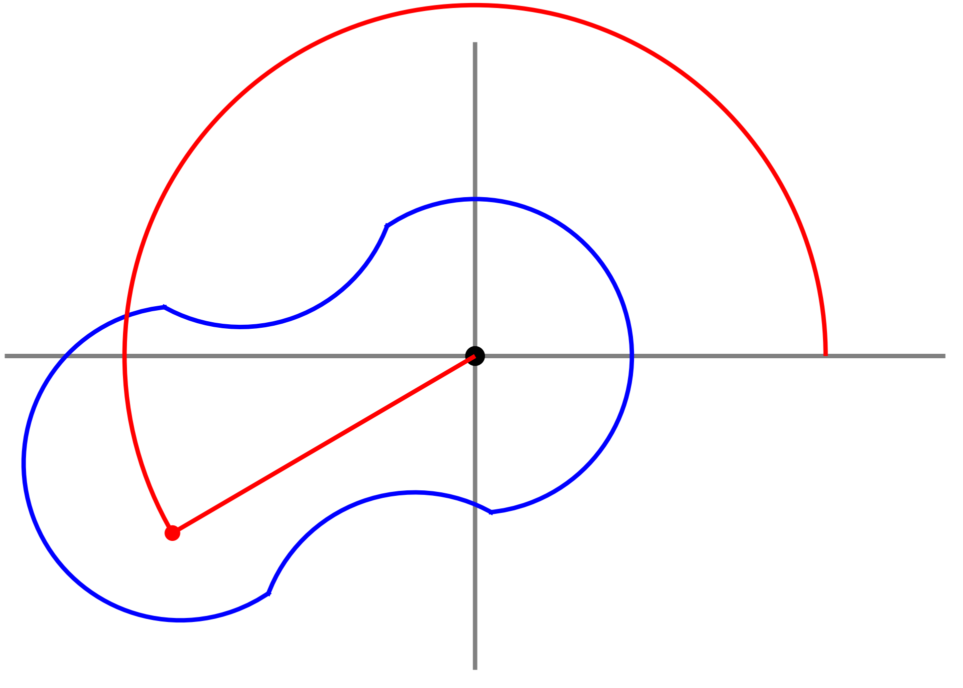

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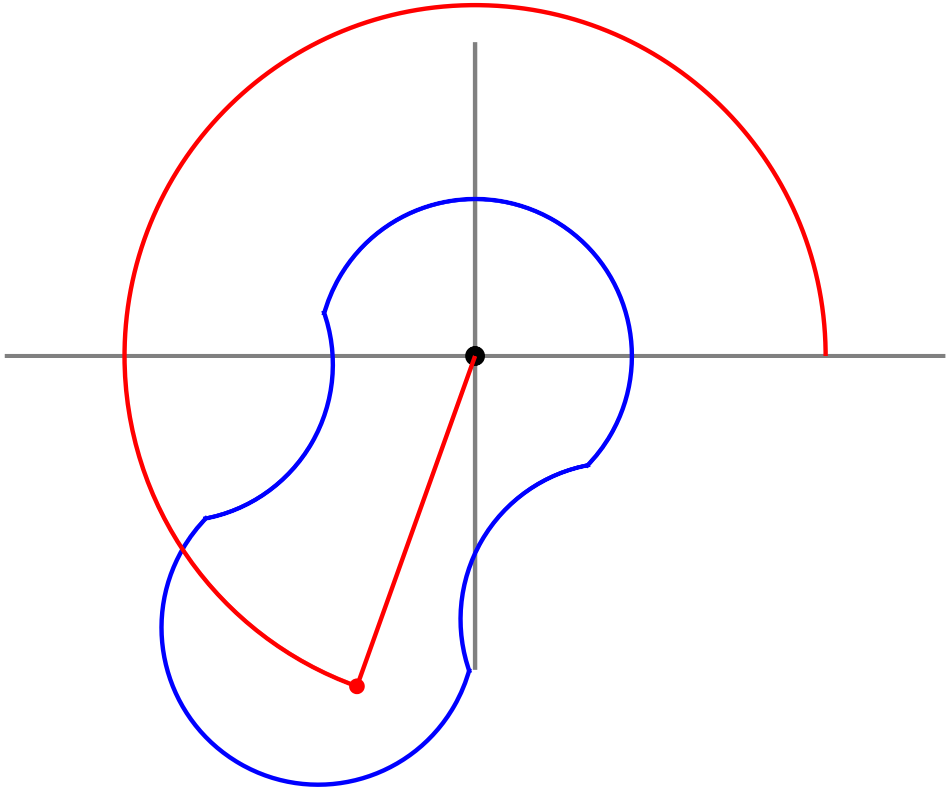

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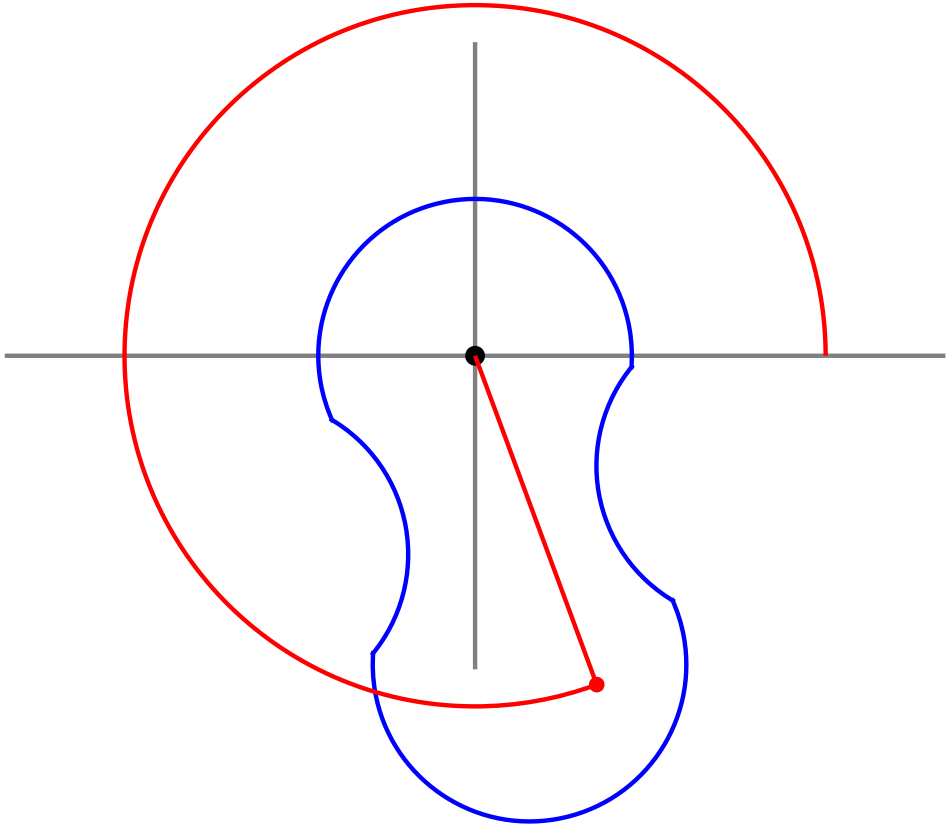

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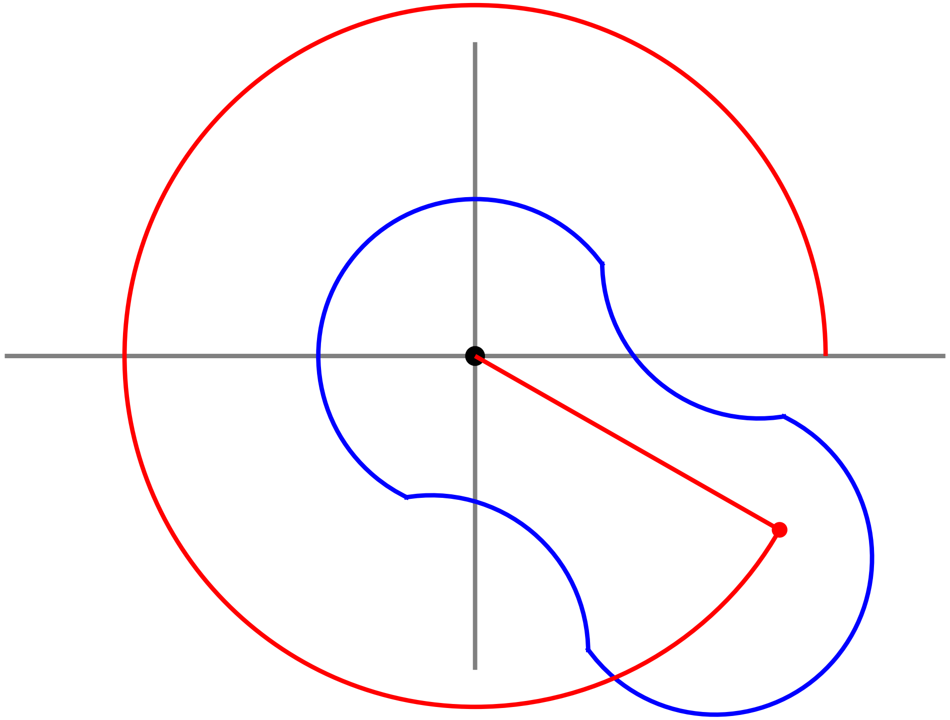

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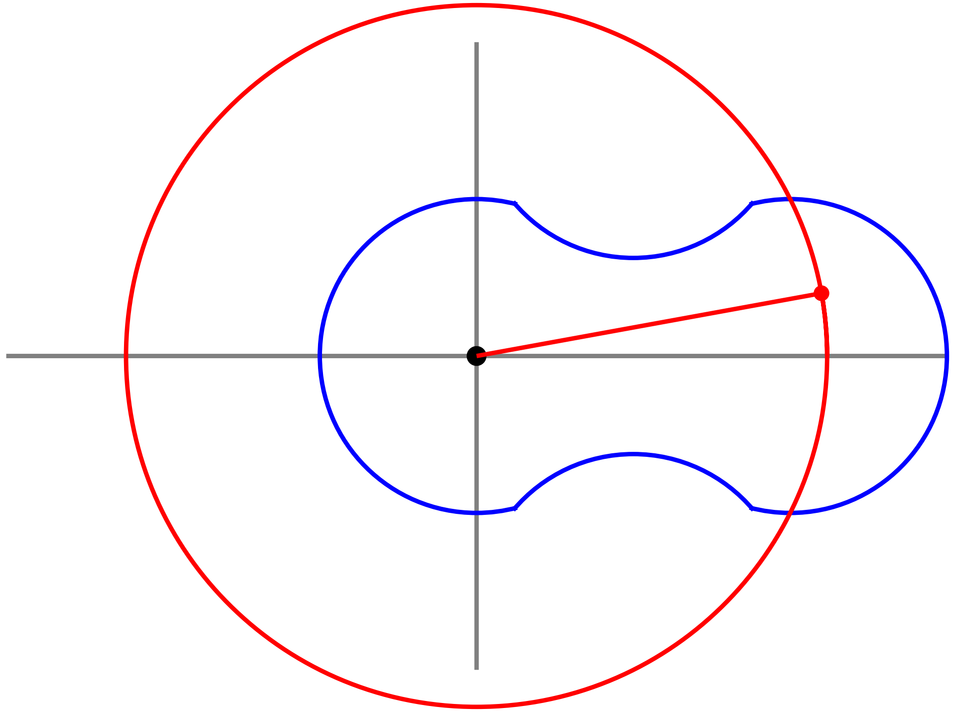

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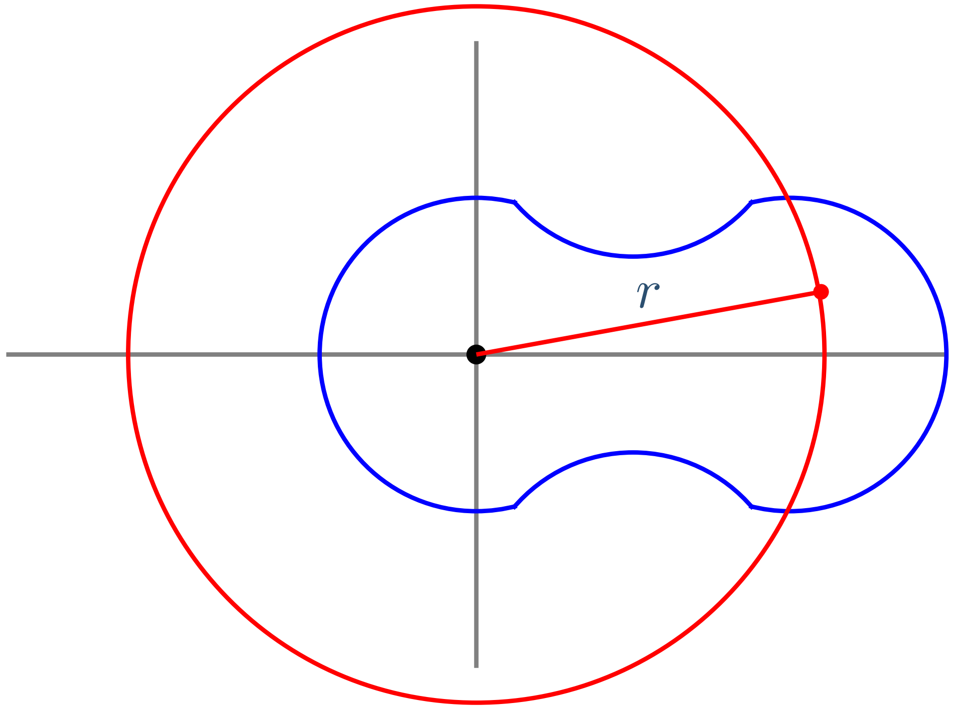

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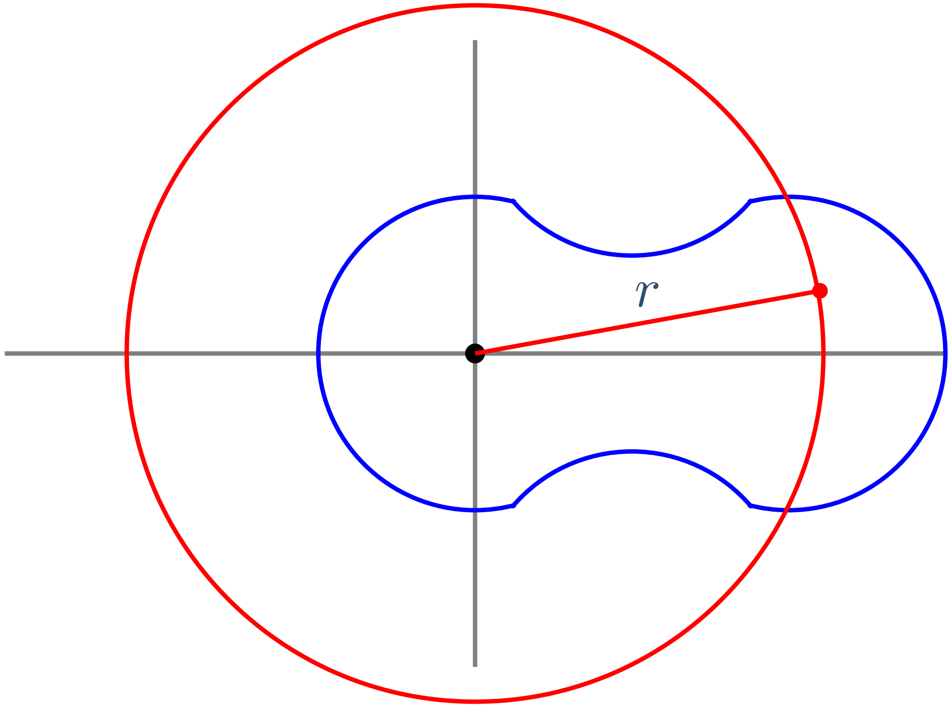

$r =$ distance from axis of rotation

Rotational Kinetic Energy

Any rotating object has a kinetic energy due to its motion.

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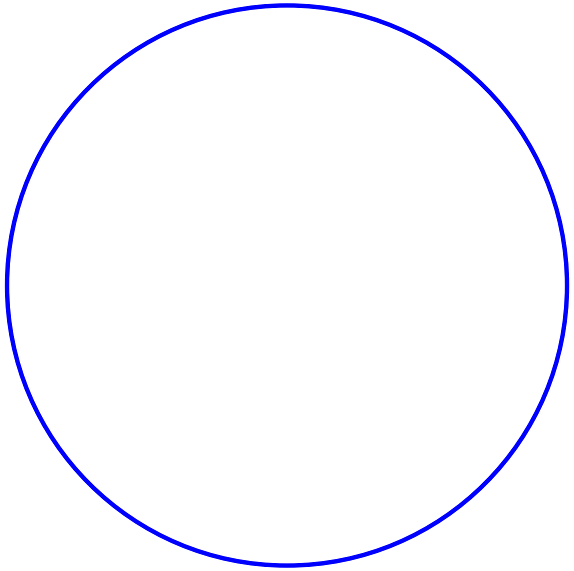

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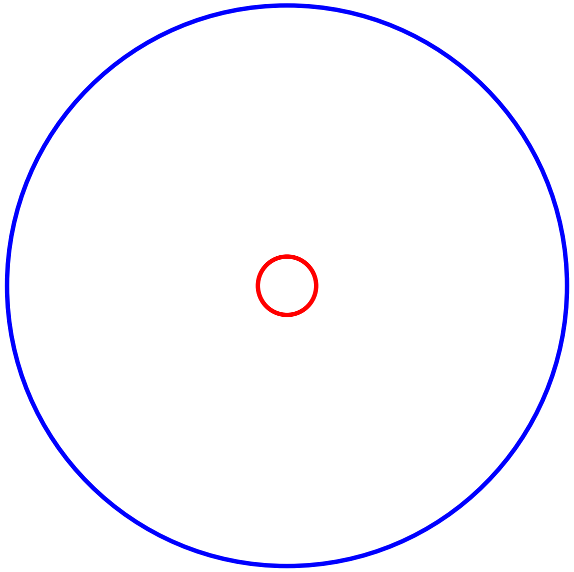

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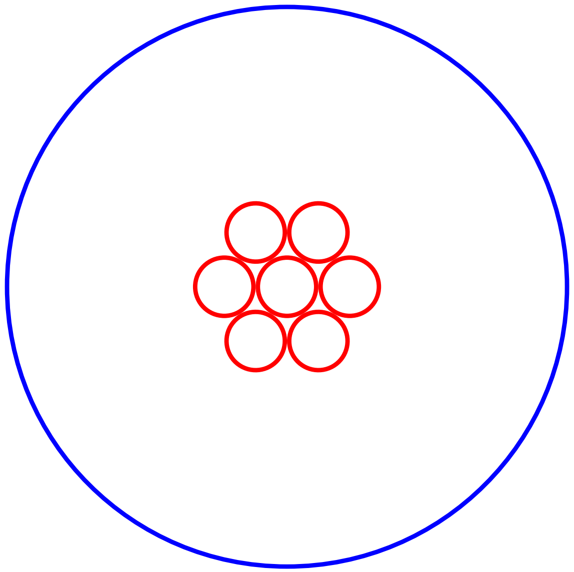

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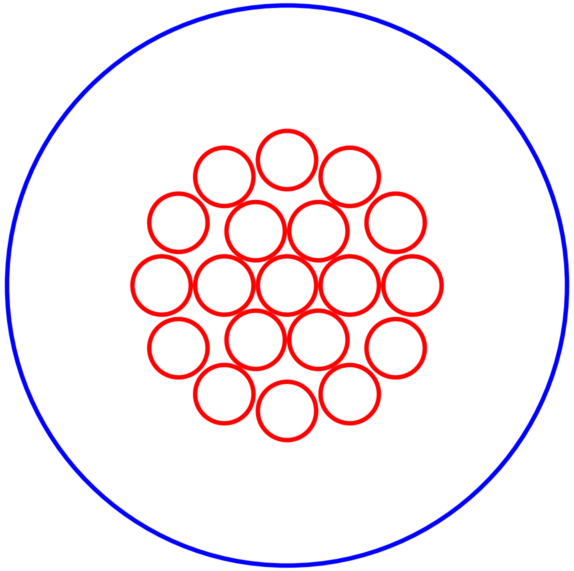

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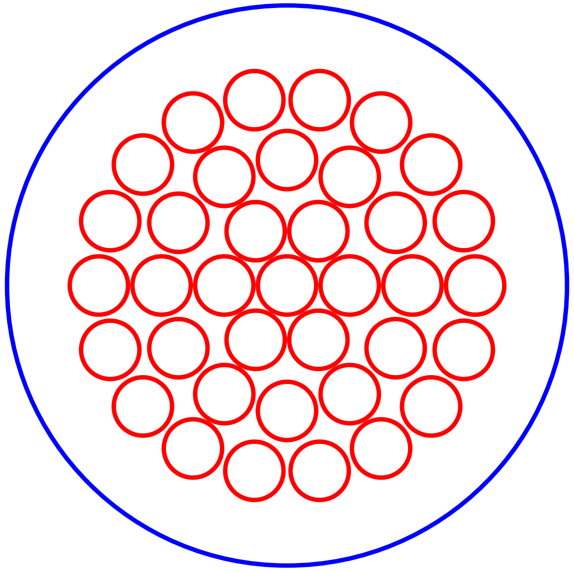

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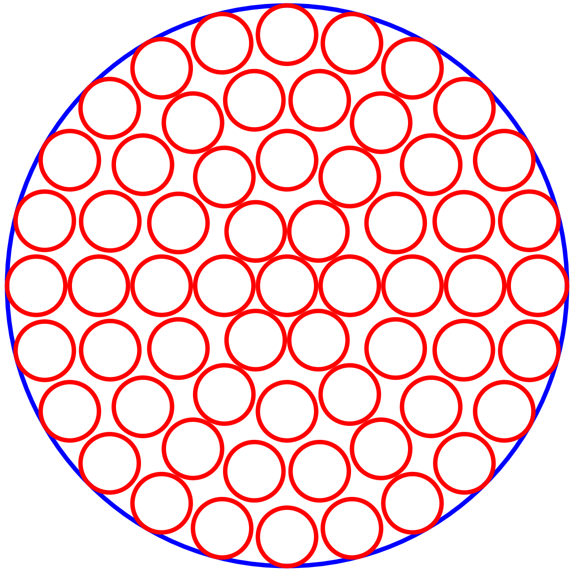

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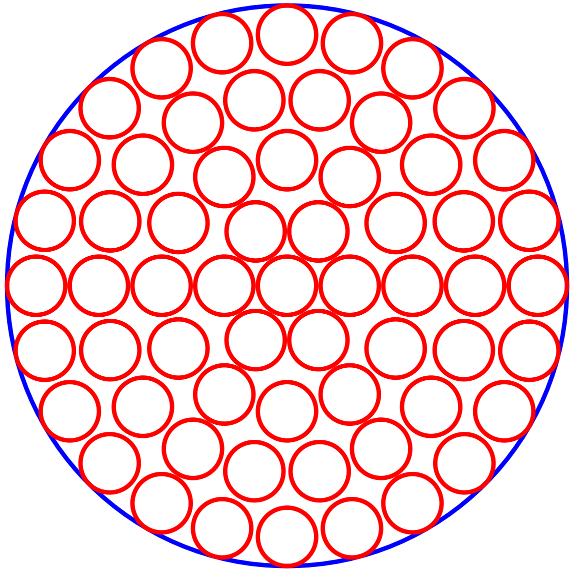
Look at the i-th piece



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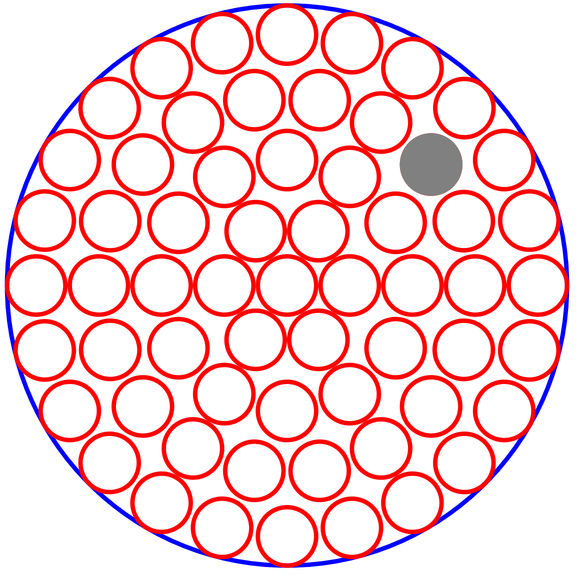

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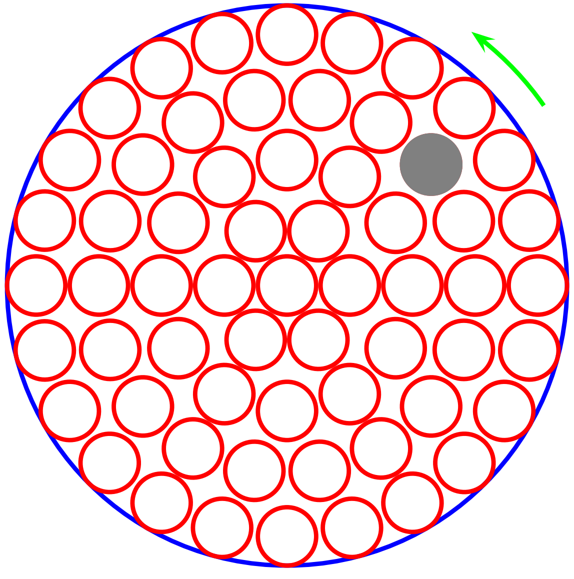

Look at the i-th piece

It has kinetic energy, $K_i = \frac{1}{2} M_i v_i^2$

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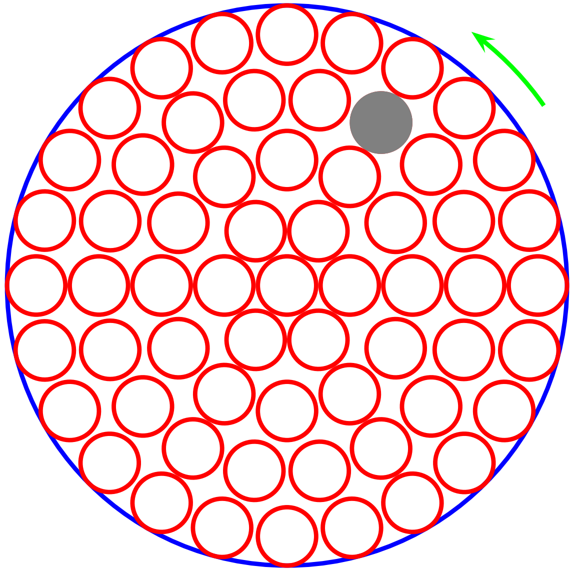

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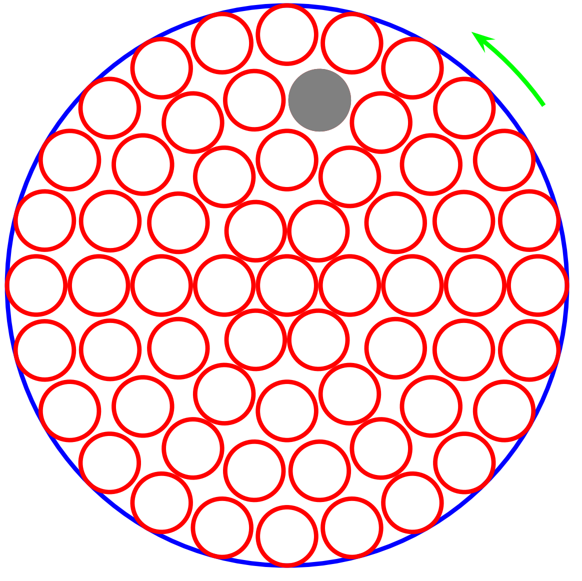

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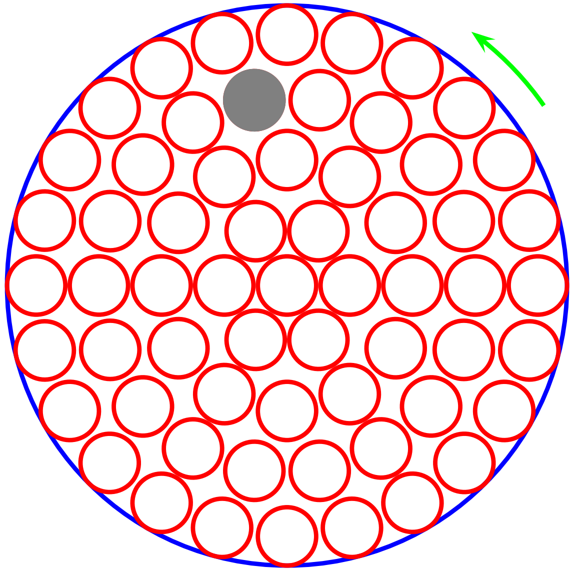

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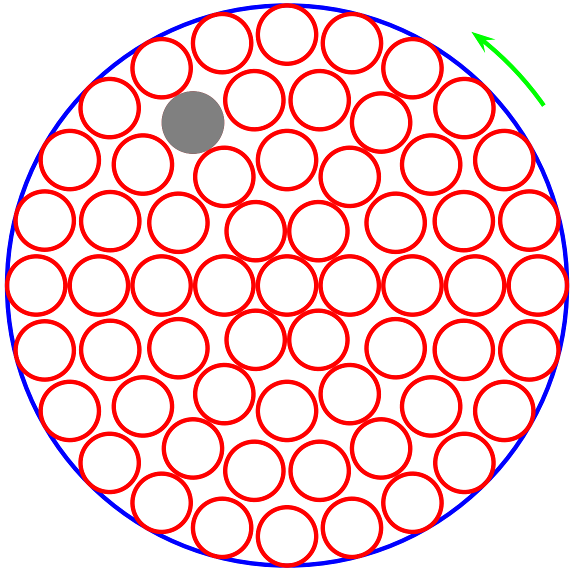

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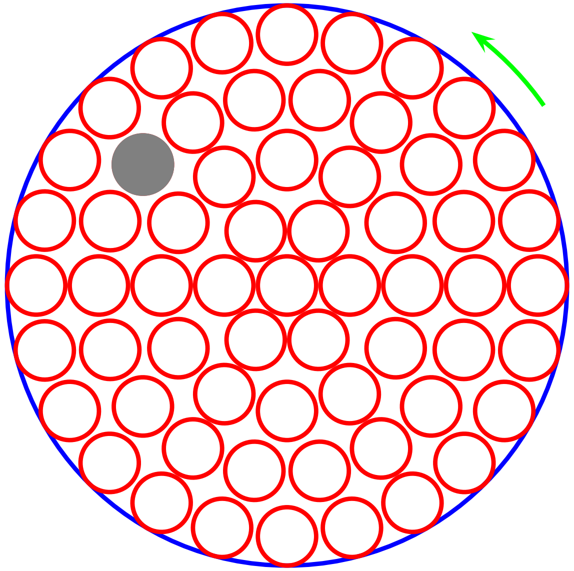

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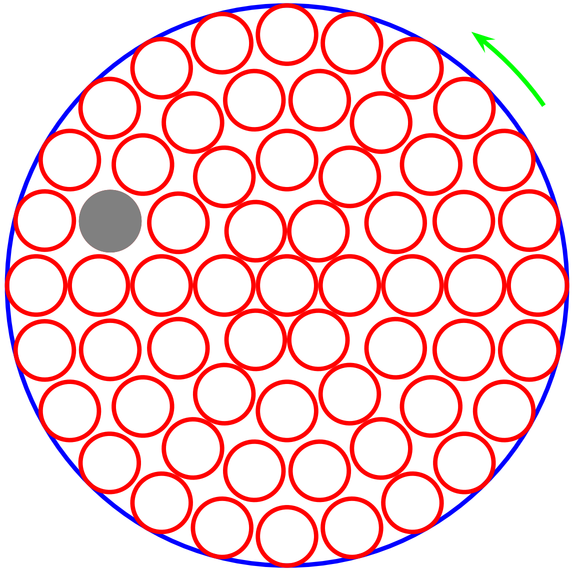

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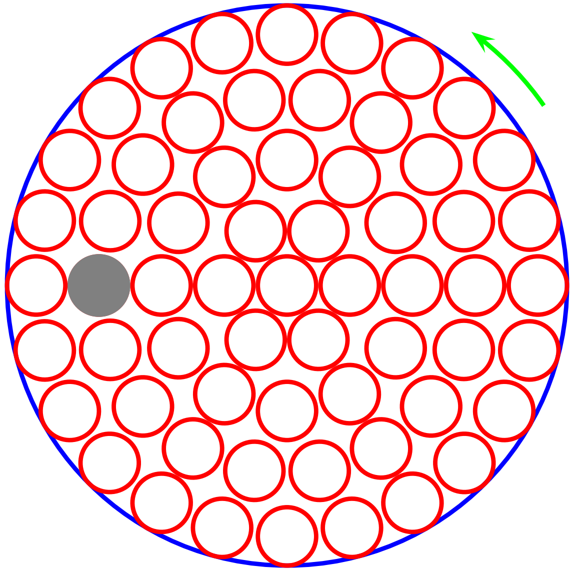

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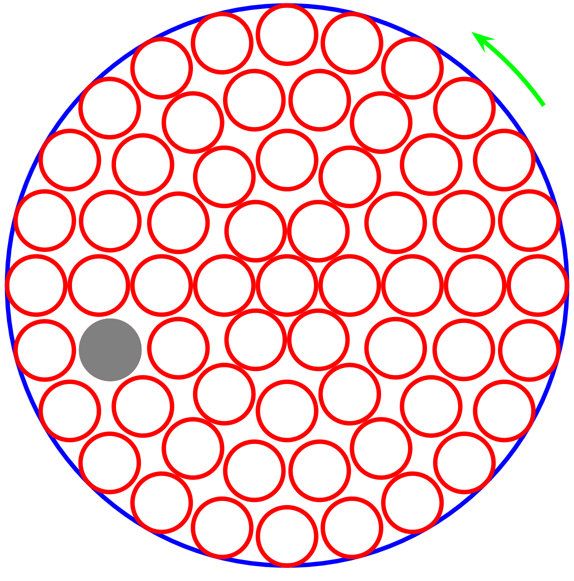

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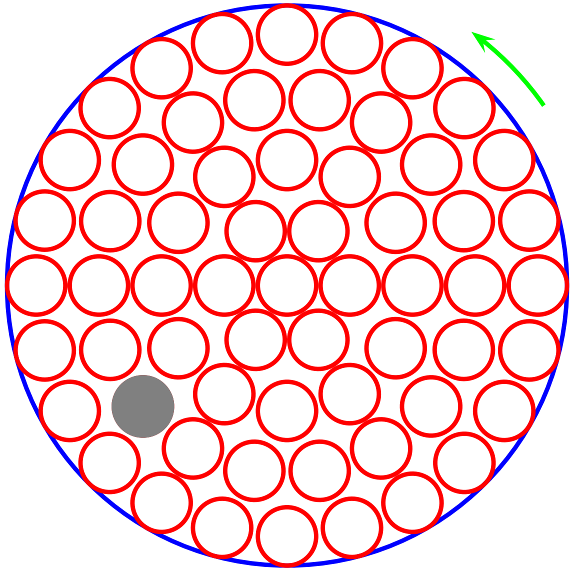

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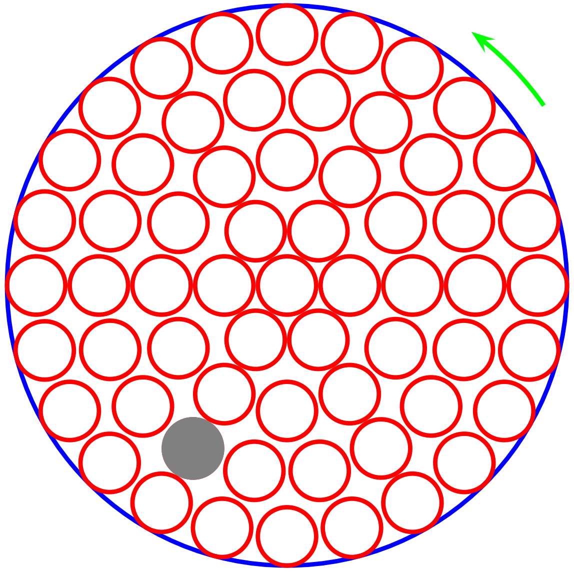

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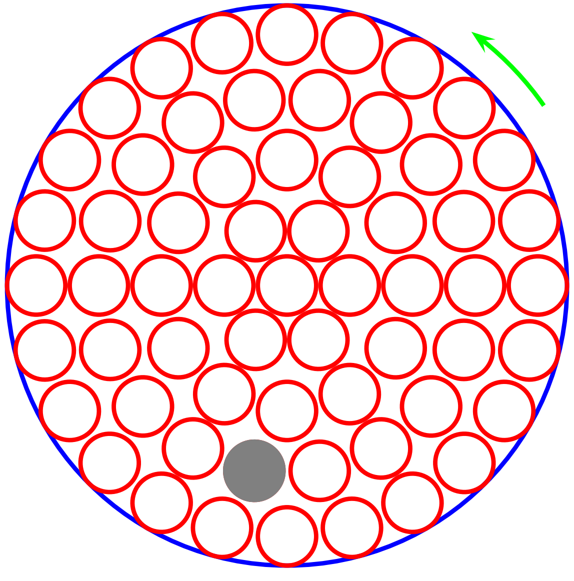

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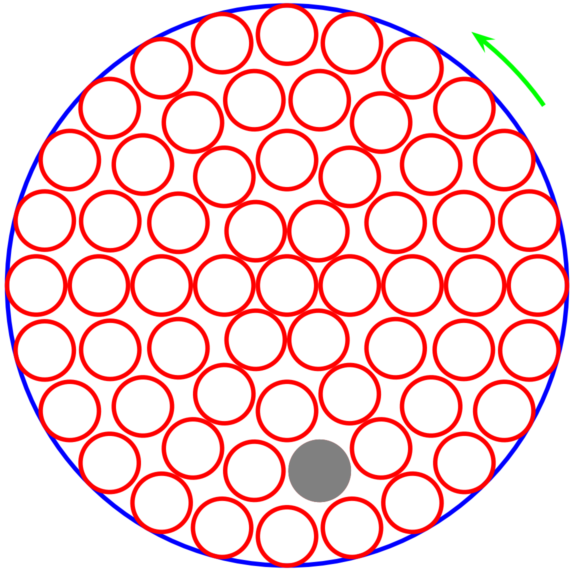

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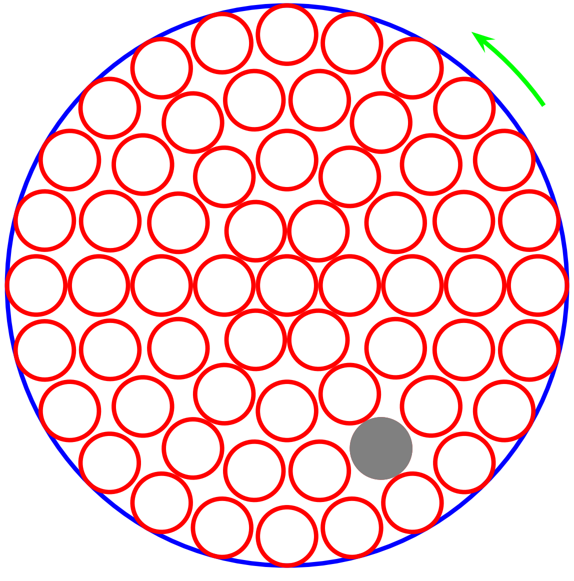

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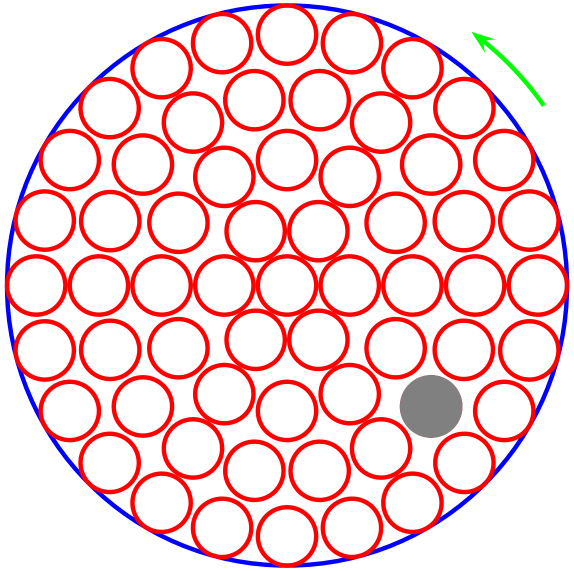

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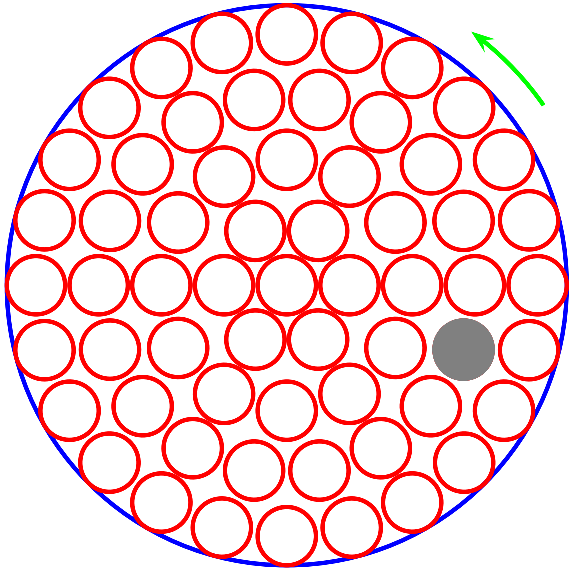

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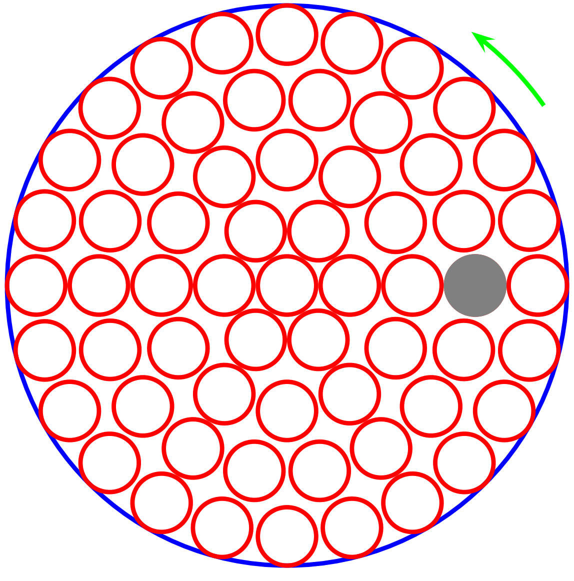

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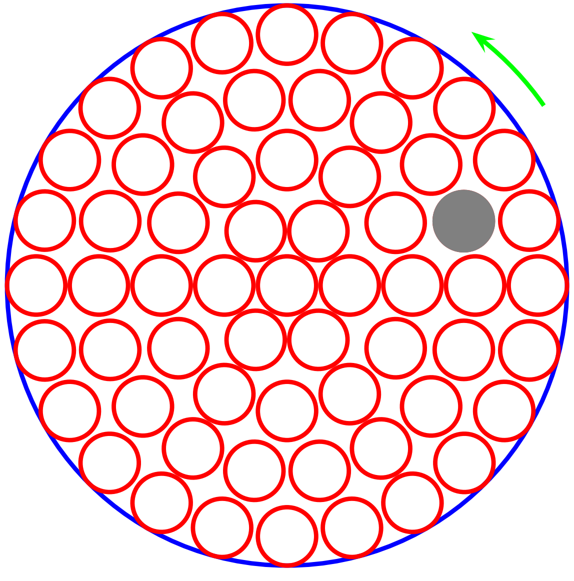

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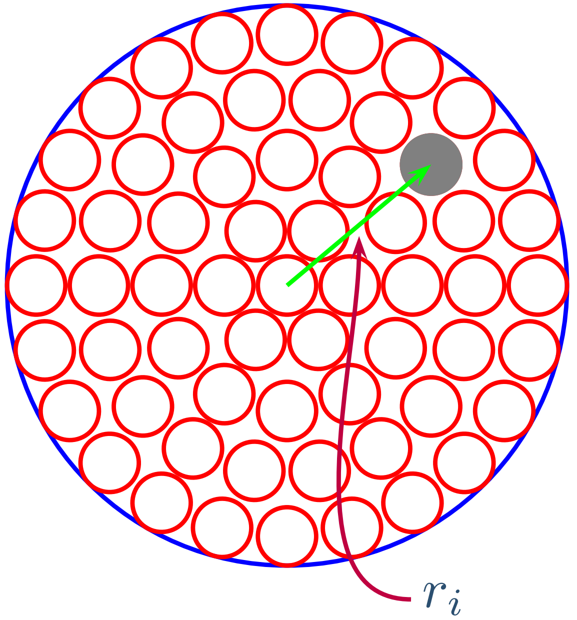

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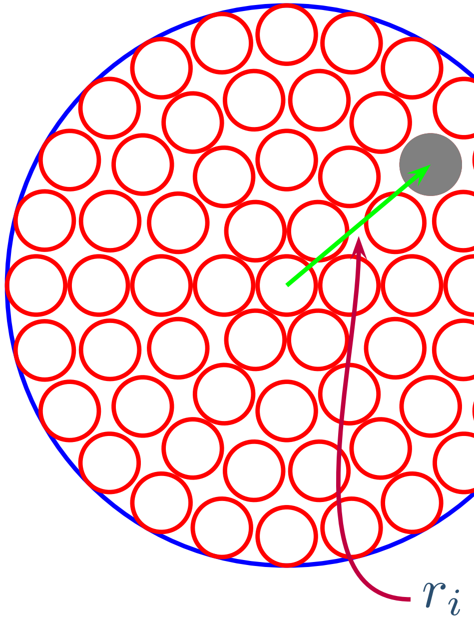

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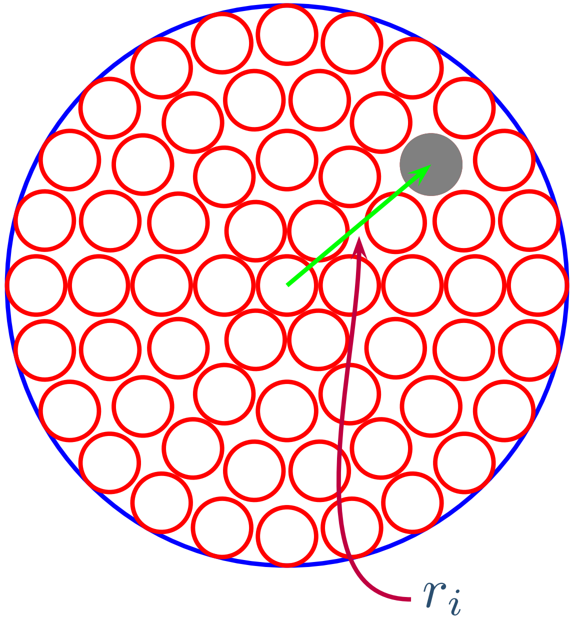

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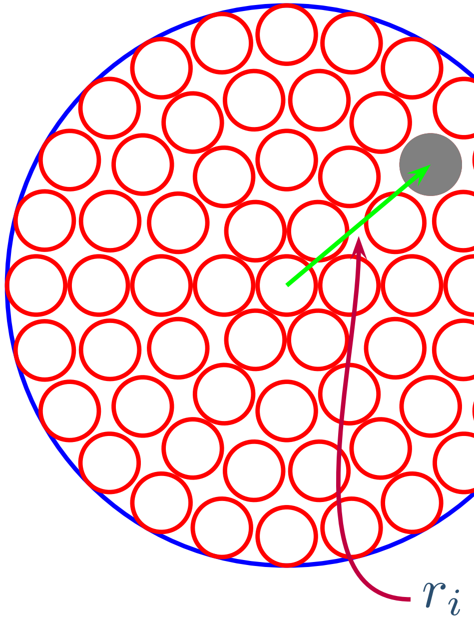

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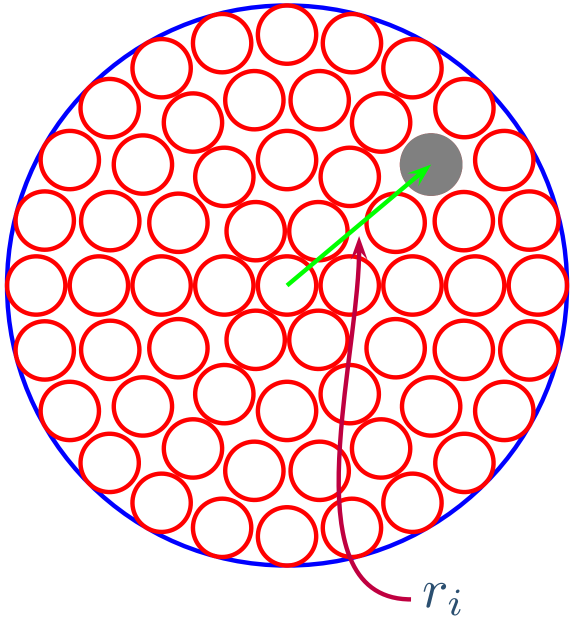

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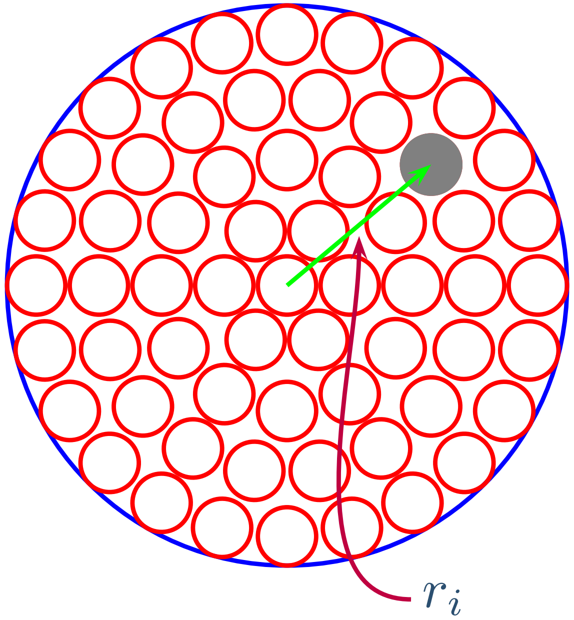

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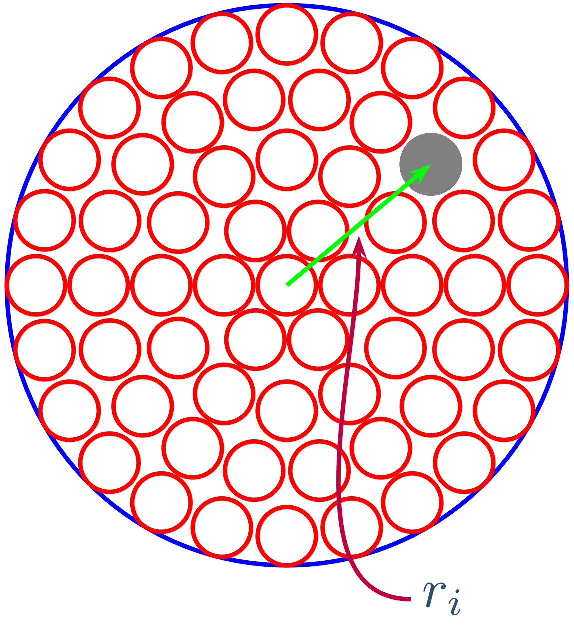

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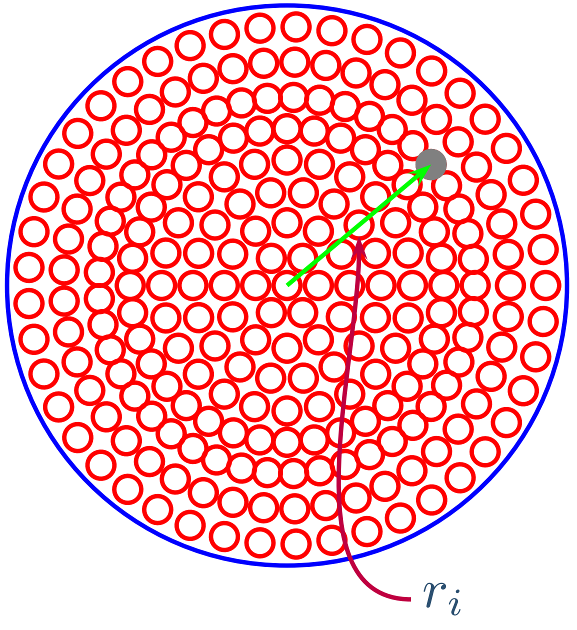

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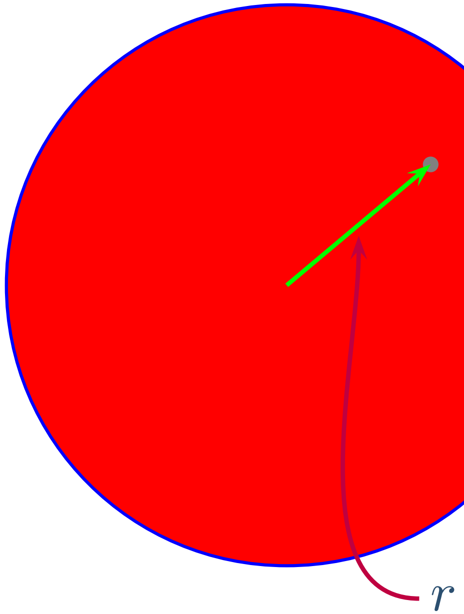

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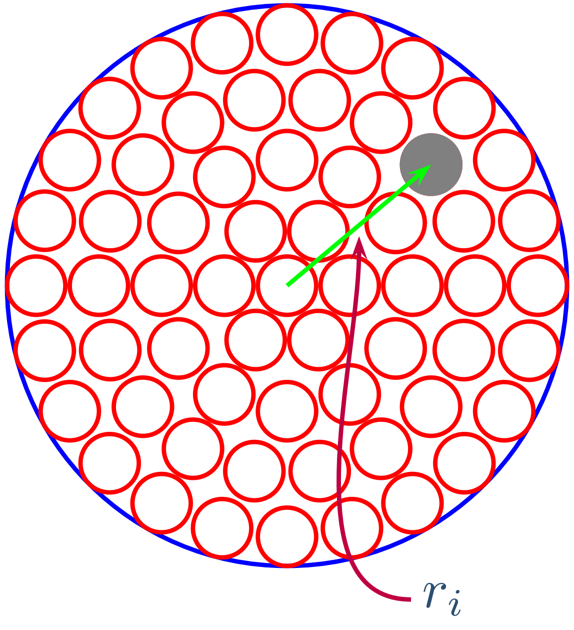

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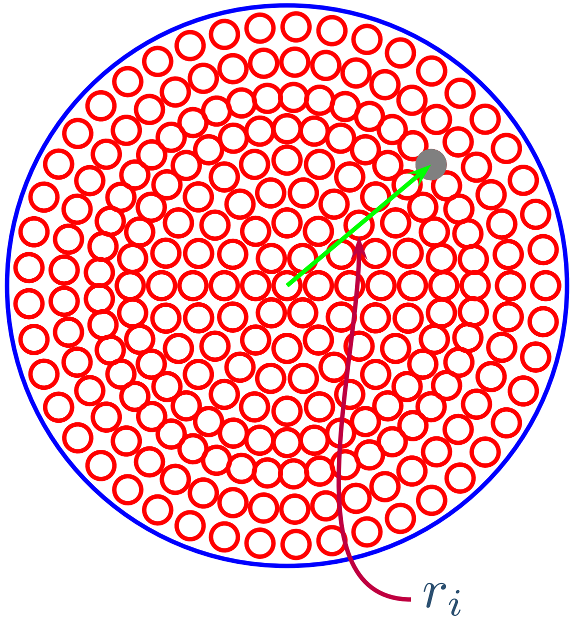

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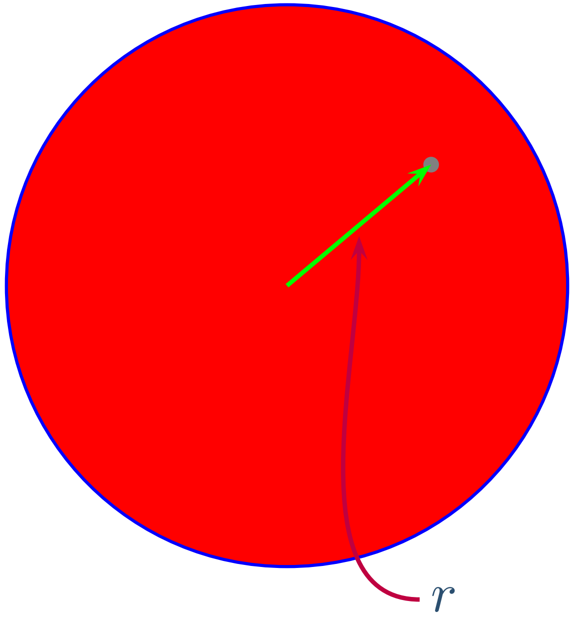

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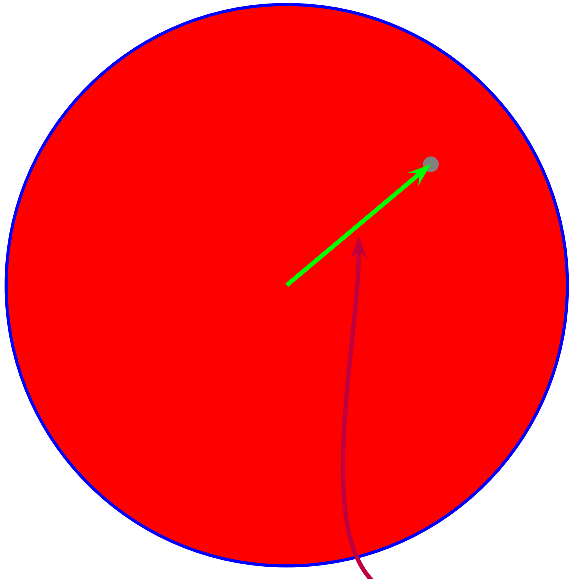

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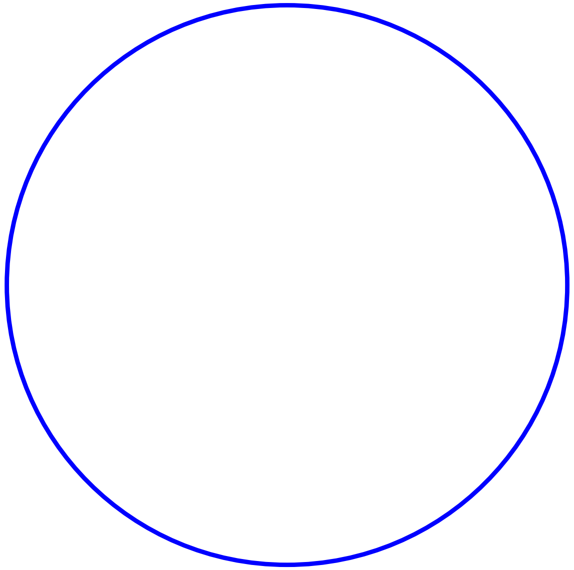

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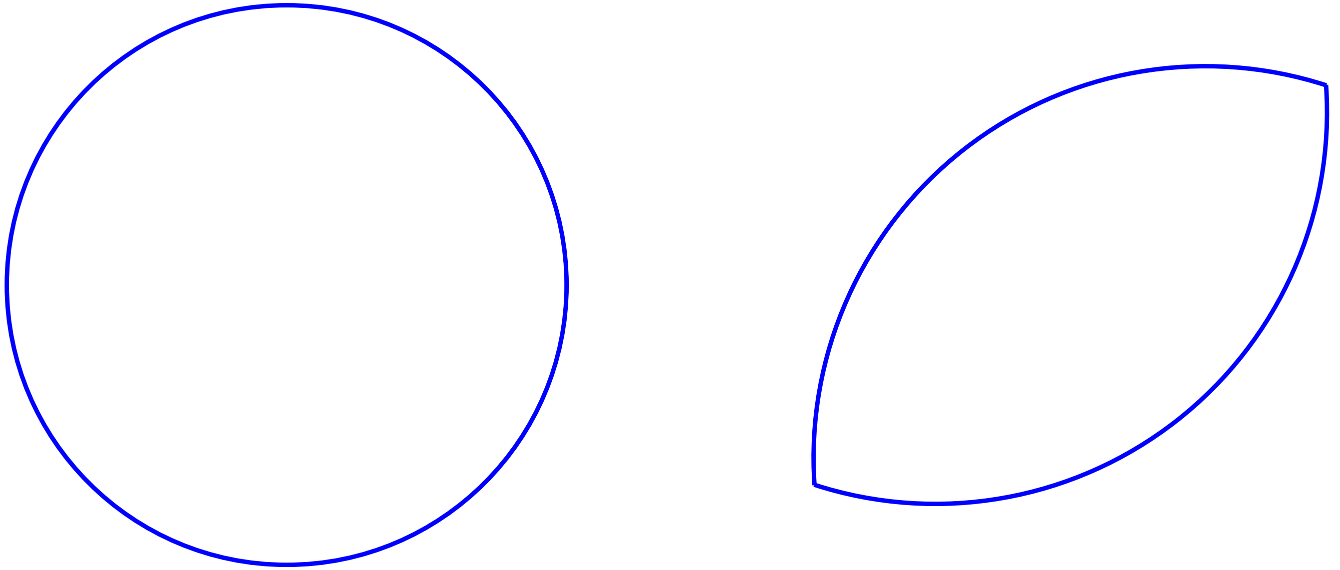

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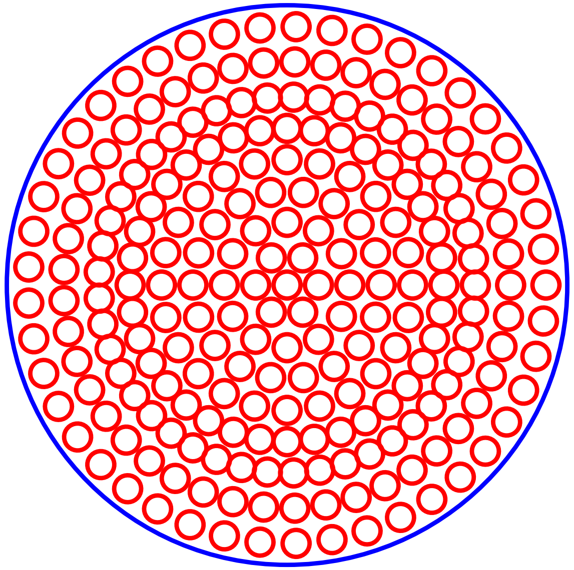

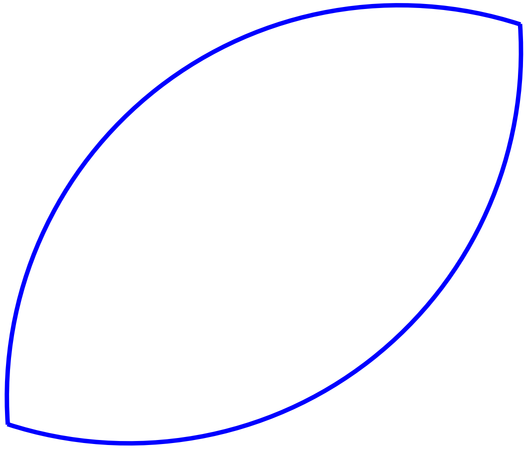

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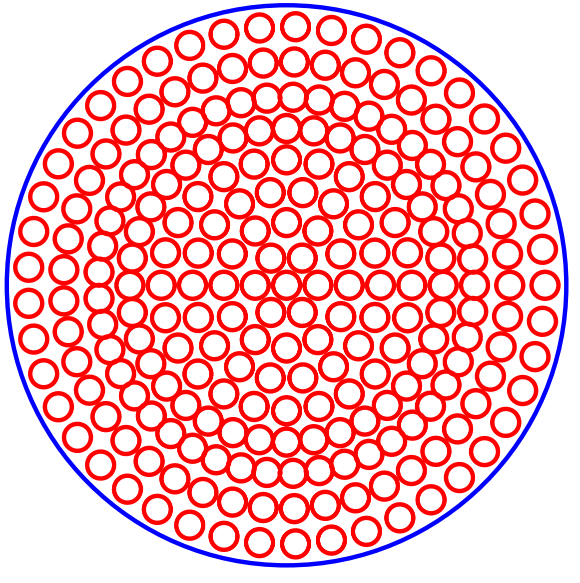

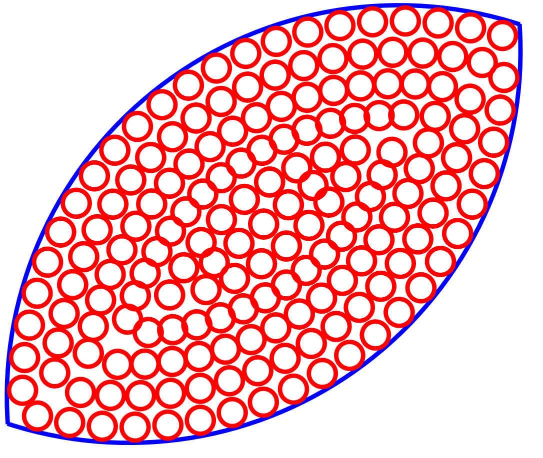

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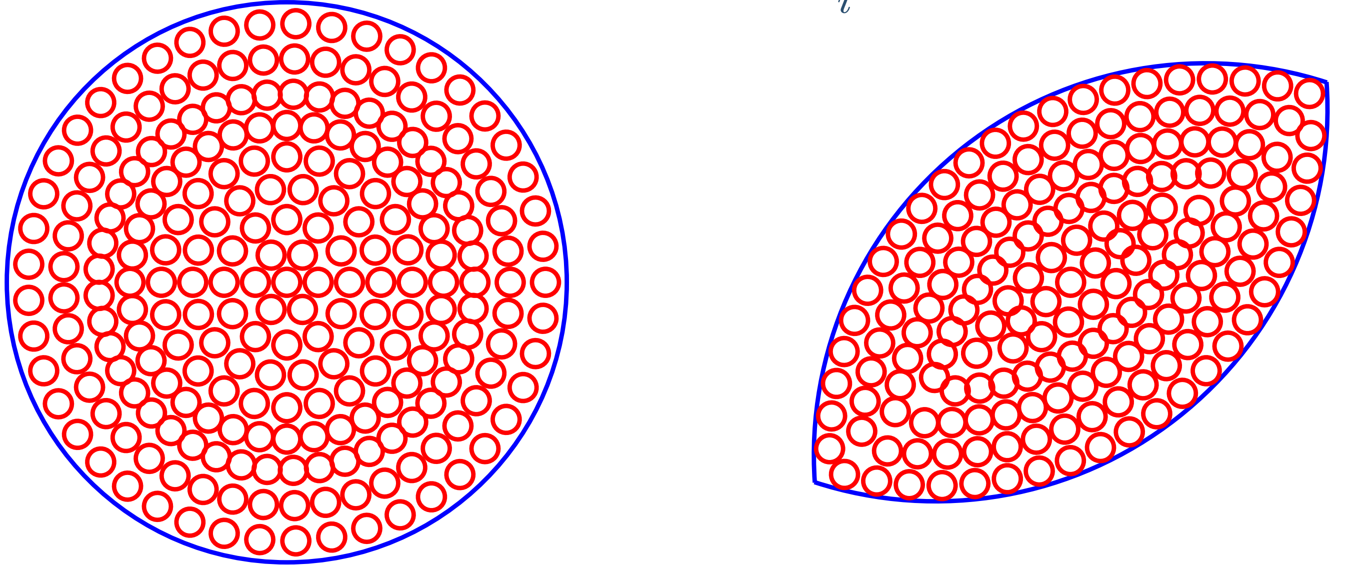

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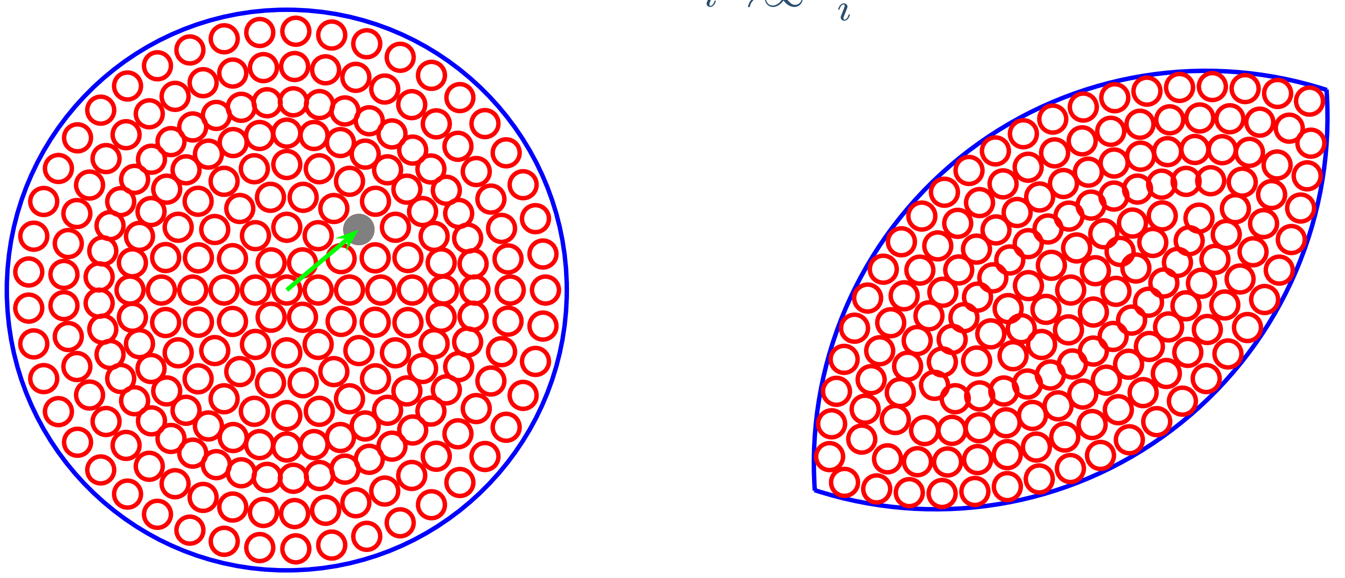

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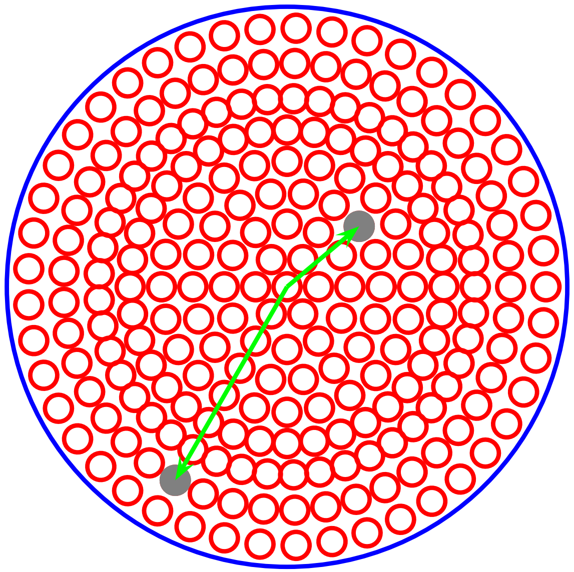

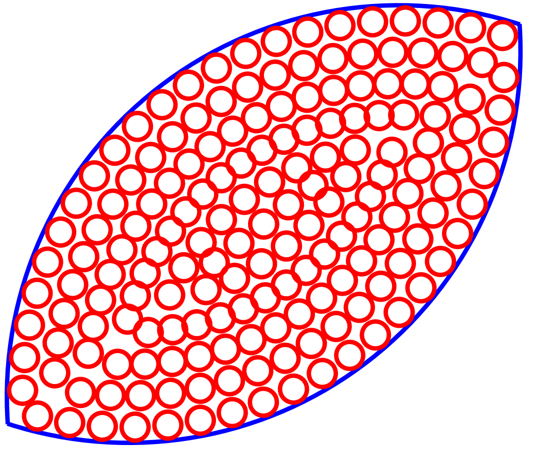

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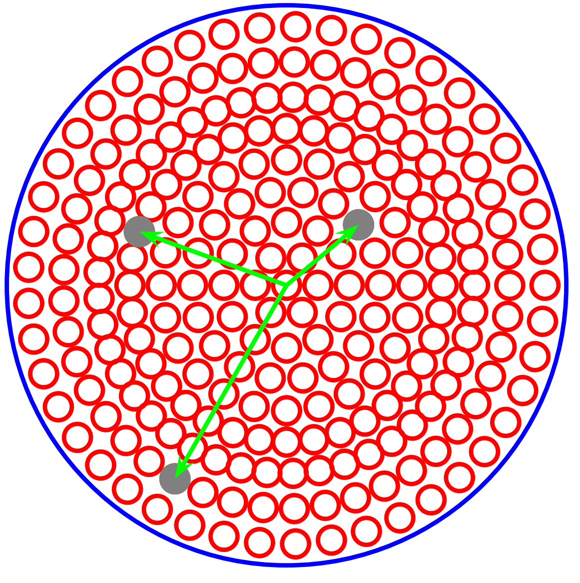

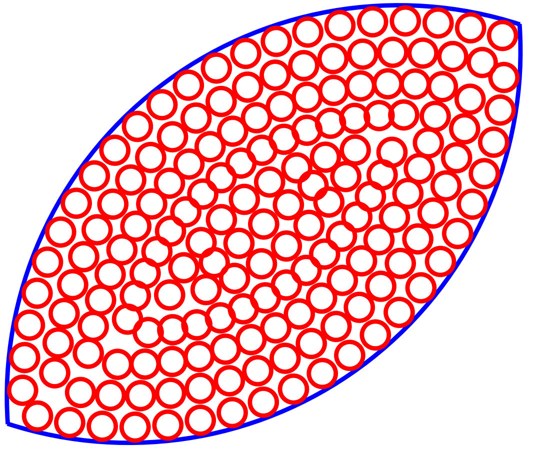

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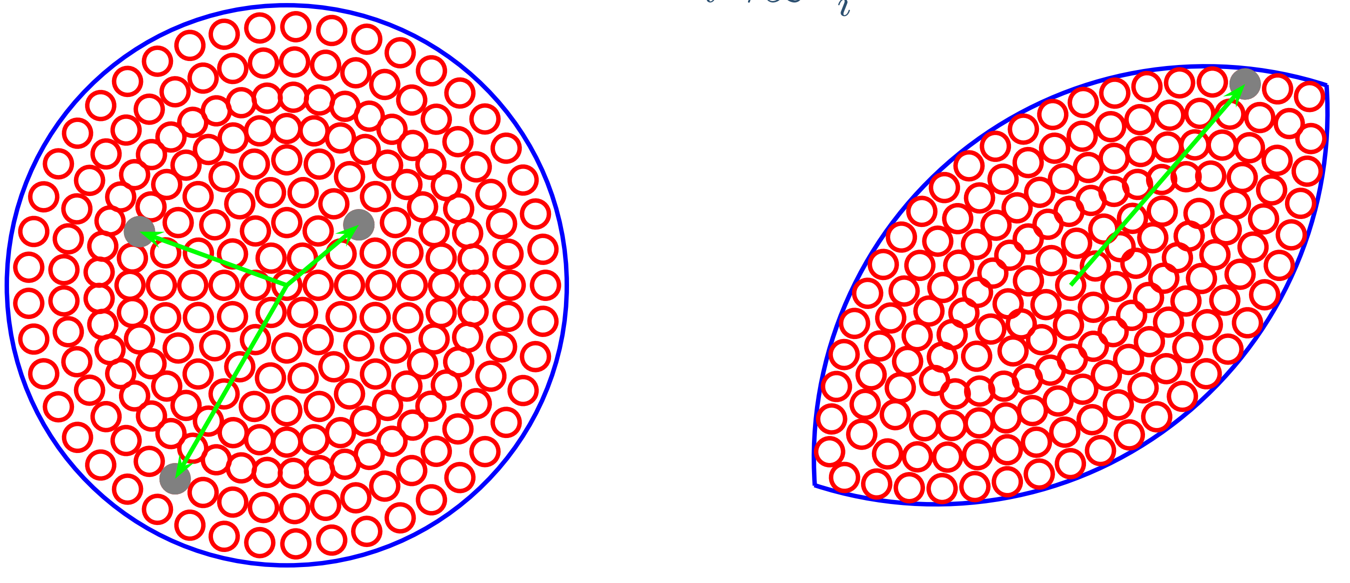

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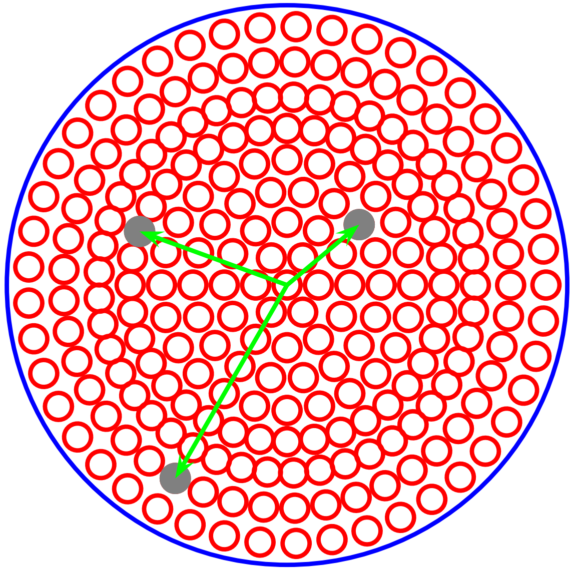

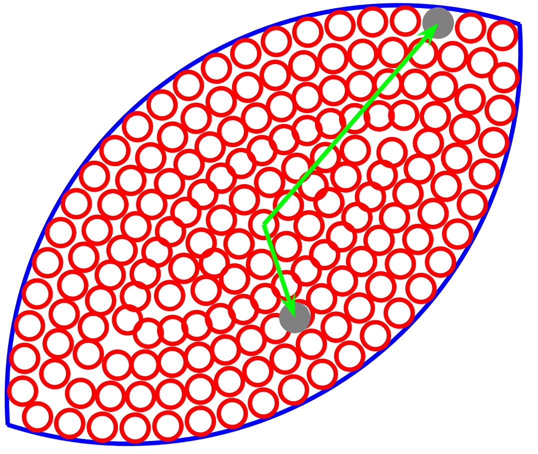

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