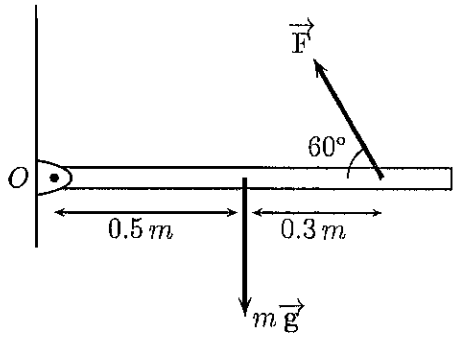
1. A uniform 6.3 kg, 1 m long slender rod is attached to a vertical wall and can pivot about the point O shown below. The rod is being held horizontal by an applied force $\vec{F}$. What is the magnitude of the force $\vec{F}$ shown above if the rod does not rotate? What are the directions of the torque exerted by $\vec{F}$ and the torque exerted by gravity?



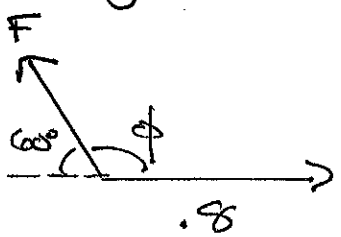
No Rotation $\Rightarrow \sum \vec{\tau} = 0 \Rightarrow \vec{\tau}_g + \vec{\tau}_F = 0$

($\vec{\tau}_g$: gravity torque; $\vec{\tau}_F$: torque of $\vec{F}$)

From RHR, $\vec{\tau}_g = \otimes$, $\vec{\tau}_F = \odot$ $\Rightarrow \vec{\tau}_g + \vec{\tau}_F = 0$

i.e. $\tau_g - \tau_F = 0 \Rightarrow \tau_g = \tau_F$.

$\tau_g = XMg = (.5m)(6.3kg)(9.8 m/s^2) = 30.87 N\cdot m$

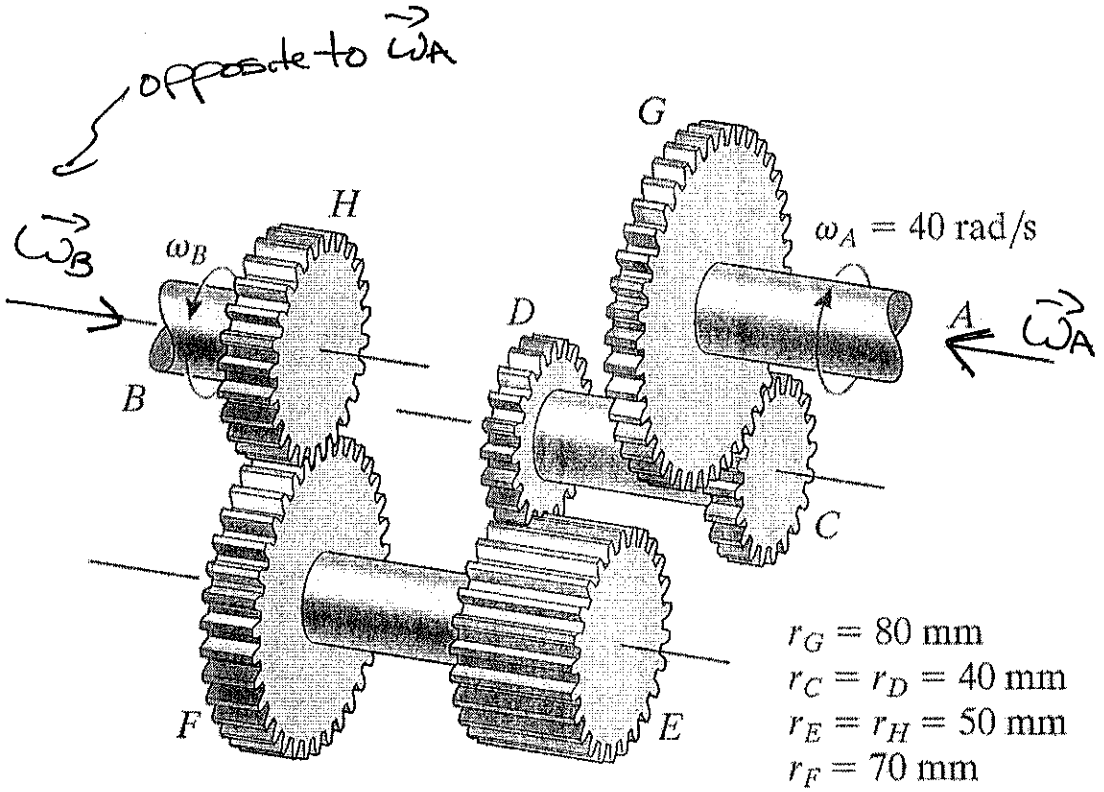


$\tau_F = (.8m)F \sin 120° = F(.69282m)$

$\therefore 30.87 N\cdot m = F(.69282m)$

$\Rightarrow \boxed{F = 44.557N = 44.6N}$

2. The reverse gear of a car is made possible through a clever arrangement of gears and axles as schematically shown below. If the radius of each gear is as given in the figure and the car's engine rotates gear A with $\omega_A = 40\,rad/s$, what is the angular *velocity* of the the "drive shaft", B?



A & G, C & D, E & F, H & B EACH ON A SAME AXLE ⇒ SAME Angular Velocity ⇒ $\omega_A = \omega_G$, $\omega_C = \omega_D$, $\omega_E = \omega_F$, $\omega_B = \omega_H$

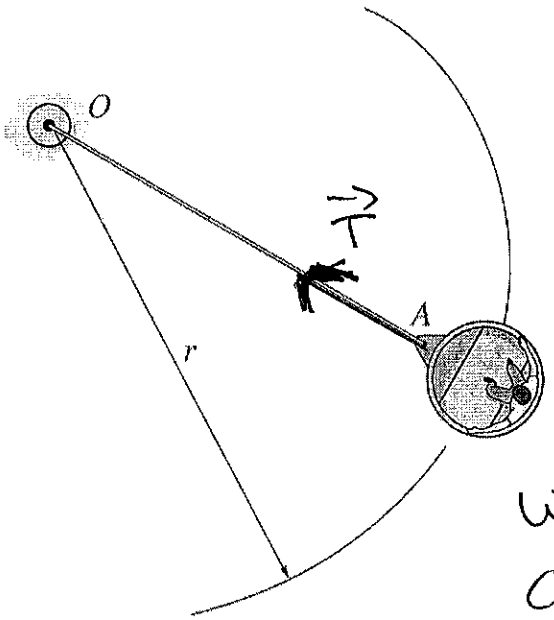
G & C, D & E, F & H IN Contact ⇒ SAME LINEAR VELOCITIES at EDGE, ⇒ $V = \omega r$ ⇒ $\omega_G r_G = \omega_C r_C$, $\omega_D r_D = \omega_E r_E$, $\omega_F r_F = \omega_H r_H$

$\omega_G = \omega_A = 40\,rad/s$ ⇒ $\omega_C = 40\,rad/s\left(\frac{80}{40}\right) = 80\,rad/s$ ⇒ $\omega_D = 80\,rad/s$

$\omega_E = 80\,rad/s\left(\frac{40}{50}\right) = 64\,rad/s$, ⇒ $\omega_F = 64\,rad/s$,

$\omega_H = 64\,rad/s\left(\frac{70}{50}\right) = 89.6\,rad/s$ ∴ $\boxed{\omega_B = 89.6\,rad/s}$

3. An amusement park ride has a car, initial speed $6\,m/s$ and distance $r = 4\,m$, traveling in a horizontal circular path. If the distance r is slowly decreased (so the car always goes around in a circle) to $1.5\,m$, how fast will the car be going? Ignore friction.



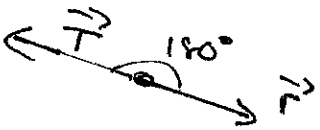
NO FRICTION SO TENSION, WEIGHT, NORMAL.

$\text{Weight}_\oplus = \otimes$

NORMAL $= \odot$

WEIGHT, NORMAL CANCEL

$\vec{T}$ EXERTS NO TORQUE

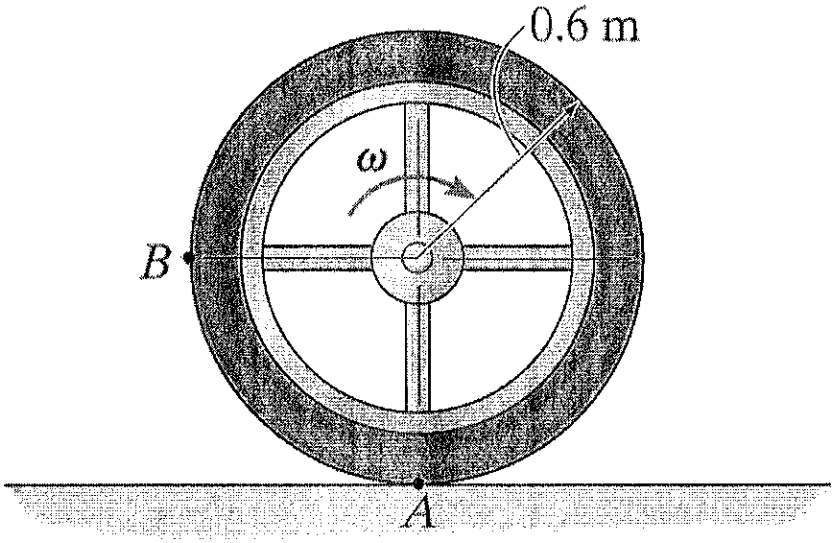
SINCE 180° BETWEEN $\vec{r}$ AND $\vec{T}$ 

THEREFORE NO NET TORQUE $\Rightarrow$ CONSERVATION OF ANGULAR MOMENTUM. ALWAYS IN CIRCLE $\Rightarrow MV_1r_1 = MV_2r_2$

$\therefore V_1r_1 = V_2r_2 \Rightarrow (6m/s)(4m) = V_2(1.5m)$

$\Rightarrow V_2 = 16m/s$

4. The 9-kg wheel shown below, when its center is given an initial speed of 4.3 m/s, rolls without slipping up a hill to a height of 6 m. What is the wheel's moment of inertia?



Rolls without slipping $\Rightarrow \frac{1}{2}MV_1^2\left(1+\frac{I}{MR^2}\right)+Mgy_1 = \frac{1}{2}MV_2^2\left(1+\frac{I}{MR^2}\right)$

$+Mgy_2$

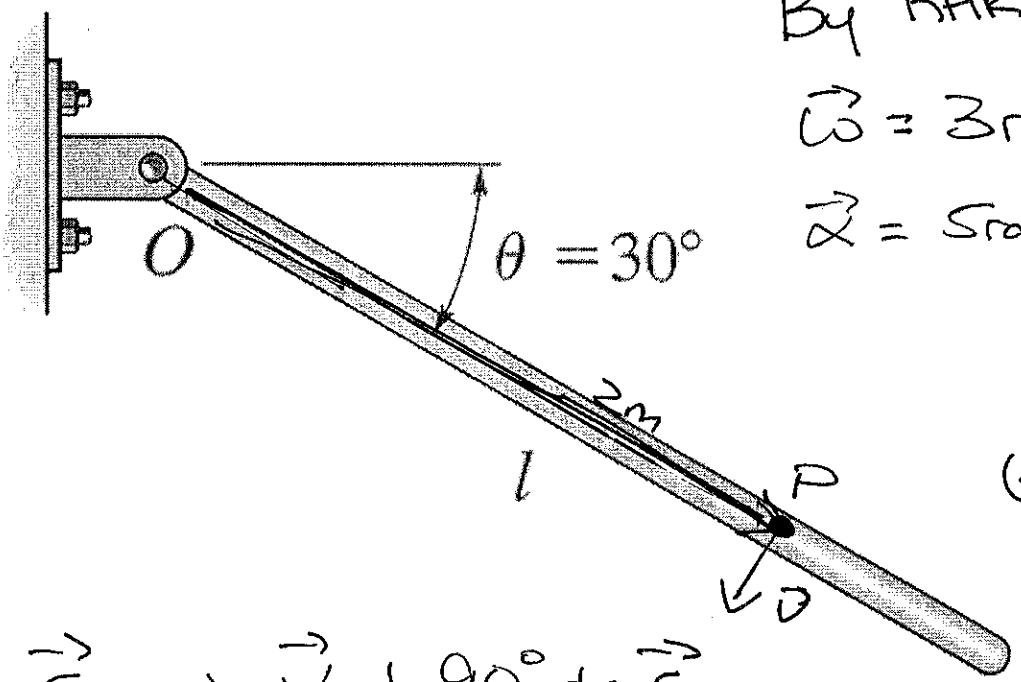
$V_1 = 4.3 m/s,\ y_1 = 0,\ V_2 = 0,\ y_2 = 6m$

$\Rightarrow \frac{1}{2}MV_1^2\left(1+\frac{I}{MR^2}\right) = Mgy_2 \Rightarrow 1+\frac{I}{MR^2} = \frac{2gy_2}{V_1^2} = \frac{2(9.8 m/s^2)(6m)}{(4.3 m/s)^2}$

$\Rightarrow 1+\frac{I}{MR^2} = 6.36 \quad \therefore \quad I = MR^2(5.36) = (9kg)(.6m)^2(5.36)$

$\Rightarrow I = 17.367\ kg\cdot m^2$

5. The $l = 2.5\text{-}m$ long stick has, at the instant shown, angular speed $\omega = 3\,rad/s$ and angular velocity $\alpha = 5\,rad/s^2$ both in the clockwise sense. What is the magnitude and direction of the angular velocity, the angular acceleration, the linear velocity, and the linear acceleration of the point at a distance $2\,m$ from O?

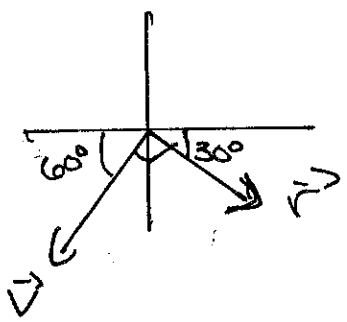


By RHR

$\vec{\omega} = 3\,rad/s$, ⊗

$\vec{\alpha} = 5\,rad/s^2$ ⊗

$\vec{V} = \vec{\omega} \times \vec{r} \Rightarrow \vec{V}$ at 90° to $\vec{r}$



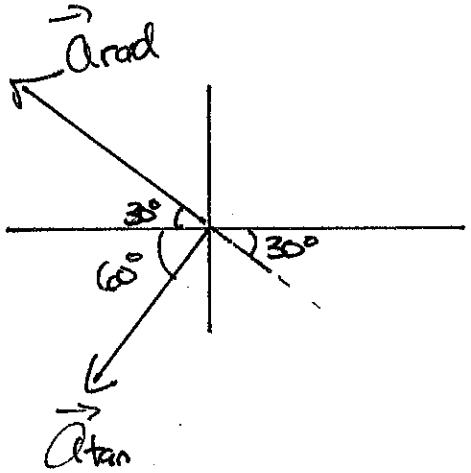
$V = \omega r = (3\,rad/s)(2m) = 6\,m/s$

$\vec{a}_{tan} = \vec{\alpha} \times \vec{r} \Rightarrow \vec{a}_{tan}$ IN SAME DIRECTION AS $\vec{V}$

$a_{tan} = \alpha r = (5\,rad/s^2)(2m) = 10\,m/s^2$

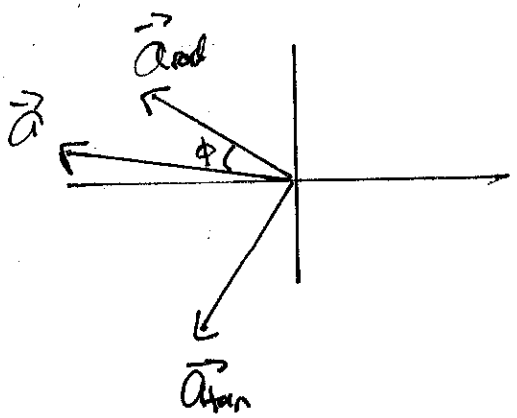
$\vec{a}_{rad} = -\omega^2 \vec{r} \Rightarrow$ opposite to $\vec{r}$

$a_{rad} = \omega^2 r = (3 rad/s)^2 (2m) = 18 m/s^2$



$a = \sqrt{a_{rad}^2 + a_{tan}^2} = \sqrt{(18 m/s^2)^2 + (10 m/s^2)^2}$

$\Rightarrow \boxed{a = 20.59 m/s^2}$



$\tan\phi = \frac{a_{tan}}{a_{rad}} \Rightarrow \phi = \tan^{-1}\left(\frac{10}{18}\right)$

$\Rightarrow \phi = 29°$

$\vec{a}_{rad}$ at 30° above −X-AXIS

$\Rightarrow \vec{a}$ IS 1° ABOVE NEGATIVE X-AXIS

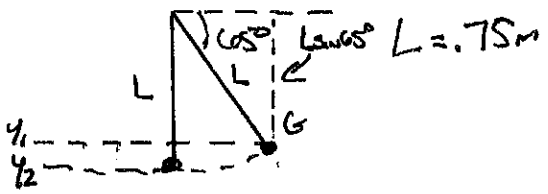
6. The 12-kg "physical-pendulum" with moment of inertia 5.5 $kg \cdot m^2$ and center of mass at G is released from rest at $\theta = 65°$. How fast will it be going when it swings through the vertical ($\theta = 90°$)?

CONSERVATION OF ENERGY:

$\frac{1}{2}I\omega_1^2 + Mgy_1 = \frac{1}{2}I\omega_2^2 + Mgy_2$

$\omega_1 = 0$, $\omega_2 = ?$

y_1, y_2 = HEIGHT OF G

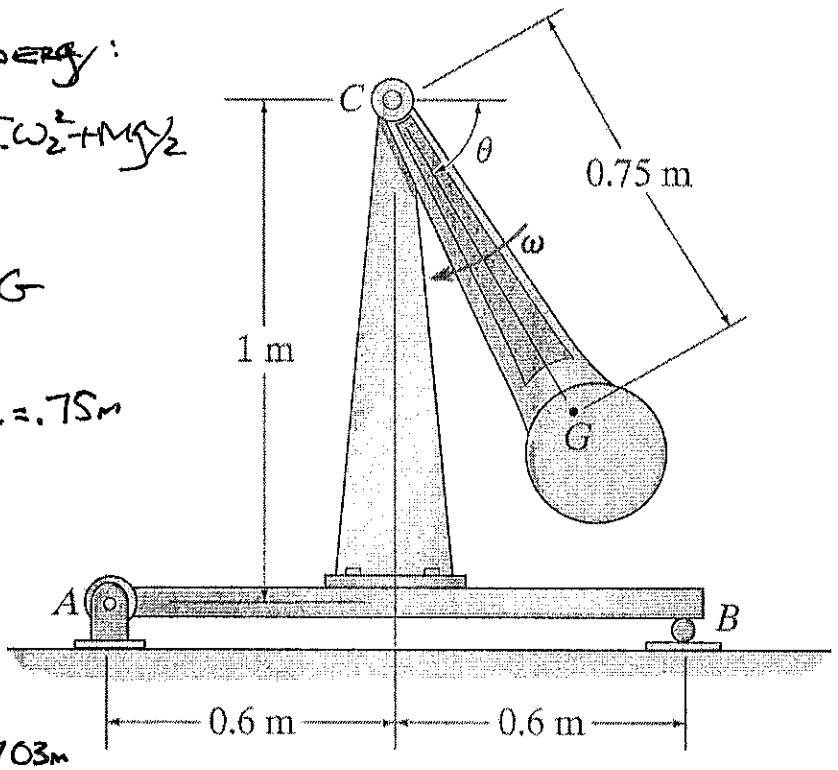


$\cos 65° = \frac{L - L\sin 65°}{L}$ $L = .75m$

$y_2 = 0$, $y_1 = L - L\sin 65°$

$y_1 = L(1 - \sin 65°) =$

$.75m\,(1 - \sin 65°) = .0703m$



7. ~~What is the angular acceleration of the physical pendulum the instant it is released from rest at $\theta = 65°$?~~ NEXT PAGE

$$Mgy_1 = \frac{1}{2}I\omega_2^2$$

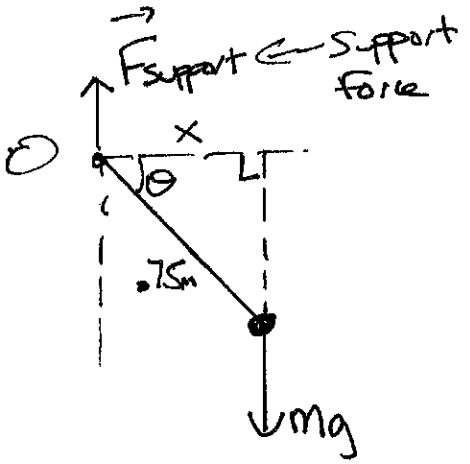
$$\Rightarrow 12kg(9.8m/s^2)(.0703m) = \frac{1}{2}(5.5kg\cdot m^2)\omega_2^2$$

$$\Rightarrow 8.264J = \frac{1}{2}(5.5kg\cdot m^2)\omega_2^2$$

$$\Rightarrow \omega_2 = \sqrt{\frac{2(8.264J)}{5.5kg\cdot m^2}} = 1.73 rad/s^2$$

7. What is ANGULAR ACCELERATION, Instant Released?

$$\sum \vec{\tau} = I\vec{\alpha}$$



Locate Weight at G

Support force at r=0

⇒ NO TORQUE

$\therefore \sum \tau = \tau_g$ ← gravity

FOR GRAVITY: $\tau_g = xMg$

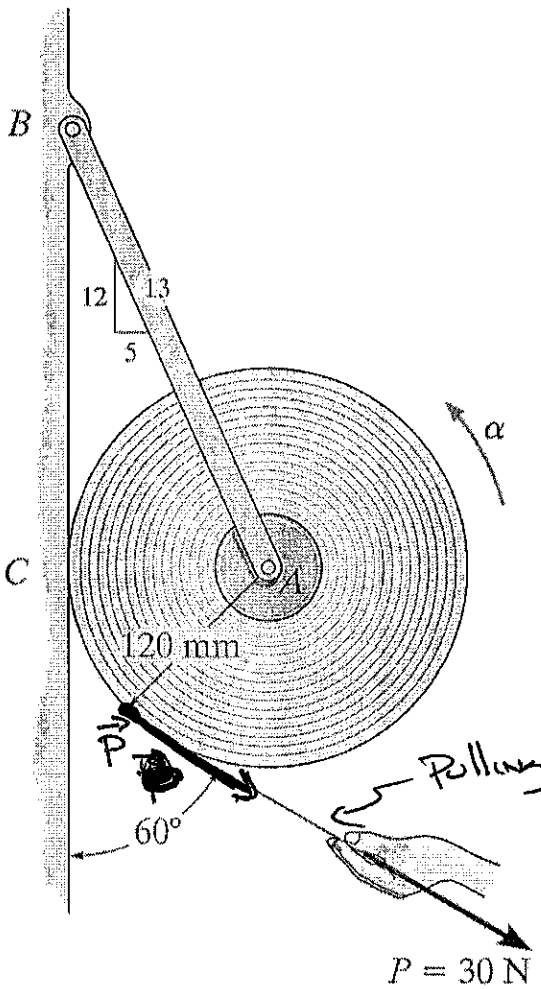
$$x = .75m \cos\theta = .75m \cos 65^\circ = .317m$$

$$\tau_g = I\alpha \Rightarrow \alpha = \frac{\tau_g}{I} = \frac{xMg}{I} = \frac{(.317m)(12kg)(9.8m/s^2)}{5.5kg\cdot m^2}$$

$$\Rightarrow \alpha = 6.77726\ rad/s^2 = 6.78\ rad/s^2$$

↑ INSERT WHEN NEEDED

8. If the roll of paper towels shown below has mass $25\,kg$ and the coefficient of kinetic friction between the wall and paper towels is $\mu_k = 0.3$, what is the force exerted by the support rod and the angular acceleration of the roll. Treat the roll as a solid cylinder, and therefore has moment of inertia, $I = \frac{1}{2}MR^2$.



Pulling force makes tension here = 30N.

So also have $\vec{P}$ tangent to Circle

Work on next page

$$f_K = \mu_K n = .3\left[\frac{5}{13}F_R - 25.98N\right]$$

$$= \frac{1.5}{13}F_R - 7.794N$$

$$\therefore \frac{12}{13}F_R + \frac{1.5}{13}F_R - 7.794N - (25kg)(9.8m/s^2) - 30N\cos 60^\circ = 0$$

$$\Rightarrow \frac{13.5}{13}F_R - 267.794N = 0$$

$$\Rightarrow F_R = \frac{(267.794N)(13)}{13.5} = 257.8757N$$

$$f_K = \frac{1.5}{13}(257.8757N) - 7.794N = 21.96N$$

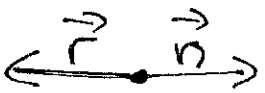
(and $n = 73.2N$)

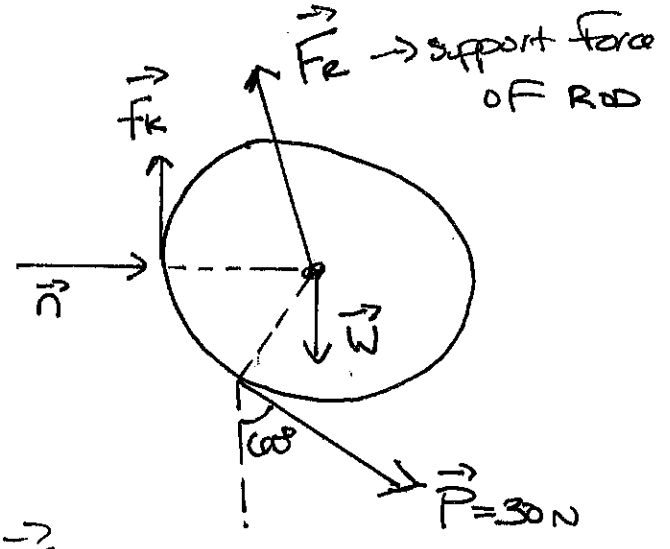
$\sum_i \vec{\tau} = I\vec{\alpha} \Rightarrow \vec{\tau}_P + \vec{\tau}_f = I\alpha$, $\vec{n}$, $\vec{\omega}$, AND $\vec{F_R}$ CAUSE NO TORQUE

($\vec{F_R}$, $\vec{\omega}$ at Center $\Rightarrow r = 0$)

$\vec{n}$ at 180° to $\vec{r}$

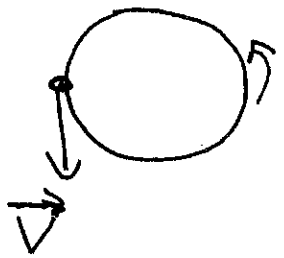
$\sin 180^\circ = 0$



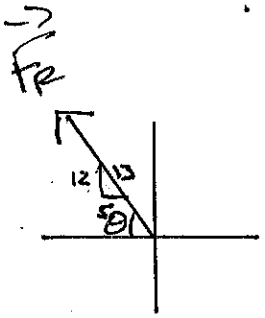


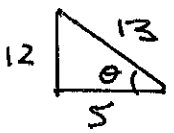
$\vec{P}$ tries to cause Counter-clockwise Rotation

⇒



So friction is up





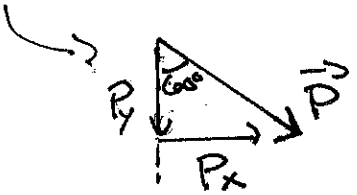
$\Rightarrow \theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62°$

last better to use $\sin\theta = \frac{12}{13}$, $\cos\theta = \frac{5}{13}$

$$F_{R,x} = -F_R\cos\theta = -\frac{5}{13}F_R, \quad F_{R,y} = +F_R\sin\theta = \frac{12}{13}F_R$$

Center at Rest $\Rightarrow \sum F_x = 0, \sum F_y = 0$

$$\sum F_x = 0 \Rightarrow -\frac{5}{13}F_R + n + P\sin 60° = 0$$

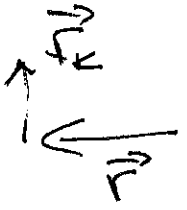


$$\Rightarrow \frac{5}{13}F_R = n + P\sin 60°$$

$$\frac{5}{13}F_R = n + 30\,N\sin 60° = n + 25.98\,N$$

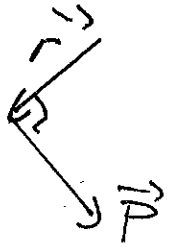
OR better yet $n = \frac{5}{13}F_R - 25.98\,N$

$$\sum F_y = 0 \Rightarrow \frac{12}{13}F_R + f_K - W - P\cos 60° = 0$$

for friction 

$\Rightarrow \vec{\tau}_f = r f_k \sin 90°, \otimes$

$\vec{\tau}_f = r f_k, \otimes$

for $\vec{P}$ 

$\vec{P}$ tAngent to circle $\Rightarrow$

$\vec{\tau}_P = r P \sin 90°, \odot = r P, \odot$

$\therefore \Sigma \tau = r P - r f_k = r(P - f_k)$

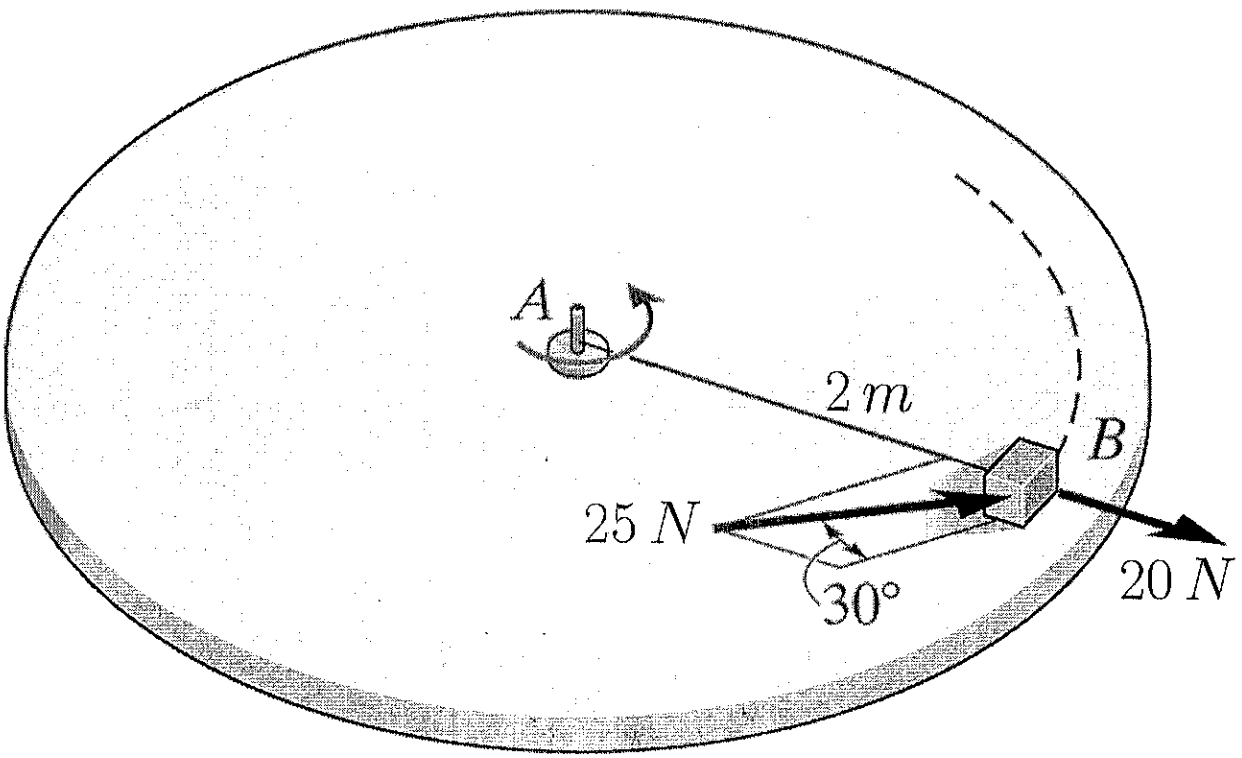
$r = 120\,mm \times \frac{1\,m}{1000\,mm} = .12m \Rightarrow \Sigma \tau = .12m(30N - 21.96N)$

$= .12m(8.04N) = .9648 N \cdot m$

$I = \frac{1}{2} M R^2 = \frac{1}{2}(25kg)(.12m)^2 = .18\,kg \cdot m^2$

$\Sigma \tau = I\alpha \Rightarrow \alpha = \frac{\Sigma \tau}{I} = \frac{.9648\,N \cdot m}{.18\,kg \cdot m^2} = 5.36\,rad/s^2$

9. Find the torque exerted by the $25\,N$ and $20\,N$ forces shown. If the torques remain constant and the 3-kg mass has an initial speed of $3.4\,m/s$, how fast will it be going after $0.25\,s$? Assume the mass continues on its 2-m circle.



For 20N force $\tau = 0$ $\vec{r}$ → 20N so $\phi = 0°$, $\sin 0° = 0$

For 25N force: tangent line

$r = 2m$, 30°, 25N, 30° $\Rightarrow$ 25N = $\vec{F}$, 30°, $\vec{r}$ $\Rightarrow$ 25N = $\vec{F}$, 30°, 60°, $\vec{r}$

$$\Rightarrow \quad \tau = rF\sin 60^\circ = (2m)(25N)\sin 60^\circ = 43.3\,N\cdot m$$

$$\Sigma\tau = 0 + 43.3\,N\cdot m = 43.3\,N\cdot m$$

Constant TORQUE $\Rightarrow$ $\Sigma\tau = \frac{\Delta L}{\Delta t}$ will work

Point Particle going on A Circle $\Rightarrow L = MVr$

$$\therefore \Sigma\tau = \frac{MV_2r_2 - MV_1r_1}{\Delta t} \qquad V_2 = ?,\ V_1 = 3.4m/s$$

$$r_1 = r_2 = 2m,\ \Delta t = .25s$$

$$\therefore \Sigma\tau = \frac{Mr(V_2 - V_1)}{\Delta t} \Rightarrow 43.3\,N\cdot m = \frac{3kg(2m)(V_2 - 3.4m/s)}{.25s}$$

$$\Rightarrow V_2 - 3.4m/s = \frac{(43.3\,N\cdot m)(.25s)}{3kg(2m)} = 1.8m/s$$

$$\Rightarrow \boxed{V_2 = 5.2m/s}$$

NOTE you Could Also USE $\Sigma\tau = I\alpha$. $I = Mr^2$ for point particle. FIND ω using $\omega = \omega_0 + \alpha t$. then Convert to LINEAR. $v = r\omega$