Five Easy Pieces

1. A 1.0 kg mass is lifted vertically two meters (on Earth) with a constant 2.3 m/s^2 acceleration. How much work is done by the lifting force, $\vec{\mathbf{F}}$?

$\sum F_y = Ma_y$

$F - 9.8N = (1kg)(2.3m/s^2) \Rightarrow F = 12.1N$

$W = \vec{F} \cdot \vec{S} = (12.1N)(2m)\cos 0^\circ = 24.2J$

(a) $W = 19.6\,J$	(b) $W = -19.6\,J$	(c) $W = 24.2\,J$	(d) $W = -24.2\,J$

2. An 80 kg man has an apparent weight of 1504 N at the bottom of a circular dip. If his speed is 6.0 m/s, what is the radius of the circle?

$\sum F_y = Ma_y$

$n - W = \frac{Mv^2}{r}$

$1504N - (80kg)(9.8m/s^2) = \frac{(80kg)(6m/s)^2}{r} \Rightarrow 720 = \frac{(80kg)(6m/s)^2}{r} \Rightarrow r = 4m$

(a) $r = 9\,m$	(b) $r = 1.26\,m$	(c) $r = 4\,m$	(d) $r = 9.8\,m$

3. A 700 kg car is traveling with a speed of 30 m/s. If 5.2 s later, its speed is 19.6 m/s, how much work, in total, was done to the car? Note the use of $kJ = kiloJoules$ to make the numbers smaller.

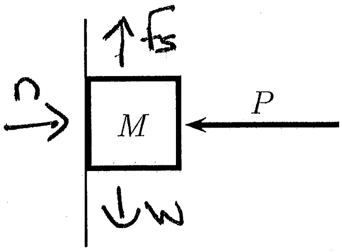
(a) $W = 1770\,kJ$	(b) $W = -35\,kJ$	(c) $W = 9.8\,kJ$	(d) $W = -180\,kJ$

$W_{TOTAL} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$= \frac{1}{2}(700kg)(19.6m/s)^2 - \frac{1}{2}(700kg)(30m/s)^2$

$= -180544J = -180.544kJ \approx -180kJ$

4. A 5.0 kg mass is being held against a vertical wall as shown. If the coefficient of static friction between the mass and the wall is .64, what is the minimum horizontal force P needed to hold mass in place?



$\sum F_x = Ma_x \Rightarrow n - P = 0 \Rightarrow n = P$

$\sum F_y = Ma_y \Rightarrow f_s - W = 0 \Rightarrow f_s = W$

Min $P \Rightarrow f_s = f_{s,MAX} = \mu_s n \Rightarrow \mu_s n = W$

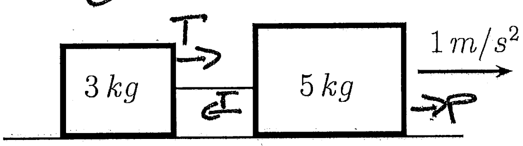
$\Rightarrow \mu_s P = Mg \Rightarrow P = \frac{Mg}{\mu_s} = 76.5625 N$

(a) $P = 49\,N$	(b) $P = 76.6\,N$	(c) $P = .64\,N$	(d) $P = 31.36\,N$

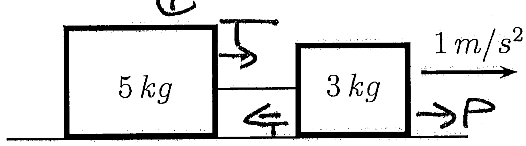
5. A 5 kg mass and a 3 kg mass are connected by a massless rope on a horizontal, frictionless surface. The masses are made to accelerate at $1\,m/s^2$ to the right as shown. In which case will the rope's tension have a larger magnitude?

$\sum F_x = Ma_x \Rightarrow T = (3kg)(1 m/s^2) = 3N$

$T = (5kg)(1 m/s^2) = 5N$



Case #1



Case #2

(a) Case # 1	(b) Case # 2
(c) The tension is the same in both cases.	(d) It is impossible to determine.

Work, Work, Work

Questions 6 through 10 refer to the following setup.

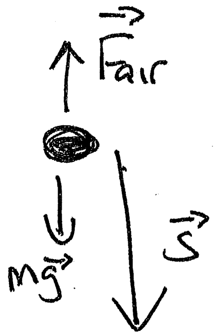
Émilie du Châtelet drops a .25 kg ball from rest, 1.6 m above a sand pit.

6. If the ball hits the sand going 3.2 m/s, how much work was done by air resistance during the ball's fall?

(a) 1.28 J	(b) −3.92 J	(c) −2.64 J	(d) −1.28 J

7. Ignoring gravity, how much work must the sand do in order to stop the 3.2 m/s ball?

(a) 1.28 J	(b) −3.92 J	(c) −2.64 J	(d) −1.28 J



$W_{TOTAL} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

$W_{TOTAL} = W_g + W_{air} = mgs + W_{air}$

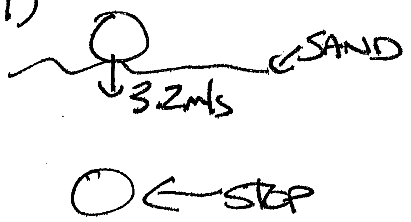
$\Rightarrow mgs + W_{air} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

$S = 1.6m,\ W_{air} = ?,\ V_2 = 3.2m/s,\ V_1 = 0$

$\Rightarrow (.25kg)(9.8m/s^2)(1.6m) + W_{air} = \frac{1}{2}(.25kg)(3.2m/s)^2$

$\Rightarrow W_{air} = 1.28J - 3.92J = -2.64J$

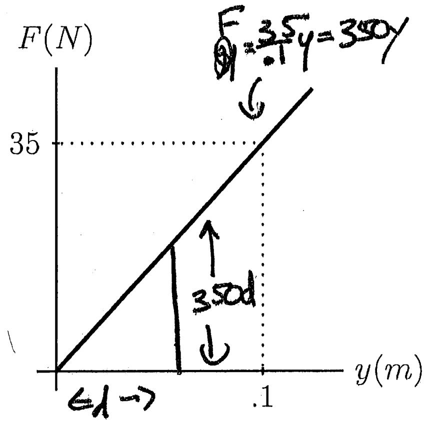
7)



$W_{TOTAL} = W_{SAND} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

$\Rightarrow W_{SAND} = 0 - 1.28J = -1.28J$

8. If the force exerted by the sand increases linearly with depth below the sand, as shown on the graph below ($y = 0$ is at the top of the sand pit and down is positive), how far will the ball sink below the surface before stopping? Again, ignore gravity in this calculation.



$\Rightarrow W_{sand} = -\text{AREA}$

$\Rightarrow -1.28J = -\frac{1}{2}(d)(350d)$

$\Rightarrow -1.28J = \frac{-350}{2}d^2$

$\Rightarrow d = \sqrt{\frac{1.28(2)}{350}} = .085524$

(a) .037 m	(b) .191 m	(c) .086 m	(d) .060 m

9. If the sand provides 24 $Watt$ of average power, estimate the time it takes for the ball to stop.

(a) .053 s	(b) .025 s	(c) 9.8 s	(d) 3 s

$P_{AV} = \frac{W}{\Delta t}$

$\Rightarrow 24\,Watt = \frac{1.28J}{\Delta t}$

$\Rightarrow \Delta t = .053s$

10. If we were to include the effect of gravity in the previous calculation, the ball would:

(a) go deeper into the sand.	(b) go the same distance into the sand.
(c) go less distance into the sand.	(d) bounce off the sand.

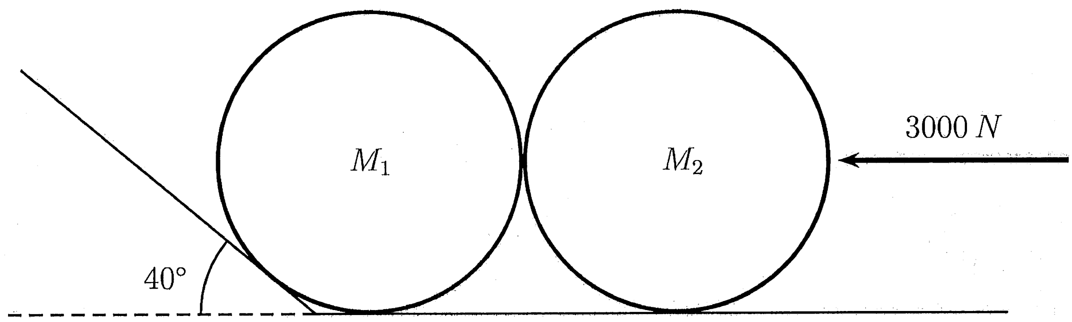
$W_g = m\vec{g} \cdot \vec{s} = mgs \cos 0° = mgs$

$\Rightarrow$ gravity doing positive work so ball would go farther

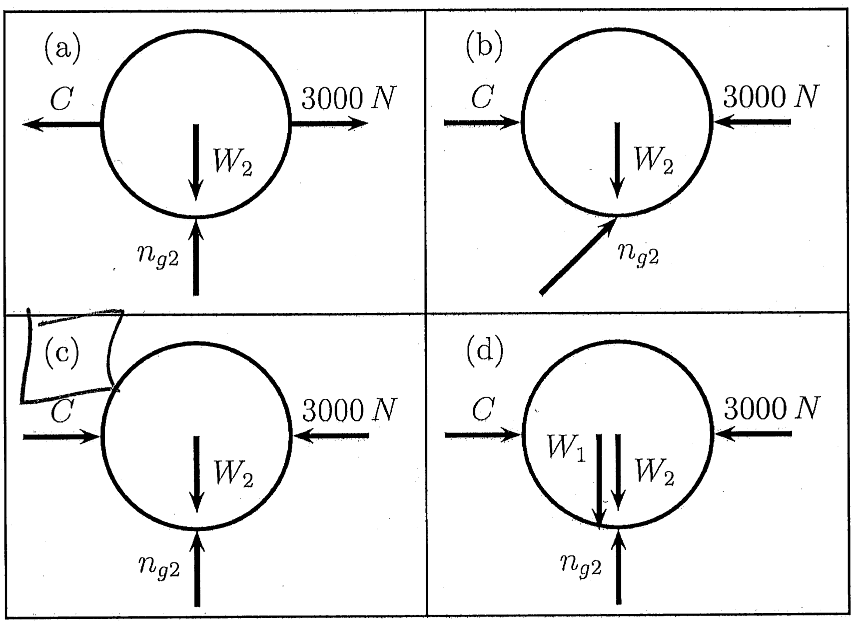
Superman Returns (to lift things)

Questions 11 through 15 refer to the following setup and figure.

The two gigantic globes (mass $M_1 = 765\,kg$ and $M_2 = 500\,kg$ respectively) from the top of the Daily Planet Headquarters have fallen down to the ground, and Superman has arrived to clean up for us (again). Miraculously, both globes are still round (in shape), lying as shown below, and have the same radius. Superman stretches to warmup and then applies a $3000\,N$ horizontal force to M_2. In all questions ignore friction.



11. The instant Superman applies his horizontal force, which of the following is the correct free body diagram for M_2? (Unlike Mastering Physics, I only care about direction!) In each case, n_{g2} is the normal force due to the ground while C is the normal force due to the contact between M_1 and M_2.



3000N Applied to M_2 ⇒ ← 3000N

If 2 pushes on 1, 1 pushes back ⇒ ⇐

Only forces on W_2 ⇒ ↓W_2 only

Horizontal surface ⇒ ↑n_{g2}

12. If Superman's force is unable to make the globes move, what is the magnitude of the force C?

(a) $C = 6000\ N$	(b) $C = 7900\ N$	(c) $C = 4900\ N$	(d) $C = 3000\ N$

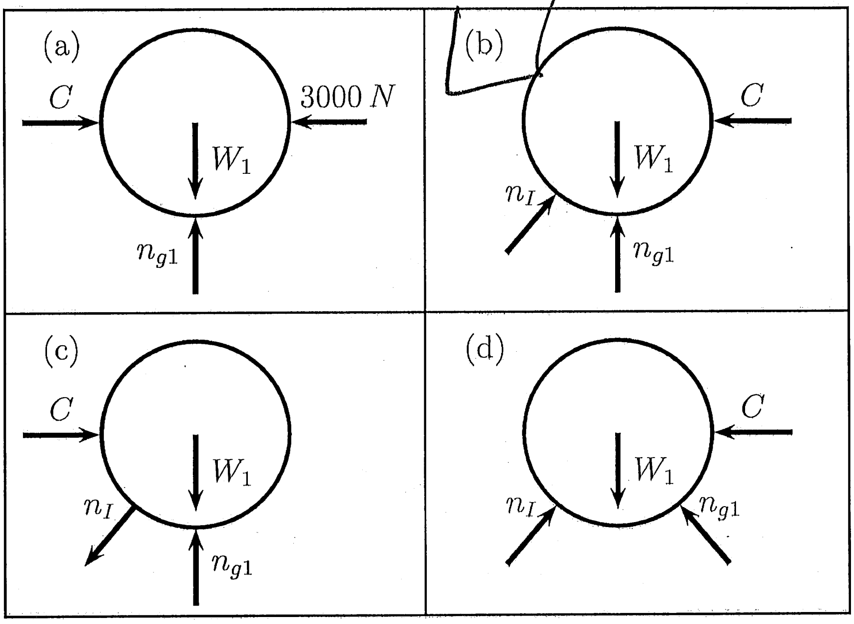
$\Sigma F_x = Ma_x$
⇒ $C - 3000N = 0$
⇒ $C = 3000N$

13. How large is the normal force due to the ground, n_{g2}?

(a) $12397\ N$	(b) $7900\ N$	(c) $4900\ N$	(d) $3000\ N$

$\Sigma F_y = Ma_y$
⇒ $n_{g2} - W_2 = 0$
⇒ $n_{g2} = W_2 = (500)(9.8)$
$= 4900N$

14. Which of the following is the correct free body diagram for M_1? In each case, n_{g1} is the normal force due to the ground, n_I is the normal force due to the incline, C is the normal force due to the contact between M_1 and M_2.



No 3000N Force on M_1! Just $\leftarrow C$

$W_1 \downarrow$ only

Horizontal $\Rightarrow \uparrow n_{g_1}$

Incline $\Rightarrow \nearrow n_I$

15. From M_1's free body diagram, we can conclude that:

(a) $n_{g1} = W_1$	(b) $n_{g1} > W_1$	(c) $n_{g1} < W_1$	(d) n_{g1} and W_1 are unrelated.

$$\sum F_y = Ma_y \Rightarrow n_{I,y} + n_{g_1} - W_1 = 0$$

($n_{I,y}$: y-Component of Incline's Normal)

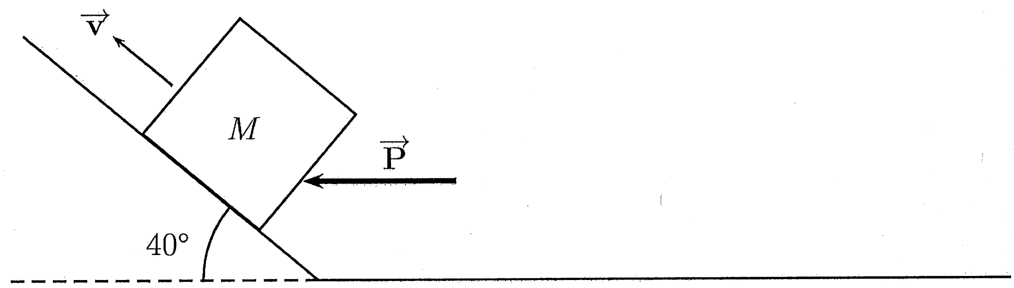
$$\Rightarrow n_{g_1} = W_1 - n_{I,y}$$

$$\Rightarrow n_{g_1} < W_1$$

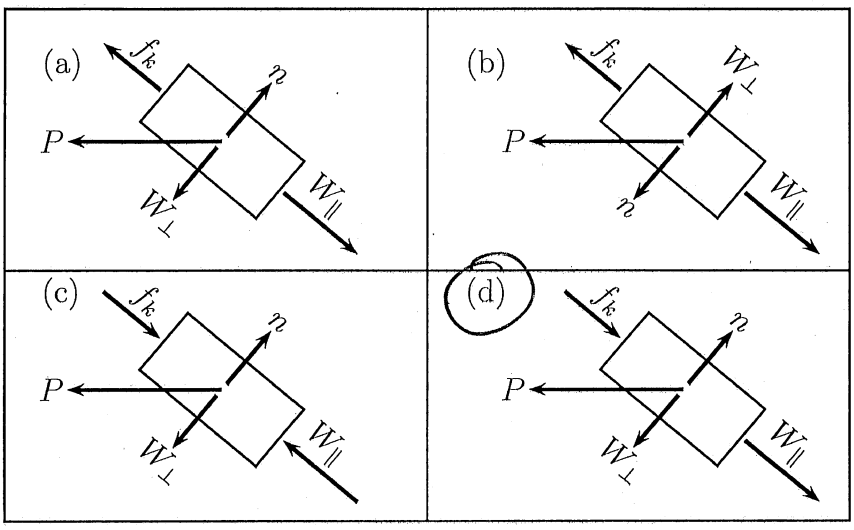
Still Lifting

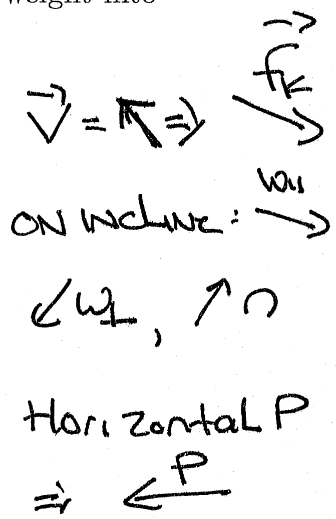
Questions 16 through 20 refer to the following setup and figure.

Tired of dealing with the two globes, Superman uses his X-ray vision (or whatever it's called) to melt them together to form one $M = 1200\,kg$, rectangular mass. Being Super-persistent, Superman continues to apply a horizontal force, $\vec{\mathbf{P}}$, to the mass even after it starts to slide up the incline.

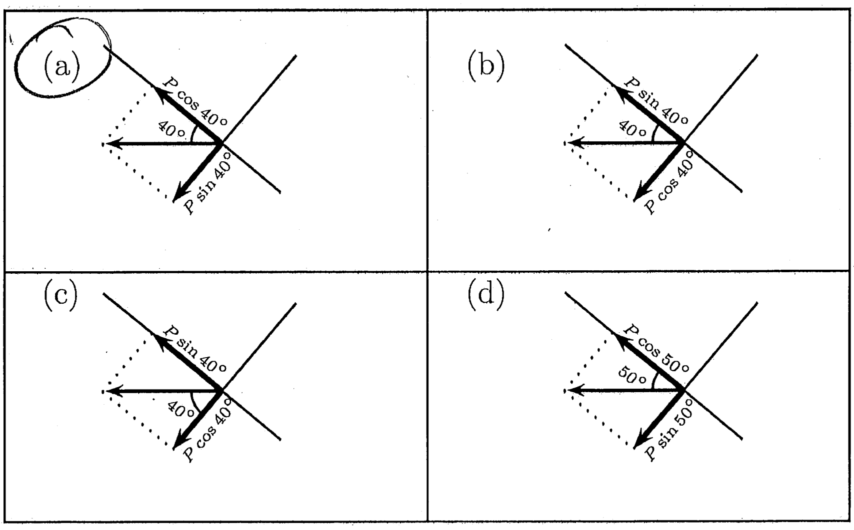


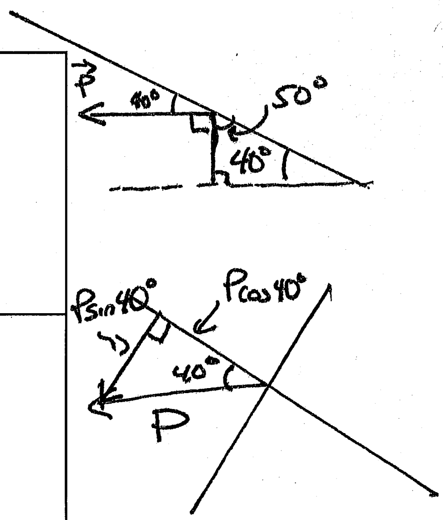
16. If we include friction, which of the following is the correct free body diagram for M? (Unlike Mastering Physics, I always split weight into components on an incline.)





17. Which of the following graphs shows how to correctly split P into its perpendicular and parallel components?





18. Which of the following is the correct listing of the parallel and perpendicular weight of the $M = 1200\,kg$ mass on the 40° incline? Note the use of $kN = kiloNewtons$ to make the numbers smaller.

$W_{\parallel} = Mg\sin 40° = (1200kg)(9.8m/s^2)\sin 40° = 7559.18N = 7.55918kN = 7.6kN$

$W_{\perp} = Mg\cos 40° = 9008.68 = 9kN$

(a) $W_{\parallel} = 9.0\,kN$, $W_{\perp} = 7.6\,kN$	(b) $W_{\parallel} = 7.6\,kN$, $W_{\perp} = 9.0\,kN$
(c) $W_{\parallel} = 11.8\,kN$, $W_{\perp} = 9.0\,kN$	(d) $W_{\parallel} = 0\,kN$, $W_{\perp} = 11.8\,kN$

19. If $P = 21\,kN$, what is the magnitude of the normal force acting on the mass?

$\Sigma F_{\perp} = Ma_{\perp} \Rightarrow$

$n - P\sin 40° - W_{\perp} = 0$

$n = P\sin 40° + W_{\perp}$

$= (21kN)\sin 40° + 9kN = 22.5kN$

(a) $n = 9.0\,kN$	(b) $n = 7.6\,kN$	(c) $n = 22.5\,kN$	(d) $n = 21\,kN$

20. If $P = 21\,kN$ and $\mu_k = .35$, what is the mass's acceleration? Use "up" the incline as the positive direction.

(a) $a = .5\,m/s^2$	(b) $a = -1.7\,m/s^2$	(c) $a = 17.5\,m/s^2$	(d) $a = .26\,m/s^2$

$\Sigma F_{\parallel} = Ma_{\parallel} \Rightarrow P\cos 40° - W_{\parallel} - f_K = Ma \quad (a_{\parallel} = a = ?)$

$f_K = \mu_K n = .35(22.5kN) = 7.875kN$

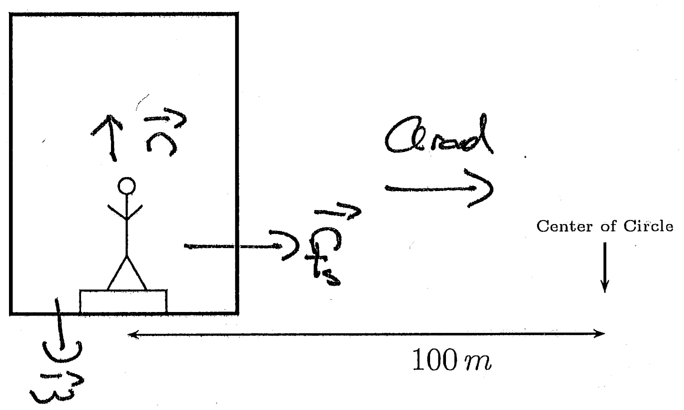
$\Rightarrow (21kN)\cos 40° - 7.6kN - 7.875kN = (1200kg)a$

$.612kN = (1200kg)a$ BUT CAN'T USE kN and get m/s²

$.612kN = 612N \Rightarrow a = \frac{612N}{1200kg} = .5099m/s^2 \approx .5m/s^2$

21. **The Great Glass Elevator**

One day finds little Charlie Bucket (mass 48 kg) and Willy Wonka riding around (on Earth) in their fabulous great glass elevator. If you've never read the book, the great glass elevator is an elevator that can move in any direction you might wish. Sometime during their trip, little Charlie Bucket and Willy Wonka turn a corner in the great glass elevator by zooming around a 100 m radius circle with a speed of 22.2 m/s. For reasons that only make sense to Willy Wonka (and your instructor), Charlie is riding in the elevator standing on a scale.



(a) If the coefficient of static friction between Charlie Bucket and his scale is .39, will he be able to remain not-sliding as he travels around this circle? Assume, as shown above, that the center of the circle is directly to the right of Charlie Bucket. (You must do a calculation of some sort to get full credit on this problem which is why there is an extra page provided.) *(10pts)*

Forces on Charlie: $\vec{n}$ up, $\vec{w}$ down, $\vec{f_s}$ right

AND NO OTHERS!

Circle's Center to right $\Rightarrow a_{rad} = a_x$ to right

$\Rightarrow \sum F_x = ma_x \Rightarrow f_s = ma_{rad} = \frac{mv^2}{r}$

$\Rightarrow f_s = \frac{(48kg)(22.2m/s)^2}{100m} = 236.5632N \Rightarrow$ Need 237N of static friction to go AROUND Circle.

$f_{s,max} = \mu_s n$ $\quad \sum F_y = ma_y. \quad a_y = 0$

$\Rightarrow n - W = 0 \Rightarrow n = W = (48kg)(9.8m/s^2)$

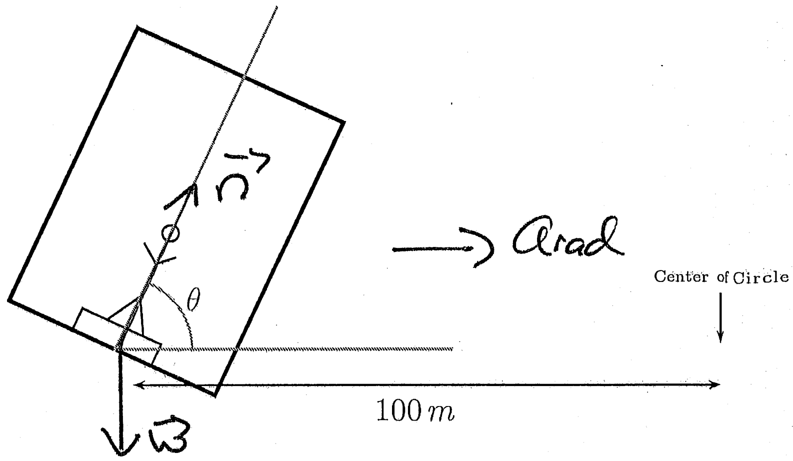
$= 470.4N$

$\Rightarrow f_{s,max} = .39(470.4N) = 183.456N$

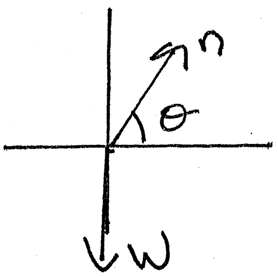
Since static friction's MAX VALUE is only 183N AND CHARLIE Needs 237N to go AROUND Circle He will SLIDE!

↳ OUCH

(b) By fiddling with some buttons, Willy Wonka discovers that he can tilt the great glass elevator to any angle that he wishes. To what angle θ should Willy Wonka tilt the elevator so that no friction is necessary for Charlie Bucket to go around a $100\,m$ radius circle (whose center is still directly to his right) with a speed of $22.2\,m/s$? What would the scale read in this case? (There's an extra page for this one too.) *(10pts)*



Now $\vec{n}$ at Angle θ, $\vec{w}$ DOWN, NO Friction. $a_{rad} = a_x$ still



$$\sum F_x = ma_x \Rightarrow n\cos\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y \Rightarrow n\sin\theta - w = 0 \Rightarrow n\sin\theta = mg$$

$$n\sin\theta = mg, \quad n\cos\theta = \frac{mv^2}{r}$$

$$\Rightarrow \frac{\not{n}\sin\theta}{\not{n}\cos\theta} = \frac{\not{m}g}{\not{m}v^2/r} \Rightarrow \tan\theta = \frac{gr}{v^2} \Rightarrow \theta = \tan^{-1}\left(\frac{gr}{v^2}\right)$$

$$\theta = \tan^{-1}\left(\frac{9.8\,m/s^2 \cdot 100\,m}{(22.2\,m/s)^2}\right) = \tan^{-1}(1.988) \Rightarrow \boxed{\theta = 63.3^\circ}$$

$$n\sin\theta = mg \Rightarrow n = \frac{mg}{\sin\theta} = \frac{(48\,kg)(9.8\,m/s^2)}{\sin 63.3^\circ} \Rightarrow \boxed{n = 526.5\,N = 527\,N}$$

$$\Rightarrow m = \frac{526.5}{9.8} = 53.7\,kg$$