PHYS 160 - MECHANICAL WAVES, CHAPTER 15

MECHANICAL WAVE - THE PROPAGATION OF ENERGY THROUGH A MEDIUM.

EXAMPLES - WATER WAVE (THE MEDIUM IS WATER), OR SOUND (THE MEDIUM IS USUALLY AIR BUT ANY ELASTIC MATERIAL WILL WORK).

LIGHT IS AN EXAMPLE OF A NON-MECHANICAL WAVE. IT REQUIRES NO MEDIUM, i.e., LIGHT CAN PROPAGATE THROUGH A VACUUM.

AS THE WAVE PROPAGATES EACH POINT OF THE MEDIUM UNDERGOES PERIODIC MOTION (USUALLY DAMPED). THE DIRECTION OF THE PERIODIC MOTION DETERMINES THE TYPE OF WAVE.

TRANSVERSE - MEDIUM OSCILLATES PERPENDICULAR TO PROPAGATION DIRECTION.

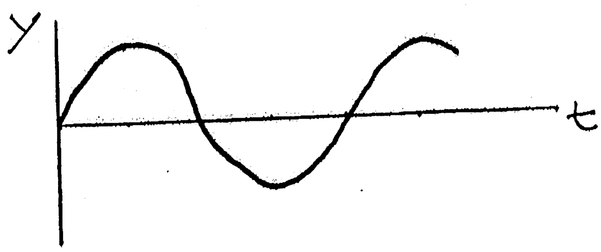
LONGITUDINAL - MEDIUM OSCILLATES PARALLEL TO PROPAGATION DIRECTION. (SOUND IS LONG.)

ROLLING WAVE - COMBINATION OF TRANSVERSE AND LONGITUDINAL (OCEAN WAVES)

PERIODIC WAVES → NON-DAMPED MEDIUM MOTION IN WHICH EVERY POINT UNDERGOES SIMPLE HARMONIC MOTION OF THE SAME FREQUENCY AND AMPLITUDE.

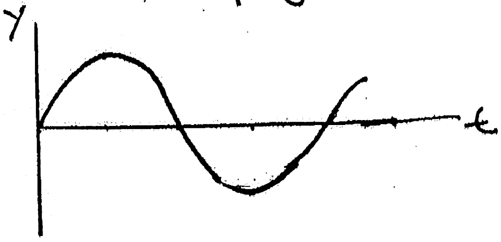
⇒ FOR ANY POINT, THE HEIGHT* OF THE MEDIUM IS $y = A\cos(\omega t + \phi)$

* WE'RE DOING A TRANSVERSE WAVE.

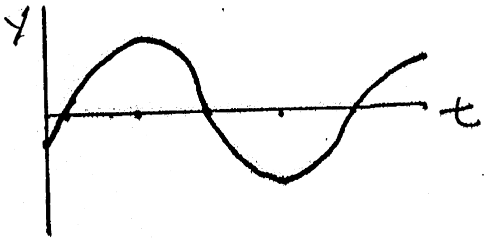


BECAUSE THE WAVE PROPAGATES, ADJACENT LOCATIONS ARE <u>NOT</u> IN PHASE.

FOR $X = X_0$

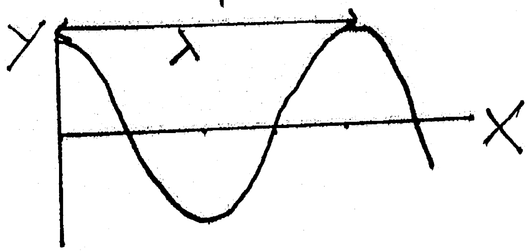


FOR $X = X_0 + \Delta X$



$\Rightarrow$ ϕ IS A FUNCTION OF THE PROPAGATION DIRECTION X.

IF WE PLOT THE HEIGHT FOR <u>DIFFERENT</u> <u>LOCATIONS</u> <u>AT A SINGLE TIME</u>, WE GET ANOTHER COSINE/SINE FUNCTION.



<u>PROPAGATION</u> $\Rightarrow$ OSCILLATION IN BOTH TIME AND SPACE.

λ = WAVELENGTH = DISTANCE BETWEEN PEAKS, i.e., DISTANCE OVER WHICH WAVE REPEATS. UNIT: meter.

SINE/COSINE HAVE A PERIOD OF 2π RAD $\Rightarrow$ $\phi \overset{\text{PROP. TO}}{\propto} \left(\frac{2\pi}{\lambda}\right)X$ s.t. $(X+\lambda)\left(\frac{2\pi}{\lambda}\right) = \frac{2\pi X}{\lambda} + 2\pi$

FOR A WAVE PROPAGATING IN THE +X DIRECTION, $y(X_0)$ SHOULD BE "AHEAD" OF $y(X_0+\Delta X)$ $\Rightarrow$ $y(X_0+\Delta X)$ NEEDS TO BE SHIFTED TO THE RIGHT (SEE y VS. t PLOTS ABOVE) $\Rightarrow$ $\phi = -\frac{2\pi X}{\lambda}$.

$$\boxed{\frac{2\pi}{\lambda} = k}$$

K = WAVE NUMBER
UNIT: rad/m

$$\Rightarrow y = A\cos(\omega t - KX) = A\cos(-(-\omega t + kx)) = A\cos(-(KX - \omega t))$$

COSINE IS AN EVEN FUNCTION $\Rightarrow \cos(-\theta) = \cos\theta$

$$\Rightarrow \boxed{y = A\cos(kx - \omega t)}$$

$y(x,t) \rightarrow$ WAVE FUNCTION. 2D FUNCTION WHICH GIVES OSCILLATION IN TIME AND SPACE.

NOTE: WORKS FOR WAVES WITH $y(0,0) = A$.

<u>WAVE SPEED</u>: AS THE ENERGY PROPAGATES THROUGH THE MEDIUM, IT DOES SO WITH A VELOCITY V.

$$y = A\cos(kx - \omega t) \Rightarrow y\left(x + \frac{2\pi}{k}, t\right) = y\left(x, t + \frac{2\pi}{\omega}\right)$$

$\frac{2\pi}{k} = \lambda$, $\frac{2\pi}{\omega} = T$ $\Rightarrow$ THE DISTURBANCE PROPAGATES OVER ONE WAVELENGTH (DISTANCE) DURING ONE PERIOD (TIME)

$$\Rightarrow \boxed{V = \frac{\lambda}{T} = \lambda f} \quad \text{OR} \quad V = \frac{\frac{2\pi}{k}}{2\pi/\omega} \Rightarrow \boxed{V = \frac{\omega}{k}}$$

<u>EXAMPLE</u>: A WAVE TABLE IS OSCILLATED AT A FREQUENCY OF 3Hz, AND WITH AN AMPLITUDE OF .35m. IF THE WAVE SPEED IS .4m/s, what is THE DIFFERENCE IN HEIGHTS AT X = .15m AND X = .2m at t = 2s? ASSUME $y(0,0) = .35m$.

$y = A\cos(kx - \omega t)$. $A = .35m$. $k = \frac{2\pi}{\lambda}$. $V = \lambda f \Rightarrow \lambda = \frac{.4m/s}{3Hz} = \frac{.4m/s}{3/s} = .133m$

$\omega = 2\pi f = 2\pi(3Hz) = 2\pi(3/s) = 6\pi$ RAD/s. $k = \frac{2\pi}{.133m} = 15\pi$ RAD/m

$y = .35m \cos(15\pi x - 6\pi t)$

$y_1 = y(X = .1m, t = 2s) = .35m \cos(15\pi(.15) - 6\pi(2)) = .35m \cos(-9.75\pi) = .707$

$y_2 = y(X = .2m, t = 2s) = .35m \cos(15\pi(.2) - 6\pi(2)) = .35m \cos(-9\pi) = -.35m$

$\Rightarrow \Delta y = 1.057m$

INTERFERENCE - WHEN TWO OR MORE WAVES PASS THROUGH THE SAME REGION AT THE SAME TIME.

All THE WAVES TRY TO MAKE THE MEDIUM OSCillaTE. THE NET EFFECT IS given BY THE SUM OF THE WAVEFUNCTIONS (IN 2D THIS would BE THE VECTOR SUM)

PRINCIPLE OF SUPERPOSITION: $\boxed{y(x,t) = y_1(x,t) + y_2(x,t)}$

y_1 = FIRST WAVE

y_2 = SECOND WAVE

CONSTRUCTIVE INTERFERENCE OCCURS when THE TWO WAVES ARE IN PHASE. FOR TWO WAVES AT A point X AT A TIME t

$$y = A_1 \cos(k_1 x + \omega_1 t) + A_2 \cos(k_2 x + \omega_2 t)$$

IF $k_1 x + \omega_1 t = k_2 x + \omega_2 t + 2\pi n \Rightarrow y = (A_1 + A_2)\cos(k_1 x + \omega_1 t)$ → THE Amplitude IS INCREASED.

$n = 0, 1, 2, \ldots$

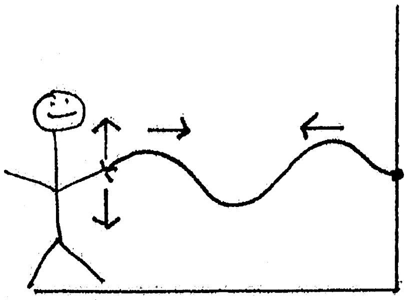
DESTRUCTIVE INTERFERENCE OCCURS WHEN THE TWO WAVES ARE OUT OF PHASE. IF $k_1 x + \omega_1 t = k_2 x + \omega_2 t + \pi n$. $n = 1, 3, 5, \ldots$

$\Rightarrow y = (A_1 - A_2)\cos(k_1 x + \omega_1 t)$ → THE TOTAL Amplitude IS DECREASED

IF $A_1 = A_2$ THEN $y = 0$ → TOTAL CANCEllATION.

INTERFERENCE CAN BE SEEN IN A STANDING WAVE.

A string with ONE END FIXED. IF THE FREE END IS OSCillATED SINUSOIDAlly, THE OUTGOING WAVES INTERFERE WITH THE ONES REFLECTED OFF THE FIXED END.



AT A POINT X: $y = y_1 + y_2$

$y_1 \rightarrow$ OUTGOING WAVE

$y_2 \rightarrow$ REFLECTED WAVE

FOR A WAVE PROPAGATING TO THE RIGHT: $y_1 = A\cos(kx - \omega t)$

A WAVE PROPAGATING TO THE LEFT: $A\cos(kx + \omega t)$

UPON REFLECTION THERE IS A PHASE SHIFT OF π (SEE BOOK FOR EXPLANATION)

$\Rightarrow y_2 = A\cos(kx + \omega t + \pi) = -A\cos(kx + \omega t)$

$\Rightarrow y = A\cos(kx - \omega t) - A\cos(kx + \omega t) = A[\cos(kx - \omega t) - \cos(kx + \omega t)]$

$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$

$\Rightarrow y = A[\cos kx \cos \omega t + \sin kx \sin \omega t - \cos kx \cos \omega t - (-\sin kx \sin \omega t)]$

$\Rightarrow \boxed{y = 2A \sin kx \sin \omega t}$

SINCE THE ROPE IS FIXED ITS at the FAREND MUST ALWAYS BE ZERO.

IF FAREND IS AT $x = L$ (ITS LENGTH) $\Rightarrow y = 2A \sin kL \sin \omega t$

$\Rightarrow \sin kL = 0 \Rightarrow kL = n\pi \quad n = 1, 2, 3, \ldots$

$\Rightarrow \frac{2\pi L}{\lambda} = n\pi \Rightarrow \boxed{L = \frac{n\lambda}{2}} \quad n = 1, 2, 3, \ldots$

$\Downarrow$

$\boxed{\lambda = \frac{2L}{n}}$

$n = 1$ IS CALLED THE FUNDAMENTAL

$n = 2, 3, \ldots$ ARE CALLED THE HARMONICS

MAKES MORE SENSE. $\rightarrow$ ONLY CERTAIN WAVELENGTHS ARE ALLOWED