PHYS 160 – PERIODIC MOTION, CHAPTER 13

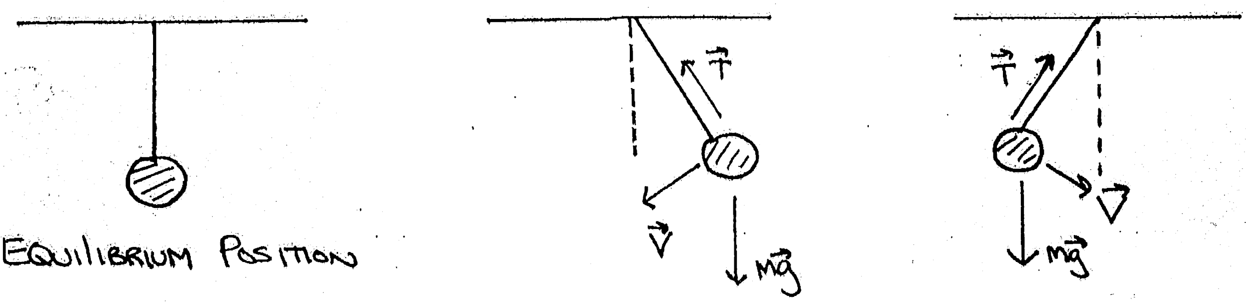
PERIODIC MOTION OR OSCILLATION IS MOTION WHICH IS REPEATED AND RHYTHMIC. (ALWAYS TAKING THE SAME AMOUNT OF TIME BEFORE REPEATING).

EXAMPLES – PENDULUM, A MASS "BOBBING" UP AND DOWN ON A SPRING, etc.

FOR ANY OBJECT TO OSCILLATE THERE MUST BE A RESTORING FORCE.

RESTORING FORCE – PUSHES OR PULLS THAT RETURN OBJECT TO IT'S EQUILIBRIUM POSITION.

FOR A PENDULUM, THE RESTORING FORCE IS A COMBINATION OF STRING'S TENSION AND GRAVITY.



THE PENDULUM IS ALWAYS MOVING TOWARDS ITS EQUILIBRIUM POSITION BECAUSE OF THE RESTORING FORCE. IT DOESN'T STOP AT ITS EQUIL. POSITION BECAUSE OF ITS INERTIA.

TERMS

AMPLITUDE – MAXIMUM DISTANCE FROM EQUILIBRIUM. DENOTED BY A. UNIT = METER OR RADIAN. WITHOUT FRICTION, OBJECTS OSCILLATE BETWEEN A AND –A INDEFINITELY.

CYCLE – ONE COMPLETE ROUND TRIP. e.g. GOING FROM A to –A AND BACK TO A.

PERIOD – TIME FOR ONE CYCLE. DENOTED BY T. UNIT = SECOND, BUT SOMETIMES WE CALL IT SECOND/CYCLE. WITHOUT FRICTION, THE PERIOD IS CONSTANT.

OSCILLATIONS ARE RYTHMIC IN THAT THE PERIOD IS THE SAME FOR STARTING A CYCLE AT ANY POINT IN THE MOTION. WE CAN START COUNTING AT ∅ (EQUILIBRIUM), A, –A, OR ANYWHERE IN BETWEEN AND GET THE SAME PERIOD.

FREQUENCY - NUMBER OF CYCLES PER TIME. DENOTED BY f. UNIT = HERTZ (Hz),
1 Hz = 1 cycle/sec. FREQUENCY TELLS US HOW OFTEN THE OBJECT OSCILLATES.

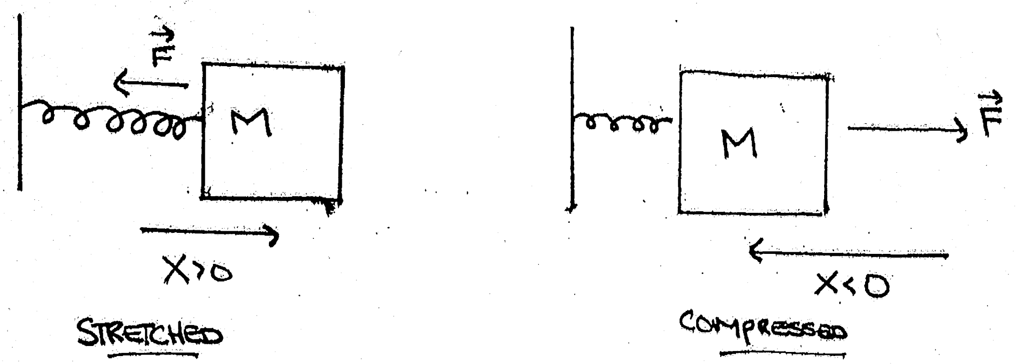
FREQUENCY AND PERIOD ARE INVERSELY RELATED $\Rightarrow$ $\boxed{f = \frac{1}{T}}$

ANGULAR FREQUENCY, $\boxed{\omega = 2\pi f}$ UNIT = RAD/S. NOT NECESSARILY RELATED TO ROTATION.

$f = \frac{1}{T} \Rightarrow \boxed{\omega = \frac{2\pi}{T}}$

SIMPLE HARMONIC MOTION - SPRINGS AND SPRING-LIKE OBJECTS MAKE THE SIMPLEST PERIODIC MOTION (WE CALL IT HARMONIC).

TO FIND MOTION OF AN OBJECT, WE NEED THE FORCE ON THE OBJECT



$F = -kX$

WITHOUT FRICTION: $F_{NET} = -KX \Rightarrow Ma = -KX \Rightarrow a = \frac{-k}{M}X$

$\Rightarrow \boxed{\frac{d^2x}{dt^2} = \frac{-k}{M}x}$ EQUATION OF MOTION FOR A "HARMONIC OSCILLATOR" (SIMPLE HARMONIC MOTION)

FUNCTION WHOSE SECOND DERIVATIVE IS EQUAL TO THE NEGATIVE OF ITSELF IS SINE OR COSINE. THE GENERAL SOLTN. IS

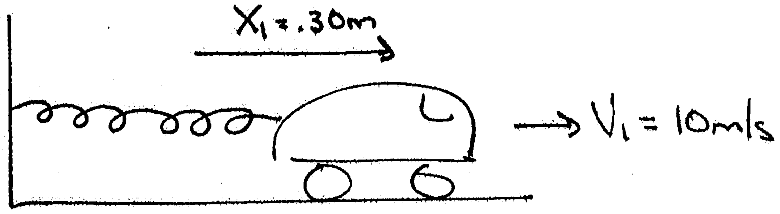
$\boxed{X = A\cos(\omega t + \phi)}$ A AND ϕ ARE CONSTANTS DETERMINED BY INITIAL POSITION AND VELOCITY. (INITIAL CONDITIONS). A = AMPLITUDE. ϕ = PHASE ANGLE.

We call ω the ANGULAR FREQUENCY BECAUSE ωt is AN ANGLE.

$V = \frac{dx}{dt} \Rightarrow \boxed{V = -A\omega \sin(\omega t + \phi)}$

$\frac{d^2X}{dt^2} = -A\omega^2 \cos(\omega t + \phi) = -\omega^2 X \Rightarrow \boxed{\omega = \sqrt{\frac{k}{M}}}$

EXAMPLE – A 2Kg TOY CAR IS ATTACHED TO A $K = 50N/m$ SPRING. IF THE SPRING IS STRETCHED BY .30m AND THE CAR IS GIVEN AN INITIAL VELOCITY OF 10m/s, FIND THE POSITION OF THE CAR AS A FUNCTION OF TIME (IGNORING FRICTION).



MASS ON A SPRING $\Rightarrow X = A\cos(\omega t + \phi)$, $V = -A\omega \sin(\omega t + \phi)$

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{50N/m}{2kg}} = \sqrt{25 \frac{Kg\,m/s^2/m}{kg}} = 5/s = 5RAD/s$$

A AND ϕ ARE FOUND FROM INITIAL CONDITIONS:

$$X_1 = X(t=0) = A\cos\phi, \quad V_1 = V(t=0) = -A\omega\sin\phi$$

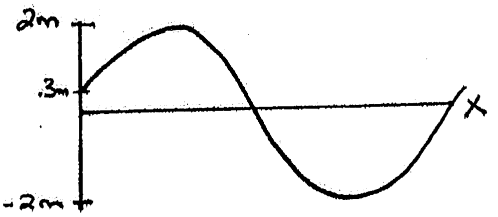
$$\Rightarrow \frac{V_1}{X_1} = \frac{-A\omega\sin\phi}{A\cos\phi} \Rightarrow \frac{V_1}{X_1} = -\omega\tan\phi \Rightarrow \phi = \tan^{-1}\left(\frac{-V_1}{\omega X_1}\right)$$

$$\phi = \tan^{-1}\left(\frac{-10m/s}{5/s \cdot .3m}\right) \Rightarrow \phi = \tan^{-1}\left(\frac{-20}{3}\right) \Rightarrow \phi = -1.42\,RAD \quad (\text{NOT } 81.4^\circ)$$

→ YOUR CALCULATOR MUST BE IN RADIAN MODE!

$$X_1 = A\cos\phi \Rightarrow A = \frac{.3m}{\cos(-1.42)} = 2.02m \approx 2m$$

$$X = 2m\cos(5RAD/s\,t - 1.42RAD) = 2m\cos(5t - 1.42)$$ → EASIER TO WRITE



- AT WHAT TIMES IS THE CAR AT ITS MAXIMUM VALUE? WHAT IS ITS PERIOD?

$X = A = 2m \Rightarrow 2m = 2m \cos(5t - 1.42) \Rightarrow \cos(5t - 1.42) = 1$

$\Rightarrow 5t - 1.42 = 2\pi n$ ($n = 0, 1, 2, 3, \ldots$ ANY INTEGER)

$\Rightarrow t = \dfrac{2\pi n + 1.42}{5/s} \Rightarrow t = .284s, 1.54s, 2.80s, \ldots$

$T = 1.54s - .284s = 1.26s.$

OR WE COULD USE THAT $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{5/s} = 1.26s.$

$f = \frac{1}{T} = .796Hz$

THE <u>ENERGY</u> FOR ANY HARMONIC OSCILLATOR IS THE ELASTIC ENERGY. ($F = -kx$ IS THE ONLY FORCE).

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$; $x = A\cos(\omega t + \phi)$, $v = -A\omega \sin(\omega t + \phi)$

$\Rightarrow E = \frac{1}{2}m\left(A^2\omega^2 \sin^2(\omega t + \phi)\right) + \frac{1}{2}k\left(A^2\cos^2(\omega t + \phi)\right)$

$\omega^2 = \frac{k}{m} \Rightarrow m\omega^2 = k^2$

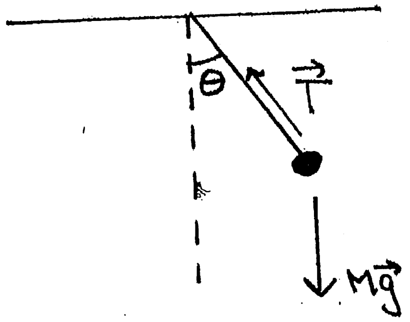
$\Rightarrow E = \frac{1}{2}kA^2\left(\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)\right)$

$\Rightarrow \boxed{E = \frac{1}{2}kA^2}$

ENERGY OF THE SYSTEM DETERMINES THE AMPLITUDE (OR DEPENDING ON YOUR POINT OF VIEW, VICE-VERSA).

PENDULUMS – FORCE ON PENDULUM DEPENDS ON WHETHER THE STRING/CONNECTOR HAS MASS.

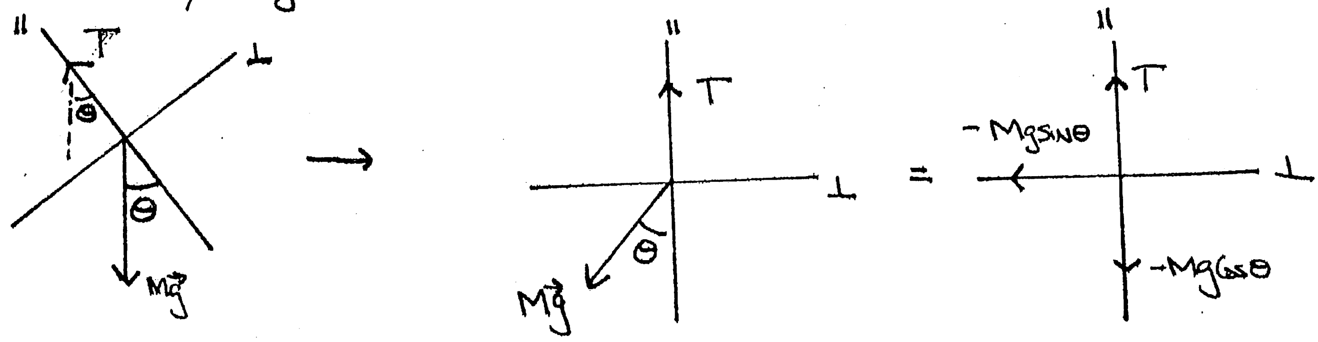
SIMPLE PENDULUM – MASSLESS STRING OF LENGTH L. THE MASS MOVES IN AN ARC, i.e., PART OF A CIRCLE.



FORCES ON MASS: WEIGHT DOWN, TENSION ALONG STRING.

FOR CIRCULAR MOTION, ONE OF THE CO-ORDINATES MUST POINT TOWARDS THE CENTER OF THE CIRCLE.

$\Rightarrow$ THE FREE BODY DIAGRAM LOOKS LIKE:



($\parallel$ = PARALLEL TO STRING,
$\perp$ = PERP. TO STRING)

$F_{NET,\parallel} = T - Mg\cos\theta = Mv^2/R$ (NOT OF INTEREST HERE, BUT THOUGHT I SHOULD PUT IT FOR COMPLETENESS).

$F_{NET,\perp} = -Mg\sin\theta = Ma_{\perp} \Rightarrow a_{\perp} = -g\sin\theta$

$a_\perp = a_{tan} = \alpha L$ (FROM CHAPTER 9)

$\Rightarrow \alpha L = -g \sin\theta \Rightarrow \alpha = -\frac{g}{L}\sin\theta.$

$\alpha = \frac{d\omega}{dt}$, $\omega = \frac{d\theta}{dt} \Rightarrow \alpha = \frac{d^2\theta}{dt^2} \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = \frac{-g}{L}\sin\theta}$ TRUE EQN FOR A PENDULUM.

TO GET SIMPLE HARMONIC MOTION, WE USE A VERY USEFUL TRICK CALLED THE SMALL ANGLE APPROXIMATION. FOR "SMALL" ANGLES, $\sin\theta \approx \theta$. THIS IS TRUE FOR $\theta < 34°$

THIS APPROXIMATION COMES FROM THE TAYLOR SERIES (AKA THE POWER SERIES) FOR SINE.

$\sin\theta = \theta - \frac{1}{3!}\theta^3 + \frac{1}{5!}\theta^5 - \frac{1}{7!}\theta^7 + \ldots$. FOR θ SMALL, $\sin\theta \approx \theta$.

$\Rightarrow \frac{d^2\theta}{dt^2} = \frac{-g}{L}\theta \Rightarrow \boxed{\theta = \theta_0 \cos(\omega t + \phi),\ \omega = \sqrt{g/L}}$

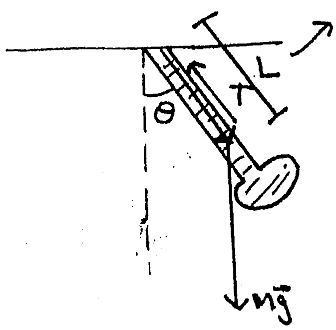
EXAMPLE – WHAT IS THE DIFFERENCE IN PERIOD OF A $L = .75m$ simple PENDULUM ON THE MOON AND ON THE EARTH. $g = 1.6 m/s^2$ ON THE MOON.

ON EARTH: $\omega = \sqrt{\frac{9.8 m/s^2}{.75m}} = \sqrt{13.07/s^2} = 3.6/s$ $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = 1.74s$

MOON: $\omega = \sqrt{\frac{1.6 m/s^2}{.75m}} = 1.46/s \Rightarrow T = 4.30s$

$\Rightarrow \Delta T = 4.30s - 1.74s = 2.56s$

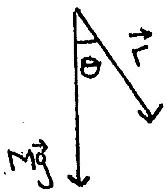
PHYSICAL PENDULUM – A REAL PENDULUM WITH MASSIVE CONNECTOR. TO FIND THE CHANGE IN θ, WE NEED THE TORQUE $\tau = \vec{r} \times \vec{F}$



$\vec{r}$ HAS MAGNITUDE L

FORCES ACTING ON CENTER OF MASS. TENSION OPPOSITE TO $\vec{r}$, $M\vec{g}$ DOWN.

$\vec{r} \times \vec{T} = 0$ SINCE $\vec{r}$ AND $\vec{T}$ ARE 180° APART.



$\tau = r M g \sin\theta$

$\vec{\tau}$ POINTS INTO PAGE. WE MAKE THIS THE NEGATIVE DIRECTION.

$r = L$

$\Rightarrow \tau = -L M g \sin\theta$

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2}$$

(I = MOMENT OF INERTIA FOR CENTER OF MASS ABOUT THE ROTATION AXIS THROUGH THE PAGE, i.e., THE Z-AXIS).

$$I\frac{d^2\theta}{dt^2} = -MgL\sin\theta \Rightarrow \boxed{\frac{d^2\theta}{dt^2} = -\frac{MgL}{I}\sin\theta}$$

TRUE EQUATION FOR A PHYSICAL PENDULUM.

FOR SMALL ANGLES, $\sin\theta \approx \theta \Rightarrow \theta = \theta_0\cos(\omega t + \phi)$ $\boxed{\omega = \sqrt{\frac{MgL}{I}}}$