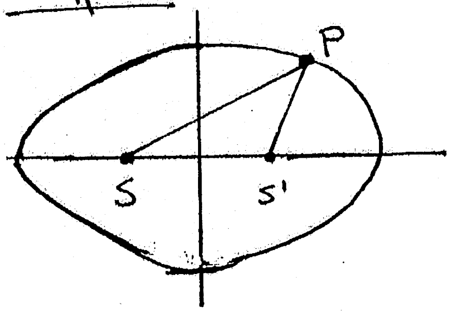
# PHYS 160 - GRAVITY II, CHAPTER 12

KEPLER'S LAWS - BEFORE NEWTON, THE WORK ON ASTRONOMICAL MOTION HAD all BEEN EMPIRICAL (OBSERVATIONAL). USING THE DATA OF THE DANISH ASTRONOMER TYCHO BRAHE (1546-1601), THE GERMAN MATHEMATICIAN JOHANNES KEPLER (1571-1630) WAS ABLE TO DEDUCE (BUT NOT EXPLAIN) LAWS ABOUT PLANETARY MOTION.

Kepler's LAWS:

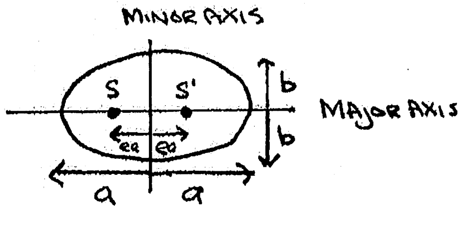
1) EACH PLANET MOVES ON AN ellipse WITH THE SUN AT ONE FOCUS
2) THE IMAGINARY line FROM THE SUN TO A PLANET SWEEPS OUT EQUAL AREAS IN EQUAL TIMES.
3) THE PERIOD OF THE PLANETS IS PROPORTIONAL TO THE SEMI-MAJOR AXIS TO THE 3/2 POWER.

Ellipse - OVAL




S AND S' = foci (Plural of focus)

ellipse = All points such that $|SP| + |S'P| = \text{CONSTANT}$




$a$ = SEMI-MAJOR AXIS, $b$ = SEMI-MINOR AXIS

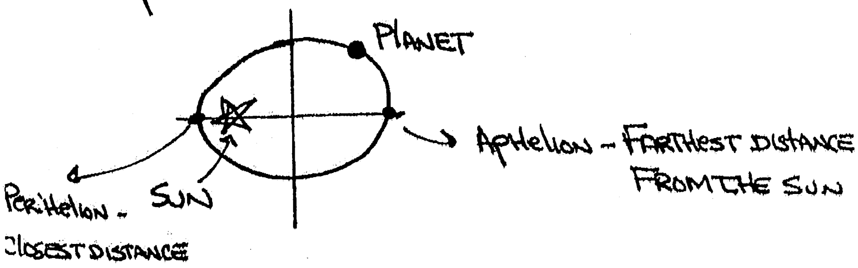
$e$ = ECCENTRICITY. $e = 0$ IS A CIRCLE, only one focus AT THE CENTER

EQUATION OF Ellipse: $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

IN Polar Co-ordinates: $r = \frac{a(1-e^2)}{1+e\cos\theta}$

As $e$ INCREASES, the ellipse BECOMES MORE Elongated

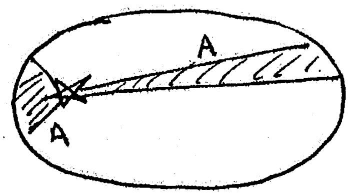
# Kepler's First Law:



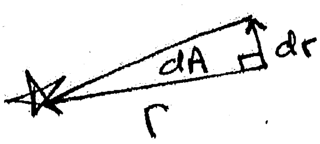

MOST PLANETARY ORBITS ARE NEARLY CIRCULAR.

| Planet | $e$ |
|---|---|
| Mercury | .206 |
| Venus | .007 |
| EARTH | .017 |
| MARS | .093 |
| Jupiter | .048 |
| Saturn | .054 |
| URANUS | .047 |
| Neptune | .009 |
| PLUTO | .249 |

# Kepler's 2ND LAW




IF THESE TWO SHAPES HAVE THE SAME AREA, THE PLANET SPENDS THE SAME AMOUNT OF TIME FOR THOSE PARTS OF THE ORBIT. THIS TELLS US THAT PLANETS GO FASTEST WHEN THEY ARE NEAREST THE SUN.

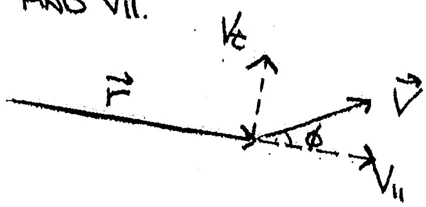



REMEMBER FROM ROTATIONAL MOTION THAT $dr$ IS PERPENDICULAR TO $r$.

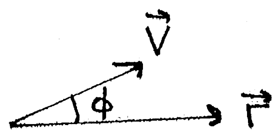
$$dA = \frac{1}{2} r\,dr \Rightarrow \frac{dA}{dt} = \frac{1}{2} r \frac{dr}{dt} = \frac{1}{2} r V_t .$$

$V_t$ = TANGENTIAL VELOCITY

ON A CIRCLE $\vec{V}$ IS ALWAYS TANGENTIAL TO $r \Rightarrow$ ONLY HAVE $V_t$. ON AN ELLIPSE, WE HAVE $V_t$ AND $V_{\parallel}$.




$V_t = V \sin\phi$. $\phi$ IS ALSO ANGLE BETWEEN $\vec{V}$ AND $\vec{r}$




$$\Rightarrow \frac{dA}{dt} = \frac{1}{2} r V \sin\phi = \frac{1}{2M} M r V \sin\phi = \frac{1}{2M} \left| \vec{r} \times M\vec{V} \right| \Rightarrow \boxed{\frac{dA}{dt} = \frac{L}{2M}}$$

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F}.$$ GRAVITATIONAL FORCE IS ALWAYS ATTRACTIVE.

$$|\vec{r} \times \vec{F}| = rF\sin 180° = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0$$

**FOR PLANETARY MOTION, THE ANGULAR MOMENTUM IS CONSTANT**

$$\frac{dA}{dt} = \frac{L}{2M} \Rightarrow \boxed{\frac{dA}{dt} = \text{CONSTANT}} \Rightarrow \Delta A = \text{CONSTANT} \times \Delta t$$

SAME $\Delta t \Rightarrow$ SAME $\Delta A$

FOR A POINT PARTICLE (LIKE A PLANET), $L = Mvr\sin\phi \Rightarrow$ AS $r$ INCREASES, $v$ MUST DECREASE AND VICE-VERSA.

remember $\vec{r}$ AND $\vec{v}$ NOT ALWAYS AT 90° ON AN ELLIPSE.

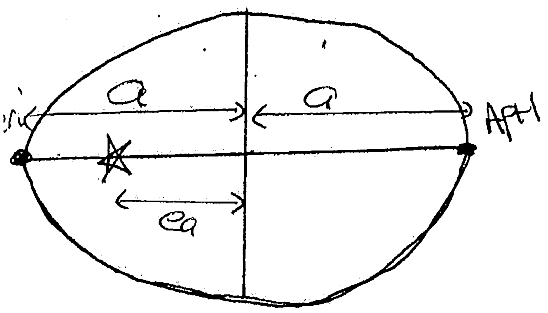
KEPLER'S 3RD LAW:

USING NEWTON'S LAWS AND NEWTON'S LAW OF GRAVITY (AND A WHOLE BUNCH OF DIFFERENTIAL EQUATIONS), NEWTON WAS ABLE TO SHOW THAT THE ONLY CLOSED ORBITS ARE CIRCLES AND ELLIPSES. THE ONLY OPEN ORBITS ALLOWED WITH GRAVITY ARE PARABOLAS AND HYPERBOLAS.

HE WAS ALSO ABLE TO SHOW THAT $\boxed{T = \frac{2\pi a^{3/2}}{\sqrt{GM_s}}}$ $M_s$ = MASS OF SUN

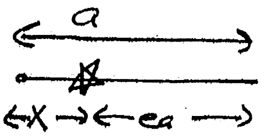
---

EXAMPLE 12.35



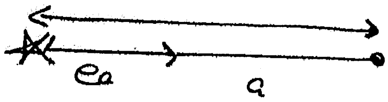

SHOW THAT SUN-PLANET DISTANCE AT APHELION IS $(1+e)a$ AND $(1-e)a$ AT PERIHELION

PERi:




$X + ea = a \Rightarrow X = a(1-e)$

APH:




$X = ea + a = a(1+e)$

b) $a_{PLUTO} = 5.92 \times 10^{12}\,m$, $e_{pluto} = .248$

$a_{NEPTUNE} = 4.5 \times 10^{12}\,m$, $e_{NEPTUNE} = .01$

FARTHEST NEPTUNE : $a_{PH} = 4.5 \times 10^{12}\,m(1+.01) = 4.545 \times 10^{12}\,m$

Closest PLUTO: $PERi = 5.92 \times 10^{12}\,m(1-.248) = 4.45 \times 10^{12}\,m$ $\Rightarrow$ THEY cross

c) PLUTO AT PERiHelion IN 1989, WHEN will THIS occur AgAIN?

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_{SUN}}} = \frac{2\pi(5.92 \times 10^{12}\,m)^{3/2}}{[6.67 \times 10^{-11}\,Nm^2/kg^2 \cdot 1.99 \times 10^{30}\,kg]^{1/2}} = 7.855 \times 10^9\,s$$

$$7.855 \times 10^9\,s \times \frac{h}{3600\,s} \times \frac{day}{24h} \times \quad = 249\,yrs$$

$1989 + 249 = 2238$