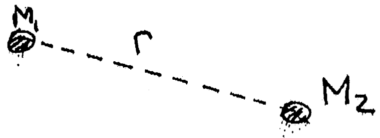
PHYS 100: GRAVITY, CHAPTER 12

<u>NEWTON'S LAW OF GRAVITATION</u> — EVERY OBJECT WITH MASS EXERTS A FORCE ON EVERY OTHER OBJECT WITH MASS. THE MAGNITUDE OF THIS FORCE IS GIVEN BY

$$\boxed{F_g = \frac{G M_1 M_2}{r^2}}$$

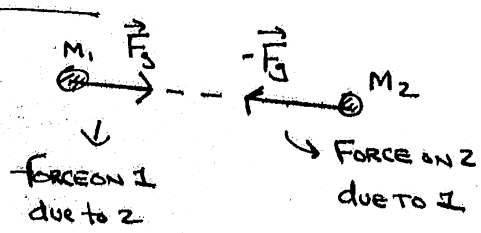


r = SEPERATION BETWEEN MASSES (AS MEASURED FROM THE CENTER FOR SPHERICALLY SYMMETRIC objects).

$$\boxed{G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}}$$

UNIVERSAL GRAVITATIONAL CONSTANT (EXPERIMENTALLY MEASURED).

<u>DIRECTION</u>: THIS FORCE IS ALWAYS ATTRACTIVE $\Rightarrow$ TOWARDS THE OTHER MASS.



<u>EXAMPLE</u>: WHAT IS THE FORCE ON THE MOON DUE TO THE EARTH?

M_1 = EARTH MASS = 5.97×10^{24} kg, M_2 = MOON MASS = 7.35×10^{22} kg

r = EARTH-MOON DISTANCE = 3.84×10^{8} m

$$F_g = \frac{(6.67 \times 10^{-11}\,Nm^2/kg^2)(5.97 \times 10^{24}\,kg)(7.35 \times 10^{22}\,kg)}{(3.84 \times 10^{8}\,m)^2} = 2 \times 10^{20}\,N$$

- WHAT IS ACCELERATION OF MOON due TO THIS FORCE?

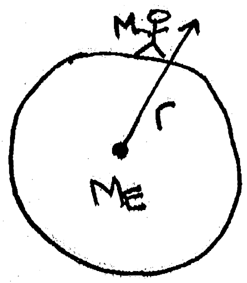
$$F_g = M_2 a_2 \Rightarrow a_2 = \frac{2 \times 10^{20}\,N}{7.35 \times 10^{22}\,kg} = 2.7 \times 10^{-3}\,m/s^2$$

- WHAT IS ACCELERATION OF EARTH?

$$F_g = M_1 a_1 \Rightarrow a_1 = \frac{2 \times 10^{20}\,N}{5.97 \times 10^{24}\,kg} = 3.3 \times 10^{-5}\,m/s^2$$

↳ EARTH DOESN'T MOVE AS MUCH.

Weight – The force on object due to gravity. To relate what we did before (Mg) to Newton's law of gravity $\frac{GM_1M_2}{r^2}$, we need to realize that compared to Earth's radius, we don't change our height much.



r measured from center of Earth.

$$F_g = \frac{GM_EM}{r^2}. \qquad r = R_E + \Delta r. \qquad R_E = \text{Earth's radius} = 6.38\times10^6\,m$$

So $\Delta r \ll R_E$

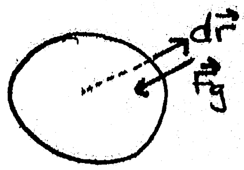
$$F_g = \frac{GM_EM}{(R_E+\Delta r)^2} \approx \left(\frac{GM_E}{R_E^2}\right)M. \qquad \frac{GM_E}{R_E^2} = \frac{(6.67\times10^{-11}\,Nm^2/kg^2)(5.97\times10^{24}\,kg)}{(6.38\times10^6\,m)^2}$$

$$= 9.782\,m/s^2 = 9.8\,m/s^2$$

For spacecraft, we have to use Newton's law of gravity to find force on it.

Potential Energy – We found $U = Mgy$ from $F_g = Mg$. The actual value is different.

Need the work done by gravity: $W_g = \int_{r_1}^{r_2} \vec{F}_g \cdot d\vec{r}$



$d\vec{r}$ and $\vec{F}_g$ are in opposite directions $\Rightarrow W_g = -\int_{r_1}^{r_2} F_g\,dr = -\int_{r_1}^{r_2} \frac{GM_1M_2}{r^2}\,dr$

$$= -GM_1M_2\int_{r_1}^{r_2}\frac{dr}{r^2} = -GM_1M_2\left(-\frac{1}{r}\right)\Bigg|_{r_1}^{r_2} = \frac{GM_1M_2}{r_2} - \frac{GM_1M_2}{r_1} = -\frac{GM_1M_2}{r_1} - \left(-\frac{GM_1M_2}{r_2}\right)$$

$$W_g = -\Delta U = U_1 - U_2 \Rightarrow \boxed{U = \frac{-GM_1M_2}{r}}$$ Notice $\frac{1}{r}$ instead of $\frac{1}{r^2}$

Escape Speed – Minimum speed to escape a planet's gravity. To escape $\Rightarrow F_g = 0$. Theoretically this doesn't occur until $r = \infty$. As $r \to \infty$, $U \to 0$.

$$K_1 + U_1 = K_2 + U_2. \qquad K_1 = \frac{1}{2} M_2 V_1^2, \; U_1 = \frac{-GM_1M_2}{R}.$$

M_1 = Mass of planet
R = Radius of planet.
(We assume you start from surface).

$K_2 = 0$ (just barely make it), $U_2 = 0$

$$\Rightarrow \frac{1}{2} \cancel{M_2} V_1^2 - \frac{GM_1\cancel{M_2}}{R} = 0 \Rightarrow \boxed{V_1 = \sqrt{\frac{2GM_1}{R}}}$$

Example – What is the escape speed for Earth?

$$V = \left[2 \frac{(6.67 \times 10^{-11} Nm^2/kg^2)(5.97 \times 10^{24} kg)}{6.38 \times 10^6 m} \right]^{1/2} = 11200 m/s \quad (25000 mph)$$

Good ole $U = Mgy$ is recovered by setting $r = R_E + y \Rightarrow$

$$U = \frac{-GM_EM}{R_E + y} = \frac{-GM_E M}{R_E(1 + y/R_E)} = \frac{-GM_EM}{R_E}(1 + y/R_E)^{-1}.$$

$$y \ll R_E \Rightarrow y/R_E \ll 1. \quad (1+x)^{-1} \approx 1 - x \text{ for } x \ll 1$$

$$\Rightarrow U \approx \frac{-GM_EM}{R_E}(1 - y/R_E) = \underbrace{\cancel{\frac{-GM_EM}{R_E}}}^{\text{constant}} + \frac{GM_E}{R_E^2}My = gMy = Mgy$$

Satellites – The Earth is not flat! It has a curvature of about 8000 to 5 (For every 8000m horizontal, the Earth drops by 5m)

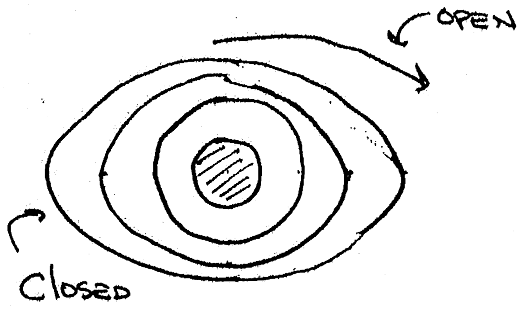
Fire a projectile horizontally, i.e. with $V_{oy} = 0$

$X = V_{ox}t, \quad y = -\frac{1}{2}gt^2.$

After 1 sec, $y = -4.9m \approx -5m$. So if $V_{ox} = 8000 m/s$ (or greater) in 1 sec, the projectile will go $X = 8000m$ and never hit the ground!

SATELLITE - ANY PROJECTILE WITH SUFFICIENT HORIZONTAL VELOCITY TO "MISS" THE GROUND. SATELLITES USE GRAVITY TO STAY IN ORBIT. SATELLITES ARE PLACED IN ORBIT SO HIGH ABOVE THE GROUND TO AVOID MOUNTAINS AND (ACTUALLY) TO AVOID AIR RESISTANCE. 8000 m/s = 18000 mph SO ANY NORMAL MATERIAL WOULD GET SO HOT IT WOULD PROBABLY EXPLODE.

ORBITS COME IN TWO TYPES - CLOSED (CIRCLES AND ELLIPSES) AND OPEN (DON'T RETURN TO STARTING POINT). AS V_{0x} INCREASES, ORBITS CHANGE FROM CIRCULAR TO ELLIPTICAL, TO OPEN.



CIRCULAR ORBIT - GRAVITY PROVIDES CENTRIPETAL FORCE. FOR A SATELLITE OF MASS M AROUND A PLANET M_p AT RADIUS r (AS MEASURED FROM CENTER OF PLANET)

$$\frac{GM_pM}{r^2} = \frac{Mv^2}{r} \Rightarrow \boxed{V = \sqrt{\frac{GM_p}{r}}}$$ SPEED IN CIRCULAR ORBIT

$$E = K + U = \frac{1}{2}MV^2 - \frac{GM_pM}{r}.$$ CONSTANT $r \Rightarrow$ CONSTANT $U \Rightarrow$ CONSTANT K

$\Rightarrow$ $\boxed{\text{SPEED IS CONSTANT IN CIRCULAR MOTION}}$

$$E = \frac{1}{2}M\left(\frac{GM_p}{r}\right) - \frac{GM_pM}{r} = \frac{GM_pM}{2r} - \frac{GM_pM}{r} \Rightarrow \boxed{E = \frac{-GM_pM}{2r}}$$ ENERGY OF CIRCULAR ORBIT

SINCE V IS CONSTANT, WE CAN ALSO FIND THE PERIOD. $V = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{V}$

$$\Rightarrow T = \frac{2\pi r}{\sqrt{\frac{GM_p}{r}}} \Rightarrow \boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM_p}}}$$