PHYS 160: <u>ANGULAR MOMENTUM</u>, CHAPTER 10

WE WANT ANGULAR MOMENTUM ($\vec{L}$) to BE ROTATIONAL EQUIVALENT TO MOMENTUM ($\vec{p}$)

$$\frac{d\vec{p}}{dt} = \vec{F} \Rightarrow \boxed{\frac{d\vec{L}}{dt} = \vec{\tau}}$$

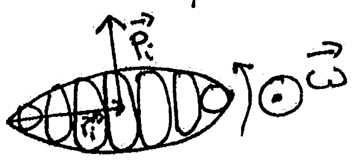
WE GET A CLUE FROM $\vec{\tau} = \vec{r}\times\vec{F}$ to try $\boxed{\vec{L} = \vec{r}\times\vec{p}}$

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r}\times\vec{p}) = \frac{d\vec{r}}{dt}\times\vec{p} + \vec{r}\times\frac{d\vec{p}}{dt} = \vec{V}\times\vec{p} + \vec{r}\times\vec{F}$$

$$= \vec{V}\times M\vec{V} + \vec{\tau}.$$ $\vec{V}\times M\vec{V} = \emptyset$ BECAUSE $\phi = 0^\circ$ ($|\vec{A}\times\vec{B}| = AB\sin\phi$)

UNIT OF $\vec{L}$: $m \cdot kg \cdot m/s = kg \cdot m^2/s$

$\vec{L} = \vec{r}\times\vec{p}$ WORKS for A POINT PARTICLE (SOMETHING WITH A SINGLE VALUE OF $\vec{V}$). For objects WITH INFINITELY MANY VALUES OF $\vec{V}$, SPLIT INTO <u>MANY</u> single-$\vec{V}$ ~~objects~~ Pieces.



$\vec{V}_i$ IS TANGENTIAL to $\vec{r}_i \Rightarrow \vec{P}_i = M\vec{V}_i$ IS tangent to $\vec{r}_i$

$\Rightarrow L_i = r_i p_i \sin\phi_i = r_i p_i \sin 90^\circ = r_i M_i V_i$

$V_i = r_i\omega \Rightarrow L_i = M_i r_i^2 \omega \Rightarrow L = (\sum_i M_i r_i^2)\omega = I\omega$

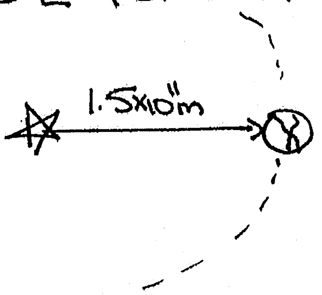
From RHR $\vec{L}_i$ IS IN SAME DIRECTION AS $\vec{\omega}$ (OUT OF PAGE for THIS DRAWING).

$\Rightarrow \vec{L} = \sum_i \vec{L}_i$ Also points in direction of $\vec{\omega}$

$$\Rightarrow \boxed{\vec{L} = I\vec{\omega}}$$

<u>Example</u> 10.36

Find L for Earth circling sun:

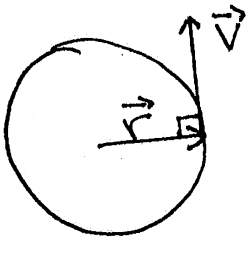


$M = 5.97\times10^{24}$ kg

$r = 1.5\times10^{11}$ m (From Appendix F)

At this scale, the Earth can be treated as point particle $\Rightarrow \vec{L} = \vec{r}\times\vec{p} = \vec{r}\times M\vec{v}$

On a circle, $\vec{r}$ and $\vec{v}$ always tangent.



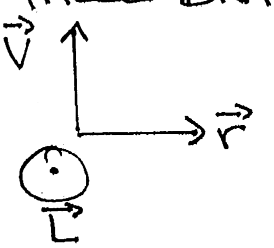
$L = rMv\sin 90° = rMv$ (usually written as $L = Mvr$)

To find V, assume circular orbit $\Rightarrow V = \frac{2\pi r}{T}$. $T = 365\text{ days} = 3.15\times10^{7}$s

$$V = \frac{2\pi(1.5\times10^{11}\text{m})}{3.15\times10^{7}\text{s}} = 29900\text{ m/s}$$

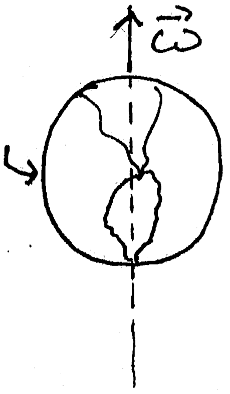
$$\Rightarrow L = (5.97\times10^{24}\text{kg})(29900\text{m/s})(1.5\times10^{11}\text{m}) = 2.68\times10^{40}\text{kgm}^2/\text{s}$$

Given these drawings $\vec{L}$ would be out of page.



$\vec{L} = \vec{r}\times M\vec{v} = M\vec{r}\times\vec{v} \Rightarrow \vec{L} = \odot$

b) FIND L FOR ROTATION ABOUT THE POLES. TREAT AS UNIFORM SPHERE.



HERE EARTH HAS INFINITELY MANY $\vec{v}$'s $\Rightarrow L = I\omega$

$$\omega = \frac{1\text{ Rev}}{24\text{ h}} \times \frac{2\pi\text{ rad}}{\text{rev}} \times \frac{\text{h}}{3600\text{s}} = 7.27\times10^{-5}\text{ rad/s}$$

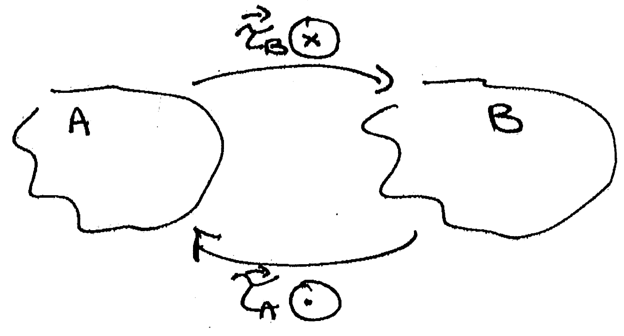
For uniform sphere: $I = \frac{2}{5}MR^2$. $R = 6.38\times10^{6}\text{m}$ (Appendix F)

$$\Rightarrow I = \frac{2}{5}(5.97\times10^{24}\text{kg})(6.38\times10^{6}\text{m})^2 = 9.72\times10^{37}\text{ kg}\cdot\text{m}^2$$

$$L = (9.72\times10^{37}\text{ kg}\cdot\text{m}^2)(7.27\times10^{-5}\text{ rad/s}) = 7.07\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}$$

For drawing above, $\vec{L}$ points up ($\uparrow$)

CONSERVATION OF ANGULAR MOMENTUM - IN THE ABSENCE OF EXTERNAL TORQUE, THE TOTAL ANGULAR MOMENTUM IS CONSTANT.

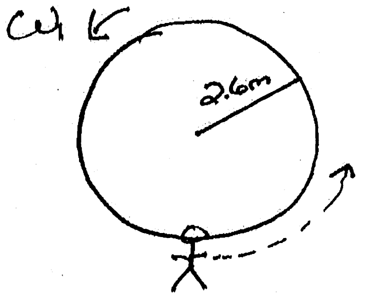


3RD LAW FOR ROTATION:
IF A EXERTS A TORQUE ON B, ($\vec{\tau}_B$)
B EXERTS AN EQUAL BUT OPPOSITE TORQUE ON A, ($\vec{\tau}_A$)

$$\vec{\tau}_A = -\vec{\tau}_B \Rightarrow \vec{\tau}_A + \vec{\tau}_B = \phi \Rightarrow \frac{d\vec{L}_A}{dt} + \frac{d\vec{L}_B}{dt} = 0 \Rightarrow \frac{d}{dt}(\vec{L}_A + \vec{L}_B) = \phi$$

$$\Rightarrow \vec{L}_A + \vec{L}_B = \text{Constant.}$$

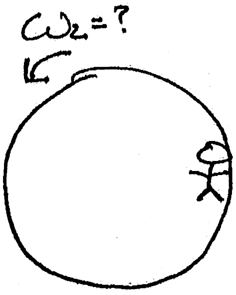
Example: A 60Kg man runs along side a 155Kg, 2.6m radius Merry go round which is rotating at 4 rad/s with a speed of 3.4m/s. If both the man and Merry go round are circling counterclockwise (as viewed from above), what is the angular velocity if the man jumps onto the outer edge of the Merry go round?



$M_{MGr} = 155Kg$, $R = 2.6m$

$M = 60Kg$, $V = 3.4m/s$

$\omega_1 = 4\ rad/s$

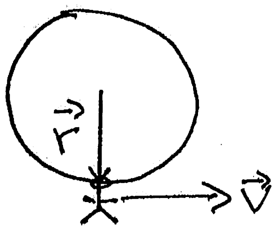


BEFORE

AFTER

$$\vec{L}_{TOTAL} = \vec{L}_{MGR} + \vec{L}_{MAN} = I\vec{\omega}_1 + \vec{r}\times\vec{p}$$

MGR rotating counterclockwise $\Rightarrow$ $\vec{\omega}_1$ out of page $\Rightarrow$ $\vec{L}_1$ out of page.



$\vec{r}\times\vec{p} = \vec{r}\times M\vec{v} = M(\vec{r}\times\vec{v}) \Rightarrow \vec{r}\times\vec{p}$ out of page.

$|\vec{r}\times\vec{p}| = MrV\sin 90° = MrV$. Man right next to MGR $\Rightarrow r = 2.6m$

$\Rightarrow L_{TOTAL} = I\omega_1 + MrV$.

For solid cylinder: $I = \frac{1}{2}M_{MGr}R^2 = \frac{1}{2}(155Kg)(2.6m)^2 = 523.9\ Kg\cdot m^2$

$\Rightarrow L_{TOTAL} = 523.9\ Kg \cdot m^2 (4 rad/s) + 60Kg(2.6m)(3.4m/s) = 2626\ Kg \cdot m^2/s$

AFTER: $\vec{L}_{TOTAL} = I_{TOTAL}\, \vec{\omega}_2 \Rightarrow L_{TOTAL} = I_{TOTAL}\, \omega_2$

$I_{TOTAL} = I_{MGR} + I_{MAN}$. MAN IS POINT PARTICLE $\Rightarrow I_{MAN} = Mr^2$.

MAN JUMPS ONTO OUTER EDGE $\Rightarrow r = 2.6m$

$\Rightarrow I_{TOTAL} = 523.9\ Kg \cdot m^2 + (60Kg)(2.6m)^2 = 929.5 Kg \cdot m^2$

$\Rightarrow L_{TOTAL} = 929.5 Kg \cdot m^2\, \omega_2$

CONSERVATION: $2626 Kg \cdot m^2/s = (929.5 Kg \cdot m^2)\omega_2 \Rightarrow \omega_2 = 2.8\ rad/s$

<u>NOTE</u>: CONSERVATION OF ANGULAR MOMENTUM ALSO OCCURS WITH A SINGLE OBJECT! IF AN OBJECT CHANGES ITS SHAPE, IT ALSO CHANGES ITS MOMENT OF INERTIA. THE ANGULAR MOMENTUM MUST REMAIN THE SAME (ASSUMING THERE ARE NO TORQUES ACTING ON THE OBJECT), $L = I\omega$

$\Rightarrow \boxed{I_1 \omega_1 = I_2 \omega_2}$ OR $\boxed{MV_1 r_1 = MV_2 r_2}$ FOR POINT PARTICLE.