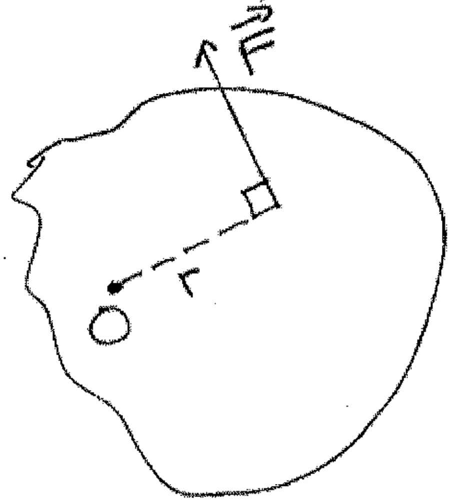
ROTATIONAL DYNAMICS CH 10

DYNAMICS – WHY objects MOVE (LINEARLY)

ROTATIONAL DYNAMICS – WHY objects ROTATE.

FORCES CAUSE LINEAR MOTION. TORQUE (τ) – CAUSE ROTATION.

TORQUE INVOLVES FORCE (YOU MUST PUSH ON SOMETHING TO MAKE IT ROTATE). BUT TORQUE DIFFERS FROM FORCE IN THAT ITS VALUE DEPENDS ON WHERE THE FORCE IS BEING APPLIED.



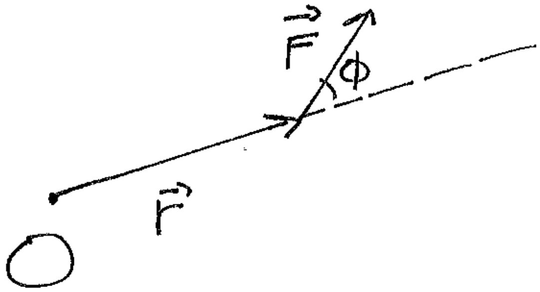
O = POINT THROUGH WHICH THE AXIS OF ROTATION PASSES, i.e., THE POINT WHICH MOVES THE LEAST.

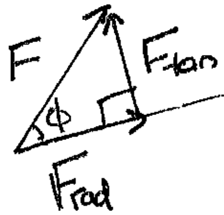
r = distance from AXIS OF ROTATION (O) to POINT WHERE FORCE IS BEING APPLIED. r IS ALSO CALLED THE LEVER AR

For $\vec{F}$ perpendicular AS SHOWN: $\tau = rF$ UNIT = N·m (WHICH IS NOT CALLED A Joule)

For $\vec{F}$ NOT PERP. TO lever ARM, THE COMPONENT OF $\vec{F}$ PERP. TO LEVER ARM DETERMINES TORQUE.

Replace r WITH $\vec{r}$ = VECTOR pointing from O to POINT WHERE $\vec{F}$ IS APPLIED



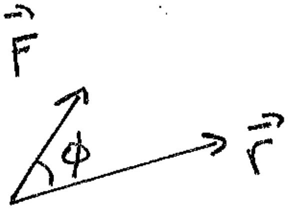


F_{rad} = Component Parallel to $\vec{r}$
(CAUSES NO TORQUE)

F_{tan} = Component perp. to $\vec{r}$

$$F_{tan} = F\sin\phi$$

By DEFINITION, F_{RAD} AND $\vec{r}$ ARE parallel $\Rightarrow$ Angle BETWEEN $\vec{r}$ AND $\vec{F}$ Also ϕ

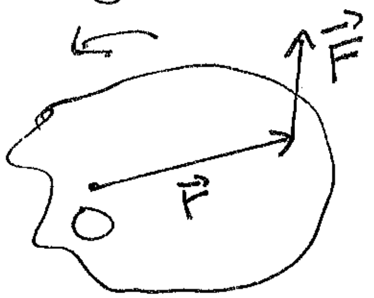


$$\boxed{\tau = rF\sin\phi}$$

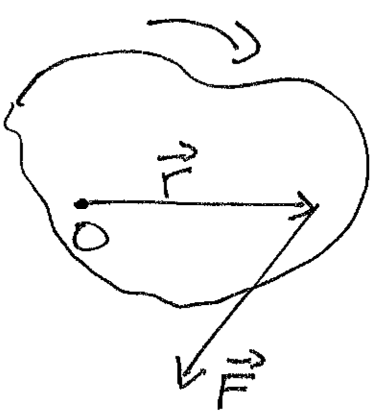
($\sin\phi \Rightarrow \vec{r}\times\vec{F}$ or $\vec{F}\times\vec{r}$ for direction of $\vec{\tau}$)

<u>DIRECTION OF $\vec{\tau}$</u> TORQUES CREATE ROTATIONS $\Rightarrow$ $\vec{\tau}$ AND $\vec{\alpha}$ IN SAME DIRECTION.

AN Object STARTING from REST HAS $\vec{\alpha}$ AND $\vec{\omega}$ IN SAME DIRECTION.



STARTING FROM REST, THIS $\vec{F}$ WOULD CAUSE COUNTER-clockwise ROTATION. Using RHR, $\vec{\omega}$ = OUT OF page (⊙) $\Rightarrow$ $\vec{\alpha}$ = OUT OF page.
Using RHR $\vec{r}\times\vec{F}$ = OUT OF PAGE.



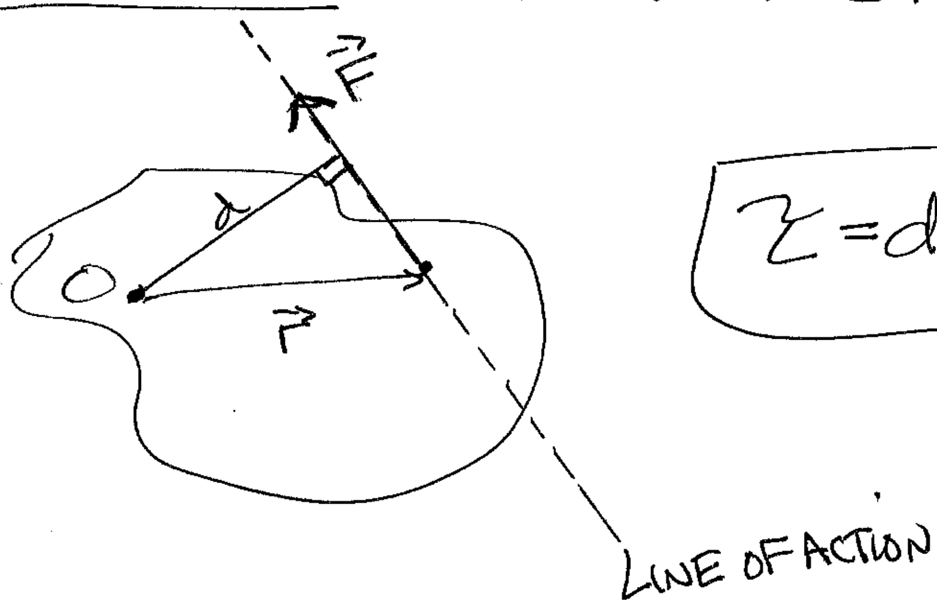
STARTING from REST, $\vec{F}$ WOULD CAUSE clockwise ROTATION $\Rightarrow$ $\vec{\omega}$ = INTO Page (⊗) $\Rightarrow$ $\vec{\alpha}$ = INTO.

RHR $\Rightarrow$ $\vec{r}\times\vec{F}$ = INTO PAGE. $\Rightarrow$ $\boxed{\vec{\tau} = \vec{r}\times\vec{F}}$

PERPENDICULAR DISTANCE → THE CALCULATION OF TORQUE CAN BE SIMPLIFIED IN SOME CASES BY USING THE PERPENDICULAR DISTANCE, d.

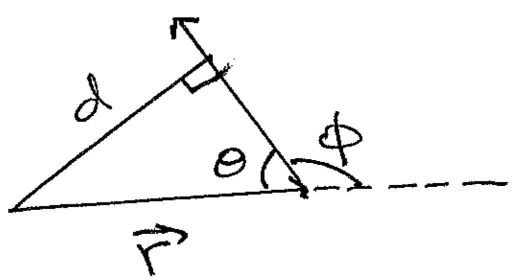
PERPENDICULAR DISTANCE → DISTANCE FROM AXIS OF ROTATION TO FORCE'S LINE OF ACTION, THAT IS PERPENDICULAR TO THE LINE OF ACTION.

LINE OF ACTION → LINE IN SAME DIRECTION AS $\vec{F}$.



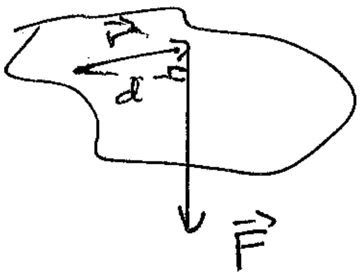
$$\boxed{\tau = dF}$$

Proof:



$$\tau = rF\sin\phi$$
$$= rF\sin(180^\circ - \theta)$$
$$= rF\sin\theta = (r\sin\theta)F$$
$$\Rightarrow \tau = dF$$

PERPENDICULAR DISTANCE IS ESPECIALLY HELPFUL FOR VERTICAL FORCES LIKE GRAVITY.

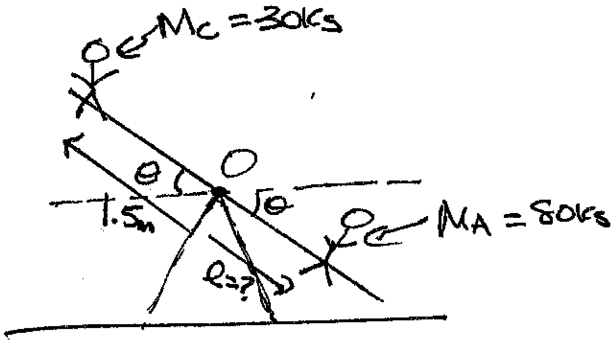


$d = x$ FOR VERTICAL FORCES

$\Rightarrow \tau = xF$

1st LAW FOR ROTATION $\rightarrow$ An object at rest stays at rest, AN object IN UNIFORM ROTATION Stays IN UNIFORM Rotation IF the net torque ACTING ON IT IS ZERO.

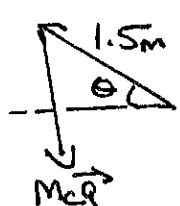
EX: A 30kg CHILD sits ON THE END OF A 3m-long SEESAW. Where MUST AN 80kg ADULT sit to Keep the SEESAW balanced at $\theta = 30°$?



$$\sum \vec{\tau} = 0$$

$$\sum \vec{\tau} = \vec{\tau}_c + \vec{\tau}_A = 0$$

$\vec{\tau}_c$:



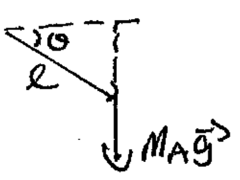
$\tau_c = (1.5m \cos 30°) M_c g$

$= (1.299m)(30kg)(9.8)$

$= 381.9172 \, N \cdot m$

FROM RHR: $\vec{\tau}_c = (1.5m \cos 30°) M_c g$, $\odot$

$\vec{\tau}_A$



$\vec{\tau}_A = l \cos\theta M_A g$, $\otimes$

$\sum \tau = \tau_A - \tau_c = 0$

$\Rightarrow \tau_A = \tau_c \Rightarrow l \cos 30° M_A g = 1.5m \cos 30° M_c g$

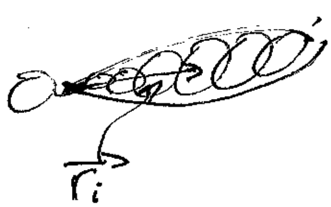
$$\tau_A = \tau_C \Rightarrow l \cos 30^\circ M_A g = 1.5m \cos 30^\circ M_C g$$

$$\Rightarrow l = 1.5m \frac{M_C}{M_A} = 1.5m\left(\frac{30kg}{80kg}\right)$$

$$\Rightarrow \boxed{l = 1.5m\left(\frac{3}{8}\right) = .5625m}$$

NOTICE RESULT IS TRUE FOR ANY ANGLE θ.

NEWTON'S 2ND LAW FOR ROTATION : Break Rotating object into many small pieces:

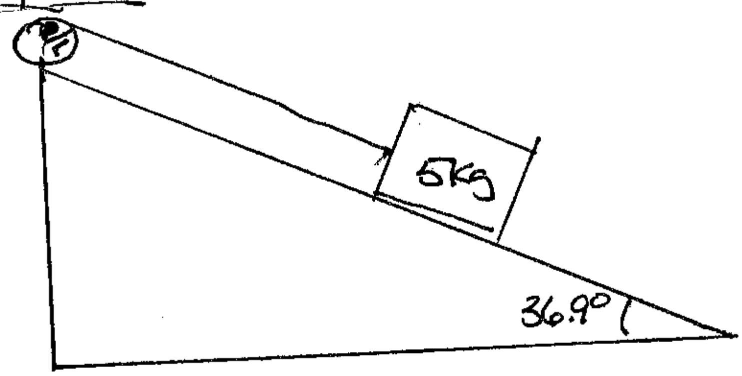


$$\tau_i = r_i F_i \sin\phi_i = r_i F_{i,tan}. \quad F_{tan} = Ma_{tan} = M\alpha r$$

$$\Rightarrow \sum_i \tau_i = r_i (M_i \alpha r_i) = \left(\sum_i M_i r_i^2\right)\alpha \rightarrow I\alpha$$

$\vec{\tau}$ AND $\vec{\alpha}$ HAVE SAME DIRECTION $\Rightarrow \boxed{\sum \vec{\tau} = I\vec{\alpha}}$

Example 10.70 $\rightarrow$ MASS CONNECTED TO A Flywheel $\Rightarrow$ MASSIVE pulley



$M_{Flywheel} = 25kg$

$I = 0.5\ kg \cdot m^2$

$r = 0.2m,\ \mu_K = 0.25$

What is tension & Acceleration?

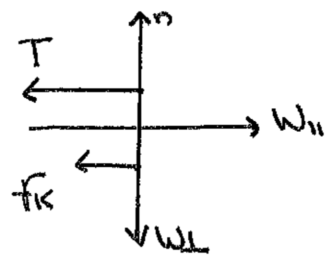
STEPS ARE SAME AS ANY NEWTON'S LAW PROBLEM, BUT MUST INCLUDE FLYWHEEL.

FORCES ON MASS: $\vec{T}$ UP INCLINE, $\vec{f}_K$ UP INCLINE, $W_\perp$, $W_\parallel$, $\vec{n}$

FLYWHEEL: $\vec{T}$ DOWN INCLINE, ($\vec{W}$ DOWN, $\vec{n}$ UP)

NOT IMPORTANT BECAUSE ACT AT $r=0$
$\Rightarrow$ EXERT NO TORQUE

FREE BODY DIAGRAM FOR 5kg MASS:



$F_{NET,\perp} = n - W_\perp = Ma_\perp$. $a_\perp = 0 \Rightarrow n = W_\perp = Mg\cos 36.9°$

$F_{NET,\parallel} = W_\parallel - T - f_K = Ma_\parallel$. $f_K = \mu_K n$

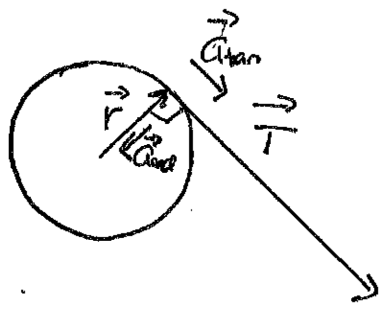
$\Rightarrow Mg\sin 36.9° - T - \mu_K Mg\cos 36.9° = Ma_\parallel$

$\Rightarrow Mg(\sin 36.9° - \mu_K \cos 36.9°) - T = Ma_\parallel$

$\Rightarrow 5kg(9.8 m/s^2)(\sin 36.9° - .25\cos 36.9°) - T = (5kg)a_\parallel$

$\Rightarrow 19.6N - T = (5kg)a_\parallel$

"FREE BODY DIAGRAM" FOR FLYWHEEL:



$\tau = rT\sin 90° = rT = I\alpha$

FLYWHEEL HAS TWO ACCELERATIONS: $\vec{a}_{rad}$, $\vec{a}_{tan}$

$\vec{a}_{rad}$ TOWARDS CENTER, $\vec{a}_{tan}$ 90° to circle

$\vec{\tau} = \vec{r} \times \vec{T} \Rightarrow \vec{\tau}$ = INTO PAGE. $\vec{\tau} = I\vec{\alpha} \Rightarrow \vec{\alpha}$ INTO PAGE.

$\vec{a}_{tan} = \vec{\alpha} \times \vec{r} \Rightarrow \vec{a}_{tan}$ = DOWN INCLINE (Parallel to $\vec{T}$) AND $a_{tan} = \alpha r \sin 90°$

$\Rightarrow \alpha = \dfrac{a_{tan}}{r}$

$$\Rightarrow rT = I\left(\frac{a_{tan}}{r}\right) \Rightarrow T = \frac{I}{r^2} a_{tan} = \frac{.5\,kg\cdot m^2}{(.2m)^2} a_{tan} \Rightarrow T = (12.5kg)\,a_{tan}$$

STRING IS CONNECTED TO EDGE OF FLYWHEEL $\Rightarrow a_{string} = a_{tan}$.

STRING IS ALSO CONNECTED TO 5kg MASS $\Rightarrow a_{string} = a_{\parallel}$

$$\Rightarrow a_{tan} = a_{\parallel} = a$$

$$\Rightarrow \begin{aligned} 19.6N - T &= (5kg)a \\ T &= (12.5kg)a \end{aligned} \quad \Rightarrow \quad \begin{aligned} &19.6N = (17.5kg)a \Rightarrow a = 1.12\,m/s^2 \\ &T = (12.5kg)(1.12\,m/s^2) = 14N \end{aligned}$$