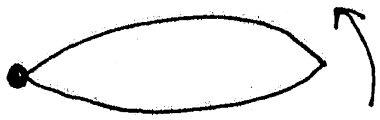
PHYS 160: ROTATIONAL ENERGY CH 9

KINETIC ENERGY $K = \frac{1}{2}MV^2$ BUT VELOCITY INCREASES AS YOU GET FARTHER FROM ROTATIONAL AXIS.

WE BREAK AN OBJECT INTO SMALLER COMPACT PARTICLES. EACH ONE IS SMALL ENOUGH TO TREAT AS HAVING A SINGLE VELOCITY, e.g.,



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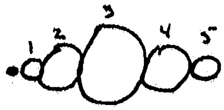
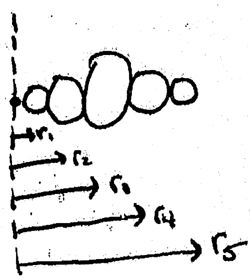


$K = K_1 + K_2 + K_3 + \ldots$

$K = \sum_i K_i$ (SUM OF INDIVIDUAL KINETIC ENERGIES IS TOTAL)

$\Rightarrow K = \sum_i \frac{1}{2} M_i V_i^2 = \sum_i \frac{1}{2} M_i (r_i \omega)^2$

r_i: DISTANCE FROM ROT. AXIS TO CENTER OF "ith" PIECE



$K = \sum_i \frac{1}{2} M_i r_i^2 \omega^2 = \frac{1}{2} \left(\sum_i M_i r_i^2 \right) \omega^2$

$\boxed{K = \frac{1}{2} I \omega^2}$ $\boxed{I = \sum_i M_i r_i^2}$ MOMENT OF INERTIA

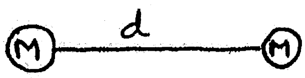
UNIT: Kgm^2

Moment of inertia is rotational analog to mass. It is more complicated than mass! Its value depends on <u>mass</u>, <u>shape</u> and which <u>axis</u> the object is rotating about.

For discrete objects, we use $I = \sum_i m_i r_i^2$

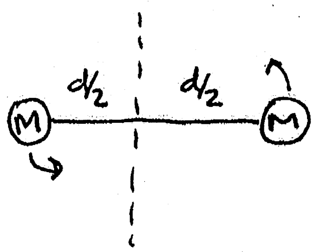
Objects with point particles which are fixed into a shape.

<u>Example</u> What is I of a dumbbell about the following axes?

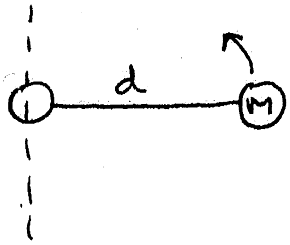


Dumbbell = two equal masses connected by a massless rod of length d.

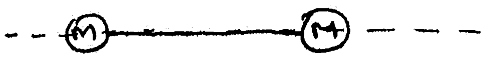
Remember r_i = length from axis to mass.



$$I = M(-d/2)^2 + M(d/2)^2 = 2Md^2/4 = \frac{Md^2}{2}$$



$$I = M(0) + Md^2 = Md^2$$



$$I = M(0) + M(0) = 0$$

Point particles have no width $\Rightarrow$ can't be rotated this way.

FOR CONTINUOUS OBJECTS, WE LET SIZE OF PIECES GO TO ZERO, BUT INCREASE NUMBER OF PIECES TO INFINITY.

$\downarrow$

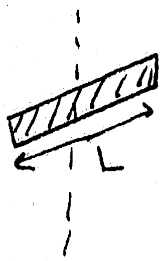
MASS SPREAD EVENLY OVER OBJECT

$$\Rightarrow \sum r_i^2 M_i \rightarrow \int r^2 \, dM \qquad dM = \text{MASS element}$$

$$dM = \underbrace{\rho}_{\text{density}} dV \quad \Rightarrow \boxed{I = \int r^2 \rho \, dV}$$

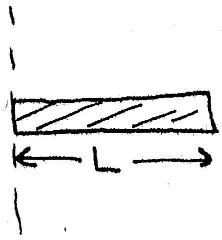
See examples p. 303-305

CERTAIN SHAPES AND AXES ARE ALREADY CALCULATED ON p. 299

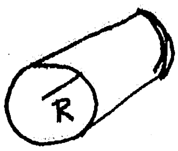


SLENDER ROD, AXIS THROUGH CENTER

$I = \frac{1}{12} ML^2$



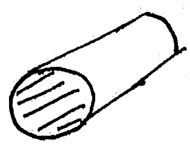
$I = \frac{1}{3} ML^2$



THIN WALLED CYLINDER

$I = MR^2$

(NOTICE, LENGTH NOT IMPORTANT)



SOLID CYLINDER

$I = \frac{1}{2} MR^2$

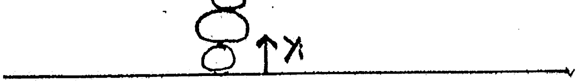
$\boxed{\text{WORK-ENERGY THM HOLDS FOR ROTATIONAL KINETIC ENERGY!}}$

$$W_{TOTAL} = \Delta K = K_2 - K_1 .$$

FOR $K_1 = 0$, $W_{TOTAL} = \frac{1}{2} I \omega^2$

LARGER I $\Rightarrow$ MORE WORK NECESSARY TO START OBJECT ROTATING

POTENTIAL ENERGY: USE A DISCRETE OBJECT



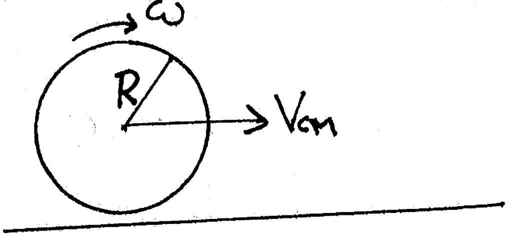
$$U = M_1 g y_1 + M_2 g y_2 + \ldots = \sum_i M_i g y_i$$
$$= g \sum_i M_i y_i$$

$$y_{CM} = \frac{\sum_i M_i y_i}{\sum_i M_i} = \frac{\sum_i M_i y_i}{M} \Rightarrow \sum_i M_i y_i = M y_{cm}$$

$$\Rightarrow \boxed{U = M g y_{CM}}$$

POTENTIAL ENERGY IS DETERMINED BY TOTAL MASS AND CENTER OF MASS'S HEIGHT.

ROLLING: WHEN SOMETHING ROLLS IT HAS ROTATIONAL AND TRANSLATIONAL ENERGY



$$K = \frac{1}{2} M V_{CM}^2 + \frac{1}{2} I \omega^2$$

WE ALWAYS ASSUME OBJECTS ROLL WITHOUT SLIPPING $\Rightarrow \boxed{V_{CM} = \omega R}$

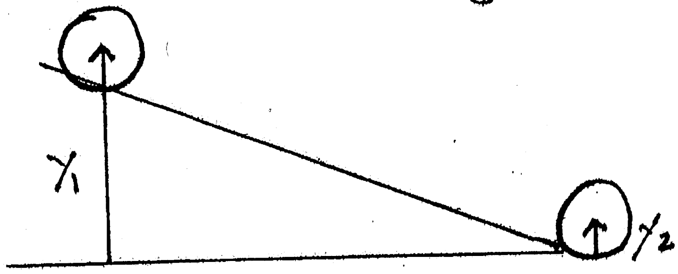
$$\Rightarrow \omega = \frac{V_{cm}}{R} \Rightarrow K = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} I \left(\frac{V_{cm}}{R}\right)^2 = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} \frac{I}{R^2} V_{cm}^2$$

$$\Rightarrow K = \frac{1}{2} M V_{cm}^2 + \frac{1}{2} M V_{cm}^2 \left(\frac{I}{MR^2}\right) \Rightarrow \boxed{K = \frac{1}{2} M V_{cm}^2 \left(1 + \frac{I}{MR^2}\right)}$$

EXAMPLE: TWO CYLINDERS ARE STARTED FROM REST AT THE TOP OF AN ALMOST FRICTIONLESS INCLINE (ENOUGH FRICTION SO THEY CAN ROLL, BUT NOT ENOUGH TO SLOW THEM DOWN) OF HEIGHT 1m. THE CYLINDERS HAVE SAME MASS AND RADIUS, BUT ONE IS SOLID THE OTHER IS HOLLOW. WHAT ARE THEIR SPEEDS AT THE BOTTOM ASSUMING THEY ROLL WITHOUT SLIPPING?

TO AVOID TOO MANY SUBSCRIPTS, LET'S DROP THE "CM" FROM OUR VARIABLES.

CONSERVATION OF ENERGY $\Rightarrow \frac{1}{2}MV_1^2\left(1+\frac{I}{MR^2}\right)+Mgy_1 = \frac{1}{2}MV_2^2\left(1+\frac{I}{MR^2}\right)+Mgy_2$



$V_1 = 0$, $y_1 = 1m+R$ (R = RADIUS),

$y_2 = R$

$$\Rightarrow \frac{1}{2}MV_2^2\left(1+\frac{I}{MR^2}\right) = Mg(y_1-y_2) = Mg\underbrace{h}_{1m} \Rightarrow V_2 = \left[\frac{2gh}{1+I/MR^2}\right]^{1/2}$$

FOR HOLLOW CYLINDER: $I = MR^2 \Rightarrow 1+\frac{I}{MR^2} = 1+\frac{MR^2}{MR^2} = 2$

$$\Rightarrow V_2 = \sqrt{\frac{2gh}{2}} = \sqrt{(9.8m/s^2)(1m)} = 3.13m/s$$

FOR SOLID CYLINDER: $I = \frac{1}{2}MR^2 \Rightarrow 1+\frac{I}{MR^2} = 1+\frac{1}{2} = \frac{3}{2}$

$$\Rightarrow V_2 = \sqrt{\frac{2gh}{3/2}} = \sqrt{\frac{4(9.8m/s^2)(1m)}{3}} = 3.61m/s$$

SOLID CYLINDER ROTATING FASTER $\Rightarrow$ IT WILL REACH BOTTOM OF INCLINE FIRST!

IF THEY WERE SLIDING, THEY WOULD REACH THE BOTTOM AT THE SAME TIME.