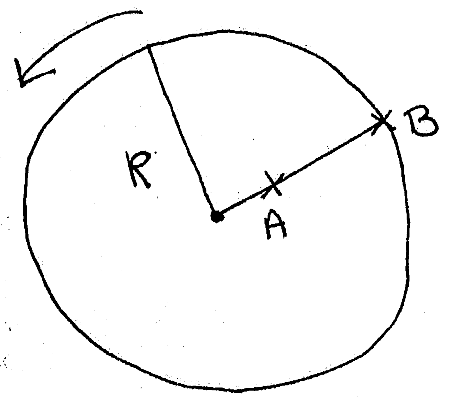
PHYS 160: ROTATIONAL MOTION CH9

ROTATING = SPINNING.

WE ALWAYS DISCUSS A RIGID BODY ⇒ DOESN'T CHANGE SHAPE

CIRCULAR OBJECTS



FOR ONE ROTATION (REVOLUTION), DISTANCE TRAVELLED BY

A: $2\pi r_A$

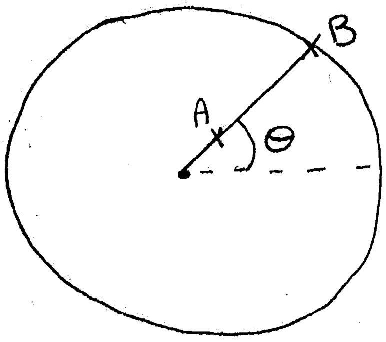
B: $2\pi r_B$

$r_B > r_A$ ⇒ B GOES FARTHER DISTANCE.

HOWEVER!! THEY TAKE THE SAME AMOUNT OF TIME TO GO AROUND.

⇒ $V_B > V_A$! POINTS FARTHER FROM CENTER HAVE LARGER SPEEDS.

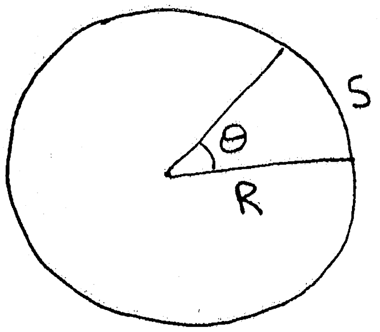
BUT ALL POINTS SPIN AT THE SAME RATE (THE DISTANCE FROM A to B STAYS SAME) ⇒



ALL POINTS (NOT JUST A AND B) TRAVEL DIFFERENT DISTANCES BUT MAKE THE SAME ANGLE.

WE NEED TO DISTINGUISH LINEAR VELOCITY = $\frac{\text{DISTANCE}}{\text{TIME}}$ FROM ANGULAR VELOCITY = $\frac{\text{ANGLE}}{\text{TIME}}$.

RADIANS — WE MEASURE ANGLES IN RADIANS **NOT** DEGREES.



IN RADIANS $S = R\theta$ → ARCLENGTH

(N.B. $\theta = \frac{S}{R}$ ⇒ HAS NO UNITS. BUT WE USE THE TERM "RAD" TO INDICATE AN ANGLE QUANTITY)

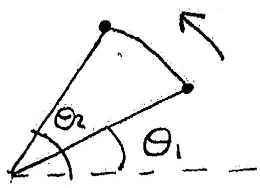
CIRCUMFERENCE IS AN ARCLENGTH FOR $\theta = 360°$

$C = 2\pi R = \theta R$ ⇒ 2π RAD $= 360°$

$1 \text{ RAD} = \frac{360°}{2\pi} = 57.3°$

$180° = \pi$ RAD, $90° = \frac{\pi}{2}$ RAD, $45° = \pi/4$ RAD, $30° = \pi/6$ RAD, etc.

ANGULAR VELOCITY



$$\omega_{AV} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

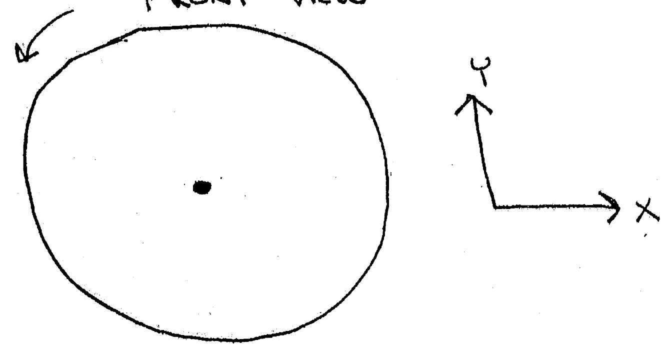
UNIT: $\frac{\text{RAD}}{S} = \frac{1}{S}$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

THE DIRECTION FOR ω IS RELATED TO THE AXIS OF ROTATION.

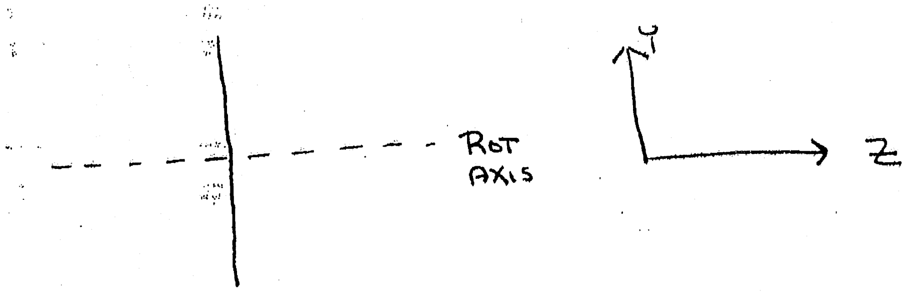
ROTATIONAL AXIS – LINE GOING THROUGH THE POINT OF ROTATING BODY WHICH MOVES THE LEAST AND IS PERP. TO MOTION.

FRONT VIEW



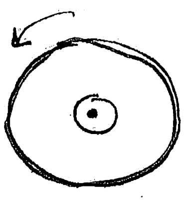
ROTATIONAL AXIS GOES THROUGH CENTER AND POINTS STRAIGHT INTO THE PAGE. WE USUALLY CALL THIS THE Z-AXIS.

SIDE VIEW

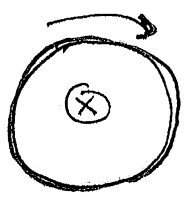


$\vec{\omega}$ POINTS ALONG THE ROTATIONAL AXIS. WE USE THE RIGHT HAND RULE (RHR) TO DETERMINE.

RHR – CURL FINGERS OF RIGHT HAND IN ~~DIRECTION~~ SENSE OF ROTATION, YOUR THUMB POINTS IN DIRECTION OF $\vec{\omega}$.



$\vec{\omega}$ = OUT OF PAGE



$\vec{\omega}$ = INTO PAGE.

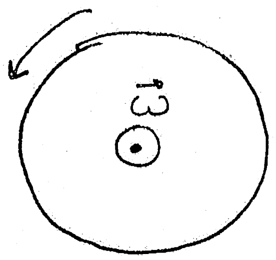
NOTATION FOR 3D DRAWING ⊗ = INTO PAGE, ⊙ = OUT OF PAGE

ANGULAR ACCELERATION

FOR $\vec{\omega}$ to CHANGE ⇒ ACCELERATION

$$\vec{\alpha}_{AV} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1} = \frac{\Delta\vec{\omega}}{\Delta t} \Rightarrow \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

UNIT: RAD/s^2



IF ω IS INCREASING ⇒ $\vec{\alpha}$ IN SAME DIRECTION AS $\vec{\omega}$

IF ω IS DECREASING ⇒ $\vec{\alpha}$ IN OPPOSITE DIRECTION TO $\vec{\omega}$

FOR CONSTANT α ⇒ KINEMATIC EQUATIONS

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

EXAMPLE A PHONOGRAPH IS SPINNING AT 45RPM (Rev. per minute). IF IT TAKES 20s to stop, WHAT IS α?

$\omega_0 = 45\text{RPM}$, $\omega = 0$, $t = 20s$.

1 Rev = 2π RAD

$$\omega_0 = \frac{45\text{Rev}}{\text{MIN}} \times \frac{2\pi\text{RAD}}{\text{Rev}} \times \frac{\text{MIN}}{60s} = 4.71\ \text{RAD/s}$$

$\omega = \omega_0 + \alpha t \Rightarrow 0 = 4.71\ \text{RAD/s} + \alpha(20s) \Rightarrow \alpha = -.236\ \text{RAD/s}^2$

– HOW MANY TIMES DID IT TURN WHILE STOPPING?

$$\theta = (4.71\ \text{RAD/s})(20s) + \tfrac{1}{2}(-.236\ \text{RAD/s}^2)(20s)^2 = 47\ \text{RAD}$$

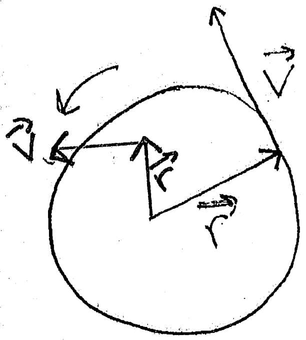
$$47\ \text{RAD} \times \frac{\text{Rev}}{2\pi\ \text{RAD}} = 7.5\ \text{Rev}$$

RELATING $\vec{V}$ & $\vec{\omega}$

FOR A CIRCLE, $S = r\theta$. S = ARCLENGTH = LINEAR DISTANCE, i.e., DISTANCE IN meters.

r = CONSTANT $\Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt} \Rightarrow$ $\boxed{V = r\omega}$

TO GET DIRECTION IS TRICKIER!



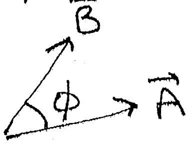
$\vec{V}$ ALWAYS TANGENTIAL TO CIRCLE $\Rightarrow 90^\circ$ to $\vec{r}$

$\vec{\omega}$ = OUT OF PAGE

WE USE THE CROSS PRODUCT $\boxed{\vec{V} = \vec{\omega} \times \vec{r}}$

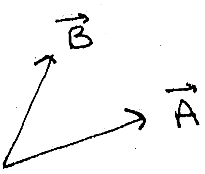
FOR ANY TWO VECTORS $\vec{A}$ AND $\vec{B}$, $\vec{A} \times \vec{B}$ IS ANOTHER VECTOR

$|\vec{A} \times \vec{B}| = AB\sin\phi$



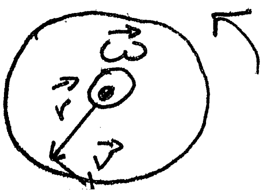
$\vec{A} \times \vec{B}$ IS PERPENDICULAR TO BOTH $\vec{A}$ AND $\vec{B}$

DIRECTION IS GIVEN BY ANOTHER RHR $\rightarrow$ TAKE FINGERS OF RIGHT HAND AND "SWEEP" $\vec{A}$ INTO $\vec{B}$, i.e., CURL YOUR FINGERS IN THE DIRECTION FROM $\vec{A}$ to $\vec{B}$ (going by smaller angle). YOUR THUMB POINTS IN DIRECTION OF CROSS PRODUCT.

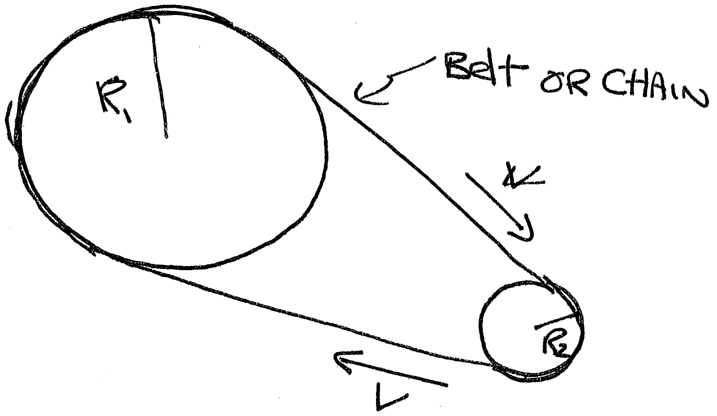


$\vec{A} \times \vec{B}$ = OUT OF PAGE
$\vec{B} \times \vec{A}$ = INTO PAGE

$\Rightarrow$ ORDER IMPORTANT!
$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$



CONNECTED ROTATING OBJECTS



IF BELT MOVES WITHOUT SLIPPING
⇒ SAME LINEAR SPEED FOR BOTH

$$\Rightarrow V_1 = V_2$$

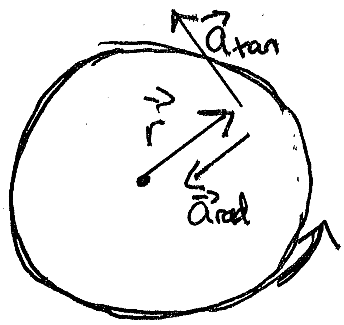
$$V_1 = \omega_1 R_1 \,,\; V_2 = \omega_2 R_2 \;\Rightarrow\; \omega_1 R_1 = \omega_2 R_2$$ ← DIFFERENT ANGULAR speeds!

$$\boxed{\omega_2 = \omega_1 \left(\frac{R_1}{R_2}\right)}$$

ACCELERATION – $\vec{a}$ HAS TWO COMPONENTS

a_{rad} = Centripetal = CHANGE IN DIRECTION

a_{tan} = tangential = CHANGE IN ANGULAR SPEED

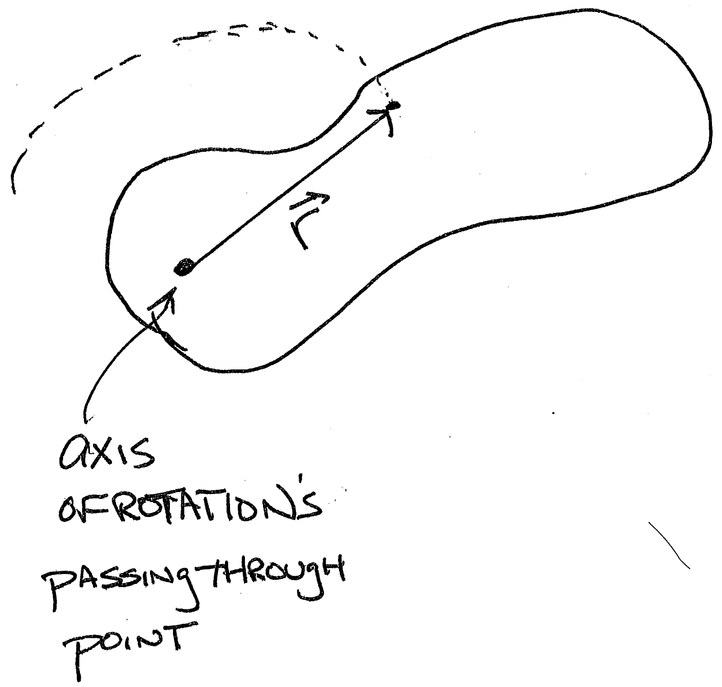


$$a_{rad} = \frac{v^2}{r} = \omega^2 r \qquad a_{tan} = \alpha r$$

$$\vec{a}_{rad} = -\omega \vec{r} \qquad \vec{a}_{tan} = \vec{\alpha} \times \vec{r}$$

$$\vec{a} = \vec{a}_{rad} + \vec{a}_{tan} \qquad a = \sqrt{a_{rad}^2 + a_{tan}^2}$$

NON-CIRCULAR OBJECTS → SET ORIGIN AT POINT WHERE AXIS OF ROTATION PASSES THROUGH OBJECT.



All points rotate on circle of radius r.

⇒ All EQUATIONS STILL APPLY.