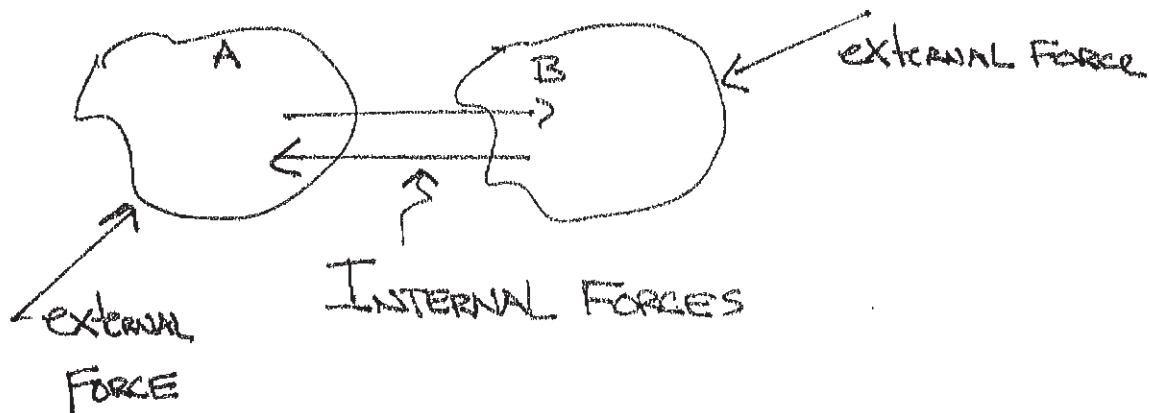


Collisions - CHAPTER 8

MOMENTUM WILL NOT BE CONSERVED WHEN THERE ARE EXTERNAL FORCES. FORCES OUTSIDE OF THE SYSTEM



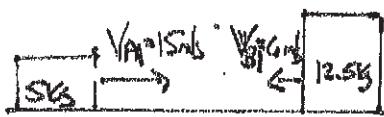
Collision → ANY STRONG INTERACTION THAT LASTS A RELATIVELY SHORT TIME. COLLISIONS ARE ASSUMED TO CONSERVE MOMENTUM BECAUSE OF THEIR SHORT DURATION. EXTERNAL FORCES DON'T GET A CHANCE TO ACT.

COLLISIONS ARE CLASSIFIED AS TO WHETHER THEY CONSERVE KINETIC ENERGY. → (JUST KINETIC!)

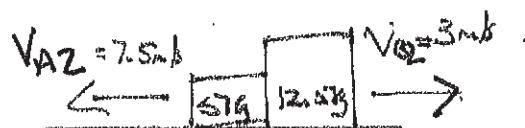
ELASTIC COLLISION → CONSERVE KINETIC ENERGY & MOMENTUM

INELASTIC COLLISION → CONSERVE MOMENTUM ONLY

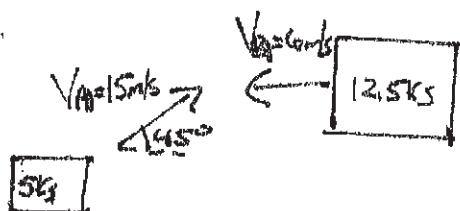
Example: ARE THE COLLISIONS FROM THE PREVIOUS LECTURE ELASTIC?



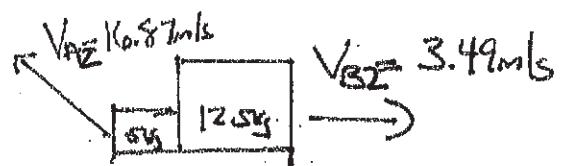
BEFORE



AFTER



BEFORE



AFTER

NEED TO CHECK, DOES THE SYSTEM CONSERVE KINETIC ENERGY,

$$\text{i.e., IF } K_1 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2 \text{ AND } K_2 = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$$

DOES $K_1 = K_2$?

$$\text{1st Collision: } K_1 = \frac{1}{2} (5\text{kg})(15\text{m/s})^2 + \frac{1}{2} (12.5\text{kg})(6\text{m/s})^2 = 787.5\text{J}$$

$$K_2 = \frac{1}{2} (5\text{kg})(7.5\text{m/s})^2 + \frac{1}{2} (12.5\text{kg})(3\text{m/s})^2 = 196.875\text{J}$$

NOT ELASTIC! "MISSING" KE CONVERTED INTO HEAT

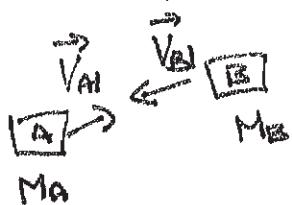
$$\text{2nd Collision: } K_1 = \frac{1}{2} (5\text{kg})(15\text{m/s})^2 + \frac{1}{2} (12.5\text{kg})(6\text{m/s})^2 = 787.5\text{J}$$

$$K_2 = \frac{1}{2} (5\text{kg})(16.87\text{m/s})^2 + \frac{1}{2} (12.5\text{kg})(3.49\text{m/s})^2 = 787.6\text{J}$$

CLOSE ENOUGH! YES, ELASTIC
(2)

Completely Inelastic Collisions - When THE COLLIDING

Objects stick together AFTER A COLLISION, IT CAN NEVER be elastic.



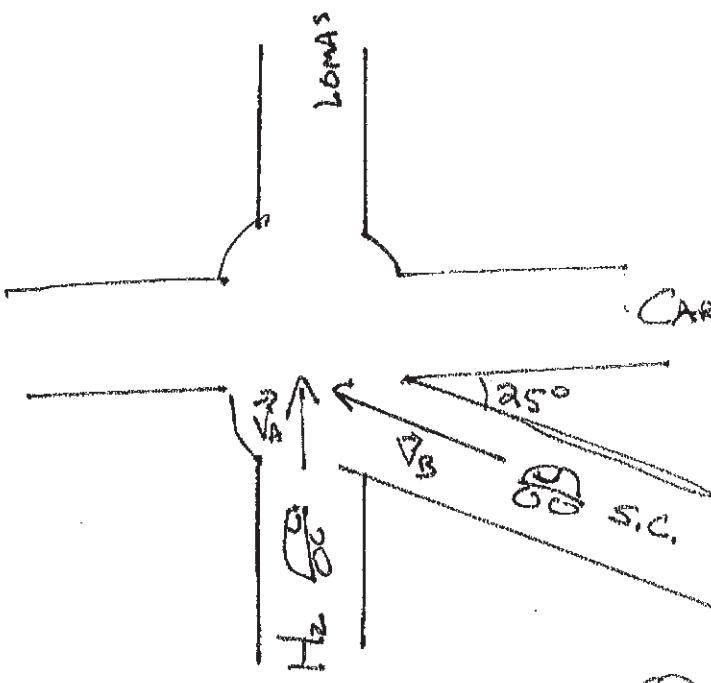
BEFORE

AFTER

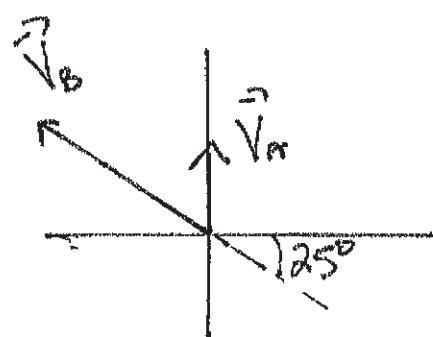
Still conserves momentum:

$$M_A \vec{V}_A + M_B \vec{V}_B = (M_A + M_B) \vec{V}_2$$

Example: A 2900kg HUMMER H₂ going 11m/s, EAST on LOMAS BLVD has a completely INELASTIC collision with a 730kg SMART CAR going 36m/s on Monte Vista Blvd (as shown). What is FINAL SPEED AND DIRECTION?



(3)



\vec{V}_A at 90°

\vec{V}_B at $180^\circ - 25^\circ = 155^\circ$

$$M_A \vec{V}_{A,i} + M_B \vec{V}_{B,i} = (M_A + M_B) \vec{V}_2 \Rightarrow M_A V_{A,i,x} + M_B V_{B,i,x} = (M_A + M_B) V_{2,x}$$

$$M_A V_{A,i,y} + M_B V_{B,i,y} = (M_A + M_B) V_{2,y}$$

$$M_A = 2900 \text{ kg}, V_{A,i,x} = 0, V_{A,i,y} = 11 \text{ m/s}$$

$$M_B = 730 \text{ kg}, V_{B,i,x} = 30 \text{ m/s} \cos 155^\circ = -27.19 \text{ m/s}$$

$$V_{B,i,y} = 30 \text{ m/s} \sin 155^\circ = 12.68 \text{ m/s}$$

$$\Rightarrow 2900 \text{ kg} (0) + 730 \text{ kg} (-27.19 \text{ m/s}) = (2900 \text{ kg} + 730 \text{ kg}) V_{2,x}$$

$$\Rightarrow -19848.7 \text{ Kg.m/s} = (3630 \text{ kg})$$

$$\Rightarrow V_{2,x} = \frac{-19848.7 \text{ Kg.m/s}}{3630 \text{ kg}} = -5.468 \text{ m/s}$$

$$2900 \text{ kg} (11 \text{ m/s}) + 730 \text{ kg} (12.68 \text{ m/s}) = (2900 \text{ kg} + 730 \text{ kg}) V_{2,y}$$

$$\Rightarrow 41156.4 \text{ Kg.m/s} = (3630 \text{ kg}) V_{2,y}$$

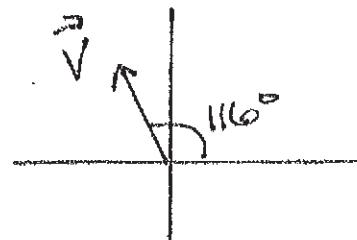
$$\Rightarrow V_{2,y} = \frac{41156.4 \text{ Kg.m/s}}{3630 \text{ kg}} = 11.34 \text{ m/s}$$

$$V_2 = \sqrt{V_{2,x}^2 + V_{2,y}^2} = \sqrt{(5.468 \text{ m/s})^2 + (11.34 \text{ m/s})^2} = 12.6 \text{ m/s}$$

$$\Theta = \tan^{-1} \left(\frac{11.34}{5.468} \right) = -64^\circ + 180^\circ = 116^\circ$$

\swarrow
WRONG QUADRANT

(4)



Elastic Collisions:

ELASTIC COLLISIONS CONSERVE MOMENTUM AND ENERGY. THEY OCCUR WHEN THERE IS NO FRICTION BETWEEN THE COLLIDING OBJECTS AND NO DEFORMATION (CHANGE OF SHAPE). BILLIARD COLLISIONS ARE VERY CLOSE TO BEING ELASTIC.

FOR AN ELASTIC COLLISION, WE MUST HAVE :

$$M_A \vec{V}_{A1} + M_B \vec{V}_{B1} = M_A \vec{V}_{A2} + M_B \vec{V}_{B2}$$

$$\text{AND } \frac{1}{2} M_A V_{A1}^2 + \frac{1}{2} M_B V_{B1}^2 = \frac{1}{2} M_A V_{A2}^2 + \frac{1}{2} M_B V_{B2}^2$$

One Dimension

IN 1D, THE VELOCITIES HAVE JUST ONE COMPONENT, e.g. $x \Rightarrow V_{A1,x} = V_{A1}$ etc.

$$\Rightarrow M_A V_{A1} + M_B V_{B1} = M_A V_{A2} + M_B V_{B2}$$

$$\frac{1}{2} M_A V_{A1}^2 + \frac{1}{2} M_B V_{B1}^2 = \frac{1}{2} M_A V_{A2}^2 + \frac{1}{2} M_B V_{B2}^2$$

IF V_{A1}, V_{B1}, M_A, M_B ARE KNOWN \Rightarrow 2 UNKNOWN AND 2 EQUATIONS
 \Rightarrow THERE IS ONLY ONE WAY TO HAVE AN ELASTIC COLLISION.

IT CAN BE SHOWN (SEE APPENDIX) THAT TO MAKE A 1D COLLISION ELASTIC, WE REQUIRE THAT :

$$V_{A2} - V_{B2} = -(V_{A1} - V_{B1})$$

$V_A - V_B = \text{RELATIVE VELOCITY}$

$$V_{A2} - V_{B2} = -(V_{A1} - V_{B1}) \Rightarrow V_{A2} - V_{B2} = -V_{A1} + V_{B1}$$

FROM MOMENTUM: $M_A V_{A2} + M_B V_{B2} = M_A V_{A1} + M_B V_{B1}$

$$M_A V_{A2} + M_B V_{B2} = M_A V_{A1} + M_B V_{B1}$$

$$M_B [V_{A2} - V_{B2} = -V_{A1} + V_{B1}]$$

$$V_{A2} (M_A + M_B) = V_{A1} (M_A - M_B) + 2M_B V_{B1}$$

$$\Rightarrow V_{A2} = \frac{V_{A1} (M_A - M_B) + 2M_B V_{B1}}{(M_A + M_B)}$$

V_{A2} FOR 1D ELASTIC
COLLISION

$$M_A V_{A2} + M_B V_{B2} = M_A V_{A1} + M_B V_{B1}$$

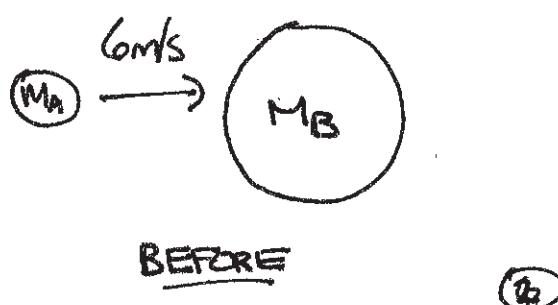
$$-M_A [V_{A2} - V_{B2} = -V_{A1} + V_{B1}]$$

$$V_{B2} (M_A + M_B) = 2M_A V_{A1} + V_{B1} (M_B - M_A)$$

$$\Rightarrow V_{B2} = \frac{V_{B1} (M_B - M_A) + 2M_A V_{A1}}{(M_A + M_B)} \Rightarrow V_{B2} = \frac{-V_{B1} (M_A - M_B) + 2M_A V_{A1}}{(M_A + M_B)}$$

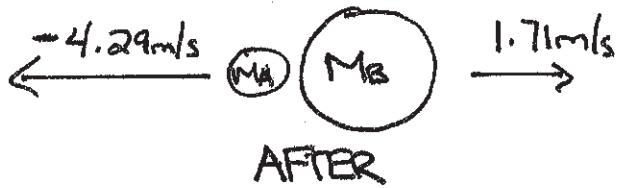
V_{B2}
ELASTIC
COLLISION

EXAMPLE: $M_A = 5\text{kg}$, $V_{A1} = 6\text{m/s}$, $M_B = 30\text{kg}$, $V_{B1} = 0$



$$\text{SINCE } V_{B1} = 0 \Rightarrow V_{A2} = \frac{V_{A1}(M_A - M_B)}{(M_A + M_B)} = \frac{6\text{m/s}(5\text{kg} - 30\text{kg})}{(5\text{kg} + 30\text{kg})} = -4.29\text{m/s}$$

$$V_{B2} = \frac{2M_A V_{A1}}{(M_A + M_B)} = \frac{2(5\text{kg})(6\text{m/s})}{(5\text{kg} + 30\text{kg})} = 1.71\text{m/s}$$



$$\text{Notice: } V_{A1} - V_{B1} = 6\text{m/s}, \quad V_{A2} - V_{B2} = -4.29\text{m/s} - 1.71\text{m/s} = -6\text{m/s}$$

TWO-DIMENSIONAL ELASTIC COLLISIONS

WITH COMPONENTS, MOMENTUM CONSERVATION BECOMES 2 EQUATIONS

$$M_A V_{A1,x} + M_B V_{B1,x} = M_A V_{A2,x} + M_B V_{B2,x}$$

$$M_A V_{A1,y} + M_B V_{B1,y} = M_A V_{A2,y} + M_B V_{B2,y}$$

ENERGY (BEING A SCALAR) IS STILL ONE EQUATION:

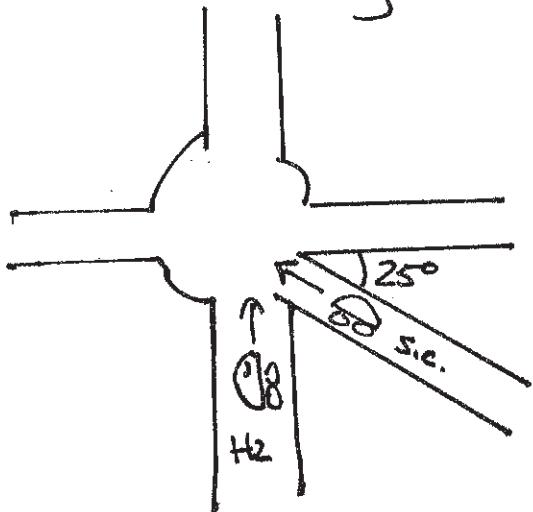
$$\frac{1}{2} M_A (V_{A1,x}^2 + V_{A1,y}^2) + \frac{1}{2} M_B (V_{B1,x}^2 + V_{B1,y}^2) = \frac{1}{2} M_A (V_{A2,x}^2 + V_{A2,y}^2) + \frac{1}{2} M_B (V_{B2,x}^2 + V_{B2,y}^2)$$

THERE ARE 4 UNKNOWNSS ($V_{A2,x}$, $V_{A2,y}$, $V_{B2,x}$, $V_{B2,y}$) BUT
ONLY 3 EQUATIONS \Rightarrow THERE ARE INFINITELY MANY WAYS
A 2D ELASTIC COLLISION CAN OCCUR!



WE MUST RESTRICT THE PROBLEM, i.e., GIVE ADDITIONAL INFORMATION, FOR THERE TO BE A SOLUTION.

EXAMPLE : A HUMMER (MASS 2900kg) going 11m/s HAS A COLLISION WITH A 730kg SMART CAR GOING 30m/s...



IF THE COLLISION IS ELASTIC AND THE HUMMER HAS A SPEED OF 9m/s AFTER, WHAT IS SPEED OF SMART CAR AND WHAT DIRECTION FOR BOTH?

SINCE WE ARE TOLD THAT THE COLLISION IS ELASTIC, WE CAN USE

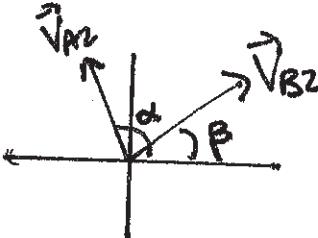
$$\frac{1}{2} M_A V_{A1}^2 + \frac{1}{2} M_B V_{B1}^2 = \frac{1}{2} M_A V_{A2}^2 + \frac{1}{2} M_B V_{B2}^2$$

$$\Rightarrow \frac{1}{2} (2900\text{kg}) (11\text{m/s})^2 + \frac{1}{2} (730\text{kg}) (30\text{m/s})^2 = \frac{1}{2} (2900\text{kg}) (9\text{m/s})^2 + \frac{1}{2} (730\text{kg}) V_{B2}^2$$

$$\Rightarrow V_{B2} = 32.54\text{m/s}$$

TO FIND DIRECTIONS, WE USE MOMENTUM CONSERVATION

$$M_A V_{A1,x} + M_B V_{B1,x} = M_A V_{A2,x} + M_B V_{B2,x}, \quad M_A V_{A1,y} + M_B V_{B1,y} = M_A V_{A2,y} + M_B V_{B2,y}$$

ASSUME:  THEN: $2900\text{kg}(11\text{m/s})\cos(\alpha) + 730\text{kg}(30\text{m/s})\cos(25^\circ) = 2900\text{kg}(9\text{m/s})\cos(x) + 730\text{kg}(32.54\text{m/s})\cos(\beta)$

$$2900\text{kg}(11\text{m/s})\sin(\alpha) + 730\text{kg}(30\text{m/s})\sin(25^\circ) = 2900\text{kg}(9\text{m/s})\sin(x) + 730\text{kg}(32.54\text{m/s})\sin(\beta)$$

I. TRIPLE DOG DARE YOU TO SHOW THAT $\alpha = 92.6^\circ$, $\beta = 141.74^\circ$ IS THE SOLUTION!
(MORE OR LESS)

CENTER OF MASS

FOR A COLLECTION OF PARTICLES M_1 at \vec{r}_1 , M_2 at \vec{r}_2 , M_3 at \vec{r}_3 , ...

WE DEFINE THE CENTER OF MASS TO BE:

$$\vec{r}_{cm} = \frac{M_1 \vec{r}_1 + M_2 \vec{r}_2 + M_3 \vec{r}_3 + \dots}{M_1 + M_2 + M_3 + \dots} = \frac{\sum_i M_i \vec{r}_i}{\sum_i M_i}$$

$$\vec{r}_{cm} = \frac{\sum_i M_i \vec{r}_i}{M}, \quad \vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M} (M_1 \vec{v}_1 + M_2 \vec{v}_2 + M_3 \vec{v}_3 + \dots) = \frac{\vec{p}_{total}}{M}$$

$$\Rightarrow \vec{p}_{total} = M \vec{v}_{cm}$$

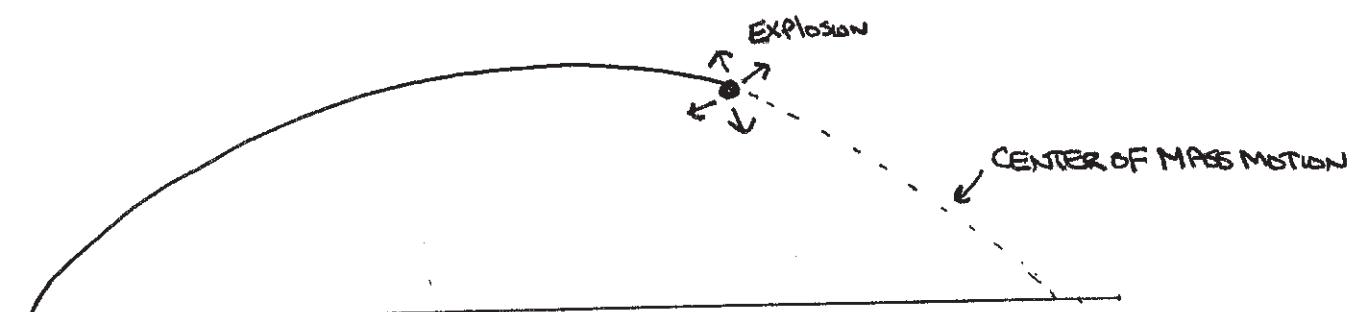
A COLLECTION OF PARTICLES BEHAVE LIKE A SINGLE OBJECT LOCATED AT THE CENTER OF MASS. THIS IS WHAT ALLOWS US TO TREAT LARGE OBJECTS LIKE CARS OR TRAINS AS A SINGLE POINT.

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{1}{M} \frac{d\vec{p}_{total}}{dt} = \frac{1}{M} \vec{F}_{total} \Rightarrow \vec{F}_{total} = M \vec{a}_{cm}$$

FORCES CAN BE SPLIT INTO EXTERNAL (ACTING ON PARTICLES INDEPENDENTLY) AND INTERNAL (ACTING AMONG THE CONSTITUENT PARTICLES)

$$\vec{F}_{total} = \sum \vec{F}_{ext} + \underbrace{\sum \vec{F}_{int}}_{\text{∅ BY 3rd LAW}} \Rightarrow \boxed{\vec{F}_{ext} = M \vec{a}_{cm}}$$

CENTER OF MASS'S MOTION DETERMINED BY EXTERNAL FORCES ONLY.



APPENDIX : SHOWING $V_{A2} - V_{B2} = -(V_{A1} - V_{B1})$ FOR 1D ELASTIC COLLISION.

AS WE FOUND IN LECTURE THIS REQUIREMENT ALONG WITH MOMENTUM CONSERVATION LEADS TO $V_{A2} = \frac{V_{A1}(M_A - M_B) + 2M_B V_{B1}}{(M_A + M_B)}$, $V_{B2} = \frac{-V_{B1}(M_A - M_B) + 2M_A V_{A1}}{(M_A + M_B)}$

SO I'LL SHOW THAT THESE VELOCITIES GIVE CONSERVATION OF

$$\text{KINETIC ENERGY} : \frac{1}{2} M_A V_{A1}^2 + \frac{1}{2} M_B V_{B1}^2 = \frac{1}{2} M_A V_{A2}^2 + \frac{1}{2} M_B V_{B2}^2$$

$$\text{OR EQUIVALENTLY } M_A V_{A2}^2 + M_B V_{B2}^2 = M_A V_{A1}^2 + M_B V_{B1}^2$$

$$M_A V_{A2}^2 + M_B V_{B2}^2 = M_A \left[\frac{V_{A1}(M_A - M_B) + 2M_B V_{B1}}{(M_A + M_B)} \right]^2 + M_B \left[\frac{-V_{B1}(M_A - M_B) + 2M_A V_{A1}}{(M_A + M_B)} \right]^2$$

$$= \frac{M_A}{(M_A + M_B)^2} \left[V_{A1}^2 (M_A - M_B)^2 + 4M_B(M_A - M_B)V_{A1}V_{B1} + 4M_B^2 V_{B1}^2 \right]$$

$$+ \frac{M_B}{(M_A + M_B)^2} \left[V_{B1}^2 (M_A - M_B)^2 - 4M_A(M_A - M_B)V_{A1}V_{B1} + 4M_A^2 V_{A1}^2 \right]$$

$$= \frac{1}{(M_A + M_B)^2} \left[(M_A - M_B)^2 (M_A V_{A1}^2 + M_B V_{B1}^2) + 4M_A M_B (M_A - M_B) V_{A1} V_{B1} - 4M_A M_B (M_A - M_B) V_{A1} V_{B1} + 4M_A M_B^2 V_{B1}^2 + 4M_A^2 M_B V_{A1}^2 \right]$$

$$= \frac{1}{(M_A + M_B)^2} \left[M_A V_{A1}^2 ((M_A - M_B)^2 + 4M_A M_B) + M_B V_{B1}^2 ((M_A - M_B)^2 + 4M_A M_B) \right]$$

$$= \frac{(M_A - M_B)^2 + 4M_A M_B}{(M_A + M_B)^2} \left[M_A V_{A1}^2 + M_B V_{B1}^2 \right]. \quad \left. \begin{aligned} (M_A - M_B)^2 + 4M_A M_B &= M_A^2 - 2M_A M_B + M_B^2 \\ &\quad + 4M_A M_B \end{aligned} \right\} = M_A^2 + 2M_A M_B + M_B^2$$

$$= \frac{(M_A + M_B)^2}{(M_A + M_B)^2} \left[M_A V_{A1}^2 + M_B V_{B1}^2 \right] = M_A V_{A1}^2 + M_B V_{B1}^2. \quad \left. \begin{aligned} &= (M_A + M_B)^2 \end{aligned} \right\}$$

WOW!
IT WORKED.