

MOMENTUM AND IMPULSE, CHAPTER 8

RE-EXPRESS NEWTON'S 2ND LAW IN ITS ORIGINAL FORM:

$$\vec{F}_{NET} = M\vec{a} = M \frac{d\vec{v}}{dt} = \frac{d}{dt}(M\vec{v})$$

MOMENTUM: $\vec{p} = M\vec{v}$

$$\vec{F}_{NET} = \frac{d\vec{p}}{dt}$$

MOMENTUM IS A MEASURE OF HOW "DIFFICULT" IT IS TO STOP A MOVING OBJECT. OBJECTS WITH LARGE MASS OR VELOCITY (OR BOTH) HAVE LARGE MOMENTA.

UNIT OF \vec{p} : $\text{kgm/s} \rightarrow \text{NO FANCY NAME}$

\vec{p} IS A VECTOR. OFTEN USE ITS COMPONENTS:

$$P_x = MV_x, P_y = MV_y, P_z = MV_z$$

IMPULSE

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{NET} dt$$

UNIT: $\text{Ns} = \text{kgm/s}^2(\text{s}) = \text{kgm/s} \rightarrow \text{MOM. UNIT}$

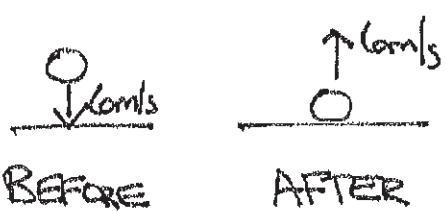
$$\vec{J} = \int_{t_1}^{t_2} \vec{F}_{NET} dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt} dt = \int_{P_1}^{P_2} d\vec{p} = \vec{P}_2 - \vec{P}_1 \Rightarrow \vec{J} = \Delta \vec{p}$$

FOR AVERAGE FORCE: $\vec{J} = \int_{t_1}^{t_2} \vec{F}_{AVG} dt = \vec{F}_{AVG} \int_{t_1}^{t_2} dt = \vec{F}_{AVG} \Delta t$

$$\vec{J} = \vec{F}_{AVG} \Delta t$$

(1)

Example: A 5kg ball is dropped and hits the ground with a speed of 6m/s. If it bounces with a speed of 6m/s in opposite direction, what impulse is imparted to the ball?



$$\text{JUST BEFORE BOUNCE: } \vec{v} = 6\text{m/s DOWN}$$

$$\Rightarrow V_y = -6\text{m/s}$$

$$\vec{p} = m\vec{v} \Rightarrow p_y = mv_y = 5\text{kg}(-6\text{m/s}) = -30\text{kg}\cdot\text{m/s}$$

$$\text{JUST AFTER BOUNCE: } \vec{v} = 6\text{m/s UP}$$

$$\Rightarrow V_y = +6\text{m/s} \Rightarrow p_y = +30\text{kg}\cdot\text{m/s}$$

$$\Rightarrow \Delta p_y = 30\text{kg}\cdot\text{m/s} - (-30\text{kg}\cdot\text{m/s}) = 60\text{kg}\cdot\text{m/s}$$

- If bouncing ball is in contact with the ground for 0.1s, what is average force on ball?

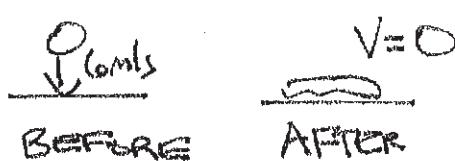
$$\vec{\Delta p} = \vec{F}_{\text{av}} \Delta t \Rightarrow \Delta p_y = F_{\text{av},y} \Delta t \Rightarrow 60\text{kg}\cdot\text{m/s} = F_{\text{av},y} (0.1\text{s}). F_{\text{av},y} = \frac{60\text{kg}\cdot\text{m/s}}{0.1\text{s}}$$

$$\Rightarrow F_{\text{av},y} = 600\text{N}$$

- What is avg. force if bounce time doubled, i.e., $\Delta t = 0.2\text{s}$

$$\Rightarrow 60\text{kg}\cdot\text{m/s} = F_{\text{av},y} (0.2\text{s}) \Rightarrow F_{\text{av},y} = 300\text{N} \rightarrow \text{shorter collisions require more force}$$

- What if sky of clay hits Earth? Clay doesn't bounce.

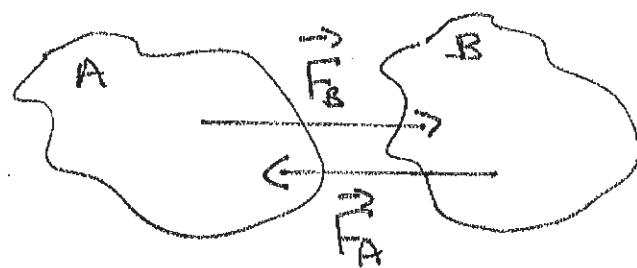


$$\Delta p_y = 0 - (-30\text{kg}\cdot\text{m/s}) = 30\text{kg}\cdot\text{m/s}$$

$$\text{For } \Delta t = 0.1\text{s}, F_{\text{av},y} = 300\text{N} \rightarrow \begin{matrix} \text{Bouncing} \\ \text{doubles} \\ \text{Force} \end{matrix}$$

(2)

CONSERVATION OF MOMENTUM



By 3rd LAW: $\vec{F}_A = -\vec{F}_B$
 $\Rightarrow \vec{F}_A + \vec{F}_B = 0$

$$\vec{F}_A = \frac{d\vec{p}_A}{dt}, \quad \vec{F}_B = \frac{d\vec{p}_B}{dt}. \quad \vec{F}_A + \vec{F}_B = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d}{dt}(\vec{p}_A + \vec{p}_B)$$

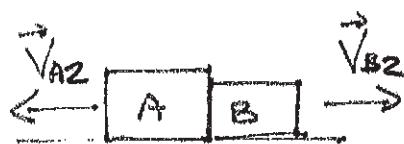
$$\vec{F}_A + \vec{F}_B = 0 \Rightarrow \frac{d}{dt}(\vec{p}_A + \vec{p}_B) = 0 \Rightarrow \vec{p}_A + \vec{p}_B = \text{Constant}$$

Call $\vec{p}_A + \vec{p}_B = \vec{p}_{\text{TOTAL}}$ $\Rightarrow \Delta \vec{p}_{\text{TOTAL}} = 0$

TOTAL MOMENTUM
DOESN'T
CHANGE.



BEFORE

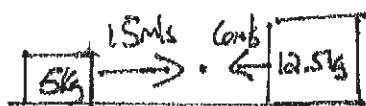


AFTER

CONSERVATION: $M_A \vec{v}_{A1} + M_B \vec{v}_{B1} = M_A \vec{v}_{A2} + M_B \vec{v}_{B2}$

NOTICE THE VECTOR ADDITION \Rightarrow COMPONENTS.

Example: A 5kg mass with $\vec{V}_{A1} = 15\text{m/s}$ to right hits a 12.5kg mass with $\vec{V}_{B1} = 6\text{m/s}$ to left. If 12.5kg mass bounces with a $\vec{V}_{B2} = 3\text{m/s}$ to right, what must the 5kg's velocity be?



BEFORE



AFTER

$$M_A \vec{V}_{A1} + M_B \vec{V}_{B1} = M_A \vec{V}_{A2} + M_B \vec{V}_{B2} \Rightarrow M_A V_{A1,x} + M_B V_{B1,x} = M_A V_{A2,x} + M_B V_{B2,x}$$

$$M_A V_{A1,y} + M_B V_{B1,y} = M_A V_{A2,y} + M_B V_{B2,y}$$

$$V_{A1,x} = 15\text{m/s}, V_{A1,y} = 0 \quad V_{B1,x} = -6\text{m/s}, V_{B1,y} = 0$$

$$V_{A2,x} = ?, V_{A2,y} = ? \quad V_{B2,x} = 3\text{m/s}, V_{B2,y} = 0$$

$$\Rightarrow 5\text{kg}(15\text{m/s}) + 12.5\text{kg}(-6\text{m/s}) = 5\text{kg} V_{A2,x} + 12.5\text{kg}(3\text{m/s})$$

$$\Rightarrow 75\text{kg.m/s} - 75\text{kg.m/s} = 5\text{kg} V_{A2,x} + 37.5\text{kg.m/s}$$

$$\Rightarrow 0 = 5\text{kg} V_{A2,x} + 37.5\text{kg.m/s} \Rightarrow V_{A2,x} = \frac{-37.5\text{kg.m/s}}{5\text{kg}} = -7.5\text{m/s}$$

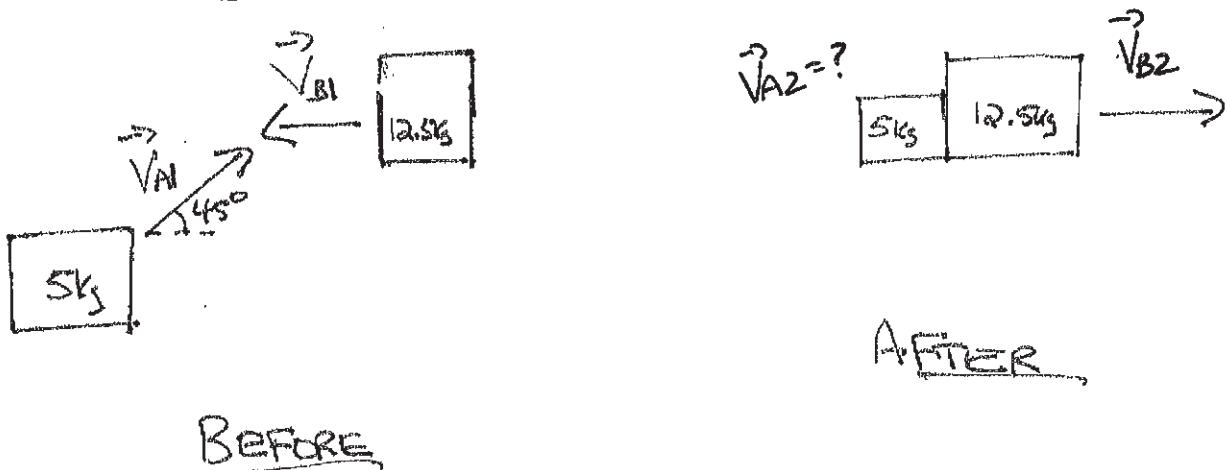
$$5\text{kg}(0) + 12.5\text{kg}(0) = 5\text{kg} V_{A2,y} + 12.5\text{kg}(0) \Rightarrow V_{A2,y} = 0$$

$$\Rightarrow \vec{V}_{A2} = 7.5\text{m/s to left}$$

(4)

Example : $M_A = 5\text{kg}$, $\vec{V}_{A1} = 15\text{m/s at } 45^\circ$, $M_B = 12.5\text{kg}$, $\vec{V}_{B1} = 6\text{m/s left}$.

IF $\vec{V}_{B2} = 3.49\text{m/s right}$, WHAT IS $\vec{V}_{A2} = ?$



$$M_A V_{A1,x} + M_B V_{B1,x} = M_A V_{A2,x} + M_B V_{B2,x}$$

$$M_A V_{A1,y} + M_B V_{B1,y} = M_A V_{A2,y} + M_B V_{B2,y}$$

$$V_{A1,x} = 15\text{m/s} \cos 45^\circ = 10.6\text{m/s}, \quad V_{A2,x} = ?$$

$$V_{A1,y} = 15\text{m/s} \sin 45^\circ = 10.6\text{m/s} \quad V_{A2,y} = ?$$

$$V_{B1,x} = -6\text{m/s}, \quad V_{B1,y} = 0$$

$$V_{B2,x} = 3.49\text{m/s}, \quad V_{B2,y} = 0$$

$$\Rightarrow 5\text{kg}(10.6\text{m/s}) + 12.5\text{kg}(-6\text{m/s}) = 5\text{kg}V_{A2,x} + 12.5\text{kg}(3.49\text{m/s})$$

$$\Rightarrow -22\text{ kg.m/s} = 5\text{kg}V_{A2,x} + 43.625\text{kg.m/s}$$

$$\Rightarrow V_{A2,x} = \frac{-65.625\text{kg.m/s}}{5\text{kg}} = -13.125\text{m/s}$$

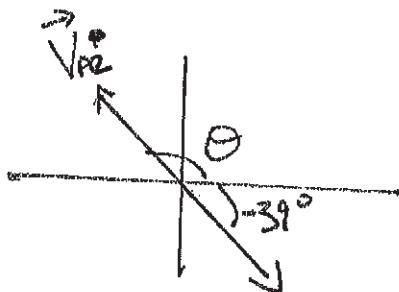
$$5\text{kg}(10.6\text{m/s}) + 0 = 5\text{kg}V_{A2,y} + 0 \Rightarrow V_{A2,y} = 10.6\text{m/s}$$

$$V_{A2,x} = -13.125\text{m/s}, V_{A2,y} = 10.6\text{m/s}$$

$$V_{A2} = \sqrt{V_{A2,x}^2 + V_{A2,y}^2} = \sqrt{(13.125\text{m/s})^2 + (10.6\text{m/s})^2} = 16.87\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{V_{A2,y}}{V_{A2,x}}\right) = \tan^{-1}\left(\frac{10.6}{-13.125}\right) = -38.925^\circ = \cancel{-39^\circ}$$

WRONG QUADRANT



$$\theta = 180^\circ - 39^\circ = 141^\circ$$

(6)