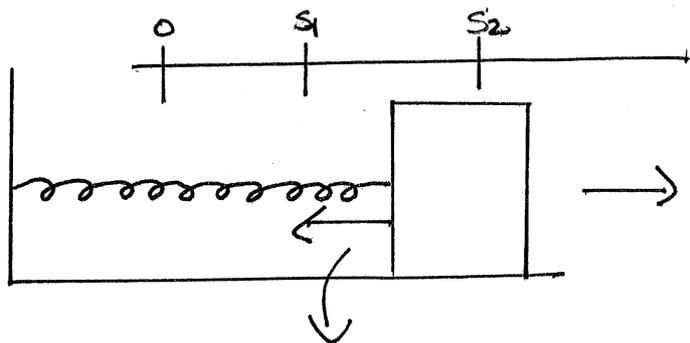


PHYS 1710 - ENERGY CONTINUED, CHAPTER 7

ELASTIC POTENTIAL ENERGY - WORK DONE BY A SPRING

WHEN WE CALCULATED $W = \frac{1}{2}KS_2^2 - \frac{1}{2}KS_1^2 \rightarrow$ WORK DONE TO A SPRING



$\rightarrow F = Ks \rightarrow$ FORCE ON SPRING
DUE TO MASS \Rightarrow WORK
DONE TO SPRING

$F = -Ks \rightarrow$ FORCE ON MASS
DUE TO SPRING \Rightarrow WORK
DONE BY SPRING

$$F = -Ks \Rightarrow W_{el} = -\left(\frac{1}{2}KS_2^2 - \frac{1}{2}KS_1^2\right) = -\Delta U_{el}$$

$$W_{el} = -\Delta U_{el}$$

$$U_{el} = \frac{1}{2}KS^2$$

ELASTIC POTENTIAL ENERGY

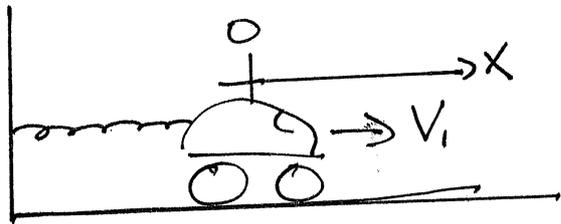
FOR SPRINGS AND OTHER FORCES:

$$W_{TOTAL} = W_{el} + W_{OTHER}$$

$$W_{TOTAL} = \Delta K, W_{el} = -\Delta U_{el} \Rightarrow \Delta K = -\Delta U_{el} + W_{OTHER}$$

$$\Rightarrow \frac{1}{2}MV_1^2 + \frac{1}{2}KS_1^2 + W_{OTHER} = \frac{1}{2}MV_2^2 + \frac{1}{2}KS_2^2$$

EXAMPLE: A 2kg Toy CAR IS ATTACHED TO A $k=200\text{N/m}$ SPRING. IF $v_1=5\text{m/s}$ AND FRICTION CAN BE IGNORED, HOW FAR DOES IT GO BEFORE STOPPING?



FORCES ON CAR:
SPRING FORCE, NORMAL, GRAVITY

SAME Stretching AND x-direction

NORMAL AND GRAVITY DO NO WORK BECAUSE 90° to motion

$$\Rightarrow \frac{1}{2} M v_1^2 + \frac{1}{2} k x_1^2 = \frac{1}{2} M v_2^2 + \frac{1}{2} k x_2^2$$

$$v_1 = 5\text{m/s}, x_1 = 0 \text{ (INITIALLY UNSTRETCHED)}$$

$$v_2 = 0, x_2 = ?$$

$$\frac{1}{2} (2\text{kg}) (5\text{m/s})^2 = \frac{1}{2} (200\text{N/m}) x_2^2 \Rightarrow 25\text{J} = 100\text{N/m} x_2^2$$

$$\Rightarrow x_2 = \sqrt{\frac{25\text{J}}{100\text{N/m}}} = .5\text{m}$$

$$\hookrightarrow \text{J/N/m} = \frac{\text{Jm}}{\text{N}} = \frac{\text{N}\cdot\text{m}^2}{\text{N}}$$

- HOW FAR FROM WALL MUST THE CAR START NOT TO HIT IT ON THE WAY BACK?

CAR WILL COMPRESS SPRING $\Rightarrow x_2 < 0$

WE CAN STILL START AT $v_1=5\text{m/s}, x_1=0$

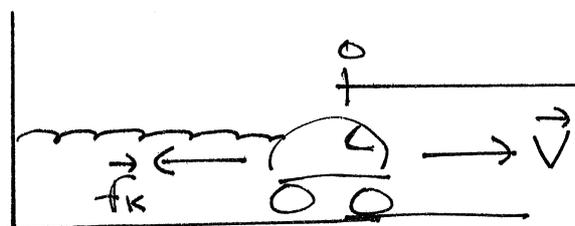
$$v_2 = 0, x_2 = ?$$

I WAS SLOPPY LAST TIME: $25\text{J} = 100\text{N/m} x_2^2 \Rightarrow x_2^2 = \pm \sqrt{\frac{25\text{J}}{100\text{N/m}}} = \pm .5\text{m}$

(2)

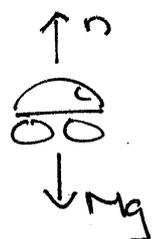
EXAMPLE: HOW FAR DOES THE CARGO IF $V_1 = 5 \text{ m/s}$,
 $K = 200 \text{ N/m}$, AND $\mu_k = .5$?

FRICTION \Rightarrow $W_{\text{OTHER}} = W_f \Rightarrow \frac{1}{2} M V_1^2 + \frac{1}{2} K X_1^2 + W_{\text{OTHER}} = \frac{1}{2} M V_2^2 + \frac{1}{2} K X_2^2$



$$W_f = -f_k x \quad (f_k x \cos 180^\circ)$$

$$= -\mu_k n x$$



$$n = Mg \Rightarrow W_f = -\mu_k Mg x$$

$$V_1 = 5 \text{ m/s}, X_1 = 0, V_2 = 0, X_2 = X = ?$$

$$\Rightarrow \frac{1}{2} (2 \text{ kg}) (5 \text{ m/s})^2 + 0 - .5 (2 \text{ kg}) (9.8 \text{ m/s}^2) X_2 = 0 + \frac{1}{2} (200 \text{ N/m}) X_2^2$$

$$\Rightarrow 25 \text{ J} - 9.8 \text{ N} X_2 = 100 \text{ N/m} X_2^2$$

$$\Rightarrow 100 X_2^2 + 9.8 X_2 - 25 = 0 \Rightarrow X_2 = \frac{-9.8 \pm \sqrt{9.8^2 - 4(100)(-25)}}{2(100)}$$

$$\Rightarrow X_2 = .454 \text{ m} \text{ OR } -.549 \text{ m}$$

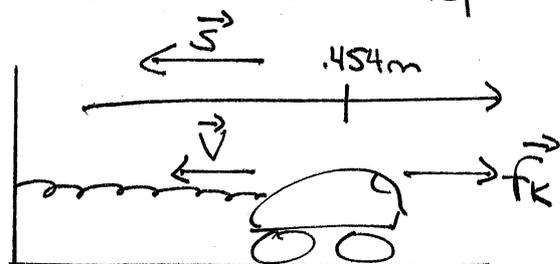
- HOW FAR BACK DOES IT GO?

WE HAVE TO BE VERY CAREFUL! FRICTION HAS "STOLEN" SOME OF THE ENERGY. WE DON'T HAVE $E = \frac{1}{2} M V_1^2 = 25 \text{ J}$

WE HAVE, $E = \frac{1}{2} kx_2^2$ LEFT. $E = \frac{1}{2} (200 \text{ N/m})(.454 \text{ m})^2 = 20.6 \text{ J}$

SO FOR TRIP BACK, WE HAVE $20.6 \text{ J} + W_{\text{OTHER}} = \frac{1}{2} Mv_3^2 + \frac{1}{2} kx_3^2$

TO FIND WHERE IT STOPS $\Rightarrow v_3 = 0$



$$W_{\text{OTHER}} = W_f = f_k x \cos 180^\circ$$

$$= -f_k x = -\mu_k m g x \rightarrow \text{WORK DONE BY FRICTION ALWAYS NEG.}$$

$X = \text{DISTANCE TRAVELLED} \Rightarrow \text{POSITIVE NUMBER}$, $X = |x_3 - .454 \text{ m}|$

$$x_3 < .454 \text{ m} \Rightarrow x_3 - .454 \text{ m} < 0 \Rightarrow |x_3 - .454 \text{ m}| = -(x_3 - .454 \text{ m}) = .454 \text{ m} - x_3$$

$$\Rightarrow W_{\text{OTHER}} = -.5(2 \text{ kg})(9.8 \text{ m/s}^2)(.454 \text{ m} - x_3)$$

$$\Rightarrow 20.6 \text{ J} - 9.8 \text{ N} (.454 \text{ m} - x_3) = \frac{1}{2} (200 \text{ N/m}) x_3^2$$

$$\Rightarrow 20.6 \text{ J} - 4.45 \text{ J} + 9.8 \text{ N} x_3 = 100 \text{ N/m} x_3^2$$

$$\Rightarrow 100 x_3^2 - 9.8 x_3 - 16.15 = 0 \Rightarrow x_3 = .454 \text{ m} \text{ OR } -.356 \text{ m}$$

↓
DOES NOTHING - NOT AN OPTION

$$\Rightarrow \underline{\underline{x_3 = -.356 \text{ m}}}$$

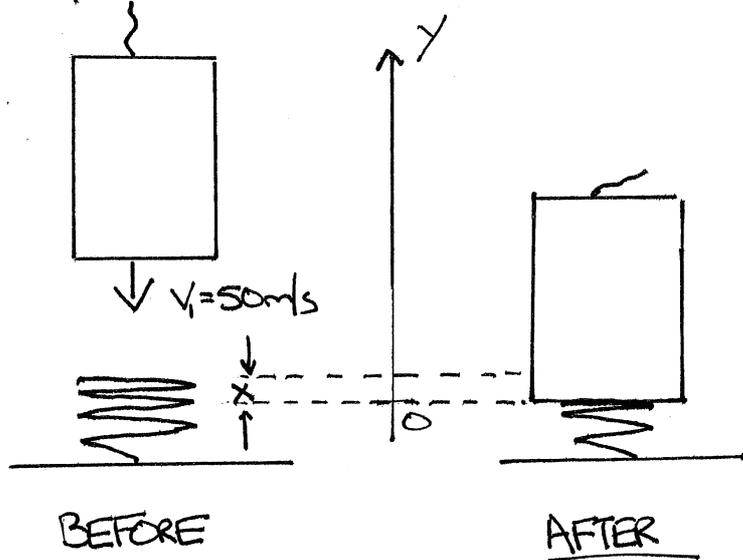
THE MOST GENERAL PROBLEM INVOLVES GRAVITY, SPRINGS, AND OTHERS

$$\Rightarrow W_{\text{TOTAL}} = W_g + W_{e1} + W_{\text{OTHER}}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \Delta K & -\Delta U_g & -\Delta U_{e1} \end{array}$$

$$\Rightarrow \frac{1}{2} M V_1^2 + M g y_1 + \frac{1}{2} K S_1^2 + W_{\text{OTHER}} = \frac{1}{2} M V_2^2 + M g y_2 + \frac{1}{2} K S_2^2$$

EXAMPLE: A RUNAWAY ^{15kg} ELEVATOR HITS A $K = 3000 \text{ N/m}$ SPRING GOING 50 m/s . HOW FAR DOES IT COMPRESS THE SPRING?



S = STRETCHING DISTANCE
 y = HEIGHT ABOVE GROUND

$$S_1 = 0, S_2 = S = ?$$

$$y_1 = S, y_2 = 0$$

$$V_1 = 50 \text{ m/s}, V_2 = 0$$

NO FRICTION $\Rightarrow W_{\text{OTHER}} = 0$

$$\frac{1}{2} M V_1^2 + M g y_1 + \frac{1}{2} K S_1^2 = \frac{1}{2} M V_2^2 + M g y_2 + \frac{1}{2} K S_2^2 \Rightarrow$$

$$\frac{1}{2} (15 \text{ kg}) (50 \text{ m/s})^2 + 15 \text{ kg} (9.8 \text{ m/s}^2) S = \frac{1}{2} (3000 \text{ N/m}) S^2$$

$$\Rightarrow 1500 S^2 - 147 S - 18750 = 0 \Rightarrow S = 3.58 \text{ m}, -3.47 \text{ m}$$

WE USE $S = 3.58 \text{ m}$ BECAUSE OF CHOICE THAT $y_1 = S > 0$

(5)