

Phys 110 - ENERGY CHAPTER 7

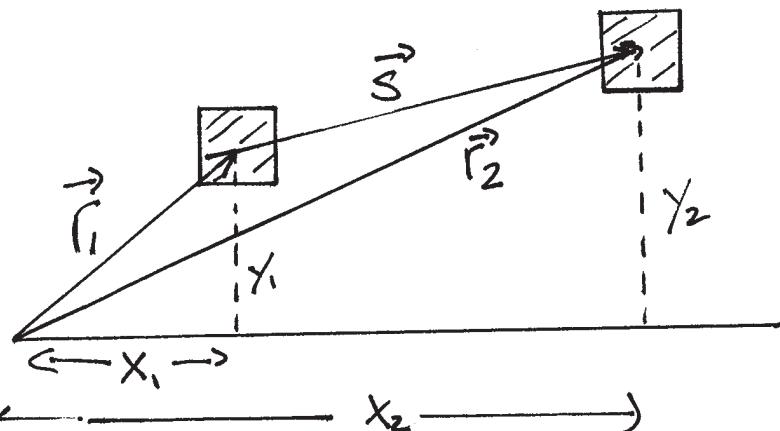
WHEN WE LIFT AN OBJECT, IT ACQUIRES THE ABILITY TO GAIN KINETIC ENERGY AT A LATER TIME (WHEN WE LET GO). IT HAS STORED ENERGY FOR LATER USE.

POTENTIAL ENERGY - STORED ENERGY, i.e., ENERGY THAT CAN BE CONVERTED INTO KINETIC ENERGY.

THE BOOK CALLS POTENTIAL ENERGY TO BE ENERGY DUE TO POSITION. THIS IS OFTEN TRUE BUT IT'S NOT THE ONLY TYPE OF POTENTIAL ENERGY. FOOD HAS POTENTIAL ENERGY INDEPENDENT OF ITS LOCATION.

GRAVITATIONAL POTENTIAL ENERGY - STORED ENERGY DUE TO GRAVITY WHICH DEPENDS ONLY ON OBJECT'S HEIGHT.

WE NEED TO FIND WORK DONE BY GRAVITY (AND NOT ANY OTHER FORCES).



$$\vec{S} = \vec{r}_2 - \vec{r}_1$$

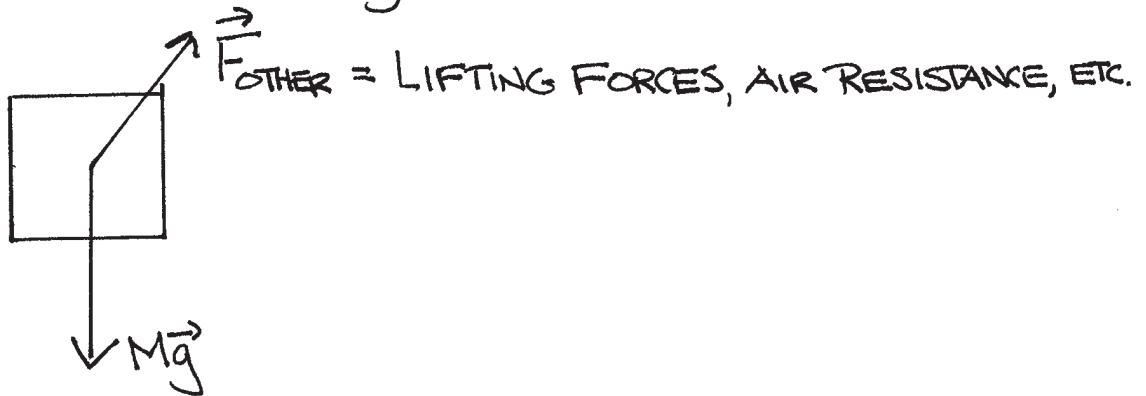
$$\vec{r}_1 = x_1 \hat{i} + y_1 \hat{j}$$

$$\vec{r}_2 = x_2 \hat{i} + y_2 \hat{j}$$

$$\Rightarrow \vec{S} = (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j}$$

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FORCES ACTING ON OBJECT:



SINCE \vec{g} IS CONSTANT, WORK DONE BY GRAVITY IS

$$W_g = M\vec{g} \cdot \vec{s}$$

REMIND YOU: $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y$

$$M\vec{g} = -Mg\hat{j} = \phi\hat{i} - Mg\hat{j}$$

$$\vec{s} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$\Rightarrow W_g = M\vec{g} \cdot \vec{s} = \phi(x_2 - x_1) - Mg(y_2 - y_1) = -Mg(y_2 - y_1)$$

$$\Rightarrow W_g = -(Mgy_2 - Mgy_1) = -(U_2 - U_1)$$

$$\Rightarrow \boxed{W_g = -\Delta U_g}$$

$$\boxed{U_g = Mg y}$$

GRAVITATIONAL
POTENTIAL ENERGY

CONSERVATION OF ENERGY - IF GRAVITY IS ONLY FORCE

DOING WORK ON A MOVING OBJECT, $W_g = W_{\text{TOTAL}}$

By WORK-ENERGY THM, $W_{\text{TOTAL}} = \Delta K$

$$W_g = -\Delta U_g$$

$\Rightarrow \underline{\Delta K = -\Delta U_g}$ AN OBJECT CAN ONLY GAIN KINETIC ENERGY ONLY BY LOSING AN EQUAL AMOUNT OF POTENTIAL ENERGY.
(PROOF THAT $U = \text{POTENTIAL}$)

$$\Delta K = K_2 - K_1, -\Delta U_g = -(U_2 - U_1) = U_1 - U_2$$

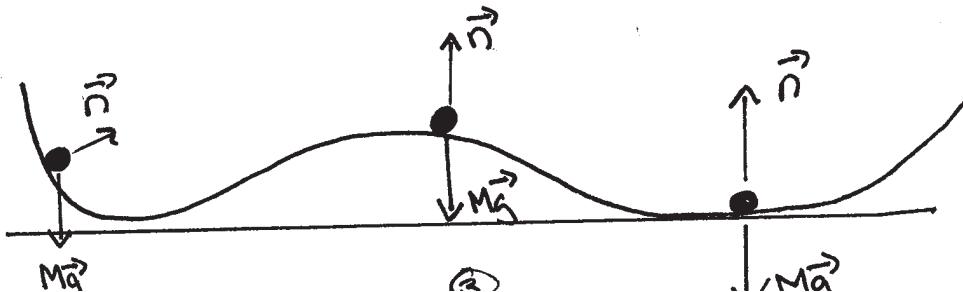
$$\Rightarrow K_2 - K_1 = U_1 - U_2 \Rightarrow K_1 + U_1 = K_2 + U_2$$

$$\Rightarrow \boxed{\frac{1}{2}MV_1^2 + MgY_1 = \frac{1}{2}MV_2^2 + MgY_2}$$

Call $E = \frac{1}{2}MV^2 + MgY$ → TOTAL MECHANICAL ENERGY.

IF GRAVITY IS ONLY FORCE DOING WORK, $E_1 = E_2$. E DOES NOT CHANGE WITH TIME $\Rightarrow E$ IS CONSERVED.

U_g CAN BE USED FOR ANY PROBLEM WITH GRAVITY. FOR FRICTIONLESS INCLINE OF ANY SHAPE, GRAVITY IS ONLY FORCE DOING WORK



AT ANY POINT, \vec{n} does NO WORK.

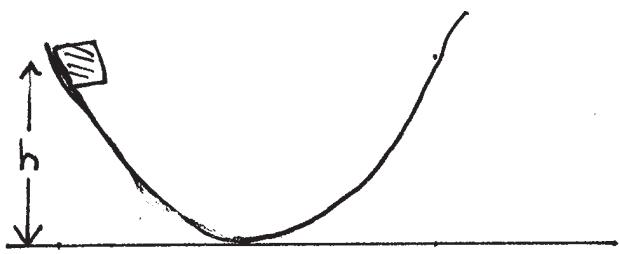
$$W_{\text{TOTAL}} = W_n + W_g = W_g$$

$$\text{SO } W_{\text{TOTAL}} = W_g$$

$$\Rightarrow E_1 = E_2 \Rightarrow$$

HEIGHT NOT LOCATION IS KEY.

EXAMPLE : A MASS SLIDES FROM REST DOWN FRICTIONLESS INCLINE AS SHOWN. WHAT IS ITS SPEED AT BOTTOM?



NO FRICTION \Rightarrow GRAVITY ONLY FORCE DOING WORK. $\Rightarrow E_1 = E_2 \Rightarrow$

$$\frac{1}{2}MV_1^2 + MgY_1 = \frac{1}{2}MV_2^2 + MgY_2$$

$$V_1 = 0 \text{ (FROM REST)}$$

$$Y_1 = h$$

$$V_2 = ?$$

$$Y_2 = 0 \text{ (BOTTOM)}$$

$$0 + MgY = \frac{1}{2}MV_2^2 + 0 \Rightarrow V_2 = \sqrt{2gh} \quad (\text{NOTICE THAT } M \text{ ALWAYS CANCELS})$$

ENERGY IS CONSERVED AT ALL POINTS.

- WHAT IS MASS'S SPEED AT HEIGHT $h/2$?



$$V_1 = 0$$

$$Y_1 = h$$

$$V_2 = ?$$

$$Y_2 = h/2$$

$$0 + MgH = \frac{1}{2}MV_2^2 + MgY_2$$

$$\Rightarrow V_2^2 = 2g(h - \underbrace{h/2}_{\Delta Y})$$

$$\Rightarrow V_2 = \sqrt{gh}$$

- HOW HIGH DOES MASS RISE ON OTHER SIDE?

$$V_1 = 0$$

$$V_2 = 0$$

$$Y_1 = h$$

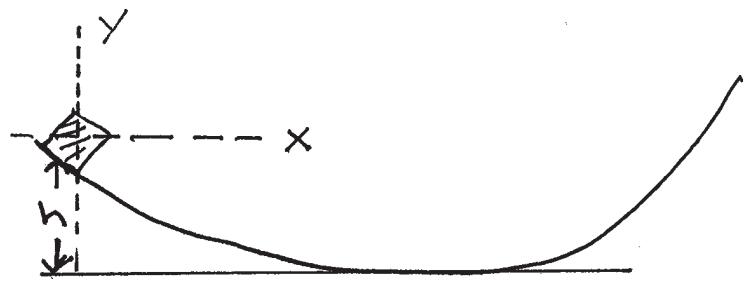
$$Y_2 = ?$$

$$0 + MgH = 0 + MgY_2$$

$$\Rightarrow Y_2 = h$$

SETTING THE ZERO: FOR GRAVITY, THE CHOICE OF $y=0$ IS ARBITRARY.
SET IT WHEREVER YOU LIKE.

EXAMPLE AT WHAT HEIGHT DOES A STARTING FROM REST SLIDING OBJECT HAVE A SPEED OF \sqrt{gh} ?



USE DIFFERENT COORDINATES

$$V_1 = 0$$

$$X_1 = 0$$

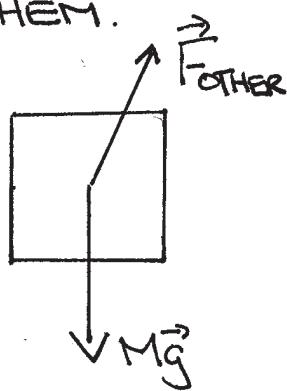
$$V_2 = \sqrt{gh}$$

$$Y_2 = ?$$

$$\frac{1}{2}MV_1^2 + MgY_1 = \frac{1}{2}MV_2^2 + MgY_2 \Rightarrow 0 + 0 = \frac{1}{2}M(\sqrt{gh})^2 + MgY_2$$

$$\Rightarrow Y_2 = -h/2 \quad (\text{h/2 below starting})$$

OTHER FORCES: IF OTHER FORCES ACT ON MOVING OBJECT (e.g. friction), WE HAVE TO FIND FORCES ACTING ON OBJECT AND WORK DONE BY THEM.



$$W_{\text{TOTAL}} = W_g + W_{\text{OTHER}}$$

$$W_{\text{TOTAL}} = \Delta K, \quad W_g = -\Delta U_g$$

$$W_{\text{OTHER}} = W_{\text{OTHER}} \quad (\text{BEST WE CAN DO!})$$

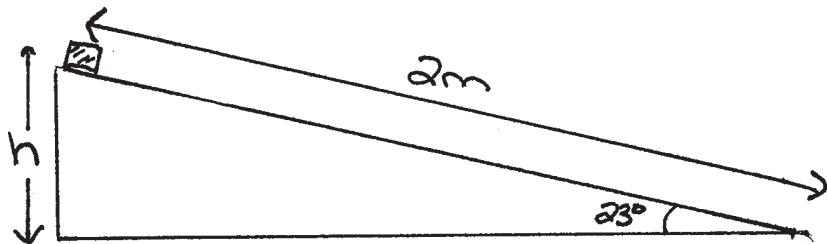
$$\Rightarrow \Delta K = -\Delta U_g + W_{\text{OTHER}} \Rightarrow K_1 + U_1 + W_{\text{OTHER}} = K_2 + U_2$$

$$\boxed{\frac{1}{2}MV_1^2 + MgY_1 + W_{\text{OTHER}} = \frac{1}{2}MV_2^2 + MgY_2}$$

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EXAMPLE A MASS SLIDES DOWN A 23° , 2m LONG INCLINE.

IF $V_1 = 5 \text{ m/s}$ AND $\mu_k = .6$, WHAT IS ITS SPEED AT BOTTOM?



$$V_1 = 5 \text{ m/s}$$

$$y_1 = h = (2 \text{ m}) \sin 23^\circ$$

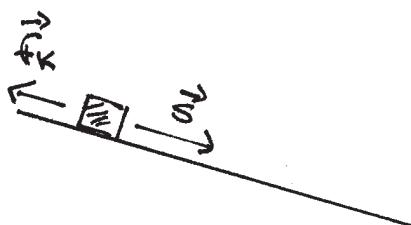
$$V_2 = ?$$

$$y_2 = 0$$

$W_{\text{OTHER}} = W_f = \text{WORK DONE BY FRICTION}$

KINETIC friction constant for THIS INCLINE ($f_k = \mu_k n$. n is constant)

$$\Rightarrow W_f = f_k \cdot s$$



$$f_k = \mu_k n, s = 2 \text{ m}$$

$$W_f = f_k s \cos 180^\circ$$

$= -f_k s$ (Work done by friction is

Always Negative)

$$f_k = \mu_k n = \mu_k W_L = \mu_k Mg \cos 23^\circ \Rightarrow W_f = -\mu_k Mg \cos 23^\circ (s) = -\mu_k M g s \cos 23^\circ$$

$$\frac{1}{2} M V_1^2 + M g y_1 + W_f = \frac{1}{2} M V_2^2 + M g y_2$$

$$\Rightarrow \frac{1}{2} M (5 \text{ m/s})^2 + M (9.8 \text{ m/s}^2) (2 \text{ m}) \sin 23^\circ - (.6) M (9.8 \text{ m/s}^2) (2 \text{ m}) \cos 23^\circ = \frac{1}{2} M V_2^2 + 0$$

$$\Rightarrow V_2 = \sqrt{(5 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(2 \text{ m})(\sin 23^\circ - .6 \cos 23^\circ)} = 4.32 \text{ m/s}$$

$$\text{WITH NO FRICTION: } V_2 = \sqrt{(5 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(2 \text{ m}) \sin 23^\circ} = 6.35 \text{ m/s}$$

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