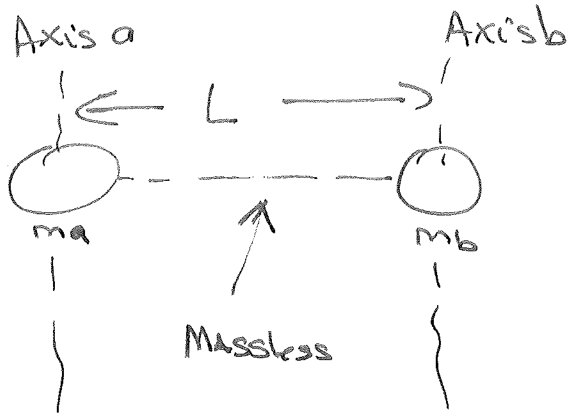
Physics 160

Extra Credit #23

Moment of Inertia

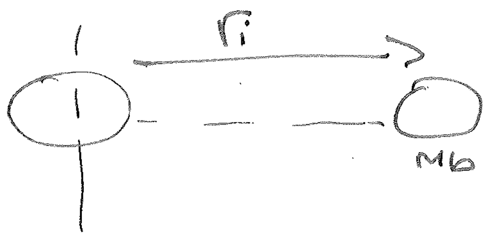


When Rotated about a ⇒ I_a

WHEN ROTATED About b ⇒ I_b

$$\frac{I_a}{I_b} = 3$$

When Rotated about a, Ma is zero Distance from Axis

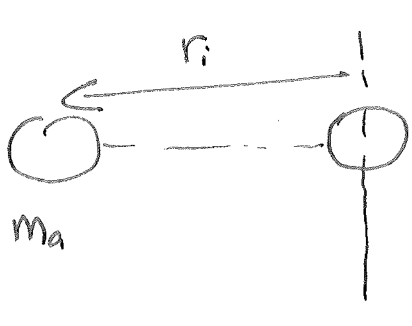


$I = \sum m_i r_i^2$

only one mass, Mb at $r_i = L$

$\Rightarrow I_a = M_b L^2$

When Rotated about b, Mb is at zero



$I = \sum m_i r_i^2 \Rightarrow I_b = M_a L^2$

$$\frac{I_a}{I_b} = \frac{M_b \cancel{L^2}}{M_a \cancel{L^2}} = \frac{M_b}{M_a} \Rightarrow \frac{M_b}{M_a} = 3 \Rightarrow \frac{M_a}{M_b} = \frac{1}{3}$$

9.51 Scaling factor $f \Rightarrow$ Volume & mass increased by f^3

All moments of inertia have the form $I = \overset{\text{Constant}}{c}\underset{\text{mass}}{M}\overset{}{\underset{\text{length}}{L^2}}$

Scaling factor $\Rightarrow$ Length increases by f

$$\therefore \; I = c(Mf^3)(fL)^2 = cMf^3f^2L^2 = cML^2 \underline{f^5}$$

$f = \frac{1}{48}$ HAS $K = 2.5\,J$

Full scale? $K = \frac{1}{2}I\omega^2$, SAme $\omega \Rightarrow$ Scale model

HAD $K_{model} = \left(\frac{1}{48}\right)^5 K_{FULL\ size} \Rightarrow K_{FULL\ SIZE} = \dfrac{K_{model}}{\left(\frac{1}{48}\right)^5}$

$$\Rightarrow K_{FULL\ SIZE} = K_{model}(48)^5 = (2.5\,J)(48)^5 = \underline{\underline{6.37 \times 10^8\,J}}$$

9.45

$R = 1.26\,m,\ m = 67\,kg$

but a_{rad} on Rim can't exceed $3480\,m/s^2$

$a_{rad} = \omega^2 r \Rightarrow 3480\,m/s^2 = \omega_{MAX}^2 R = \omega_{MAX}^2 (1.26\,m)$

$\Rightarrow \omega_{MAX} = \sqrt{\dfrac{3480\,m/s^2}{1.26\,m}} = 52.553\,\dfrac{}{s}$ ← RADIANS when lacking unit

$\therefore\ \omega_{MAX} = 52.553\,rad/s \Rightarrow \omega_{MAX}^2 = 2761.9\,rad^2/s^2$

$K_{MAX} = \frac{1}{2} I\, \omega_{MAX}^2$

Uniform Solid Disk Rotated about Center $\Rightarrow I = \frac{1}{2} m R^2$

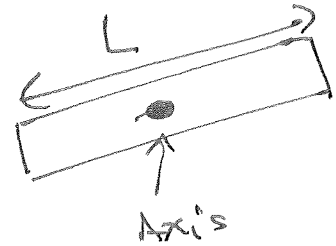
$\therefore\ I = \frac{1}{2}(67\,kg)(1.26\,m)^2 = 53.1846\,kg \cdot m^2$

$\therefore\ K_{MAX} = \frac{1}{2}(53.1846\,kg \cdot m^2)(2761.9\,rad^2/s^2) = 7345\,\cancel{rad^2}\,\frac{kg \cdot m^2}{s^2}$

INCONVENIENT AGAIN!

$\frac{kg \cdot m^2}{s^2} = J \Rightarrow K_{MAX} = 7340\,J$

9.36



Propeller Rotated about Center

$L = 2.17\,m$

$M = 109\,Kg$

$\omega = 2000\,RPM$

$K = \frac{1}{2} I \omega^2$

Can't use RPM, It's not the Correct fake unit. Has to be rad/s

$$\omega = \frac{2000\,rev}{min} \times \frac{2\pi\,rad}{rev} \times \frac{min}{60\,s} = 209.4\,rad/s$$

Thin Rod Rotated about Center $\Rightarrow I = \frac{1}{12} ML^2$

$$\Rightarrow I = \frac{1}{12}(109\,kg)(2.17\,m)^2 = 42.7725\,kg \cdot m^2$$

$$K = \frac{1}{2}(42.7725\,kg \cdot m^2)(209.4\,rad/s)^2 = 937752\,J$$
$$= 938000\,J$$

$J = \frac{kg \cdot m^2}{s^2} \Rightarrow$ INCONVENIENT!

b) Suppose $M = .75(109\text{kg}) = 81.75\text{ kg}$

but same Kinetic Energy. what ω?

$K = \frac{1}{2} I \omega^2$

$I = \frac{1}{12}(81.75\text{ kg})(2.17\text{m})^2 = 32.079\text{ kg}\cdot\text{m}^2$ ← 75% of its ORIGINAL VALue

$\Rightarrow \omega^2 = \frac{2K}{I} = \frac{2(937752\text{J})}{32.079\text{ kg}\cdot\text{m}^2} = 58464/\text{s}^2 = 58464\frac{\text{rad}}{\text{s}^2}$

↑

$\frac{1}{.75} = \frac{4}{3}$ of its ORiginAL

$\therefore \omega = \sqrt{58464\text{rad/s}} = 241.79\text{rad/s} \times \frac{\text{Rev}}{2\pi\text{ rad}} \times \frac{60\text{s}}{\text{min}} \cong 2308.96$

$= 2310\text{ RPM}$

If you prefer: ω had to increase by A factor of $\sqrt{\frac{4}{3}}$

so we could skip back to $\omega_0 = 2000\text{RPM}$ & simply multiply by $\sqrt{\frac{4}{3}}$