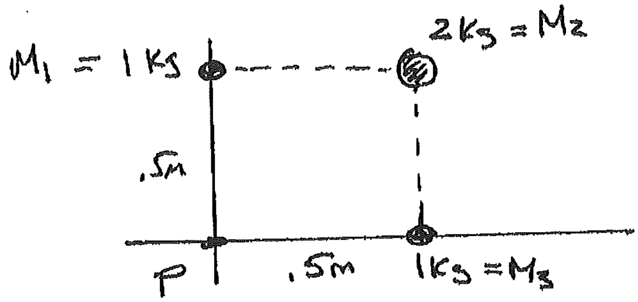
Physics 160, HW#11

Mastering Physics: 7 problems from Chapters 13 & 14

Written: 13.77

13.43



A $M_4 = .015\,kg$ MASS IS PLACED AT ORIGIN, what is NET Force on it?

LABEL M_1, M_2, M_3 AS SHOWN ABOVE. $\Rightarrow$ Net Force on M_4 IS

$$\Sigma \vec{F}_4 = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$

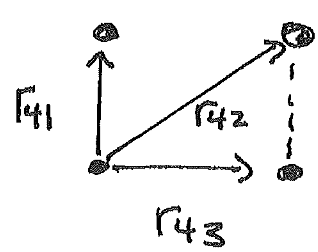
~~F~~ Force ~~Force~~ on #4 due to #1

Force on #4 due to #2

Force on #4 due to #3

NEWTONS LAW OF GRAVITY $\Rightarrow$ $F_{41} = \dfrac{GM_4M_1}{r_{41}}$, $F_{42} = \dfrac{GM_4M_2}{r_{42}}$

$F_{43} = \dfrac{GM_4M_3}{r_{43}}$



$r_{41} = r_{43} = .5m$

$r_{42} = \sqrt{(.5m)^2 + (.5m)^2}$

$= \sqrt{.25m^2 + .25m^2} = \sqrt{.5m^2}$

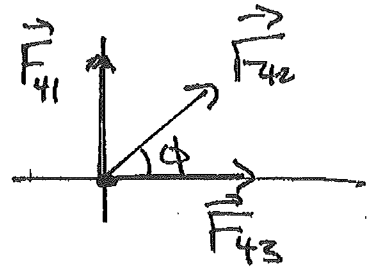
$$M_1 = M_3 = 1\,\text{kg} \Rightarrow F_{41} = F_{43} = \frac{(6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2})(.015\,kg)(1\,kg)}{(.5\,m)^2}$$

$$\Rightarrow F_{41} = F_{43} = 4\times10^{-12}\,N$$

$$F_{42} = \frac{(6.67\times10^{-11}\,N\cdot m^2/kg^2)(.015\,kg)(2\,kg)}{(\sqrt{.5\,m^2})^2} = \frac{(6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2})(.015\,kg)(2\,kg)}{.5\,m^2}$$

$$\Rightarrow F_{42} = 4\times10^{-12}\,N$$

Gravity is Attractive:



$$\phi = \tan^{-1}\left(\frac{.5}{.5}\right) = 45^\circ$$

$$\sum F_{4,x} = F_{41,x} + F_{42,x} + F_{43,x} = 0 + (4\times10^{-12}\,N)\cos 45^\circ + 4\times10^{-12}\,N$$

$$\Rightarrow \sum F_{4,x} = 6.828\times10^{-12}\,N$$

$$\sum F_{4,y} = F_{41,y} + F_{42,y} + F_{43,y} = 4\times10^{-12}\,N + (4\times10^{-12}\,N)\sin 45^\circ + 0 = 6.828\times10^{-12}\,N$$

$$\sum F_4 = \sqrt{\sum F_{4,x}^2 + \sum F_{4,y}^2} = \sqrt{(6.828\times10^{-12}\,N)^2 + (6.828\times10^{-12}\,N)^2} = 9.66\times10^{-12}\,N$$

$$\theta = \tan^{-1}\left(\frac{\sum F_{4,y}}{\sum F_{4,x}}\right) = \tan^{-1}(1) = 45^\circ$$

Part C: IF .015 kg particle is released from rest 300 m from others, How fast will it be going at origin?

Gravity only force doing work $\Rightarrow \frac{1}{2} M_4 V_1^2 + U_{g,1} = \frac{1}{2} M_4 V_2^2 + U_{g,2}$

Compared to .5 m, 300 m $\approx$ INFINITY $\Rightarrow U_{g,1} = 0,\ V_1 = 0$

Particle has potential Energy due to M_1, M_2, AND M_3.

$U_g = \frac{-G M_a M_b}{r}$. Energy A scalar $\Rightarrow U_{g,2} = \frac{-G M_4 M_1}{r_{41}} - \frac{G M_4 M_2}{r_{42}} - \frac{G M_4 M_3}{r_{43}}$

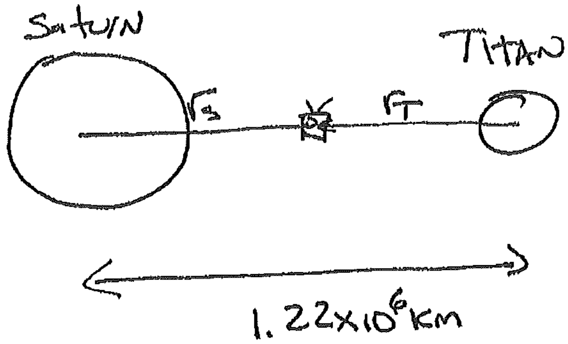
$$\therefore 0 = \frac{1}{2} \cancel{M_4} V_2^2 - \frac{G \cancel{M_4} M_1}{r_{41}} - \frac{G \cancel{M_4} M_2}{r_{42}} - \frac{G \cancel{M_4} M_3}{r_{43}}$$

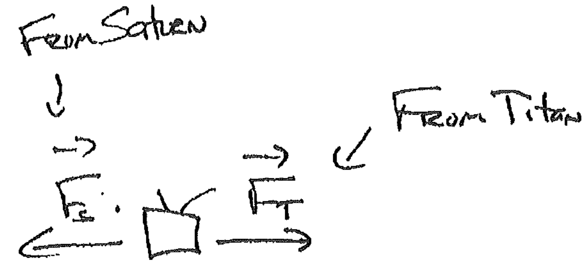
$$\Rightarrow \frac{1}{2} V_2^2 = \frac{G M_1}{r_{41}} + \frac{G M_2}{r_{42}} + \frac{G M_3}{r_{43}} \Rightarrow V_2^2 = 2G\left(\frac{M_1}{r_{41}} + \frac{M_2}{r_{42}} + \frac{M_3}{r_{43}}\right)$$

$$= 2(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2})\left(\frac{1 kg}{.5 m} + \frac{2 kg}{\sqrt{.5 m^2}} + \frac{1 kg}{.5 m}\right) = 2(6.67 \times 10^{-11})(6.8284)$$

$$\Rightarrow V_2^2 = 9.109 \times 10^{-10}\ m^2/s^2 \Rightarrow V_2 = \cancel{3}\ 3.02 \times 10^{-5}\ m/s$$

13.46





$$\sum \vec{F} = 0$$

$$\Rightarrow \vec{F}_T = -\vec{F}_S$$

$$\Rightarrow F_T = F_S$$

Let Huygens Probe have mass M: Newton's Law $\Rightarrow$

$$\frac{\cancel{G}\cancel{M}M_T}{r_T^2} = \frac{\cancel{G}\cancel{M}M_S}{r_S^2} \Rightarrow \frac{M_T}{r_T^2} = \frac{M_S}{r_S^2}$$

$$\Rightarrow r_T^2 = r_S^2\left(\frac{M_T}{M_S}\right) \Rightarrow r_T^2 = r_S^2\left(\frac{1.35\times10^{23}\,\text{kg}}{5.68\times10^{26}\,\text{kg}}\right) = r_S^2\left(2.37\times10^{-4}\right)$$

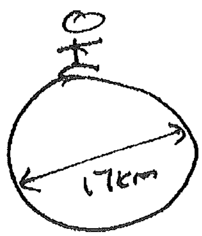
Appendix F

$$\Rightarrow r_T = r_S\sqrt{2.37\times10^{-4}} = r_S(.0154)$$

$$r_T + r_S = 1.22\times10^{6}\,\text{km} \Rightarrow r_S(.0154) + r_S = 1.22\times10^{6}\,\text{km}$$

$$\Rightarrow r_S = \frac{1.22\times10^{6}\,\text{km}}{1.0154} = 1.2\times10^{6}\,\text{km} \quad \therefore\ r_T = .0185\times10^{6}\,\text{km} = 18500\,\text{km}$$

Weight on Neutron Star



$W^{\text{eight}}_{\text{on Earth}} = 665N$

$W = Mg$ NEAR SURFACE

$\Rightarrow M = \dfrac{W}{g} = \dfrac{665N}{9.8 m/s^2} = 67.857 kg$

ON "SURFACE" ~~of~~ of NEUTRON Star $g = \dfrac{GM}{R^2}$

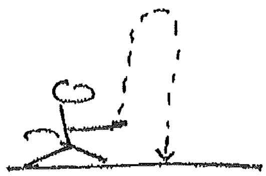
M = MASS OF Neutron star $= 1.99 \times 10^{30} kg$

R = RADIUS $= \dfrac{17 km}{2} = 8.5 km = 8500 m$

$$\Rightarrow g = \frac{(6.67\times10^{-11} N\cdot m^2/kg^2)(1.99\times10^{30} kg)}{(8500m)^2} = 1.837\times10^{12} m/s^2$$

$$\therefore W = (67.857 kg)(1.837\times10^{12} m/s^2) = 1.24\times10^{14} N$$

13.58



$M = 2.5\,kg$, $V_0 = 12\,m/s$, RETURNS IN 6s

NO ATMOSPHERE ON MONGO.

$$\Rightarrow y = y_0 + V_0 t - \tfrac{1}{2} g t^2$$

$$y = y_0 = 0,\ V_0 = 12\,m/s,\ t = 6\,s$$

$$\Rightarrow 0 = 12\,m/s(6s) - \tfrac{1}{2} g (6s)^2 \Rightarrow g = 4\,m/s^2$$

OR $V = V_0 + gt$, $V = 0$, $V_0 = 12\,m/s$, $t = \frac{6s}{2} = 3s \Rightarrow g = 4\,m/s^2$

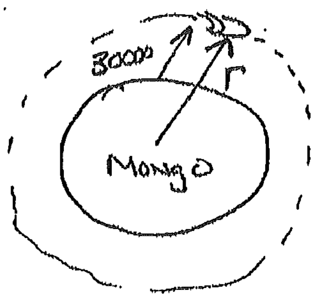
$$g = \frac{GM_M}{R_M^2}$$ M_M = MONGO'S MASS R_M = MONGO'S RADIUS

CIRCUMFERENCE $= 2\times10^5\,km = 2\times10^8\,m$

$$C = 2\pi R_M \Rightarrow R_M = \frac{2\times10^8\,m}{2\pi} = 3.18\times10^7\,m$$

$$M_M = \frac{gR_M^2}{G} = \frac{(4\,m/s^2)(3.18\times10^7\,m)^2}{6.67\times10^{-11}\,N\cdot kg^2/m^2} = \boxed{6.08\times10^{25}\,kg = M_M}$$

b) HOW MANY HOURS FOR CIRCULAR ORBIT 30000Km ABOVE SURFACE?



$$r = 30000\text{km} + R_M$$

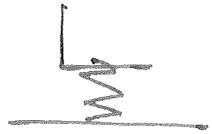
$$= 3.0\times10^{4}\times 1000\text{m} + 3.18\times10^{7}\text{m}$$

$$= 3\times10^{7}\text{m} + 3.18\times10^{7}\text{m} = 6.18\times10^{7}\text{m}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{G M_M}} = \frac{2\pi(6.18\times10^{7}\text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(6.06\times10^{25}\text{kg})}}$$

$$\Rightarrow T = 47996\text{s} \times \frac{1\text{h}}{3600\text{s}} = \underline{13.3\text{ hour}}$$

14.15



$M_{chair} = 42.5\,kg$, $T = 1.3\,s$

WITH PERSON, $T = 2.54\,s$

MASS ON SPRING $\Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

$T_{CHAIR} = 2\pi\sqrt{\frac{M_{chair}}{K}}$, $T_{person} = 2\pi\sqrt{\frac{M_{chair} + M_{person}}{K}}$

chair only

CHAIR & PERSON

$\therefore \frac{T_{CHAIR}}{\sqrt{M_{CHAIR}}} = \frac{T_{PERSON}}{\sqrt{M_{CHAIR} + M_{person}}}$ Since they both EQUAL $\frac{2\pi}{\sqrt{K}}$

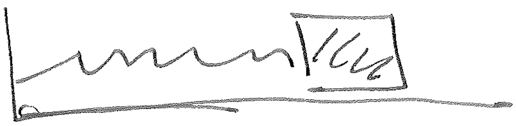
$\Rightarrow \frac{T^2_{CHAIR}}{M_{chair}} = \frac{T^2_{person}}{M_{chair} + M_{person}} \Rightarrow M_{chair} + M_{person} = \frac{T^2_{person}}{T^2_{CHAIR}} M_{CHAIR}$

$\Rightarrow M_{person} = M_{CHAIR}\left(\frac{T^2_{person}}{T^2_{CHAIR}} - 1\right) = 42.5\,kg\left(\frac{2.54^2}{1.3^2} - 1\right) = 42.5\,kg\,(2.8175)$

$= 119.7\,kg$

$= 120\,kg$

14.11



$m = 1.65\,kg$

$K = 350\,N/m$

At $t=0$, $x_0 = 0$

$V_0 = -11.2\,m/s$

$$x = A\cos(\omega t + \phi)$$

$$V = -\omega A\sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{350\,N/m}{1.65\,kg}} = 14.56\,rad/s = 14.6\,rad/s$$

at $t=0$ $\quad x_0 = 0 \Rightarrow 0 = A\cos\phi \Rightarrow \phi = \pm\pi/2$

$V_0 = -11.2\,m/s \Rightarrow -11.2\,m/s = -(14.6\,rad/s)\,A\sin\phi$

$\Rightarrow \phi$ must be $+\pi/2$ otherwise we get wrong direction for V_0

Also $\sin\pi/2 = 1 \Rightarrow -11.2\,m/s = -14.6\,rad/s\; A \Rightarrow A = \frac{-11.2\,m/s}{-14.6\,rad/s} = \underline{0.767\,m}$

$\therefore\; x = A\cos(\omega t + \phi) = 0.767\,m\cos(14.6t + \pi/2)$

Remembering that $\cos(\theta + \pi/2) = -\sin\theta \;\therefore\; x = -0.767\,m\,\sin(14.6t)$

Gravity on Another Planet

104 full swings in 142s

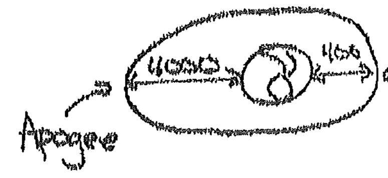
$$\Rightarrow T = \frac{142s}{104} = 1.365s$$

On any planet $T = 2\pi\sqrt{\frac{l}{g}}$ $\quad l = 46cm = 0.46m$

$$\Rightarrow T^2 = \left(2\pi\sqrt{\frac{l}{g}}\right)^2 = \frac{4\pi^2 l}{g} \Rightarrow g = \frac{4\pi^2 l}{T^2}$$

$$g = \frac{4\pi^2(\cancel{1.365s}\ 0.46m)}{(1.365s)^2} = 9.741\, m/s^2$$

13.77



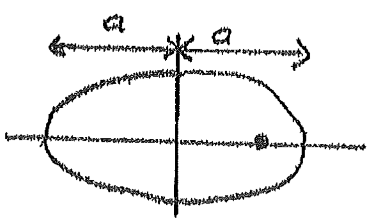
NOT TO SCALE OBVIOUSLY

$400\text{km} = 4\times10^5\text{m}$

$4000\text{km} = 4\times10^6\text{m}$

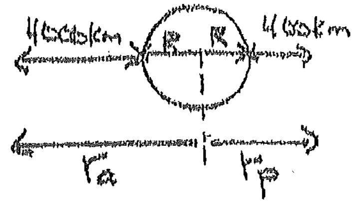
a) WHAT IS PERIOD? $T = \dfrac{2\pi a^{3/2}}{\sqrt{GM_E}}$

EARTH IS AT ONE FOCUS:



TOTAL DISTANCE IS $2a$.

HAVE TO INCLUDE EARTH'S RADIUS



$r_p = R + 400\text{km}$

$= 6.38\times10^6\text{m} + 4\times10^5\text{m} = 6.78\times10^6\text{m}$

$r_a = R + 4000\text{km}$

$= 6.38\times10^6\text{m} + 4\times10^6\text{m}$

$= 10.38\times10^6\text{m}$

$$r_a + r_p = 2a \Rightarrow a = \frac{10.38\times10^6\text{m} + 6.78\times10^6\text{m}}{2} = 8.58\times10^6\text{m}$$

$$\Rightarrow T = \frac{2\pi(8.58\times10^6\text{m})^{3/2}}{\left[(6.67\times10^{-11}\frac{\text{Nm}^2}{\text{kg}^2})(5.97\times10^{24}\text{kg})\right]^{1/2}} \Rightarrow \boxed{T = 7913\text{s} = 2.2\text{h}}$$

b) $L = Mvr$ IS CONSERVED $\Rightarrow Mv_a r_a = Mv_p r_p$

$$\Rightarrow \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{10.38\times10^6}{6.78\times10^6} \Rightarrow \boxed{\frac{v_p}{v_a} = 1.53}$$

c) $E = \frac{1}{2}MV^2 - \frac{GM_EM}{r}$ IS CONSERVED

$\Rightarrow \frac{1}{2}MV_a^2 - \frac{GM_EM}{r_a} = \frac{1}{2}MV_p^2 - \frac{GM_EM}{r_p}$

$\Rightarrow \frac{1}{2}V_a^2 - \frac{1}{2}V_p^2 = \frac{GM_E}{r_a} - \frac{GM_E}{r_p} = GM_E\left(\frac{1}{r_a} - \frac{1}{r_p}\right)$

$V_p = 1.53V_a \Rightarrow \frac{1}{2}V_a^2 - \frac{1}{2}(1.53V_a)^2 = GM_E\left(\frac{1}{r_a} - \frac{1}{r_p}\right)$

$\Rightarrow \frac{1}{2}(1-1.53^2)V_a^2 = GM_E\left(\frac{1}{r_a} - \frac{1}{r_p}\right) \Rightarrow V_a^2 = \frac{2GM_E}{(1-1.53^2)}\left(\frac{1}{r_a} - \frac{1}{r_p}\right)$

$\Rightarrow V_a^2 = \frac{2(6.67\times10^{-11}Nm^2/kg^2)(5.97\times10^{24}kg)}{(1-1.53^2)}\left(\frac{1}{10.38\times10^6 m} - \frac{1}{6.78\times10^6 m}\right)$

$= (-5.94\times10^{14})(-5.12\times10^{-8})m^2/s^2 = 3.04\times10^7 m^2/s^2$

$\Rightarrow \boxed{V_a = 5512m/s}$ $V_p = 1.53(5512m/s) \Rightarrow \boxed{V_p = 8434m/s}$

d) TO ESCAPE EARTH $V_{es} = \sqrt{\frac{2GM_E}{r}}$

AT PERIGEE, $V_{es} = \sqrt{\frac{2GM_E}{r_p}} = 10838m/s$ (I THINK YOU KNOW THE NUMBERS BY NOW!)

IT ALREADY HAS $V_p = 8434m/s \Rightarrow \Delta V_p = V_{es} - V_p \Rightarrow \boxed{\Delta V_p = 2404m/s}$

AT APOGEE, $V_{es} = \sqrt{\frac{2GM_E}{r_a}} = 8759m/s$, $V_a = 5512m/s$

$\Delta V_a = V_{es} - V_a \Rightarrow \boxed{\Delta V_a = 3247m/s}$

EASIER TO DO AT PERIGEE.