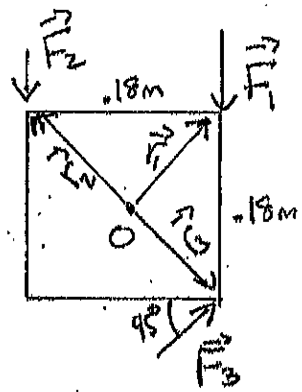
Physics 160, HW #10

Mastering: 7 problems from Chapter 10

Written: 10.86

10.3



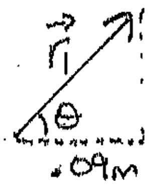
$F_1 = 18N,\ F_2 = 26N,\ F_3 = 14N$

FIND NET TORQUE

$$\sum \vec{\tau} = \vec{\tau}_1 + \vec{\tau}_2 + \vec{\tau}_3$$

$$\vec{\tau}_1 = \vec{r}_1 \times \vec{F}_1$$

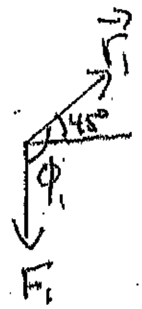
Center of square:



$r_1 = \sqrt{(.09m)^2 + (.09m)^2} = .09(\sqrt{2})\,m$

$= .127m$

$\theta = 45°$



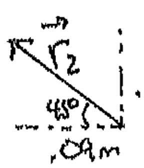
$$\tau_1 = r_1 F_1 \sin\phi_1 = .09(\sqrt{2})(18)\sin(90° + 45°)$$

$$= .09(\sqrt{2})(18)\cos 45° = .09(\sqrt{2})(18)\frac{1}{\sqrt{2}}$$

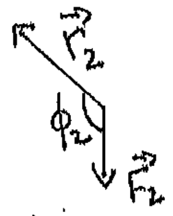
$$= (.09)(18) = 1.62\ N\cdot m$$

By RHR, $\vec{\tau}_1 = 1.62 N\cdot m$, ⊗

$\vec{\tau}_2 = \vec{r}_2 \times \vec{F}_2$



$r_2 = .09(\sqrt{2})$ ← Also could see FROM distance to edge from center EQUAL

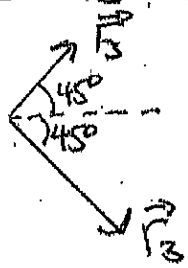


$$\tau_2 = r_2 F_2 \sin\phi_2 = .09(\sqrt{2})(26)\sin(90^\circ + 45^\circ)$$

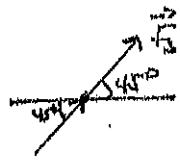
$$= .09(26) = 2.34\,\text{N}\cdot\text{m}$$

From RHR, $\vec{\tau}_2 = 2.34\,\text{N}\cdot\text{m}, \odot$

$$\vec{\tau}_3 = \vec{r}_3 \times \vec{F}_3$$



$$r_3 = .09\sqrt{2}\,\text{m}$$



$\rightsquigarrow$ opposite Angles equal, gives $\vec{F}_3$ at 45°

$$\tau_3 = r_3 F_3 \sin\phi_3 \qquad \phi_3 = 45^\circ + 45^\circ = 90^\circ$$

$$\sin 90^\circ = 1 \;\Rightarrow\; \tau_3 = r_3 F_3 = .09\sqrt{2}(14) = 1.26\sqrt{2}$$

$$= 1.7819\,\text{N}\cdot\text{m}$$

RHR $\Rightarrow$ $\vec{\tau}_3 = 1.26\sqrt{2}\,\text{N}\cdot\text{m}, \odot$

$$\sum \vec{\tau} = 1.62\,\text{N}\cdot\text{m}, \otimes + 2.34\,\text{N}\cdot\text{m}, \odot + 1.26\sqrt{2}\,\text{N}\cdot\text{m}, \odot$$

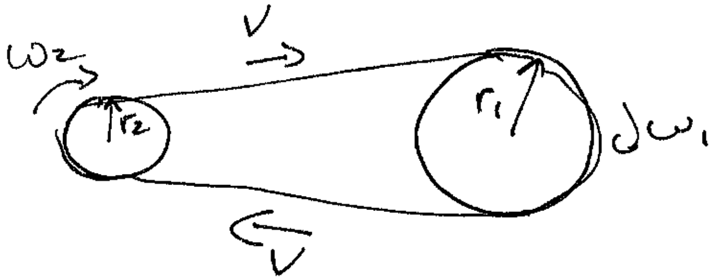
Make $\odot$ positive $\Rightarrow$ $\sum \tau = -1.62\,\text{N}\cdot\text{m} + 2.34\,\text{N}\cdot\text{m} + 1.26\sqrt{2}\,\text{N}\cdot\text{m}$

$$= 2.5019\,\text{N}\cdot\text{m}$$

$$\Rightarrow \boxed{\sum \vec{\tau} = 2.5\,\text{N}\cdot\text{m}, \odot}$$

$\odot \Rightarrow \tau$ = Counter-Clockwise

Simple Gear System



PART A: FIND $\frac{\omega_1}{\omega_2}$.

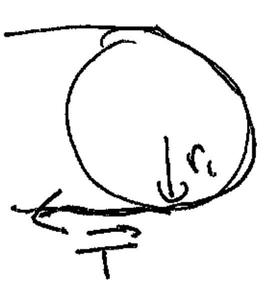
SAME LINEAR SPEEDS $\Rightarrow V_1 = V_2 \Rightarrow \omega_1 r_1 = \omega_2 r_2$

$$\Rightarrow \boxed{\frac{\omega_1}{\omega_2} = \frac{r_2}{r_1}}$$

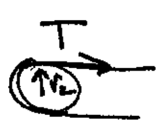
PART B: FIND $\frac{\tau_1}{\tau_2}$

THE TENSION IN THE CHAIN CAUSES THE ROTATION.

Single massless chain $\Rightarrow$ EQUAL TENSION ON #1 AND #2



$$\tau_1 = r_1 T$$



$$\tau_2 = r_2 T$$

$$\Rightarrow \boxed{\frac{\tau_1}{\tau_2} = \frac{r_1 T}{r_2 T} = \frac{r_1}{r_2}}$$

Part C

Ratio of Powers: Hopefully you read in textbook

$P = \vec{F} \cdot \vec{v}$ becomes $P = \vec{\tau} \cdot \vec{\omega}$ in the rotational case

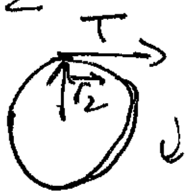
#1

$\circlearrowright \omega_1 \Rightarrow \vec{\omega}_1 = \otimes$

$\vec{r}_1 \times \vec{T} = \otimes$

$\Rightarrow P_1 = \tau_1 \omega_1$

#2



$\circlearrowright \omega_2 \Rightarrow \vec{\omega}_2 = \otimes$

$\vec{\tau}_2 = \otimes$

$\Rightarrow P_2 = \tau_2 \omega_2$

$$\Rightarrow \frac{P_1}{P_2} = \frac{\tau_1 \omega_1}{\tau_2 \omega_2} = \left(\frac{\tau_1}{\tau_2}\right)\left(\frac{\omega_1}{\omega_2}\right) = \left(\frac{r_1}{r_2}\right)\left(\frac{r_2}{r_1}\right) = 1$$

$$\Rightarrow \boxed{\frac{P_1}{P_2} = \frac{1}{1}}$$

← Gears do not create energy

Part D:

$$P_1 = 27000\,J/min\ ,\quad \omega_1 = 450\,rad/min$$

$$g = \frac{r_{front}}{r_{rear}} = \frac{r_1}{r_2} = 4$$

Find $\tau_{rear} = \tau_2$.

$$P_1 = \tau_1 \omega_1 \Rightarrow \frac{27000\,J}{\cancel{min}} = \tau_1 \left(\frac{450\,rad}{\cancel{min}}\right)$$ ← No need to convert since 'min' cancels

$$\Rightarrow \tau_1 = \frac{27000\,J}{450\,rad} = \frac{27000\,N\cdot m}{450\,\cancel{rad}} = 60\,N\cdot m$$

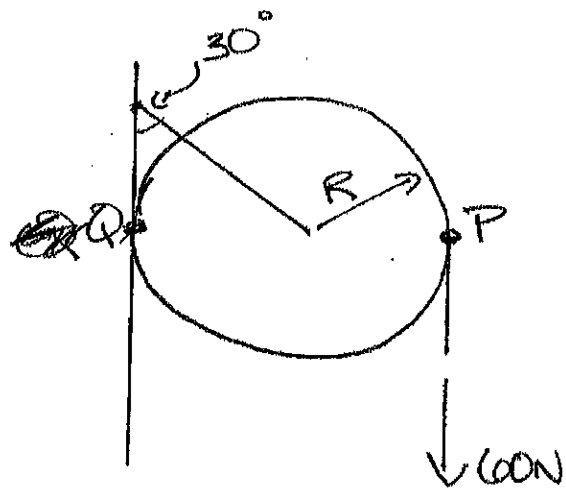
↑ inconvenient

$$\frac{\tau_1}{\tau_2} = \frac{r_1}{r_2} = g \Rightarrow \tau_2 = \frac{\tau_1}{g} = \frac{60\,N\cdot m}{4} = 15\,N\cdot m$$

Part E: g decreased to .7 but same power and ω_1

$$\Rightarrow \text{same } \tau_1 \Rightarrow \tau_2 = \frac{60\,N\cdot m}{.7} = 85.7\,N\cdot m$$ ← More torque $\left\{ \frac{\omega_1}{\omega_2} = \frac{1}{g} \Rightarrow \omega_2 = g\omega_1 \Rightarrow \omega_2 \text{ decreases} \right.$

10.69



$M = 16\,Kg$

$R = 18\,cm = .18\,m$

$I = .26\,Kg \cdot m^2$

$\mu_K = .25$

a) WHAT IS FORCE OF ROD?

b) ANGULAR ACCELERATION.

FORCES ON ROLL OF PAPER : GRAVITY : $Mg = (16kg)(9.8m/s^2)$

$mg = 156.8N$. ASSUMING UNIFORM DENSITY

$\Rightarrow m\vec{g} = 156.8N$ DOWN AND LOCATED AT CENTER.

60N FORCE DOWN. PAPER TOUCHING ROLL AT POINT LABELED P $\Rightarrow$ 60N, DOWN AT P

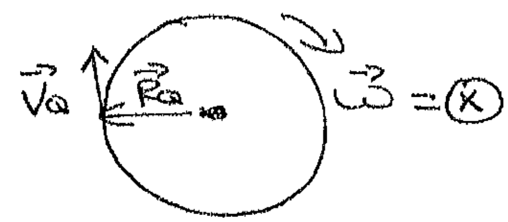
NORMAL FORCE FROM WALL $\Rightarrow \vec{n}$ to Right

FRICTION FORCE (KINETIC), $\vec{f}_K$. $f_K = \mu_K n$

BOTH NORMAL AND FRICTION AT CONTACT POINT $\Rightarrow$ Q

AT Q, SINCE THE Roll will ROTATE clockwise,

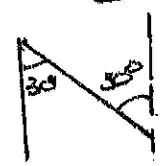
$\vec{V}_Q$ IS UPWARD

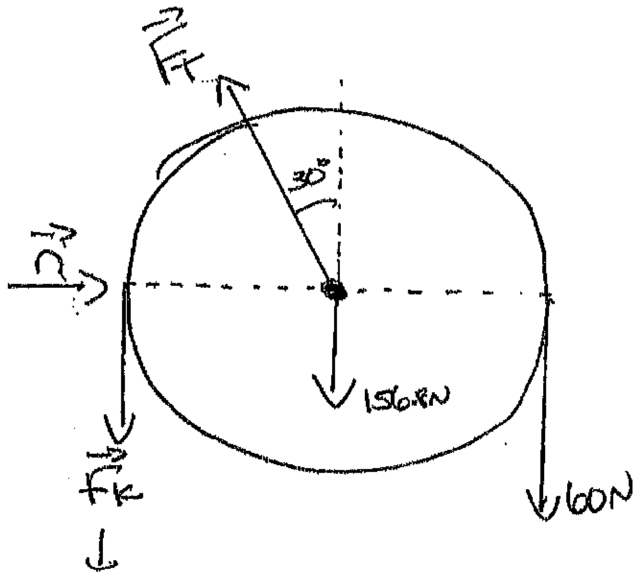


$\vec{f}_K$ opposite DIRECTION to $\vec{V}_Q$ $\Rightarrow$ $\vec{f}_K$ IS DOWN

FINALLY: BRACKET pulls (like tension) $\Rightarrow$ $\vec{F}_T$ at $30°$ (FROM vertical)

SO THE FBD IS





$f_K = \mu_K n = .25n$

$\sum F_x = Ma_x$, $\sum F_y = Ma_y$.

CENTER OF Roll NOT MOVING $\Rightarrow$ $a_x = 0$, $a_y = 0$

$$\sum F_x = 0$$

$$\Rightarrow n - F_T \sin 30° = 0$$

$$\Rightarrow n = F_T \sin 30° = \tfrac{1}{2} F_T$$

$$\sum F_y = 0 \Rightarrow F_T \cos 30° - f_K - 156.8N - 60N = 0$$

$$\Rightarrow F_T(.866) - .25n - 216.8N = 0$$

$n = \frac{1}{2} F_T = .5 F_T$

$\therefore\ F_T(.866) - .25(.5 F_T) = 216.8N$

$\Rightarrow F_T(.866 - .125) = 216.8N$

$\Rightarrow F_T(.741) = 216.8N \Rightarrow F_T = \frac{216.8N}{.741} = 292.58$

$= \underline{\underline{293N}}$

$n = .5 F_T = .5(292.58) = 146.3N$

$f_K = .25n = .25(146.3N) = 36.57N$

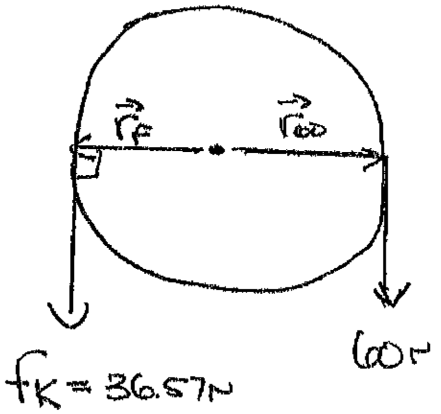
What is Ang. Acceleration? $\sum \vec{\tau} = I\vec{\alpha}$

$\vec{F}_T$, $m\vec{g}$ at center so they apply no torque.

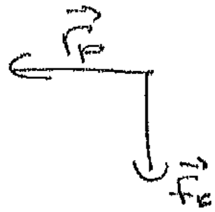
For $\vec{n}$: $\xrightarrow{\vec{n}}\xleftarrow{\vec{R}_n} \Rightarrow \xleftarrow{\vec{R}_n}\cdot\xrightarrow{\vec{n}}$ $180°$ $\sin 180° = 0$

$\Rightarrow$ No torque

THAT LEAVES $\vec{f_K}$ AND 60N



$$\vec{\tau_F} = r_F\, f_K \sin 90^\circ, \odot \quad \text{RHR}$$



$$= R f_K, \odot$$

$$= (.18m)(36.57N), \odot$$

$$= 6.5826\,N\cdot m, \odot$$

$$\vec{\tau_{60}} = r_{60}(60) \sin 90^\circ, \otimes \quad \text{RHR}$$

$$= R(60N), \otimes$$

$$= .18m(60N), \otimes$$

$$= 10.8\,N\cdot m, \otimes$$

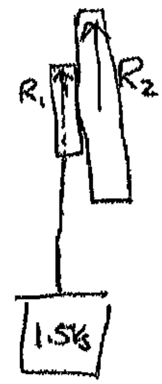
MAKING $\odot$ POSITIVE $\Rightarrow \sum \tau = \tau_F - \tau_{60}$

$$= 6.5826 - 10.8 = -4.2174\, N\cdot m$$

$$\sum \tau = I\alpha \Rightarrow \alpha = \frac{\sum \tau}{I} = \frac{-4.2174\,N\cdot m}{.26\,kg\cdot m^2} = -16.22\, rad/s^2$$

$$\Rightarrow \vec{\alpha} = \underline{16.2\, rad/s^2}, \otimes \rightarrow \text{clockwise}$$

16.71



$R_1 = 2.5\text{cm} = .025\text{m}$, $M_1 = .8\text{kg}$

$R_2 = 5\text{cm} = .05\text{m}$, $M_2 = 1.6\text{kg}$

$$I = I_1 + I_2 = \tfrac{1}{2}M_1R_1^2 + \tfrac{1}{2}M_2R_2^2$$

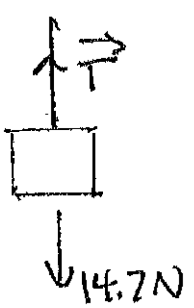
$$= \tfrac{1}{2}(.8\text{kg})(.025\text{m})^2 + \tfrac{1}{2}(1.6\text{kg})(.05\text{m})^2$$

$$= 2.25\times10^{-3}\ \text{kg}\cdot\text{m}^2 \quad (\text{just like } 9.87)$$

a) FIND ACCELERATION IF STRING is attached to M_1

FORCES ON 1.5kg: GRAVITY DOWN: $(1.5\text{kg})(9.8\text{m/s}^2) = 14.7\text{N}$

TENSION, $\vec{T}$ up



LOOKING AHEAD, MAKING DOWN POSITIVE IS A good IDEA

$$\sum F_y = Ma_y \Rightarrow -T + 14.7\text{N} = 1.5\text{kg}\ a_y$$

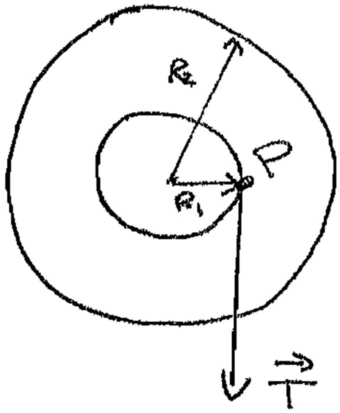
MASSLESS ROPE SO SAME TENSION

FORCES ON DISKS: GRAVITY DOWN, TENSION, $\vec{T}$ DOWN

AN UPWARDS SUPPORT FORCE BY THE AXLE.

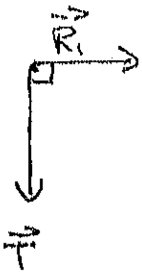
GRAVITY AND AXLE FORCE APPLIED AT CENTER SO EXERT NO TORQUE (SO I will IGNORE)

FROM THE SIDE



$$\sum \vec{\tau} = \vec{\tau}_T \text{ only}$$

$$\Rightarrow \vec{\tau}_T = I\vec{\alpha}$$



$$\vec{\tau}_T = R_1 T \sin 90^\circ, \otimes$$

$$= R_1 T, \otimes$$

$\Rightarrow \vec{\alpha} = \dfrac{R_1 T}{I}, \otimes$ SINCE MASSLESS STRING CONNECTS DISKS AND 1.5kg $\Rightarrow$ THE TANGENTIAL Acceleration at P SHOWN ABOVE AND a_y OF 1.5kg ARE THE SAME

$\vec{a}_{tan} = \vec{\alpha} \times \vec{R}_1 \Rightarrow \vec{a}_{tan} = \alpha R_1 \sin 90^\circ, \downarrow = \alpha R_1, \downarrow$ ← Down is positive

$\Rightarrow a_y = \alpha R_1 \Rightarrow \alpha = \dfrac{a_y}{R_1}$ $\therefore$ $\dfrac{a_y}{R_1} = \dfrac{R_1 T}{I} \Rightarrow \cancel{a_y = R_1^2 T}$

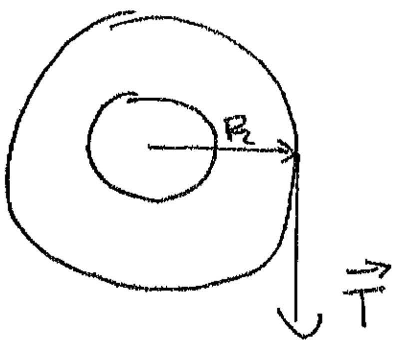
$$T = \frac{I a_y}{R_1^2} = \frac{2.25\times10^{-3}\,kg\cdot m^2}{(.025m)^2} a_y = 3.6\,kg\, a_y$$

$$-T + 14.7N = 1.5kg\, a_y \Rightarrow -3.6kg\, a_y + 14.7N = 1.5kg\, a_y$$

$\Rightarrow (1.5kg + 3.6kg)\, a_y = 14.7N$

$\Rightarrow 5.1kg\, a_y = 14.7N \Rightarrow a_y = \dfrac{14.7N}{5.1kg} = \underline{\underline{2.88 m/s^2}}$

b) REPEAT WITH STRING ON R_2



ONLY DIFFERECE IS $\tau_T = R_2 T$

$\Rightarrow T = \dfrac{I a_y}{R_2^2} = \dfrac{(2.25 \times 10^{-3} kg \cdot m^2)\, a_y}{(.05m)^2}$

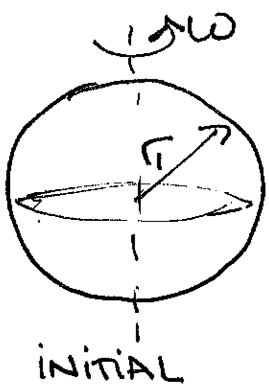
$= (.9kg)\, a_y$

$\therefore -.9kg\, a_y + 14.7N = 1.5kg\, a_y$

$\Rightarrow a_y = \dfrac{14.7N}{2.4kg} = \underline{\underline{6.125 m/s^2}}$

↳ REMEMBER THAT $\overset{1.5kg}{\wedge}$ MASS IS going FASTER when ATTACHED TO LARGER DISK. SO LARGER ACCELERATION MAKES SENSE

10.41 Neutron Star



$r_1 = 7\times10^5 km$

$\omega = \frac{1 rev}{30 days}$

$r_2 = 16 km!$

Find Final Angular Speed

Single object Angular Momentum Conservation:

$$L_1 = L_2 \Rightarrow I_1\omega_1 = I_2\omega_2$$

Solid Sphere (Before and After) $\Rightarrow I = \frac{2}{5}MR^2$

$$\Rightarrow \frac{2}{5}Mr_1^2\omega_1 = \frac{2}{5}Mr_2^2\omega_2 \Rightarrow r_1^2\omega_1 = r_2^2\omega_2$$

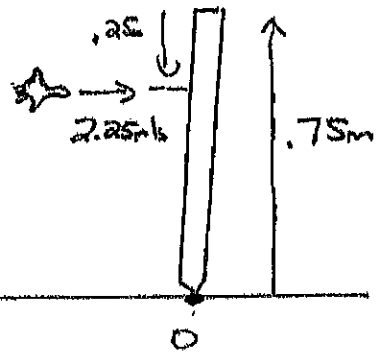
$r_1 = 7\times10^5 km = 7\times10^8 m,\quad r_2 = 16 km = 16000 m = 1.6\times10^4 m$

$$\omega_1 = \frac{1 rev}{30 day}\times\frac{2\pi rad}{rev}\times\frac{day}{24h}\times\frac{h}{3600s} = 2.42\times10^{-6} rad/s$$

$$\Rightarrow \omega_2 = \frac{r_1^2\omega_1}{r_2^2} = \frac{(7\times10^8 m)^2(2.42\times10^{-6} rad/s)}{(1.6\times10^4 m)^2} = 4640 rad/s$$

738 rev/s!!!

10.95



$M_{BIRD} = 500g = .5Kg$

$M_{Bar} = 1.5Kg$

UNIFORM Bar ROTATED ABOUT ONE END $\Rightarrow I_{Bar} = \frac{1}{3}ML^2$

$= \frac{1}{3}(1.5Kg)(.75m)^2$

$= .28125\ Kg \cdot m^2$

a) How FAST AFTER Collision?

BIRD is point particle $\Rightarrow L_{BIRD} = M_{BIRD}Vr$.

r = DISTANCE FROM O $= .75m - .25m = .5m$

Bar HAS MANY VALUES of V $\Rightarrow L_{Bar} = I_{Bar}\omega$

CONSERVATION: $M_{BIRD}V_1r_1 + I_{Bar}\omega_1 = M_{BIRD}V_2r_2 + I_{Bar}\omega_2$

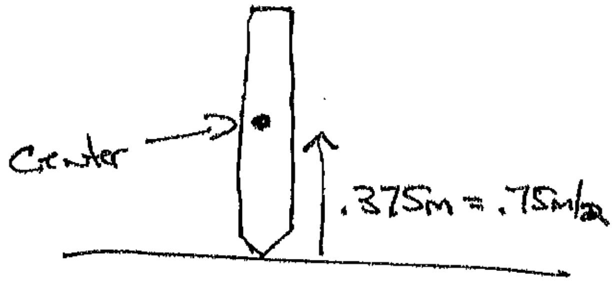
$V_1 = 2.25m/s$, $r_1 = r = .5m$, $\omega_1 = 0$, BIRD Drops to ground $\Rightarrow r_2 = 0$

$\omega_2 = ?$

$\therefore (.5Kg)(2.25m/s)(.5m) = .28125\ Kg \cdot m^2\ \omega_2$

$\Rightarrow \omega_2 = \dfrac{.5625\ Kg \cdot m^2/s}{.28125\ Kg \cdot m^2} = 2/s = \underline{2rad/s}$ INSERT when Needed

b) How Fast as it hits ground?



Bar rotating, gravity, so conservation ⇒

$$\frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}I\omega_3^2 + mgy_3$$

$\omega_2 = 2\,rad/s$, $y_2 = .375m$

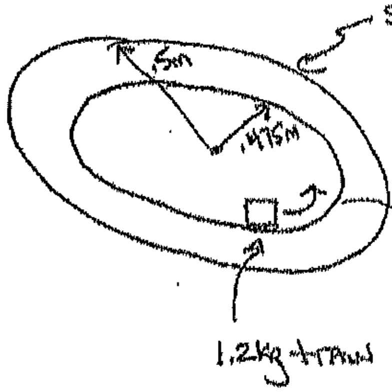
$\omega_3 = ?$, $y_3 = 0$ ← hits ground

$$\therefore \frac{1}{2}(.28125\,Kg\cdot m^2)(2\,rad/s)^2 + 1.5Kg(9.8m/s^2)(.375m) = \frac{1}{2}(.28125)\omega_3^2$$

$$\Rightarrow 6.075\,J = \frac{1}{2}(.28125\,Kg\cdot m^2)\omega_3^2$$

$$\Rightarrow \omega_3 = \sqrt{43.2/s^2} = \underline{\underline{6.57\,rad/s}}$$

10.97



SOLID plywood, $r = \frac{1m}{2} = .5m$ (stupid diameter!)
Free to rotate about center

$\omega = ?$

train track at $r = \frac{.95m}{2} = .475m$ (stupider diameter)

Train and plywood initially at rest $\Rightarrow L = 0$
Train goes counter-clockwise $\Rightarrow$ $\boxed{\text{plywood must rotate clockwise}}$ to keep $L = 0$

$L = 0 = L_{train} - L_{plywood}$ (Negative because opposite directions)

Solid plywood $\Rightarrow$ many values of v $\Rightarrow$ $L_{plywood} = I\omega$

Solid cylinder, $I = \frac{1}{2}mR^2 = \frac{1}{2}(7kg)(.5m)^2 = .875\ kg\cdot m^2$

Train has single value of v $\Rightarrow$ $L_{train} = mvr$

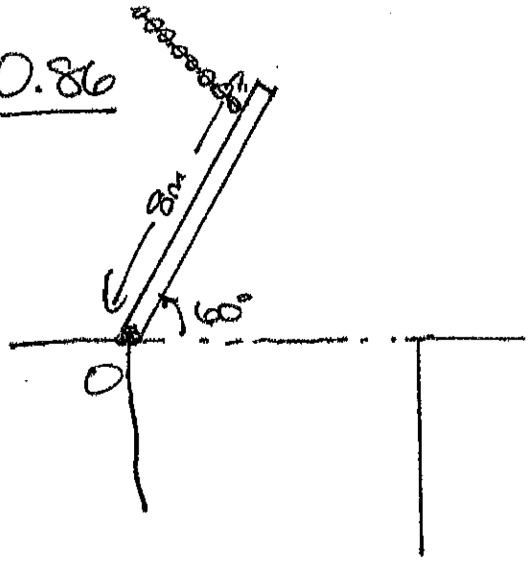
But .6m/s is speed relative to track, plywood rotating opposite to track.
$\Rightarrow v = .6m/s - v_{plywood}$. at .475m, $v_{plywood} = \omega r = \omega(.475m)$

$0 = L_{train} - L_{plywood} \Rightarrow 0 = (1.2kg)(.6m/s - .475m\times\omega)(.475m) - .875kg\cdot m^2(\omega)$

$\Rightarrow 0 = .342\ kg\cdot m^2/s - .27075\ kg\cdot m^2(\omega) - .875\ kg\cdot m^2(\omega)$

$\Rightarrow 0 = .342\ kg\cdot m^2/s - 1.14575\ kg\cdot m^2(\omega) \Rightarrow \omega = \frac{.342\ kg\cdot m^2/s}{1.14575\ kg\cdot m^2} = .2985\ rad/s \approx .3\ rad/s$

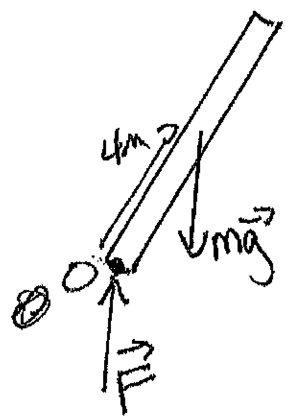
10.86



a) Find Angular Acceleration just after cable breaks

After cable breaks there is gravity and a support force at O acting on drawbridge

Uniform ⇒ Center of mass at center.



support force that is opposite to $m\vec{g}$ and keeps $a_y = 0$

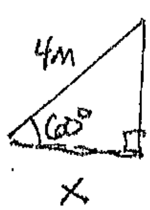
$$\sum \vec{\tau} = I\vec{\alpha}$$

$\vec{F}$ is at O ⇒ No Torque

$$\therefore \sum \vec{\tau} = \vec{\tau}_g$$

For a vertical force like gravity

$$\tau_g = x mg$$



$$x = 4\text{m} \cos 60° = 2\text{m}$$

By RHR, $\vec{\tau}_g = x m g$, ⊗

WHICH JUST TELLS US BRIDGE IS going to Rotate Clockwise (WHICH WE ALL KNEW)

$\therefore \; x m g = I\alpha$

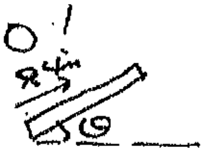
DON'T HAVE MASS VALUE, BUT WE DON'T NEED IT SINCE WE CAN WRITE $I = \frac{1}{3}ML^2$ SINCE THE DOOR IS A THIN SLAB ROTATED About ONE END.

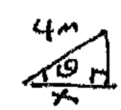
$$\Rightarrow x m g = \tfrac{1}{3} m L^2 \alpha \Rightarrow \alpha = \frac{xg}{\frac{1}{3}L^2} = \frac{3xg}{L^2} = \frac{3(2m)(9.8m/s^2)}{(8m)^2}$$

$$= .91875/s^2 = \underline{\underline{.919 rad/s^2}}$$

b) Could WE USE $\omega = \omega_0 + \alpha t$? $\longrightarrow$ NO!

AFTER Angle changes FROM 60°, SO DOES x





$x = 4m\cos\theta \Rightarrow (4m)\cos\theta \, mg = I\alpha \Rightarrow (4m)\cos\theta \, mg = \frac{1}{3}ML^2\alpha$

$\Rightarrow \alpha = \frac{(12m)(g)\cos\theta}{L^2} = (1.8375 rad/s^2)\cos\theta \Rightarrow$ $\boxed{\alpha \text{ IS NOT CONSTANT}}$

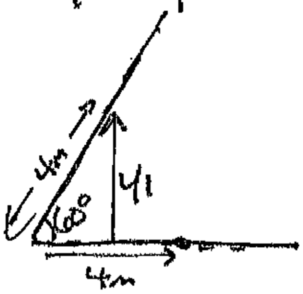
$\Rightarrow$ WON'T WORK

c) WHAT IS ω when DRAWBRIDGE IS HORIZONTAL?

GRAVITY ONLY FORCE EXERTING TORQUE $\Rightarrow$ CONSERVATION OF ENERGY WITH ROTATIONAL KINETIC ENERGY AND GRAVITATIONAL POT.

$$\therefore \ \frac{1}{2}I\omega_1^2 + mgy_1 = \frac{1}{2}I\omega_2^2 + mgy_2$$

y_2, y_2 = CENTER OF MASS HEIGHT



$$y_1 = 4m\sin 60° = 3.4641m$$

$$y_2 = 0$$

$\omega_1 = 0, \ \omega_2 = ?, \ I = \frac{1}{3}mL^2$

$$\Rightarrow mgy_1 = \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_2^2 \Rightarrow gy_1 = \frac{1}{6}L^2\omega_2^2$$

$$\Rightarrow \omega_2 = \sqrt{\frac{6gy_1}{L^2}} = \sqrt{\frac{6(9.8m/s^2)(4m)\sin 60°}{(8m)^2}} = 1.78399/s$$

$$= \underline{1.78\ rad/s}$$