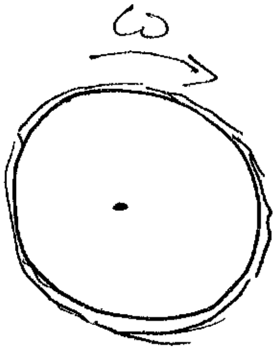
Physics 160, HW #9

Mastering: 7 problems From Chapter 9

Written: 10.80

GRINDING WHEEL



At $t=0$ $\omega_0 = 20\text{ rad/s}$

$\alpha = 25\text{ rad/s}^2$ UNTIL $t = 1.9\text{s}$

FROM THERE, WHEEL TURNS THROUGH 430 RAD AS IT STOPS

PART A: TOTAL ANGLE FROM 0 to Stop.

FIRST FIND ANGLE UNTIL $t = 1.9\text{s}$, $\theta_1 = ?$

$$\theta_1 = \cancel{\theta_0} + \omega_0 t + \tfrac{1}{2}\alpha t^2 = (20\text{ rad/s})(1.9\text{s}) + \tfrac{1}{2}(25\text{ rad/s}^2)(1.9\text{s})^2$$

$$\Rightarrow \theta_1 = 83.125\text{ rad}$$

SO $\boxed{\theta_{\text{total}} = 83.125\text{ rad} + 430\text{ rad} = 513.125\text{ rad} = 513\text{ rad}}$

PART B: AT WHAT TIME DOES WHEEL STOP

$\Rightarrow$ HOW LONG SINCE ZERO?

At $t = 1.9\text{s}$, $\omega = \omega_0 + \alpha t \Rightarrow \omega = 20\text{ rad/s} + 25\text{ rad/s}^2(1.9\text{s})$

$$\Rightarrow \omega = 67.5\text{ rad/s}$$

So For Coasting, $\omega_0 = 67.5 \text{ rad/s}$, $\omega = 0$

$\theta - \theta_0 = 430 \text{ rad}$

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

$$\Rightarrow 0 = (67.5 \text{ rad/s})^2 + 2\alpha(430 \text{ rad})$$

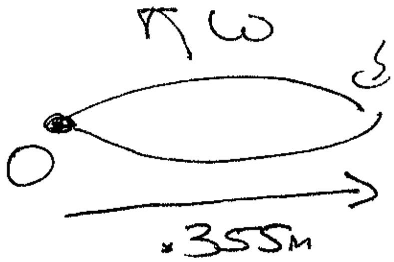
$$\Rightarrow \boxed{\alpha = \frac{-(67.5 \text{ rad/s})^2}{2(430 \text{ rad})} = -5.30 \text{ rad/s}^2 \leftarrow \text{Part C}}$$

$$\omega = \omega_0 + \alpha t \Rightarrow 0 = 67.5 \text{ rad/s} - 5.3 \text{ rad/s}^2 \, t_2$$

$$\Rightarrow t_2 = 12.74 \text{ s} \leftarrow \text{ELAPSED TIME SINCE } 1.9 \text{ s}$$

$$t = 1.9 \text{ s} + 12.74 \text{ s} = 14.64 \text{ s} = \boxed{14.6 \text{ s}}$$

Electric Ceiling Fan:



CEILING FAN BLADE → THE PROBLEM SAYS THEY'RE SUPPOSED TO BE CIRCULAR. I'VE NEVER SEEN SUCH A THING!

$\omega_0 = .24\,\text{rev/s}$

$\alpha = .88\,\text{rev/s}^2$

↳ For Circular Axis at Center ⇒ ½ × diameter

PART B: # of revolutions after .194s? ← Accidentally did this first.

KINEMATICS IS VERY ~~FLEX~~ ADAPTABLE WHEN IT COMES TO UNITS.

$$\theta = \cancel{\theta_0} + \omega_0 t + \tfrac{1}{2}\alpha t^2 = (.24\,\text{rev/s})(.194\,\text{s}) + \tfrac{1}{2}(.88\,\text{rev/s}^2)(.194\,\text{s})^2$$

will give θ in REV., $\boxed{\theta = .0631\,\text{rev}}$

PART B: $V_{tan} = V$ at $t = .194\,\text{s}$. $V = \omega r$

So we need ω, ⇒ $\omega = \omega_0 + \alpha t = .24\,\text{rev/s} + .88\,\text{rev/s}^2(.194\,\text{s})$

PART A → ⇒ $\boxed{\omega = .41072\,\text{rev/s}}$ $V = \omega r$ IS NOT ADAPTABLE

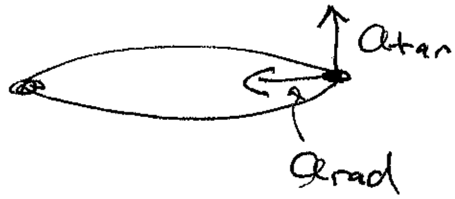
MUST USE RADIANS. $\omega = .41072\,\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}} = 2.58\,\text{rad/s}$

$$\boxed{V = \omega r = (2.58\,\cancel{\text{rad}}/\text{s})(.355\,\text{m}) = .916\,\text{m/s}}$$

↑ INCONVENIENT

PART C: MAGNITUDE OF ACCELERATION:

$$a = \sqrt{a_{rad}^2 + a_{tan}^2}$$



$$a_{rad} = \omega^2 r = (2.58\,\cancel{\text{rad}}/s)^2(.355m) = 2.363\,m/s^2$$

INCONVENIENT

$a_{tan} = \alpha r \Leftarrow$ HAVE TO USE RADIANS HERE TOO

$$\alpha = .88\,rev/s^2 \times \frac{2\pi\,rad}{rev} = 5.5292\,rad/s^2$$

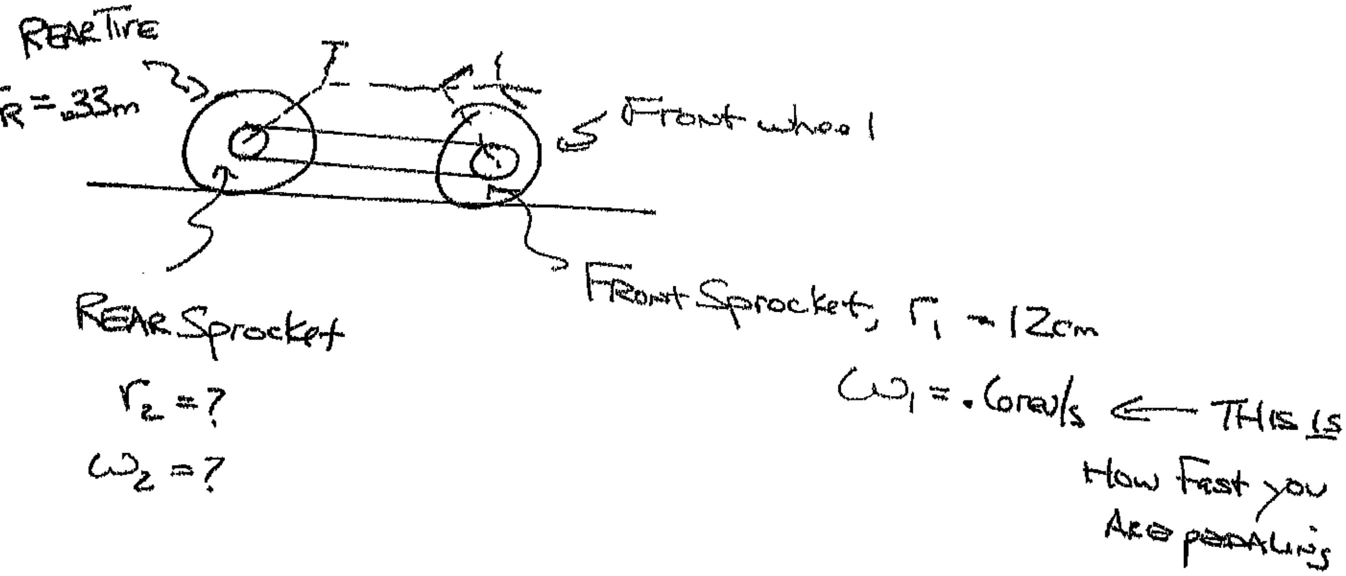
$$a_{tan} = (5.5292\,\cancel{\text{rad}}/s^2)(.355m) = 1.96\,m/s^2$$

INCONVENIENT

$$a = \sqrt{(2.363\,m/s^2)^2 + (1.96\,m/s^2)^2} = 3.0719\,m/s^2$$

$$\Rightarrow \boxed{a = 3.07\,m/s^2}$$

9.71



WE WANT $V_R = 5m/s$ FOR POINT ON RIM OF REAR TIRE.

$V = \omega r \Rightarrow V_R = \omega_R r_R \Rightarrow \omega_R = \frac{5m/s}{.33m} = 15.1515... 1/s =$ Need rad

15.1515 $\frac{rad}{s}$

BECAUSE REAR TIRE AND SPROCKET ARE ON SAME AXIS

$\omega_R = \omega_2$ (THEY HAVE SAME ANGULAR VELOCITY)

FRONT AND REAR Sprockets Connected by (NON-slipping chain)

$\Rightarrow \omega_1 r_1 = \omega_2 r_2$. Need SAME ω unit: $\omega_1 = .6\frac{rev}{s} \times \frac{2\pi rad}{rev} = 3.77 rad/s$

$\therefore 3.77 rad/s (12cm) = 15.1515 rad/s \; r_2 \Rightarrow \underline{\underline{r_2 = 2.99cm}}$

ROTATIONAL KINETIC ENERGY OF EARTH:

$M = 5.97 \times 10^{24}$ kg

$R_E = 6.38 \times 10^{6}$ m

Part A: $I = ?$, SOLID SPHERE $I = \frac{2}{5} M R^2$

$\Rightarrow I = \frac{2}{5}(5.97 \times 10^{24}\,\text{kg})(6.38 \times 10^{6}\,\text{m})^2 = 9.72 \times 10^{37}\,\text{kg}\cdot\text{m}^2$

Part B: $I < 9.72 \times 10^{37}\,\text{kg}\cdot\text{m}^2$ IN REAL LIFE BECAUSE MASS CONCENTRATED TOWARDS CENTER $\Rightarrow$ Bigger M_i at small r_i, $I \approx \sum M_i r_i^2 \Rightarrow$ smaller r_i^2, so I IS LESS THAN UNIFORM sphere.

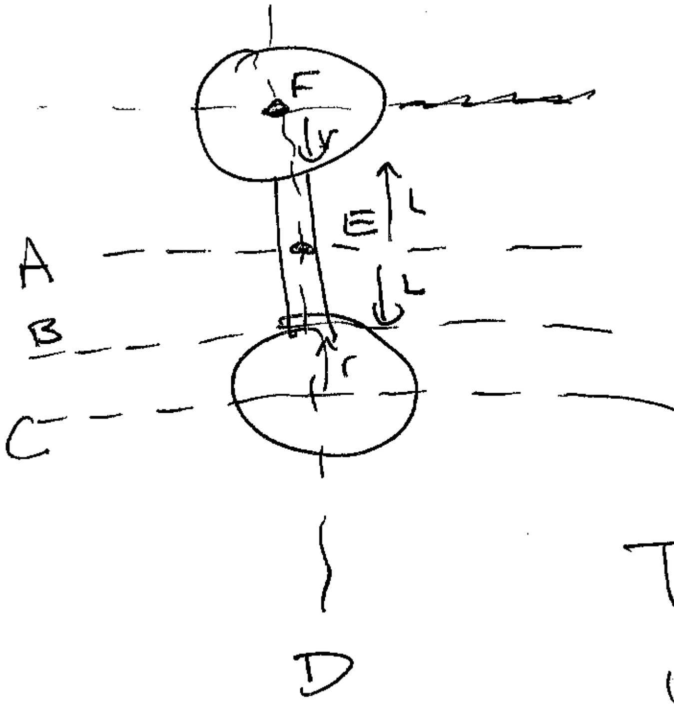
Part C: $K = \frac{1}{2} I \omega^2$. FIND ω FROM PERIOD

EARTH ROTATES 1rev IN 24h $\Rightarrow \omega = \frac{1\,\text{rev}}{24\,\text{h}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{\text{h}}{3600\,\text{s}}$

$\Rightarrow \omega = 7.27 \times 10^{-5}$ rad/s

$\therefore K = \frac{1}{2}(9.72 \times 10^{37}\,\text{kg}\cdot\text{m}^2)(7.27 \times 10^{-5}\,\text{rad/s})^2 = 2.57 \times 10^{29}\,\text{J}$

MOMENT OF INERTIA

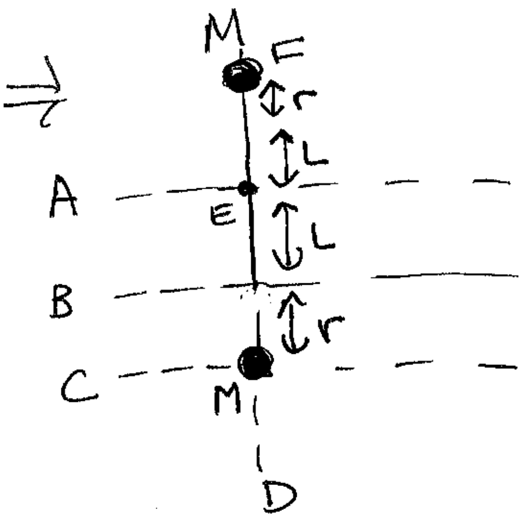


TWO SPHERES OF EQUAL SIZE

THIN ROD ≈ NEGLIGIBLE WEIGHT

TO ESTIMATE MOMENTS USE $I = \sum_i M_i r_i^2$

AND ASSUME ALL MASS OF SPHERES AT CENTER (AND ROD NEGLIGIBLE)



FOR A: $I_A = M(L+r)^2 + M(L+r)^2$

$= M\,2(L+r)^2$

$I_A = M(2L^2 + 4Lr + 2r^2)$

NOTE FOR E: SAME DISTANCES

$\Rightarrow I_E = I_A$

FOR B: $I_B = M(2L+r)^2 + Mr^2$

$= M(4L^2 + 4Lr + r^2 + r^2)$

$\Rightarrow I_B = M(4L^2 + 4Lr + 2r^2)$

$\Rightarrow I_B > I_A$

FOR C: $I_C = M(2L+2r)^2 + M(0)$

$= M4(L+r)^2 = M(4L^2 + 8Lr + 4r^2)$

$\Rightarrow I_C > I_B$

NOTE: FOR F, $I_F = M(0) + M(2L+2r)^2$

$\Rightarrow I_F = I_C$

FINALLY, FOR D $r = 0$ FOR BOTH SINCE EACH MASS IS ON THE AXIS

SO FINAL RANKING IS:

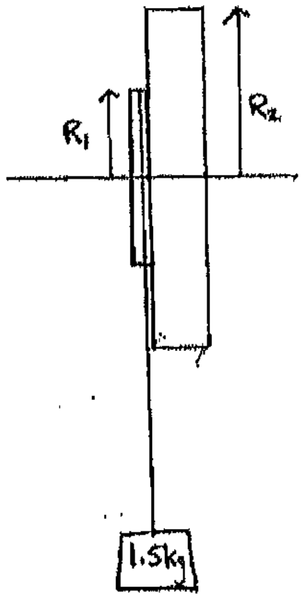
Largest   Smallest

I_F $\quad I_B \quad$ I_A $\quad I_D$

I_C $\qquad\quad$ I_E

EQUAL $\qquad\qquad$ EQUAL

9.87



$M_1 = .80\text{Kg},\ M_2 = 1.6\text{Kg}$

$R_1 = 2.5\text{cm} = .025\text{m},\ R_2 = 5\text{cm} = .05\text{m}$

TWO DISKS: $I = \frac{1}{2}MR^2$

$I_1 = \frac{1}{2}(.80\text{Kg})(.025\text{m})^2 = 2.5\times10^{-4}\text{Kgm}^2$

$I_2 = \frac{1}{2}(1.6\text{Kg})(.05\text{m})^2 = 2\times10^{-3}\text{Kgm}^2$

$I = \int \rho r^2 dV = \int \rho r^2 dV_1 + \int \rho r^2 dV_2 = I_1 + I_2$

↳ TOTAL VOLUME

$\Rightarrow I = 2.5\times10^{-4}\text{Kgm}^2 + 2\times10^{-3}\text{Kgm}^2 \Rightarrow \boxed{I = 2.25\times10^{-3}\text{Kgm}^2}$

b) $M = 1.5\text{Kg}$ IS $y_1 = 2\text{m}$ ABOVE FLOOR. WITH WHAT SPEED DOES IT HIT THE FLOOR?

CONSERVATION OF ENERGY: $\frac{1}{2}I\omega_1^2 + \frac{1}{2}MV_1^2 + Mgy_1 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}MV_2^2 + Mgy_2$.

$\omega_1 = 0,\ V_1 = 0,\ y_1 = 2\text{m},\ \omega_2 = ?,\ V_2 = ?,\ y_2 = 0 \Rightarrow Mgy_1 = \frac{1}{2}I\omega_2^2 + \frac{1}{2}MV_2^2$

THE MASS IS SIMPLY CONNECTED TO R_1, SO IT MUST HAVE THE SAME VELOCITY AS A POINT ON THE OUTSIDE OF R_1. $\Rightarrow V_2 = \omega_2 R_1 \Rightarrow \omega_2 = V_2/R_1$

$\Rightarrow Mgy_1 = \frac{1}{2}I\left(\frac{V_2}{R_1}\right)^2 + \frac{1}{2}MV_2^2 \Rightarrow Mgy_1 = \frac{1}{2}\left(I/R_1^2\right)V_2^2 + \frac{1}{2}MV_2^2$

$\Rightarrow Mgy_1 = \frac{1}{2}\left(I/R_1^2 + M\right)V_2^2 \Rightarrow V_2 = \sqrt{\frac{2Mgy_1}{I/R_1^2 + M}}$

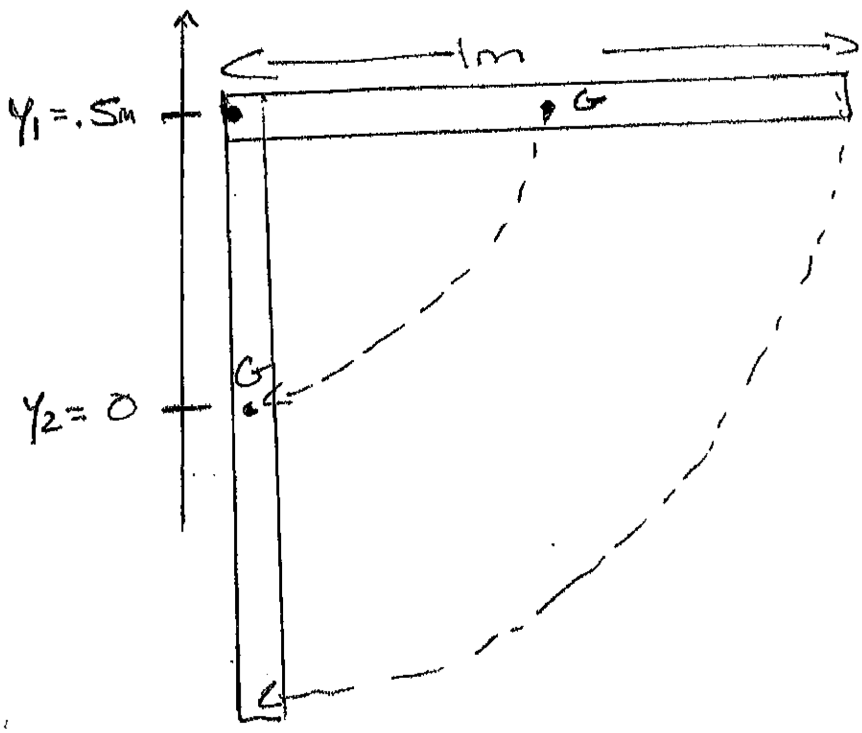
$$\Rightarrow V_2 = \left[\frac{2(1.5\text{Kg})(9.8\text{m/s}^2)(2\text{m})}{\frac{2.25\times10^{-3}\text{Kgm}^2}{(.025\text{m})^2} + 1.5\text{Kg}}\right]^{1/2} \Rightarrow \boxed{V_2 = 3.40\text{m/s}}$$

c) WHAT IF STRING IS CONNECTED TO R_2? $\Rightarrow V_2 = \omega_2 R_2 \rightarrow$ REPLACE R_1 WITH R_2.

$$V_2 = \left[\frac{2(1.5\text{Kg})(9.8\text{m/s}^2)(2\text{m})}{\frac{2.25\times10^{-3}\text{Kgm}^2}{(.05\text{m})^2} + 1.5\text{Kg}}\right]^{1/2} \Rightarrow \boxed{V_2 = 4.95\text{m/s}}$$

FASTER BECAUSE MASS IS ATTACHED TO LARGER CYLINDER $\Rightarrow$ LARGER LINEAR VELOCITY.

9.81



M = .18 Kg
Released FROM rest
Swings THROUGH
VERTICAL.

a) FIND ΔU_g → For potential Energy, All mass Acts As if Concentrated At center of mass. The problem Doesn't explicitely state, but it's A reasonable Assumption THAT A meter stick's center is at .5m = G

$\Delta U_g = mgy_2 - mgy_1$. $y_2 = 0, y_1 = .5m$

$\Rightarrow \Delta U_g = -.18kg(9.8 m/s^2)(.5m) = \underline{-.882J}$

b) ω of stick.

CONSERVATION OF ENERGY (NO FRICTION)

$\Rightarrow \Delta U_g = -\Delta K \Rightarrow \Delta K = .882\text{J}$. STICK ROTATING

$\Rightarrow \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2 = .882\text{J}$

$\omega_1 = 0 \quad \therefore \quad \frac{1}{2} I \omega_2^2 = .882\text{J}$

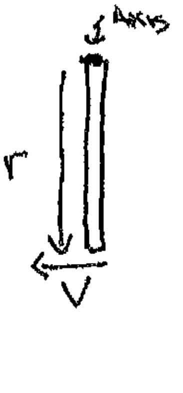
FOR SLENDER ROD ROTATED ABOUT ONE END:

$I = \frac{1}{3} ML^2 = \frac{1}{3}(.18\text{kg})(1\text{m})^2 = .06\ \text{kg}\cdot\text{m}^2$

$\therefore \quad \omega_2 = \sqrt{\dfrac{2(.882\text{J})}{.06\text{kg}\cdot\text{m}^2}} = \sqrt{29.4 \text{ }^{1}/_{s^2}} = \underline{5.42\ \text{rad/s}}$

$\hookrightarrow$ Unit: $\dfrac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{kg}\cdot\text{m}^2} = \dfrac{1}{\text{s}^2} = \dfrac{\text{rad}^2}{\text{s}^2}$ INSERT when Needed

c) LINEAR SPEED OF STICK'S END



$V = \omega r. \quad r = 1\text{m} \Rightarrow V = 5.42\text{rad/s}(1\text{m})$

$= \underline{5.42\text{m/s}}$

d) Compare with particle:

$$\frac{1}{2}\cancel{m}V_1^2 + \cancel{m}gy_1 = \frac{1}{2}\cancel{m}V_2^2 + \cancel{m}gy_2$$

$V_1 = 0$, $y_1 = 1m$, $V_2 = ?$, $y_2 = 0$

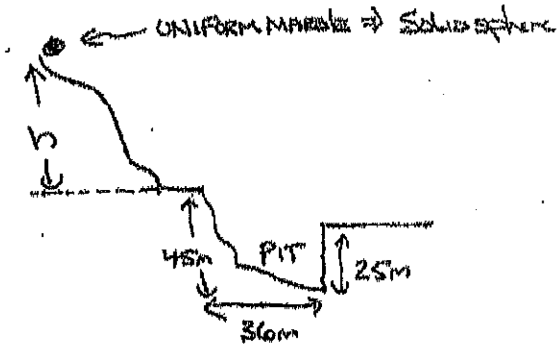
$$\Rightarrow (9.8m/s^2)(1m) = \frac{1}{2}V_2^2$$

$$\Rightarrow V_2 = \sqrt{2(9.8m/s^2)(1m)} = \underline{\underline{4.43m/s}}$$

Slower

Seems surprising since Δy was bigger for particle. But a rotating object has more "inertia" $\Rightarrow$ infinitely many particles in motion so the result is a larger speed.

10.80



a) MINIMUM h NOT TO FALL INTO PIT. Rolls WITHOUT slipping.

AT ~~END~~ EDGE OF PIT CENTER OF MARBLE going HORIZONTAL.

TO MISS PIT, $X = 36m$ WHEN $y = 25m - 45m = -20m$

IGNORE AIR RESISTANCE ⇒ projectile MOTION WITH HORIZONTAL LAUNCH

$\Rightarrow X = V_0 t, \quad y = -\frac{1}{2}gt^2$

$\Rightarrow -20m = -\frac{1}{2}(9.8m/s^2)t^2 \Rightarrow t = \sqrt{\frac{40m}{9.8m/s^2}} = 2.02s$

$\Rightarrow 36m = V_0(2.02s) \Rightarrow V_0 = 17.819 m/s$

FOR Rolling Down Hill, NO slipping $\Rightarrow K = \frac{1}{2}mv^2\left(1 + \frac{I}{MR^2}\right)$

SOLID sphere $\Rightarrow I = \frac{2}{5}MR^2 \Rightarrow K = \frac{1}{2}mv^2\left(1 + \frac{\frac{2}{5}MR^2}{MR^2}\right)$

$\Rightarrow K = \frac{1}{2}mv^2\left(1 + \frac{2}{5}\right) = \frac{1}{2}mv^2\left(\frac{7}{5}\right) = \frac{7}{10}mv^2$

$$\frac{7}{10}mv_1^2 + mgy_1 = \frac{7}{10}mv_2^2 + mgy_2$$

$V_1 = 0$, $y_1 = h = ?$, $V_2 = V_0 = 17.819\,m/s$, $y_2 = 0$

$$\Rightarrow \cancel{m}(9.8\,m/s^2)h = \frac{7}{10}\cancel{m}(17.819\,m/s)^2$$

$$\Rightarrow \boxed{h = \frac{\frac{7}{10}(17.819\,m/s)^2}{9.8\,m/s^2} = 22.68\,m = 22.7\,m}$$

b) ~~M~~ R Cancelled. Physically R cancels because Larger R ⇒ Larger I ⇒ Doesn't have to go as fast to have the same KE as a smaller marble.

c) Same problem with frictionless block ⇒ sliding ⇒ $K = \frac{1}{2}mv^2$

$$\Rightarrow m(9.8\,m/s^2)h = \frac{1}{2}m(17.819\,m/s)^2$$

$$\Rightarrow \boxed{h = \frac{\frac{1}{2}(17.819\,m/s)^2}{9.8\,m/s^2} = 16.2\,m}$$

Smaller h because when sliding all kinetic energy goes to center of mass. When rolling $KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ ⇒ Kinetic energy shared between center of mass and rotation, so need more kinetic to reach V_0.