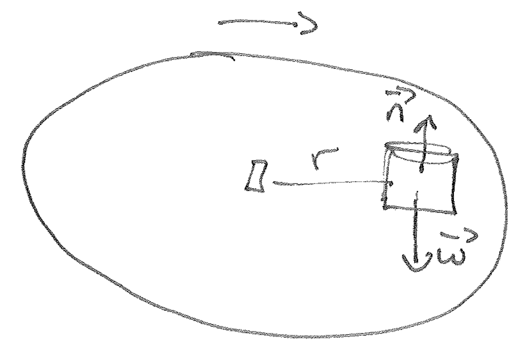
Physics 160, HW#6

Mastering Physics: 9 problems
from chapt. 5 & 6

Written: 6.73

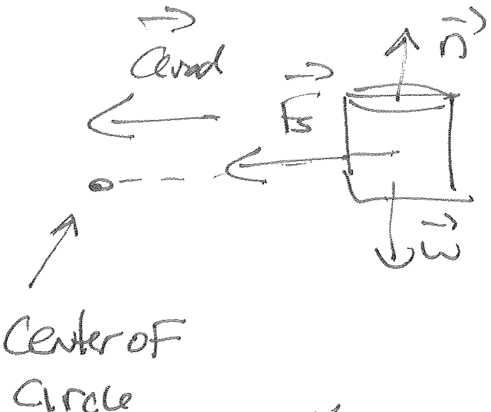
Mass on a Turntable

$M = .2\,\text{Kg}$, $\mu_s = .080$, $r = .15\,\text{m}$

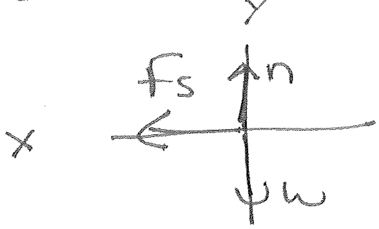


What is maximum speed of cylinder without slipping?

Forces on mass: Normal, Weight, and Static friction



Static friction must point towards center or circular motion isn't possible.



$$\sum F_x = M a_x, \quad \sum F_y = M a_y$$

$$a_x = a_{rad}, \quad a_y = 0$$

$$\sum F_y = 0 \Rightarrow n - w = 0 \Rightarrow n = w = Mg$$

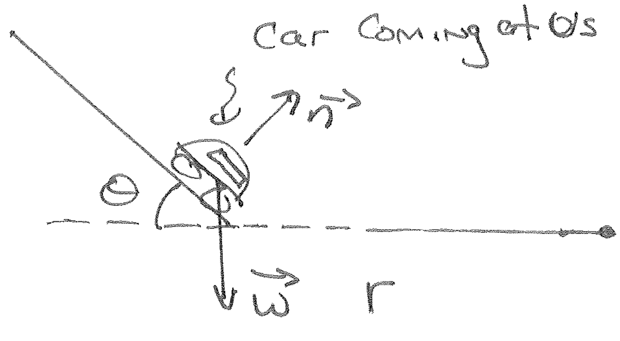
$$\sum F_x = M a_x \Rightarrow f_s = M a_{rad} = \frac{M v^2}{r}$$

At MAXIMUM SPEED $f_s = f_{s,MAX} = \mu_s n$

$$\Rightarrow \mu_s n = \frac{M V_{MAX}^2}{r} \qquad \Rightarrow \mu_s M g = \frac{M V_{max}^2}{r}$$

$$\Rightarrow V_{max} = \sqrt{\mu_s r g} = \sqrt{.08(.15m)(9.8m/s^2)} = .343 m/s$$

BANKED CURVE :



$M = 1500\,kg$

$V = \frac{50\,km}{h} \times \frac{1000\,m}{km} \times \frac{h}{3600\,s}$

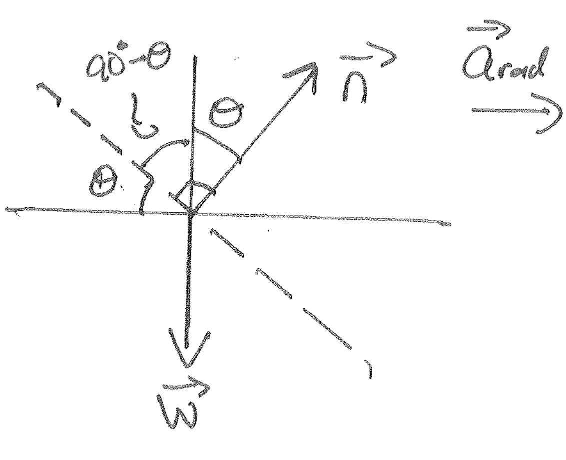
$= 13.88\bar{8}..\,m/s$

NO FRICTION

PART A: $\theta = 20°$ what is r?

FORCES ON Car : $\vec{n}$ at 90° to SURFACE, AND $\vec{w}$ DOWN

to go AROUND Circle, CAR MUST HAVE $\vec{a}_{rad}$ TOWARDS CENTER $\Rightarrow$ $\vec{a}_{rad}$ to RIGHT IN DRAWING.



NON-STANDARD Angle:

$\sin\theta = \frac{n_x}{n} \Rightarrow n_x = n\sin\theta$

$\cos\theta = \frac{n_y}{n} \Rightarrow n_y = n\cos\theta$

$$\sum F_x = Ma_x, \quad \sum F_y = Ma_y$$

$\vec{a}_{rad}$ to RIGHT $\Rightarrow a_x = a_{rad},\ a_y = 0$

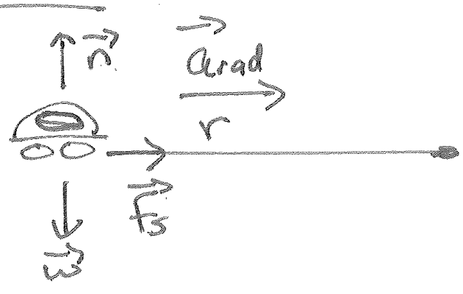
$$\therefore \sum F_y = 0, \quad \sum F_x = Ma_{rad} = \frac{Mv^2}{r}$$

$$\sum F_y = 0 \Rightarrow n\cos\theta - w = 0 \Rightarrow n = \frac{w}{\cos\theta} = \frac{Mg}{\cos\theta}$$

$$\sum F_x = \frac{Mv^2}{r} \Rightarrow n\sin\theta = \frac{Mv^2}{r} \Rightarrow \left(\frac{Mg}{\cos\theta}\right)\sin\theta = \frac{Mv^2}{r}$$

$$\Rightarrow g\tan\theta = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{g\tan\theta} = \frac{(13.888\,m/s)^2}{9.8\,m/s^2 \tan 20^\circ} = 54.1\,m$$

PART B:

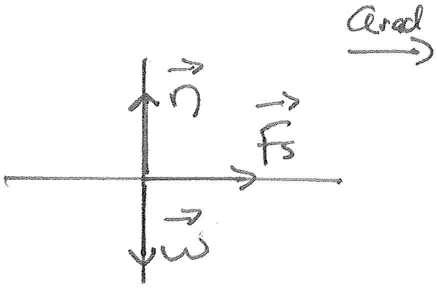


WHAT IS MINIMUM COEFFICIENT FOR CAR to have $V = 13.888\,m/s$ AND $r = 54.1\,m$

FLAT SURFACE SO $\vec{n}$ UP, $\vec{\omega}$ DOWN,

STATIC FRICTION, $\vec{f_s}$

$\vec{a}_{rad}$ still to RIGHT $\Rightarrow$ $\vec{f_s}$ MUST be to RIGHT ALSO OTHERWISE NO FORCE would be creating a_{rad}.



$$\sum F_x = Ma_x \,, \quad \sum F_y = Ma_y$$

$$a_x = a_{rad} = \frac{v^2}{r} \,, \quad a_y = 0$$

$$\sum F_y = 0 \Rightarrow n - w = 0 \Rightarrow n = w = Mg$$

$$\sum F_x = Ma_x \Rightarrow f_s = \frac{Mv^2}{r}$$

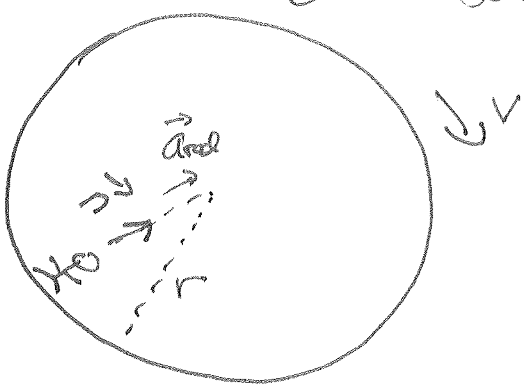
MINIMUM COEFFICIENT $\Rightarrow f_s = f_{s,MAX} = \mu_s n$

$$\therefore \mu_s n = \frac{Mv^2}{r} \Rightarrow \mu_s Mg = \frac{Mv^2}{r} \Rightarrow \mu_s = \frac{v^2}{rg} = \frac{(13.888\,m/s)^2}{(54.1\,m)(9.8\,m/s^2)}$$

$$\Rightarrow \mu_s = .364$$

5.49

"space-station"
WITH Diameter 800m $\Rightarrow r = 400m$



Only Force is Normal force $\vec{n}$

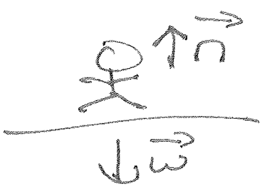
$\vec{n}$ towards Center

$$\sum \vec{F} = M\vec{a} \Rightarrow \sum F = Ma$$

$$\text{so} \quad n = Ma_{rad} \Rightarrow n = \frac{Mv^2}{r}$$

Like Always n = Apparent weight.

On Earth (or Mars)



$\sum F_y = Ma_y$. Normally $a_y = 0$

$\Rightarrow n - w = 0 \Rightarrow n = w = Mg$

So to make spacestation feel normal $n = Mg$

So $Mg = \frac{Mv^2}{r} \Rightarrow \frac{v^2}{r} = g$ (As you might have guessed $a_{rad} = g$)

$\Rightarrow V = \sqrt{rg}$, FOR EARTH gravity

$V = \sqrt{400m(9.8m/s^2)} = 62.6 m/s$ ← speed of outer edge

TO FIND F = REV per Minute.

Remember 1 rev = 1 Circumference = $2\pi r$ meters

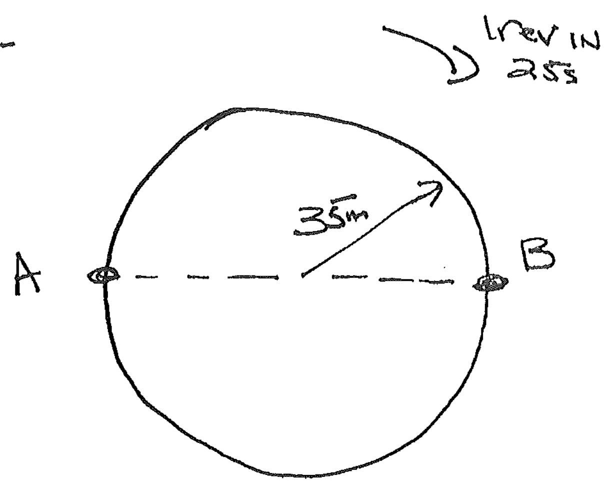
$$F = \frac{62.6m}{s} \times \frac{1 rev}{2\pi(400)m} \times \frac{60s}{min} = 1.4947 \, rev/min \approx 1.5 \, rev/min$$

FOR MARS gravity, $g = 3.7 m/s^2$

$V = \sqrt{400m(3.7m/s^2)} = 38.47 m/s$

$$38.47 m/s \times \frac{1 rev}{2\pi(400)m} \times \frac{60s}{min} = .918 \, rev/min$$

5.116



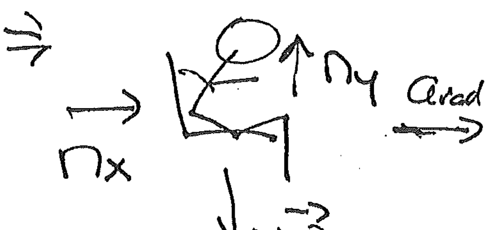
Ferris wheel Rider with $M = 85\text{kg}$

FIND MAGNitude AND DIRECTION OF NET FORCE exerted BY SEAT ON passenger at points A AND B

AT A: Center is to Right $\Rightarrow a_x = +a_{rad} = \frac{v^2}{r}$, $a_y = 0$

WE KNOW there's GRAVITY SO SEAT HAS TO PUSH UP ON PERSON to make $a_y = 0$ AND PUSH to RIGHT TO make $a_x \neq 0$.

$\Rightarrow$



n_x, n_y ARE the Components of $\vec{n}$

$$\sum F_x = Ma_x \Rightarrow n_x = \frac{Mv^2}{r}$$

$$\sum F_y = Ma_y \Rightarrow n_y - w = 0 \Rightarrow n_y = w = M$$

1 rev = ONCE AROUND = 1 Circumference $= 2\pi r$

$$\Rightarrow V = \frac{2\pi r}{25s} = \frac{2\pi(35m)}{25s} = (2.8 m/s)\pi$$

$\therefore$

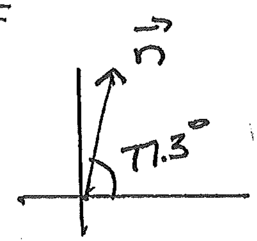
$$n_x = \frac{mv^2}{r} = \frac{(85kg)(2.8m/s)^2\pi^2}{35m} \cong (85kg)(2.21 m/s^2)$$

$$= 187.92N$$

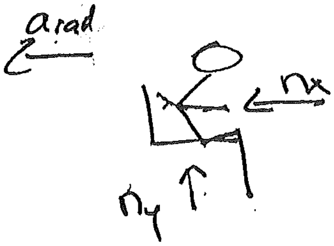
$$n_y = Mg = (85kg)(9.8m/s^2) = 833N$$

$$n = \sqrt{n_x^2 + n_y^2} = \sqrt{(187.92N)^2 + (833N)^2} = 854N$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}\left(\frac{833}{187.92}\right) \cong 77.3°$$



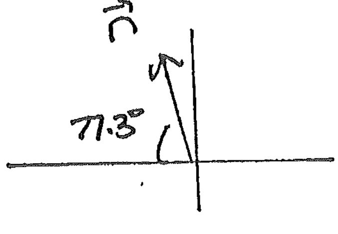
AT B, Center is to LEFT $\Rightarrow$ n_x must be to Left



Here, IN REALITY n_x would be provided BY FRICTION, OR IN THE WORST CASE SCENARIO ~~THE~~ A LAP Belt OR BAR.

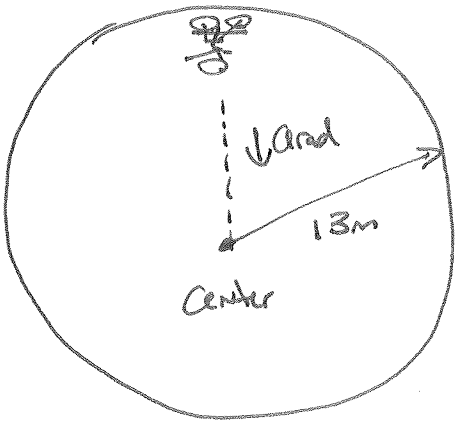
SAME V AND r $\Rightarrow$ $n_x = -187.92N$, $n_y = 833N$

$\Rightarrow$ $n = 854N$ $\quad \theta = 77.3°$ FROM $-x$ AXIS



standard Angle: $180° - 77.3° = \underline{102.7°}$

5.118

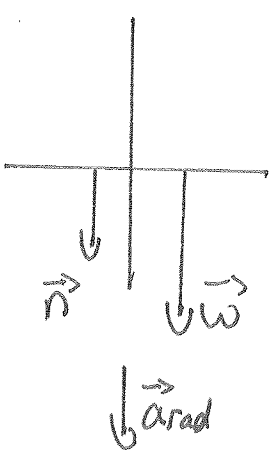


$M_{\text{physics major}} = 70\text{kg}$

$M_{\text{Motorcycle}} = 40\text{Kg}$

a) WHAT IS MINIMUM SPEED to Make it over the top?

AT TOP, forces are $\vec{w}$ DOWN AND $\vec{n}$ DOWN.



$\sum F_y = Ma_y$

Make DOWN positive

$\Rightarrow n + w = Ma_{rad}$

$\Rightarrow \underline{n + Mg = \frac{Mv^2}{r}}$

SURFACES CAN ONLY PUSH THEY CANNOT pull. Motorcycle is Below the sphere, So NORMAL Force is DOWN.

Mg is Constant, so AS V decreases so does NORMAL.

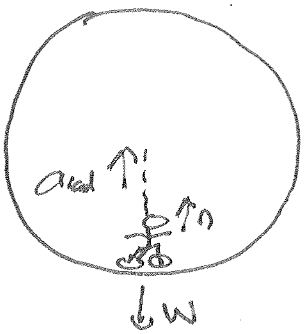
AT MINIMUM SPEED NORMAL BECOMES ZERO $\rightarrow$ Motorcycle loses Contact WITH SPHERE.

$n = 0 \Rightarrow \cancel{M}g = \frac{\cancel{M}v_{MIN}^2}{r}$

$\Rightarrow v_{MIN}^2 = rg \Rightarrow v_{MIN} = \sqrt{13m(9.8m/s^2)} = \underline{\underline{11.3m/s}}$

b) At bottom, $V = 2V_{MIN} = 22.6m/s$

WHAT is NORMAL force ON motorcycle?



$\sum F_y = Ma_y$. Make up positive

$\Rightarrow a_y = a_{rad}$.

$\therefore n - w = Ma_{rad} \Rightarrow n = w + Ma_{rad}$

$\Rightarrow n = Mg + Ma_{rad} = M(g + a_{rad}) = M\left(g + \frac{v^2}{r}\right)$

Bottom must hold up both

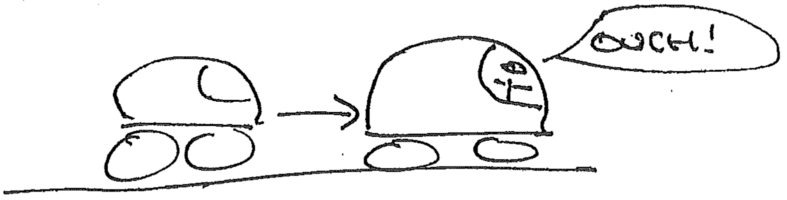
Motorcycle AND MAJOR $\Rightarrow M = M_{total} = 70kg + 40kg = 110kg$

$\therefore n = 110kg\left(9.8m/s^2 + \frac{(22.6m/s)^2}{13m}\right) = 110kg(9.8m/s^2 + 39.3m/s^2)$

$= 110kg(49.1m/s^2) = 5399.8N$

$= \underline{\underline{5400N}}$

6.69



Neck Bones
CAN withstand
8J, 10ms Collision

$10ms = 10\times10^{-3}s = .01s$

$M = 5kg$ ← HEAD ONLY

a) GREATEST SPEED DURING Collision? IF $W_{TOTAL} = 8J$

BONES BREAK. $W_{TOTAL} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$. $V_1 = 0$

SINCE INITIALLY at rest $\Rightarrow \frac{1}{2}mV_{max}^2 = W_{TOTAL}$

$\Rightarrow \frac{1}{2}(5kg)V_{MAX}^2 = 8J \Rightarrow V_{max} = \sqrt{\frac{2(8J)}{5kg}} = \sqrt{3.2m^2/s^2} = \underline{1.79m/s}$

UNIT: $\frac{J}{kg} = \frac{kg \cdot m^2/s^2}{kg} = m^2/s^2$

$1.79m/s = 1.79m/s \times \frac{1mi/h}{.447m/s} = \underline{4mi/h}$

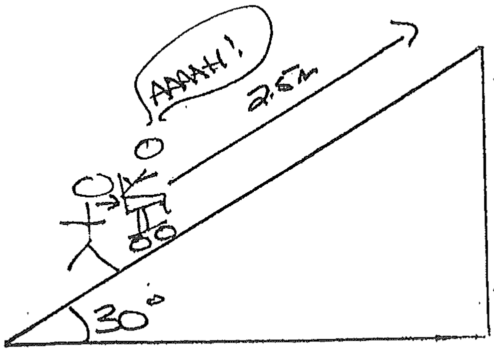
b. What IS Acceleration? → ASSUMED Constant (OF COURSE)

$V = \cancel{V_0}^{0} + at \Rightarrow a = \frac{V}{t} = \frac{1.79m/s}{.01s} = \underline{179m/s^2}$ $\frac{179m/s^2}{9.8m/s^2} = \underline{18.3\,g's}$

How large is Force ?

$$F = Ma = (5kg)(179 m/s^2) = \underline{\underline{895 N}}$$

6.84



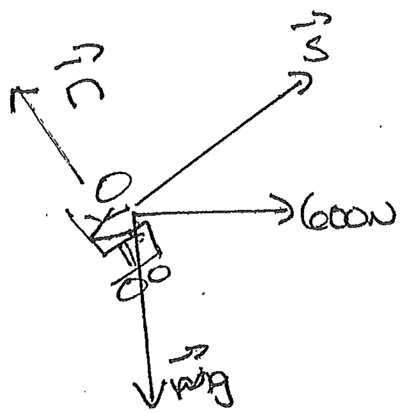
$M = 85\,\text{Kg}$

HORIZONTAL, 600N FORCE

$V_1 = 2\,\text{m/s}$

FIND speed at top.

FORCES ON CHAIR/PROFESSOR: NORMAL, Weight, 600N



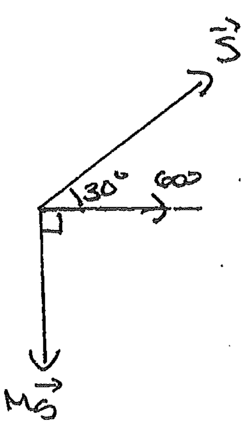
$$W_{TOTAL} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

$$W_{TOTAL} = W_n + W_g + W_{600}$$

AS ALWAYS, NORMAL DOES NO WORK

$$\Rightarrow W_g + W_{600} = \tfrac{1}{2}mV_2^2 - \tfrac{1}{2}mV_1^2$$

gravity AND 600N both constant $\Rightarrow W_g = M\vec{g} \cdot \vec{s}$, $W_{600} = \vec{F}_{600} \cdot \vec{s}$



$$\Rightarrow W_g = Mgs\cos 120° = (85\text{kg})(9.8\text{m/s}^2)(2.5\text{m})\cos 120°$$

$$= -1041.25\,\text{J}$$

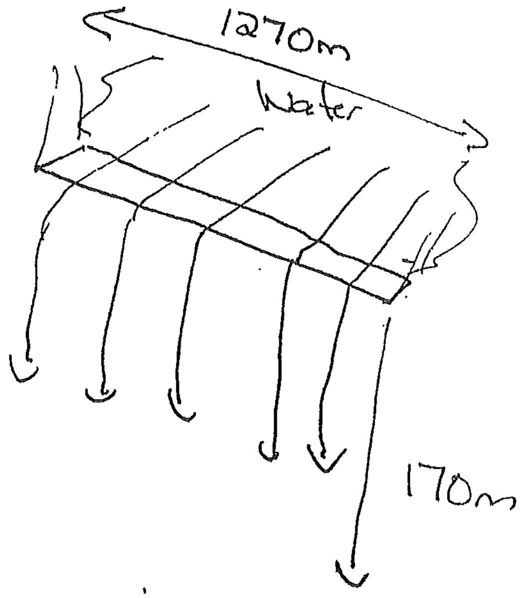
$$W_{600} = (600\text{N})(2.5\text{m})\cos 30° = 1299\,\text{J}$$

$$\therefore -1041.25J + 1299J = \frac{1}{2}(85kg)V_2^2 - \frac{1}{2}(85kg)(2m/s)^2$$

$$\Rightarrow 257.75J = \frac{1}{2}(85kg)V_2^2 - 170J$$

$$\Rightarrow V_2 = \sqrt{\frac{2(427.75J)}{85kg}} = \underline{\underline{3.17m/s}}$$

6.94



GRAND COULEE DAM generates 2000MW

↑ Mega-Watt

92% OF Work Done by gravity Converted to Electric Energy.

$1\,m^3$ of water has 1000kg of mass

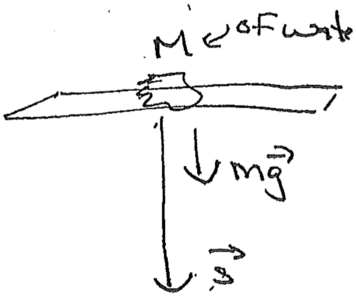
How many cubic meters of water per second?

$2000\,MWatt = 2000\times10^6\,Watt = 2\times10^9\,Watt = 2\times10^9\,J/s$

$\therefore\ .92\,W_g = 2\times10^9\,J$ EVERY SECOND. $\Rightarrow W_g = \frac{2\times10^9\,J}{.92} = 2.1739$

W_g: WORK DONE by gravity

So Gravity Needs to do $2.1739\times10^9\,J$ of work every second.



GRAVITY IS Constant FORCE $\Rightarrow W_g = m\vec{g}\cdot\vec{s} = mgs$

$s = 170m \Rightarrow 2.1739\times10^9\,J = m(9.8\,m/s^2)(170m)$

$\Rightarrow m = 1.3049\times10^6\,Kg$ of water every second

$1m^3$ of water $= 1000kg \Rightarrow Volume = 1.3049\times10^6\,kg \times \frac{1m^3}{1000kg} = 1304.9\,m^3$

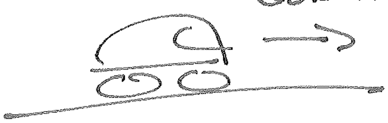
$= 1305\,m^3$

6.101

$m = 1800\,\text{kg}$

$P_{engine} = 7\,\text{hp}$

68 km/h

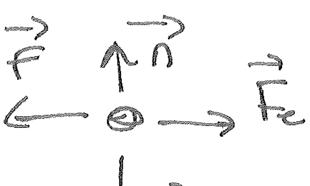


a) what is total retarding force, i.e., total friction?

forces on car: $\vec{n}$ up, $\vec{w}$ down, friction $\vec{f}$ to left

total

engine force, $\vec{F_e}$ to RIGHT ← As always, this is actually a third law reaction, but good enough



Drive at 68 km/h ⇒ constant speed ⇒ $a_x = 0$

$$\sum F_x = \overset{ma_x}{0} \Rightarrow F_e - f = 0 \Rightarrow f = F_e$$

68 km/h

$P_{engine} = \vec{F_e} \cdot \vec{V} = F_e V \cos 0°$ since both to the right

$\Rightarrow P_{engine} = F_e V$

but both P & V are in wrong units!

Fe in Newtons and V in m/s $\Rightarrow$ $N\cdot m/s = J/s = \text{Watt}$

$1\,hp = 746\,\text{Watt} \Rightarrow 7\,hp \times \frac{746\,\text{Watt}}{hp} = 5222\,\text{Watt}$

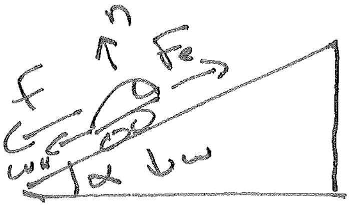
$68\frac{km}{h} \times \frac{1000\,m}{km} \times \frac{h}{3600\,s} = 18.889\,m/s$

$\therefore F_e = \frac{P_{engine}}{V} = \frac{5222\,\text{Watt}}{18.889\,m/s} = 276.458\,N \approx 276\,N$

$\therefore f = F_e = 276\,N$

b) what Power to drive up 10% grade at 68 km/h?

at 68 km/h total friction still the same $\Rightarrow$



$f = 276\,N$ still

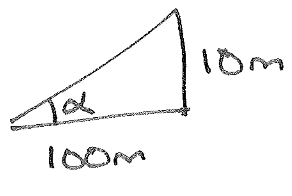
Now Engine Pushes Parallel to incline

Against friction AND $w_{\parallel}$!

$\sum F_{\parallel} = m a_{\parallel}$. $a_{\parallel} = 0$ still $\Rightarrow F_e - f - W_{\parallel} = 0$

$\Rightarrow F_e = f + W_{\parallel} = f + mg\sin\alpha$

10% grade $\Rightarrow$



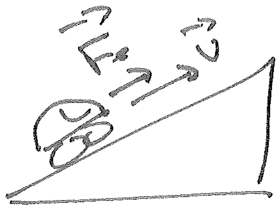
$$\tan\alpha = \frac{10\,m}{100\,m} = 0.1$$

In other words $\tan\alpha$ = grade in decimal

$$\therefore \alpha = \tan^{-1}(0.1) = 5.71^\circ$$

$$\therefore \quad F_e = f + mg\sin\alpha = 276N + (1800kg)(9.8m/s^2)\sin 5.71^\circ$$

$$= 276N + 1755N = 2031N$$



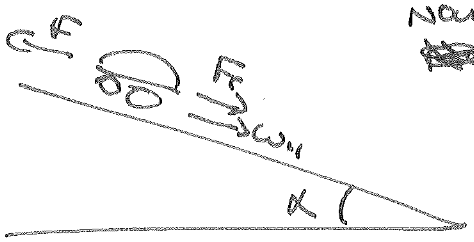
Angle Between $\vec{F_e}$ & $\vec{v}$ is still 0°

$$\Rightarrow P_{engine} = F_e V \cos 0^\circ = F_e V$$

$$\Rightarrow P_{engine} = (2031N)(18.89m/s) = 38368 \text{ watt}$$

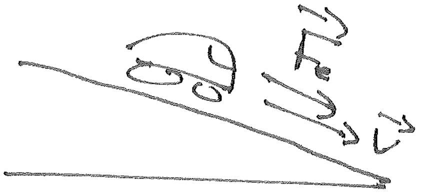
$$38368 \text{ watt} \times \frac{hp}{746 \text{ watt}} = 51.4\,hp$$

b) What Power Down 1%? $\Rightarrow \tan\alpha = 0.01 \Rightarrow \alpha = 0.573^\circ$



Now $W_{\parallel}$ help $\quad \sum F_{\parallel} = 0 \Rightarrow F_e + W_{\parallel} - f = 0$

$$\Rightarrow F_e = f - W_{\parallel} = 276N - (1800kg)(9.8m/s^2)\sin 0.573^\circ$$
$$= 276N - 176.4N = 99.6N$$



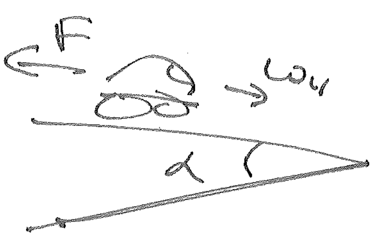
$\vec{F_e}$ and $\vec{v}$ still parallel, so $0°$ AGAIN

$$P_{engine} = F_e v \cos 0° = F_e v$$

$$\therefore P_{engine} = (99.6N)(18.889 m/s) = 1881.5 \text{ Watt}$$

$$1881.5 \text{ watt} \times \frac{hp}{746 \text{ watt}} = 2.52 hp$$

c) what grade to coast? $\Rightarrow F_e = 0$



$$\sum F_{\parallel} = 0 \Rightarrow W_{\parallel} - f = 0$$

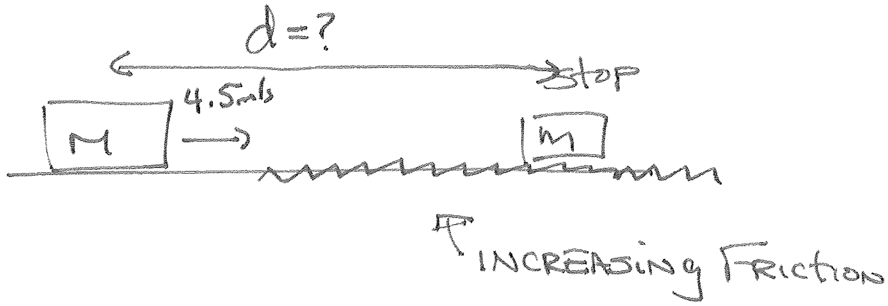
$$\Rightarrow W_{\parallel} = f \Rightarrow mg \sin\alpha = f$$

$$\Rightarrow \sin\alpha = \frac{f}{mg} = \frac{276N}{(1800kg)(9.8 m/s^2)} = 0.0156$$

$$\therefore \alpha = \sin^{-1}(0.0156) = 0.8965°$$

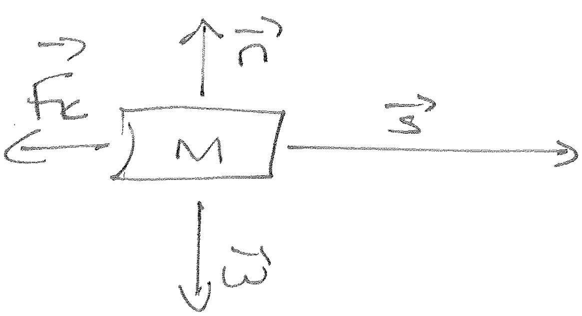
$$\tan\alpha = \text{grade} \Rightarrow \text{grade} = \tan(0.8965°) = 0.0156 \times 100$$
$$= 1.56\%$$

6.73



μ_K INCREASES FROM .1 to .6 over A distance of 12.5m

Forces ON BOX: $\vec{n}$ up, $\vec{w}$ DOWN, $\vec{f}_K$ to left



For displacement to right $\vec{n}, \vec{w}$ DO NO WORK

$\Rightarrow \vec{f}_K$ only Force Doing work

$\Rightarrow$ Work done by friction, $W_F = W_{total}$

WORK-ENERGY $\Rightarrow$ $W_F = \Delta K = \frac{1}{2}MV_2^2 - \frac{1}{2}MV_1^2$

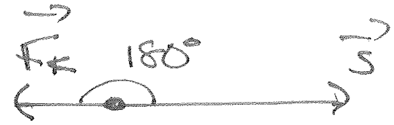
$V_2 = 0$, $V_1 = 4.5$m/s $\therefore$ $W_F = -\frac{1}{2}MV_1^2$

FRICTION IS A VARIABLE FORCE $\Rightarrow$ AREA UNDER CURVE (OR INTEGRATION IF you PREFER):

$$W_F = \int_0^d F_K \cos\phi \, dx$$

d = distance traveled = ?

For friction



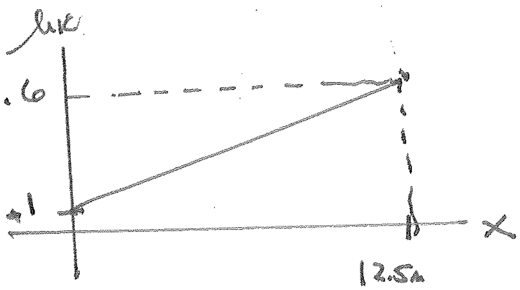
$\phi = 180^\circ$ (always)

$\cos 180^\circ = -1 \Rightarrow W_F = -\int_0^d f_K \, dx$

$f_K = \mu_K n$. $\sum F_y = M a_y$. $a_y = 0$ since no motion in y-direction $\Rightarrow n - W = 0$

$\Rightarrow n = W = Mg$

$\Rightarrow f_K = \mu_K Mg$. So need to find equation for μ_K to find f_K.



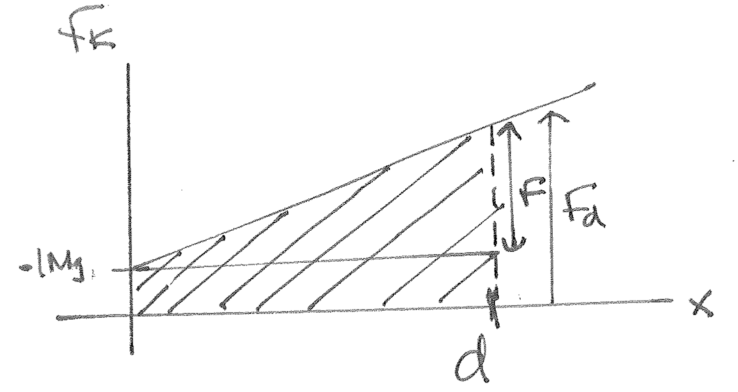
Straight line: $\mu_K = mx + b$

$m = \text{slope} = \dfrac{(.6 - .1)}{(12.5m - 0)} = \dfrac{.5}{12.5m} = .04/m$

b = y-intercept = .1

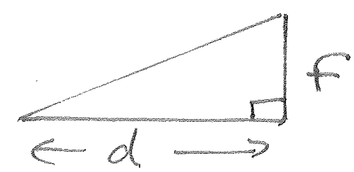
$\therefore \mu_K = (.04/m)X + .1$

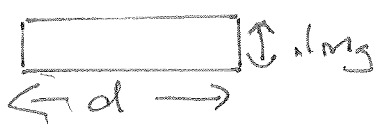
$$f_K = \mu_K Mg = (.04/m)x\,Mg + .1Mg$$



$\rightarrow$ Area $= W_F$

Area = rectangle + Triangle





$\Rightarrow A_r = (.1mg)d$

$A_t = \frac{1}{2} df$

at $x = d$,

$f_K = (.04/m)dMg + .1Mg$

$f = f_d - .1Mg = (.04/m)Mgd$

$\Rightarrow A_t = \frac{1}{2} d(.04/m)Mgd$

$= (.02/m)Mgd^2$

So $A = (.1mgd) + (.02/m)Mgd^2$

For Calculus Lovers:

$$A = \int_0^d f_K\,dx = \int_0^d \left((.04/m)Mgx + .1Mg\right)dx$$

$$= \int_0^d (.04/m)Mg\,x\,dx + \int_0^d .1Mg\,dx$$

$$= (.04/m)Mg\int_0^d x\,dx + .1Mg\int_0^d dx$$

$$= (.04/m)Mg\left(\tfrac{1}{2}x^2\right)\Big|_0^d + .1Mg\,x\Big|_0^d$$

$$= (.02/m)Mg(d^2-0) + .1Mg(d-0)$$

$$\Rightarrow A = (.02/m)Mgd^2 + .1Mgd \quad \longleftarrow \text{SAME RESULT (OF COURSE)}$$

$$W_F = -A = -\left[.1Mgd + (.02/m)Mgd^2\right] = -Mg\left[.1d + (.02)d^2\right]$$

BACK to WORK-ENERGY: $W_F = -\frac{1}{2}MV_1^2$

$$\Rightarrow +Mg\left[.1d + (.02/m)d^2\right] = +\tfrac{1}{2}MV_1^2$$

$$\Rightarrow (.02/m)d^2 + .1d = \frac{V_1^2}{2g}$$

$$\Rightarrow (.02/m)d^2 + .1d - \frac{V_1^2}{2g} = 0$$

$$\frac{V_1^2}{2g} = \frac{(4.5m/s)^2}{2(9.8m/s^2)} = 1.033m$$

$$\therefore (.02/m)d^2 + .1d - 1.033m = 0 \leftarrow \text{QUADRATIC}$$

$$d = \frac{-.1 \pm \sqrt{(.1)^2 - 4(.02/m)(1.033m)}}{2(.02/m)} = \frac{-.1 \pm \sqrt{.09264}}{.04/m}$$

$$\Rightarrow d = 5.1092m \text{ OR } \cancel{-10.1m}$$

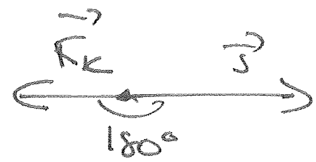
SO $\boxed{d = 5.11m}$

WHAT IS μ_K at d? $\mu_K = (.04/m)(5.1092m) + .1 = \underline{\underline{.304}}$

How far would box go if $\mu_K = .1$ (constant)?

If $\mu_K = .1$, $f_K = .1\,Mg \Rightarrow$ Constant force

So $W_f = \vec{f_K} \cdot \vec{s} = f_K\, s \cos 180°$



Always for friction

Let $s = d = ?$ $\Rightarrow W_f = -f_K d = -.1\,Mgd$

$W_f = W_{total} \Rightarrow -.1\,Mgd = \frac{1}{2}MV_2^2 - \frac{1}{2}MV_1^2$, with $V_2 = 0$

$\Rightarrow +.1\,Mgd = +\frac{1}{2}MV_1^2 \Rightarrow d = \dfrac{V_1^2}{2(.1)g} = \dfrac{(4.5\,m/s)^2}{0.2(9.8\,m/s^2)}$

$$\boxed{d = 10.3\,m}$$