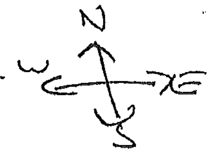
Physics 160, HW #4

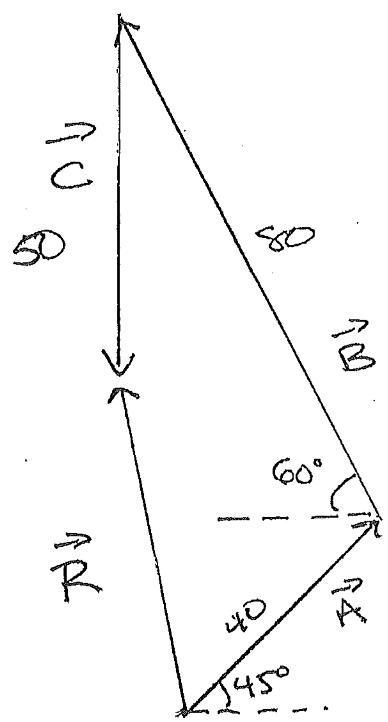
Mastering Physics: 8 problems
from chapters 1 & 3

Written Question: 3.65

1.76 40 steps at NE, 80 steps at 60° N of W, 50 steps Due South.

USE traditional 

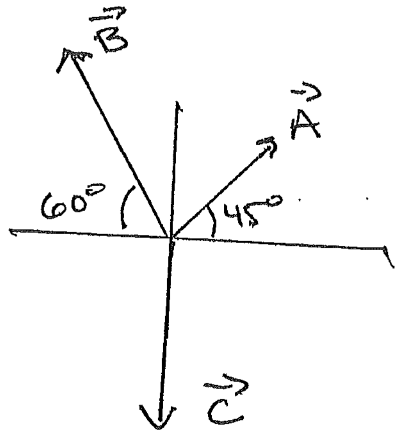
NE at 45°



$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

POINTS FROM HUT TO HIS FINAL LOCATION

$\Rightarrow -\vec{R}$ will BRING HIM BACK



$A_x = A\cos 45° = 40\cos 45° = 28.28$

$A_y = A\sin 45° = 40\sin 45° = 28.28$

USE STANDARD ANGLE $\Rightarrow \vec{B}$ at $180° - 60° = 120°$

$\Rightarrow B_x = B\cos 120° = 80\cos 120° = -40$

$B_y = B\sin 120° = 80\sin 120° = 69.282$

$\vec{C}$ STRAIGHT DOWN $\Rightarrow C_x = 0 \quad C_y = -C = -50$

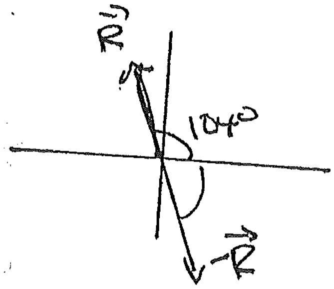
$R_x = A_x + B_x + C_x = 28.28 - 40 + 0 = -11.72$

$R_y = A_y + B_y + C_y = 28.28 + 69.282 - 50 = 47.562$

$\Rightarrow R = \sqrt{R_x^2 + R_y^2} = 48.98 \approx$ ~~50~~ 49 steps

$R_x < 0,\ R_y > 0 \Rightarrow$ 2ND QUADRANT

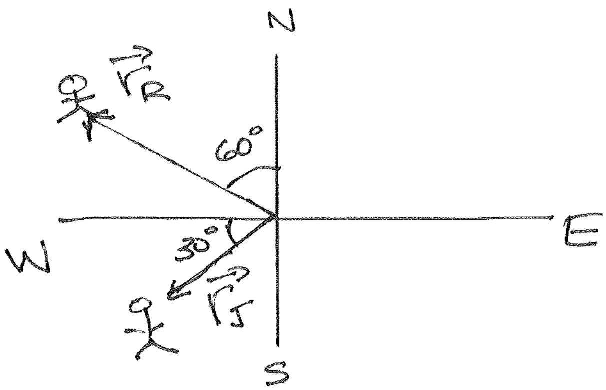
$\Rightarrow \Theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) + 180° = \tan^{-1}\left(\frac{47.562}{-11.72}\right) + 180° = 103.8 = 104°$



$\Rightarrow$ to RETURN $180° - 104° = 76°$

$\Rightarrow$ ~~50~~ 49 steps, 76° SOUTH OF EAST

PROBLEM 1.84



RICARDO: 26m, 60° W OF NORTH

⇒ 60° FROM NORTH TOWARDS West

JANE: 16m, 30° SOUTH OF West

⇒ 30° FROM West TOWARD SOUTH

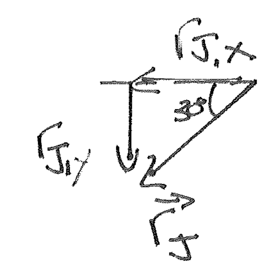
a) WHAT DISTANCE? FIND DISPLACEMENT VECTOR FROM

RICARDO to JANE ⇒ $\vec{r}_R = \vec{r}_1$ = INITIAL.

$\vec{r}_J = \vec{r}_2$ = FINAL

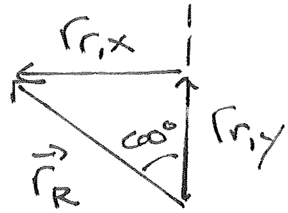
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}_J - \vec{r}_R$$

PRACTICE WITH NON-STANDARD ANGLES:



$$r_{J,x} = -r_J \cos 30° = -16\text{m} \cos 30° \doteq 13.856\text{m}$$

$$r_{J,y} = -r_J \sin 30° = -16\text{m} \sin 30° = -8\text{m}$$



$r_{r,x} = -r_r \sin 60° = -26m \sin 60° =$
$-22.51666m$

$r_{r,y} = +r_r \cos 60° = +26m \cos 60° = 13m$

$\Delta\vec{r} = \vec{r}_J - \vec{r}_r \Rightarrow \Delta r_x = \Delta X = r_{J,x} - r_{r,x}$

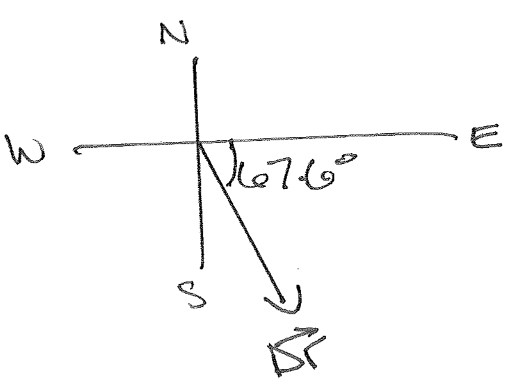
$\Rightarrow \Delta X = -13.8564m - (-22.51666m) = 8.66026m$

$\Delta r_y = \Delta y = r_{J,y} - r_{r,y} = -8m - (13m) = -21m$

$\Delta r = \sqrt{\Delta X^2 + \Delta y^2} = \sqrt{(8.66026m)^2 + (21m)^2} = \sqrt{516m} = \underline{\underline{22.7m}}$

b) WHAT Angle? $\Delta X > 0,\ \Delta y < 0 \Rightarrow 4^{th}$ QUADRANT

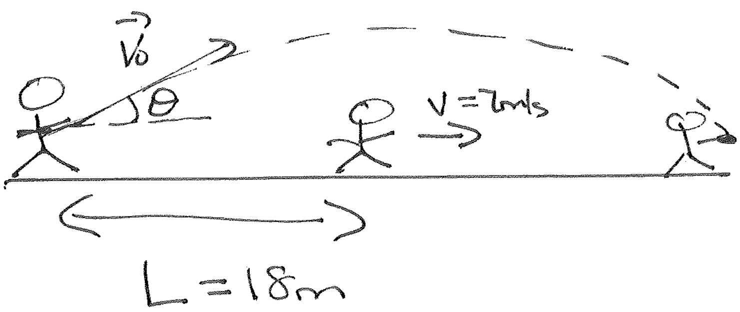
$\Rightarrow$ Calculator OK, $\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta X}\right) = \tan^{-1}\left(\frac{-21}{8.66026}\right) = \underline{\underline{-67.6°}}$



WE Could Also say: 67.6° S of E

OR EVEN $90° - 67.6° = 22.4°$ E of S

SPEED OF A SOFTBALL



CATCHES 2s AFTER BEING HIT

a, b FIND LAUNCH SPEED V_0 AND ANGLE θ

~~CATCHER~~ BASEMAN RUNS WITH CONSTANT SPEED IN STRAIGHT LINE

$\Rightarrow X_{BASEMAN} = X_0 + V_0 t + \frac{1}{2}at^2$

$X_0 = L = 18m, \; V_0 = 7m/s, \; a = 0$ (CONSTANT SPEED)

$\Rightarrow X_{BASEMAN} = 18m + 7m/s(2s) = 18m + 14m = 32m$

HE CATCHES BALL $\Rightarrow$ $x_0 = 0$, $X = 32m$ FOR BALL, AT SAME

HEIGHT $\Rightarrow y_0 = y = 0$ WHEN CAUGHT,

SO FOR BALL: $X_0 = 0, \; Y_0 = 0, \; X = 32m, \; Y = 0, \; t = 2s$

$V_{0x} = ?, \; V_{0y} = ?,$

SO SOLVE FOR V_{ox}, V_{oy} AND USE $V_0 = \sqrt{V_{ox}^2 + V_{oy}^2}$, $\theta = \tan^{-1}\left(\frac{V_{oy}}{V_{ox}}\right)$

$$X = \cancel{X_0}^{0} + V_{ox}t \Rightarrow V_{ox} = \frac{X}{t} = \frac{32m}{2s} = 16m/s$$

$$\cancel{y}^{0} = \cancel{y_0}^{0} + V_{oy}t - \frac{1}{2}gt^2 \Rightarrow V_{oy} = \frac{\frac{1}{2}gt^2}{t} = \frac{1}{2}gt = \frac{1}{2}(9.8m/s^2)(2s)$$
$$= 9.8m/s$$

$$V_0 = \sqrt{(16m/s)^2 + (9.8m/s)^2} = 18.8m/s$$

$$\theta = \tan^{-1}\left(\frac{9.8}{16}\right) = 31.5°$$

c) FIND V_x, V_y .1s BEFORE CAUGHT. CAUGHT AT 2s

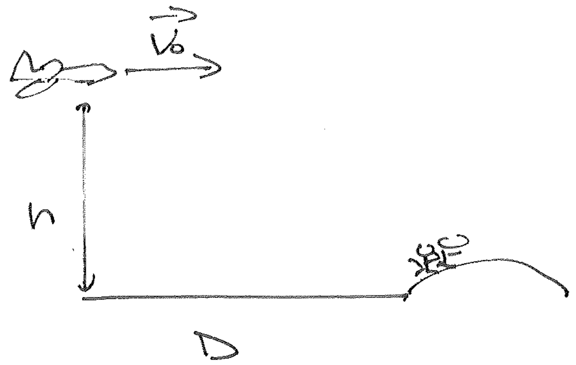
$$\Rightarrow t = 2s - .1s = 1.9s$$

$$V_x = V_{o,x} = 16m/s, \quad V_y = V_{oy} - gt = 9.8m/s - (9.8m/s^2)(1.9s) = -8.82m/s$$

d) FIND x, y .1s BEFORE CAUGHT

$$X = \cancel{X_0}^{0} + V_{ox}t \Rightarrow X = 16m/s(1.9s) = 30.4m$$
$$y = \cancel{y_0} + V_{oy}t - \frac{1}{2}gt^2 = 0 + (9.8m/s)(1.9s) - 4.9m/s^2(1.9s)^2 = .931m$$

DELIVERING A PACKAGE BY AIR:



$V_0 = 200\text{mph}, \; h = 1000\text{m}$

$\vec{V_0}$ HORIZONTAL, ~~[illegible]~~

a) How long to REACH ground?

$\vec{V_0}$ Horizontal $\Rightarrow V_{0x} = V_0,\; V_{0y} = 0 \;\Rightarrow\; y = y_0 - \frac{1}{2}gt^2$

$y = 0,\; y_0 = h = 1000\text{m} \;\Rightarrow\; 0 = h - \frac{1}{2}gt^2 \;\Rightarrow\; t = \sqrt{\frac{2h}{g}}$

$t = \sqrt{\frac{2(1000\text{m})}{9.8\text{m/s}^2}} = 14.3\text{s}$

b) D = ? IN METERS

$X = X_0 + V_{0,x}t,\quad X = D,\; X_0 = ?,\; V_{0,x} = V_0 = 200\text{mph},\; t = 14.3\text{s}$

$\Rightarrow D = V_{0x}t$, but FIRST HAVE to Convert mph to m/s

Using textbook: $1\text{mph} = .4470\text{m/s}$ $\therefore$ $200\text{mph} \times \frac{.4470\text{m/s}}{\text{mph}} = 89.4\text{m/s}$

$\therefore D = (89.4\text{m/s})(14.3\text{s}) = 1278.42\text{m} = 1280\text{m}$

c) WHAT IS SPEED IN mph when package hits ground?

SPEED $\Rightarrow V = ?$ $\qquad V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_{0x} = 200\,mph = 89.4\,m/s$

$V_y = V_{0y} - gt = 0 - 9.8\,m/s^2 (14.3\,s) = -140.14\,m/s$

$\therefore \quad V = \sqrt{(89.4\,m/s)^2 + (140.14\,m/s)^2} = 166.227\,m/s$

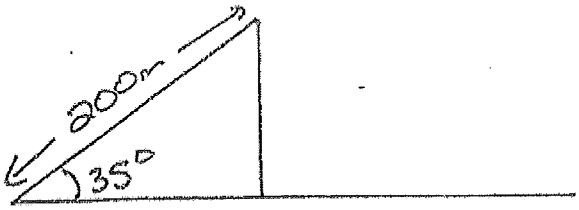
$166.227\,m/s \times \frac{mph}{.4470\,m/s} = 371.8735\ldots mph = 372\,mph$

d) WHAT WOULD make V SMALLER?

DECREASE HEIGHT BECAUSE t WOULD BE SMALLER AND SO $|V_y|$ WOULD BE SMALLER

DECREASE SPEED BECAUSE V_x WOULD BE SMALLER.

3.47



ROCKET ACCELERATED AT $1.25 m/s^2$ for 200m. THEN MOVES UNDER force of gravity only.

ON INCLINE ROCKET MOVING IN A STRAIGHT LINE, SO WE CAN USE $V^2 = V_0^2 + 2a(r - r_0)$ TO FIND SPEED WITH WHICH ROCKET LEAVES INCLINE.

$$V = ?,\ V_0 = 0,\ a = 1.25 m/s^2,\ r = 200m,\ r_0 = 0$$

$$\Rightarrow V^2 = 0 + 2(1.25 m/s^2)(200m) \Rightarrow V = \sqrt{500 m^2/s^2} = 22.36 m/s$$

AFTER LEAVING INCLINE ROCKET BECOMES A PROJECTILE WHOSE INITIAL VELOCITY IS $\vec{V}_0 = 22.36 m/s$ at $35°$. ITS INITIAL POSITION IS $X_0 = 200m \cos 35°$; $y_0 = 200m \sin 35°$

a) FIND $y = ?$ WHEN $V_y = 0$. $y = y_0 + V_{0y}t - \frac{1}{2}gt^2$. $\rightarrow$ NEED t.

$$V_y = V_{0y} - gt \Rightarrow 0 = 22.36 m/s \sin 35° - 9.8 m/s^2\, t$$

$$\Rightarrow t = 1.309s \Rightarrow y = 200m \sin 35° + 22.36 m/s \sin 35° (1.309s) - \frac{1}{2}g(1.3$$

$$\Rightarrow \boxed{y = 123.1m = 123m}$$

b) $X = ?$ WHEN $y = 0$. $X = X_0 + V_{0x}t \rightarrow$ NEED t

$$y = y_0 + V_{0y}t - \frac{1}{2}gt^2 \Rightarrow 0 = 200m \sin 35° + (22.36 m/s \sin 35°)t - \frac{1}{2}(9.8 m/s^2$$

$$\Rightarrow 0 = 114.7m + 12.825 m/s\, t - 4.9 m/s^2\, t^2$$

$$t = \frac{-12.825\,m/s \pm \sqrt{(12.825\,m/s)^2 - 4(-4.9\,m/s^2)(114.7\,m)}}{2(-4.9\,m/s^2)}$$

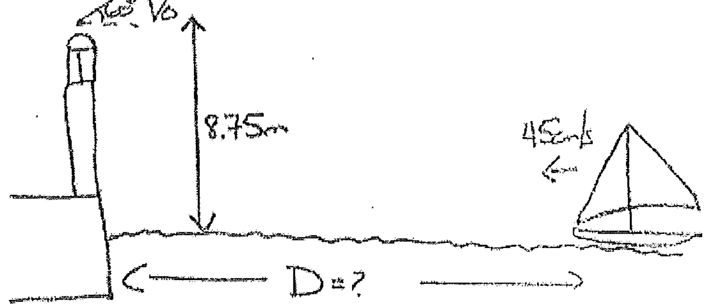
$$= \frac{-12.825\,m/s \pm \sqrt{2412.6\,m^2/s^2}}{-9.8\,m/s^2}$$

$$\Rightarrow t = -3.7s,\ \underline{\underline{6.32s}} \quad \text{use } t>0$$

$$\Rightarrow X = 200m\cos 35° + 22.36m/s\cos 35°(6.32s)$$

$$\Rightarrow \boxed{X = 279.6m = 280m}$$

3.56



$\vec{V}_0 = 15\,m/s$ at $60°$

PACKAGE THROWN TO SAILBOAT

$D = ?$ SUCH THAT $X_{package} = X_{sailboat}$

FOR PACKAGE: $X_0 = 0,\ y_0 = 8.75m,\ y = 0,\ V_{0x} = 15m/s \cos 60° = 7.5m/s$

$V_{0y} = 15m/s \sin 60° = 7.5(\sqrt{3})m/s$

$a_x = 0,\ a_y = -g$

For SAILBOAT: $X_0 = D,\ y_0 = y = 0,\ V_{0x} = -45cm/s = -.45m/s,\ V_{0y} = 0$

$a_x = a_y = 0$

$X = X_0 + V_{0x}t + \frac{1}{2}a_x t^2$ & $X_{package} = X_{sailboat} \Rightarrow 0 + 7.5m/s\,t + 0 = D - .45m/s\,t + 0$

$\Rightarrow 7.5m/s\,t = D - .45m/s\,t \Rightarrow D = (7.95m/s)t \rightarrow$ NEED t.

USE $y = y_0 + V_{0y}t - \frac{1}{2}gt^2$ OF PACKAGE TO FIND t

$0 = 8.75m + (7.5)\sqrt{3}m/s\,t - \frac{1}{2}(9.8m/s^2)t^2 \Rightarrow 0 = -4.9m/s^2\,t^2 + (7.5\sqrt{3})m/s\,t + 8.75m$

$\Rightarrow 4.9m/s^2\,t^2 - 7.5(\sqrt{3})m/s\,t - 8.75m = 0$

$\Rightarrow t = \dfrac{7.5\sqrt{3}m/s \pm \sqrt{(7.5)^2 3m^2/s^2 - 4(4.9m/s^2)(-8.75m)}}{2(4.9m/s^2)} = \dfrac{7.5(\sqrt{3})m/s \pm \sqrt{340.25m^2/s^2}}{9.8m/s^2}$

$\Rightarrow t = 3.21s$ OR $\cancel{-.556s}$ $\Rightarrow D = (7.95m/s)(3.21s) \Rightarrow \boxed{D = 25.5m}$

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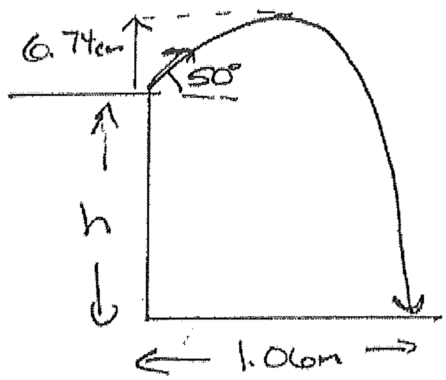

FIND INITIAL SPEED AND height.

SET ORIGIN AT LAUNCH POINT $\Rightarrow X_0 = 0, Y_0 = 0$

$6.74cm = .0674m \Rightarrow$ max height $\Rightarrow V_y = 0$ when $y = .0674$

$V_y = V_{0,y} - gt \Rightarrow 0 = V_0 \sin 50° - 9.8 m/s^2 t$

$y = y_0 + V_{0,y} t - \frac{1}{2} g t^2 \Rightarrow .0674m = 0 + V_0 \sin 50° t - \frac{1}{2}(9.8 m/s^2) t^2$

$\Rightarrow .0674m = V_0 \sin 50° t - 4.9 m/s^2 t^2$

1^{st} EQN $\Rightarrow V_0 \sin 50° = 9.8 m/s^2 t$

$\Rightarrow .0674m = (9.8 m/s^2 t) t - 4.9 m/s^2 t^2$

$\Rightarrow .0674m = (9.8 m/s^2 - 4.9 m/s^2) t^2 = 4.9 m/s^2 t^2$

$\Rightarrow t = \sqrt{\frac{.0674m}{4.9 m/s^2}} = .117s$

$\rightarrow$ UNIT: $m \times s^2/m = s^2$, $\sqrt{s^2} = s$

$$V_0 \sin 50^\circ = 9.8\,m/s^2\, t$$

$$\Rightarrow V_0 = \frac{9.8\,m/s^2(.117\,s)}{\sin 50^\circ} \Rightarrow \underline{\underline{V_0 = 1.5\,m/s}}$$

Find Height: $y = -h$ when $x = 1.06\,m$

$$y = y_0 + V_{0,y}t - \frac{1}{2}gt^2. \quad V_{0y} = V_0 \sin 50^\circ = 1.5\,m/s \sin 50^\circ$$

$$= 1.15\,m/s$$

$$V_{0,x} = V_0 \cos 50^\circ = 1.5\,m/s \cos 50^\circ = .964\,m/s$$

$-h = 0 + 1.15\,m/s\, t - \frac{1}{2}(9.8\,m/s^2)t^2$; Find t from $x = x_0 + V_{0x}t$

$$\Rightarrow 1.06\,m = 0 + .964\,m/s\, t \Rightarrow t = \frac{1.06\,m}{.964\,m/s} = 1.0996\,s$$

$$\Rightarrow -h = 1.15\,m/s\,(1.0996\,s) - 4.9\,m/s^2(1.0996\,s)^2$$

$$\Rightarrow -h = -4.66\,m \Rightarrow \underline{\underline{h = 4.66\,m}}$$

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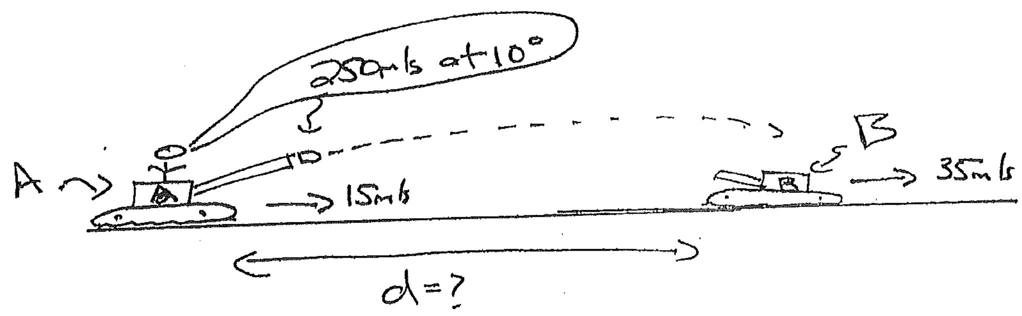

OTHER TANK HIT. HOW FAR APART INITIALLY AND WHEN HIT?

IF YOU ARE ON THE GROUND, THE SHELL IS ALREADY GOING WITH ~~THE~~ SAME VELOCITY AS THE TANK

$\Rightarrow V_{0,x} = 250\text{m/s} \cos 10° + 15\text{m/s} = 261.2\text{m/s}$

$V_{0,y} = 250\text{m/s} \sin 10° = 43.412\text{m/s}$

LET FIRST TANK BE A AND SECOND BE B.

$\therefore X_{0A} = 0, \quad X_{0B} = d = ?$

SHELL LAUNCHED FROM A $\Rightarrow X_{0S} = 0$.

SHELL HITS B $\Rightarrow X_S = X_B$.

SHELL PROJECTILE, B NOT ACCELERATING

$X_{0,S} + V_{0,xS} t = X_{0,B} + V_{0,xB} t \Rightarrow 0 + 261.2\text{m/s}\, t = d + 35\text{m/s}\, t$

$\Rightarrow 261.2\text{m/s}\, t - 35\text{m/s}\, t = d \Rightarrow 226.2\text{m/s}\, t = d$

TO FIND t USE FACT THAT SHELL AT SAME HEIGHT WHEN IT HITS B.

$$\Rightarrow y_S = y_{0,S} = 0$$

PROJECTILE $\Rightarrow y = y_0 + V_{0,y}t - \frac{1}{2}gt^2$

$$\Rightarrow 0 = 0 + 43.412\,m/s\,t - \tfrac{1}{2}(9.8\,m/s^2)t^2$$

$$\Rightarrow 0 = t[43.412\,m/s - 4.9\,m/s^2\,t]$$

$$\Rightarrow t = 0 \text{ OR } t = \frac{43.412\,m/s}{4.9\,m/s^2} = 8.86\,s$$

$$\therefore \underline{d = 226.2\,m/s(8.86\,s)} = \underline{2004\,m} = 2000\,m$$

HOW FAR APART AT $t = 8.86\,s$?

A ALSO NOT ACCELERATING $\Rightarrow X_A = X_{0,A} + 15\,m/s\,t$

$$\Rightarrow X_A = 0 + 15\,m/s(8.86\,s) = 132.9\,m$$

$$X_B = d + 35\,m/s\,t = 2004\,m + 35\,m/s(8.86\,s) = 2314.1\,m$$

$$\Rightarrow X_B - X_A = 2314.1\,m - 132.9\,m = \underline{2181.2\,m} = \underline{2180\,m}$$

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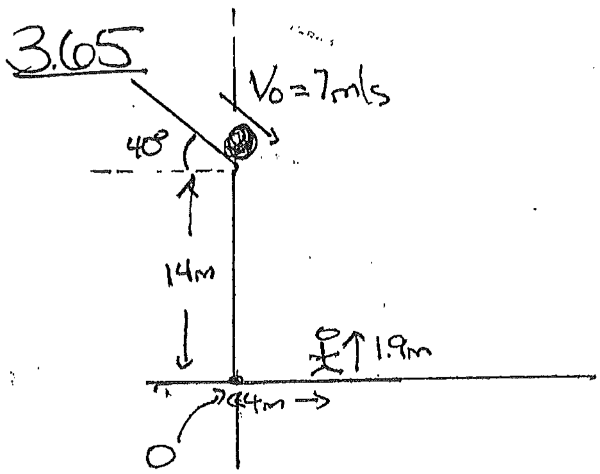


Put origin as shown

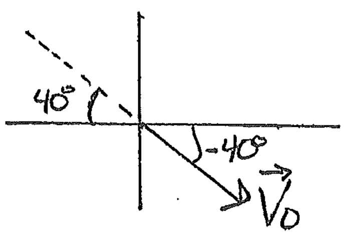
$\Rightarrow$ From picture

$x_0 = 0, \; y_0 = 14$m

$\Rightarrow$ up is positive $\Rightarrow$

$a_x = 0, \; a_y = -g$

a) How far from barn does snowball land?



$V_{0,x} = 7\text{m/s}\cos(-40°) = 5.362\text{m/s}$

$V_{0,y} = 7\text{m/s}\sin(-40°) = -4.4995\text{m/s}$

How far $\Rightarrow x = ?$ when $y = 0$

$x = x_0 + V_{0,x}t \leftarrow$ Need t

Use $y = y_0 + V_{0,y}t - \frac{1}{2}gt^2$ to find

$\therefore \; 0 = 14\text{m} - 4.4995\text{m/s}\,t - \frac{1}{2}(9.8\text{m/s}^2)t^2$

$\Rightarrow 0 = 14\text{m} - 4.4995\text{m/s}\,t - 4.9\text{m/s}^2\,t^2$

$\Rightarrow +4.9\text{m/s}^2\,t^2 + 4.4995\text{m/s}\,t - 14\text{m} = 0$

$$t = \frac{-4.4995\,m/s \pm \sqrt{(4.4995\,m/s)^2 - 4(4.9\,m/s^2)(-14\,m)}}{2(4.9\,m/s^2)}$$

$$= \frac{-4.4995\,m/s \pm \sqrt{294.6455\,m^2/s^2}}{9.8\,m/s^2} = 1.29\,s,\ -2.21\,s$$

OBVIOUSLY NOT

$$X = X_0 + V_{0,x}t = 0 + (5.362\,m/s)(1.29\,s)$$

$$\Rightarrow X = \underline{\underline{6.91698\,m = 6.92\,m}}$$

b) DRAW $x-t$, $y-t$, V_x-t, V_y-t graphs

$X = X_0 + V_{0,x}t \Rightarrow$ STRAIGHT LINE FROM $(0,0)$ to $(1.29, 6.92)$

$Y = Y_0 + V_{0,y}t - \frac{1}{2}gt^2 \Rightarrow$ PARABOLA WITH $(0,14)$ $(1.29,0)$.

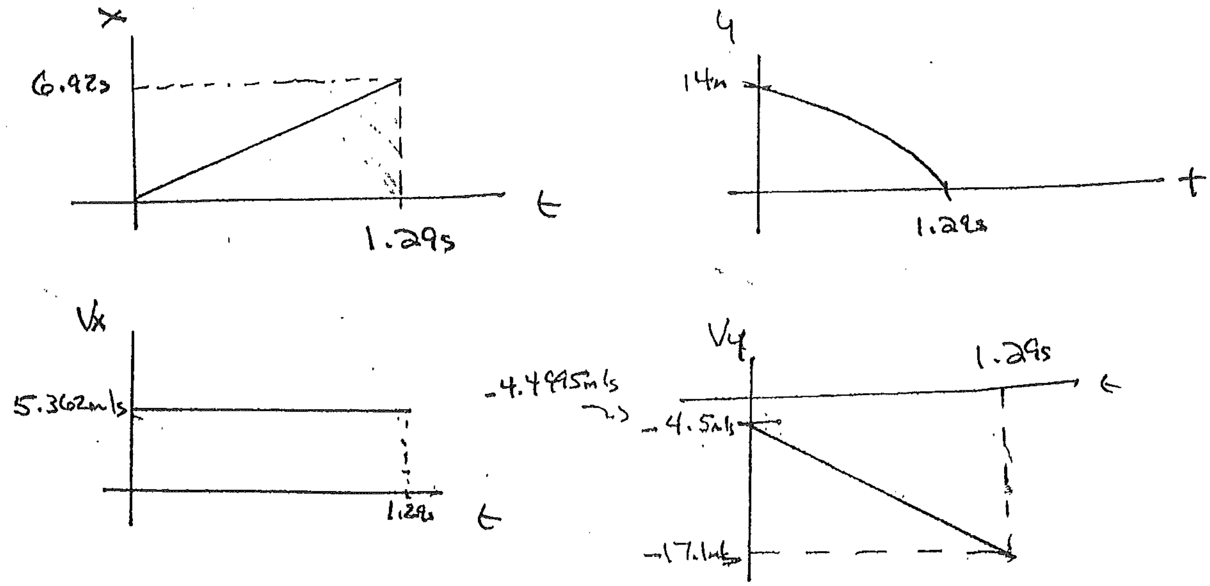
SNOWBALL ALWAYS HAS NEGATIVE VELOCITY (SEE BELOW)

$\Rightarrow$ NO CHANGE IN DIRECTION $\Rightarrow$ NO VERTEX ON PARABOLA

$V_x = V_{0,x} \Rightarrow$ HORIZONTAL LINE.

$V_y = V_{0,y} - gt \Rightarrow$ STRAIGHT LINE STARTING AT $(0, -4.4995)$

$V_y = -4.4995\,m/s - 9.8\,m/s^2(1.29\,s) = -17.1\,m/s \Rightarrow$ ENDING at $(1.29, -17.1)$



c) Will MAN BE HIT? $\Rightarrow$ WHEN SNOWBALL'S $x = 4m$

IS ITS y BETWEEN 0 AND 1.9m

$y = y_0 + V_{0,y}t - \frac{1}{2}gt^2$ $\Leftarrow$ NEED t when $x = 4m$

$\Rightarrow X = X_0 + V_{0,x}t$ to FIND $4m = 0 + (5.362 m/s)t$

$\Rightarrow t = \frac{4m}{5.362 m/s} = .746 s$ $\Rightarrow y = 14m - 4.4995 m/s(.746s) - \frac{1}{2}(9.8 m/s^2)(.746s)^2$

$\Rightarrow y = 7.92 m$ $\Leftarrow$ NO!