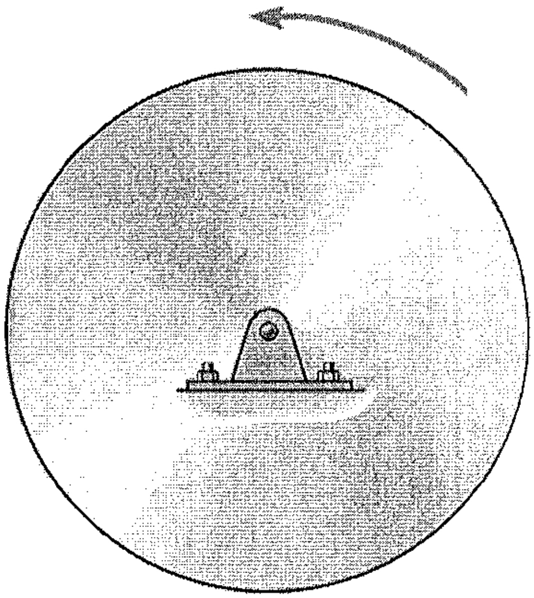
TAN

1. A grindstone is rotating in the counter-clockwise sense as shown. If its speed is decreasing, what direction is its angular acceleration?



(a) $\otimes$
(b) $\odot$
(c) *up*
(d) *Down*

By RHR

$\vec{\omega} = \odot$

Slowing Down

$\Rightarrow \vec{\alpha}$ opposite to $\vec{\omega}$

$\vec{\alpha} = \otimes$

2. A wheel, starting from rest, rotates through $6.0\,rev$ in $2.5\,s$. What is its constant angular acceleration?

(a) $15\,rad/s^2$	(b) $2.4\,rad/s^2$	(c) $1.92\,rad/s^2$	(d) $12\,rad/s^2$

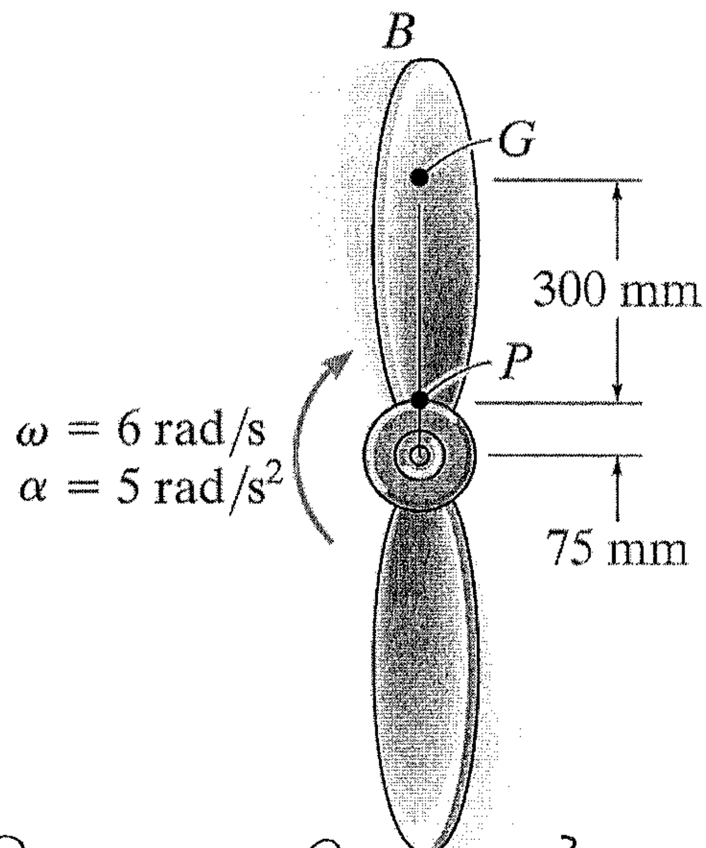
$$\theta - \theta_0 = 6\,rev \times \frac{2\pi\,rad}{rev} = 12\pi\,rad$$

rest $\Rightarrow \omega_0 = 0$, $t = 2.5s$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow 12\pi\,rad = \frac{1}{2}\alpha(2.5s)^2$$

$$\Rightarrow \alpha = \frac{24\pi\,rad}{(2.5s)^2} = 12\,rad/s^2$$

3. If the instant shown, the propeller has angular speeds and accelerations given in the figure, what is the magnitude and direction of the *linear* acceleration at the point G? Assume the propeller is rotating about its center.



(a) $13.63\, m/s^2$ at $-82°$,

(b) $9.64\, m/s^2$ at $-76.5°$,

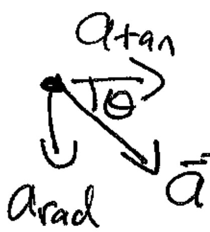
(c) $13.63\, m/s^2$ into the page, $\otimes$

(d) $13.63\, m/s^2$ at $82°$,

$a_{tan} = \alpha r, \quad a_{rad} = \omega^2 r$

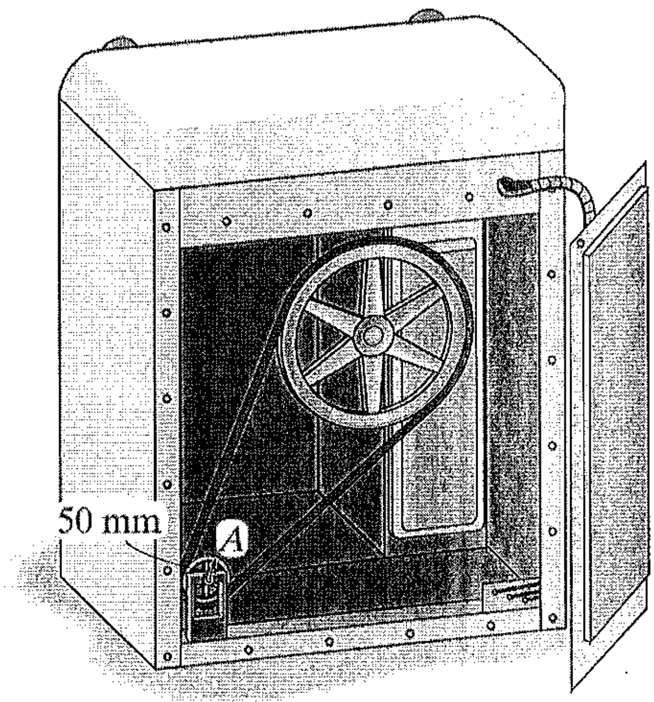
At G, $r = 75\text{mm} + 300\text{mm} = 375\text{mm} \times \frac{1\text{m}}{1000\text{mm}} = .375\text{m}$

$\therefore a_{tan} = 5\,\text{rad/s}^2(.375\text{m}) = 1.875\,\text{m/s}^2, \quad a_{rad} = (6\,\text{rad/s})^2(.375\text{m}) = 13.5\,\text{m/s}^2$

At G  $a = \sqrt{a_{tan}^2 + a_{rad}^2} = 13.63\,\text{m/s}^2$

$\theta = \tan^{-1}\left(\frac{a_{rad}}{a_{tan}}\right) = 82°$ 

4. The 50-mm radius motor at A of a clothes dryer rotates at $20\,rad/s$ and causes the larger gear (and therefore your clothes) to rotate at $5\,rad/s$. Assuming no slipping of the connecting belt, what is the radius of the larger gear?



(a) $1000\,mm$
$\boxed{\text{(b) } 200\,mm}$
(c) $50\,mm$
(d) $12.5\,mm$

CONNECTED ROTATING OBJECTS ⇒ SAME LINEAR SPEED ⇒ $\omega_A r_A = \omega_B r_B$

$$\Rightarrow (20\,rad/s)(50\,mm) = (5\,rad/s)\,r_B$$

$$\Rightarrow r_B = \frac{(20\,rad/s)(50\,mm)}{(5\,rad/s)} = 200\,mm$$

5. A hollow sphere rotated about its center has moment of Inertia, $I = \frac{2}{3}MR^2$. Which of the following expressions is the correct one for the kinetic energy of a hollow sphere that is rolling without slipping?

(a) $\frac{5}{3}Mv^2$	(b) $\frac{5}{6}Mv^2$	(c) $\frac{1}{2}Mv^2$	(d) There is not enough information to determine.

Rolling w/out slipping $K = \frac{1}{2}MV^2\left(1 + \frac{I}{MR^2}\right)$

$$\Rightarrow K = \frac{1}{2}MV^2\left(1 + \frac{\frac{2}{3}\cancel{MR^2}}{\cancel{MR^2}}\right) = \frac{1}{2}MV^2\left(1 + \frac{2}{3}\right) = \frac{1}{2}MV^2\left(\frac{5}{3}\right)$$

$$= \frac{5}{6}MV^2$$

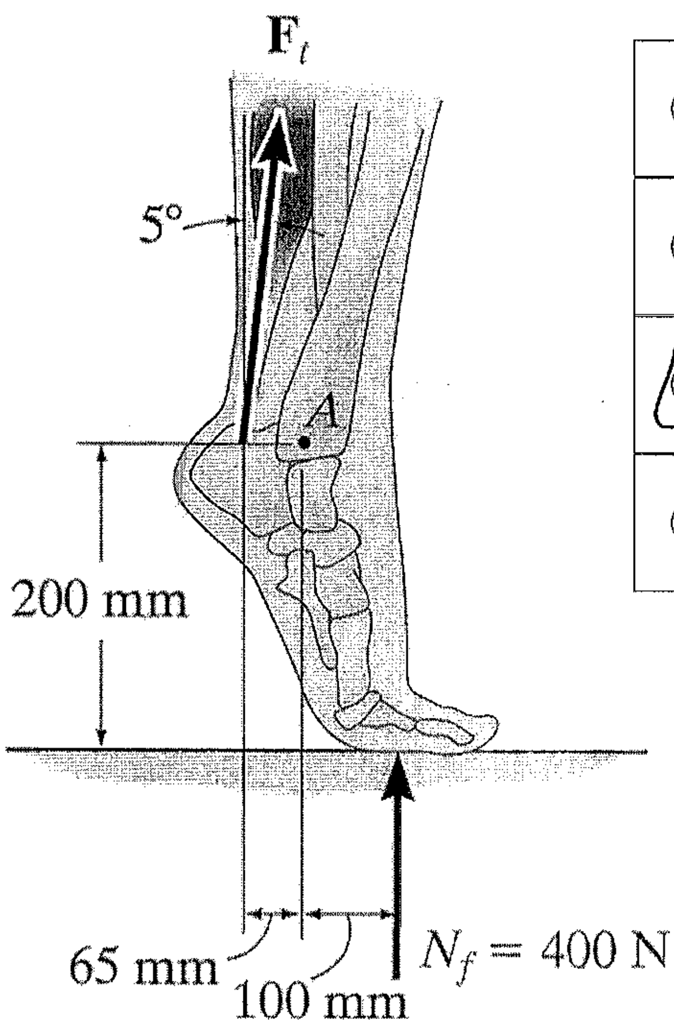
6. Water molecules in food absorb energy from a microwave oven and begin to rotate about their center at $4\,rev/s$. If a singe molecule of water has an angular momentum of $7.5 \times 10^{-46}\ kg \cdot m^2/s$, what is the moment of inertia for water rotated about its center?

(a) $9.5 \times 10^{-48}\ kg \cdot m^2$	(b) $5.97 \times 10^{-47}\ kg \cdot m^2$
(c) $2.98 \times 10^{-47}\ kg \cdot m^2$	(d) $1.88 \times 10^{-46}\ kg \cdot m^2$

$L = I\omega$ $\qquad \omega = \frac{4\,rev}{s} \times \frac{2\pi\,rad}{s} = 8\pi\ rad/s$

$$I = \frac{L}{\omega} = \frac{7.5 \times 10^{-46}\ kg \cdot m^2/s}{(8\pi\ rad/s)} = 2.98 \times 10^{-47}\ kg \cdot m^2$$

7. When a person stands on their "tip-toes", their tendons have to apply a force to prevent the foot from rotating. For the schematic foot shown below, find the magnitude of the tendon force $\vec{F}_t$ applied at 5° from the vertical necessary to make the sum of the torques about the point A zero. Ignore all other forces that would exist in reality besides the $N_f = 400\,N$ normal force. (Which is why these forces won't sum to zero.)



(a) 15800 N
(b) 7060 N
$\boxed{\text{(c) } 618\ N}$
(d) 400 N

$\sum \tau = 0$

$\Rightarrow \tau_N + \tau_{F_t} = 0$

NORMAL force is VERTICAL $\Rightarrow$

$\tau_N = x N_f$

$x = 100\,mm = .1\,m$

For F_t: 15° F_t

$r = 65\,mm = .065\,m$

$\Rightarrow \tau_{F_t} = r F_t \sin 95^\circ$

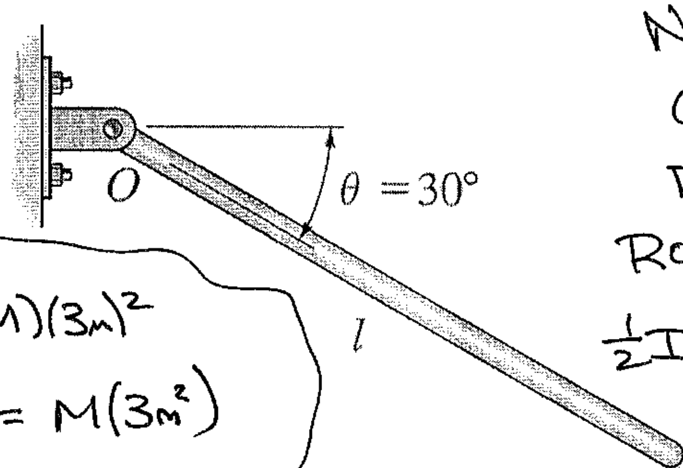
About A: F_t does clockwise

N_f does counter-clockwise

$\Rightarrow \sum \tau = x N_f - r F_t \sin 95^\circ = 0 \Rightarrow F_t = \frac{x N_f}{r \sin 95^\circ} = \frac{(.1\,m)(400\,N)}{(.065\,m)\sin 95^\circ}$

$\Rightarrow F_t = 617.73\,N$

8. A uniform thin rod of length $l = 3.00\,m$, free to rotate about one end, is started horizontally (at $\theta = 0°$) from rest. If friction can be ignored, what angular *velocity* (both speed and direction) will the rod have when it reaches the $\theta = 30°$ angle shown below? The moment of inertia for a thin rod rotated about one end is $I = \frac{1}{3}Ml^2$.



No friction ⇒ gravity only force doing work.

Rod is rotating, so

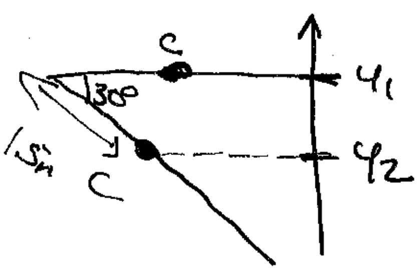
$$\frac{1}{2}I\omega_1^2 + Mgy_1 = \frac{1}{2}I\omega_2^2 + Mgy_2$$

$$I = \frac{1}{3}Ml^2 = \frac{1}{3}(M)(3m)^2 = M\left(\frac{9m^2}{3}\right) = M(3m^2)$$

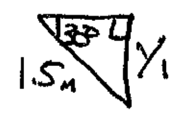
$\omega_1 = 0$ starts from rest, $\omega_2 = ?$

y_1, y_2 = height of center of mass. Uniform rod ⇒

Center at $l/2 = 1.5m$



Let $y_2 = 0$



$y_1 = 1.5m \sin 30° = .75m$

$$\therefore Mgy_1 = \frac{1}{2}I\omega_2^2 \Rightarrow M(9.8m/s^2)(.75m) = \frac{1}{2}(M)(3m^2)\omega_2^2$$

$$\Rightarrow \omega_2 = \sqrt{\frac{2(9.8m/s^2)(.75m)}{3m^2}} = 2.21\,rad/s$$

Bar rotating clockwise so RHR ⇒ $\vec{\omega}_2 = \otimes$

$$\therefore \boxed{\vec{\omega}_2 = 2.21\,rad/s, \otimes}$$