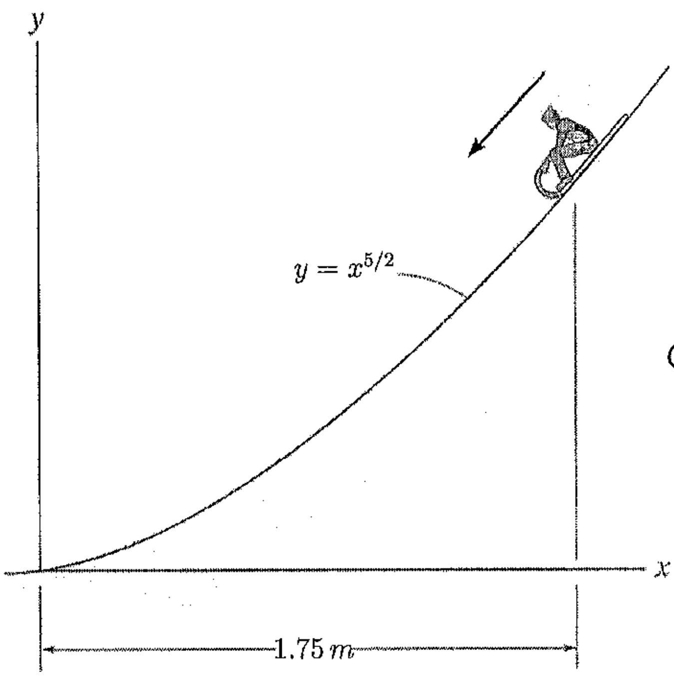
TAN

1. A boy rides a sled down an icy (and therefore frictionless) hill whose height above the ground is given by the equation $y = x^{5/2}$, where y is in meters when x is in meters. If he starts from rest at $x = 1.75\,m$, how fast will he be going at the bottom?



(a) $7.0\,m/s$	(b) $8.91\,m/s$
(c) $5.86\,m/s$	(d) $0\,m/s$

GRAVITY ONLY FORCE DOING WORK:

$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mV_2^2 + mgy_2$

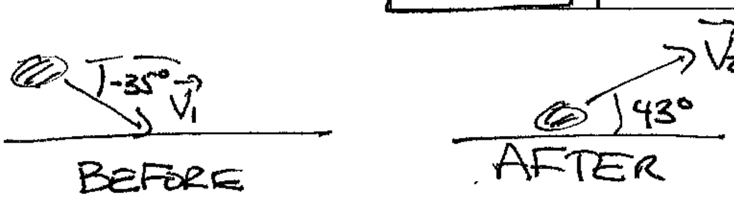
$V_1 = 0,\ y_1 = (1.75)^{5/2} = 4.05\,m$

$V_2 = ?,\ y_2 = 0$

$\Rightarrow (9.8\,m/s^2)(4.05\,m) = \frac{1}{2}V_2^2 \Rightarrow V_2 = 8.91\,m/s$

2. A 5.0-kg ball going $6.0\,m/s$ at $-35°$ hits the ground and bounces at $5.3\,m/s$ at $+43°$ (both angles are from the positive x-axis). If during its bounce, the average force on the ball has a y-component $F_{av,y} = 3500\,N$, how long was the bounce time?

(a) $0.0101\,s$	(b) $0.0126\,s$	(c) $0.0161\,s$	(d) $0.001\,s$



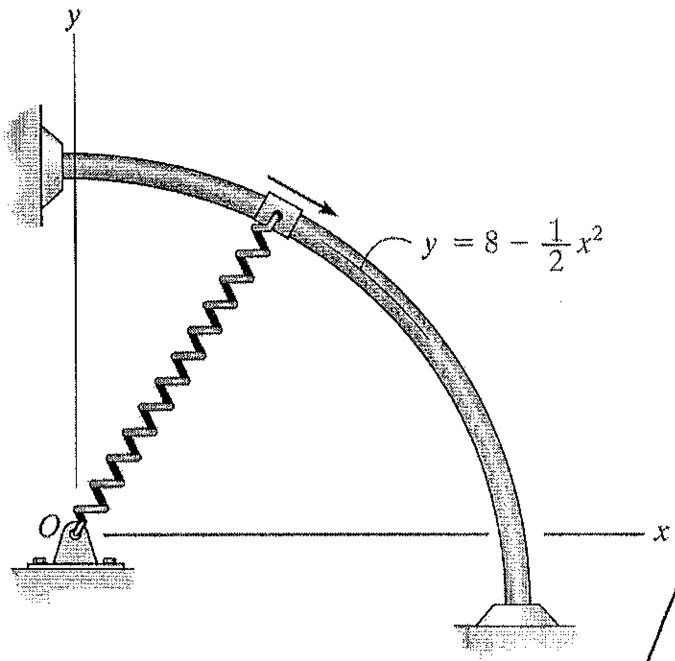
$J_y = \Delta P_y \Rightarrow (F_{AV,y})\Delta t = MV_{2,y} - MV_{1,y}$

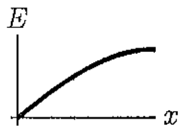
$\Rightarrow \Delta t = \frac{MV_{2,y} - MV_{1,y}}{F_{AV,y}} = \frac{M(V_{2,y} - V_{1,y})}{F_{AV,y}}$

$\Delta t = \frac{5\,kg\,(5.3\,m/s \sin 43° - 6\,m/s \sin(-35°))}{3500\,N} = \frac{5\,kg\,(3.61\,m/s - -3.44\,m/s)}{3500\,N} = \frac{5\,kg\,(7.05\,m/s)}{3500\,N}$

$= .0101\,s$

3. A $6\,kg$ collar is allowed to slide over a frictionless pole whose height above the ground obeys the parabolic equation $y = 8-(1/2)\,x^2$, where y is in meters when x is in meters. Attached to the collar is a $k = 25\,N/m$ spring. The spring, unstretched length $2\,m$, is connected such that as the collar moves, the spring is always oriented along the line connecting the point O and the collar. If the collar is started from rest at $x = 0$, which of the following graphs correctly displays the collar's total energy, E, versus position, x as it slides down the pole?



(a)


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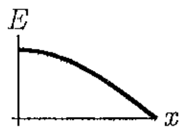
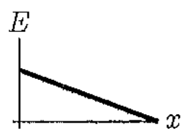


(c)


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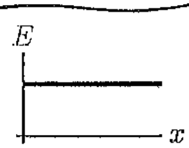
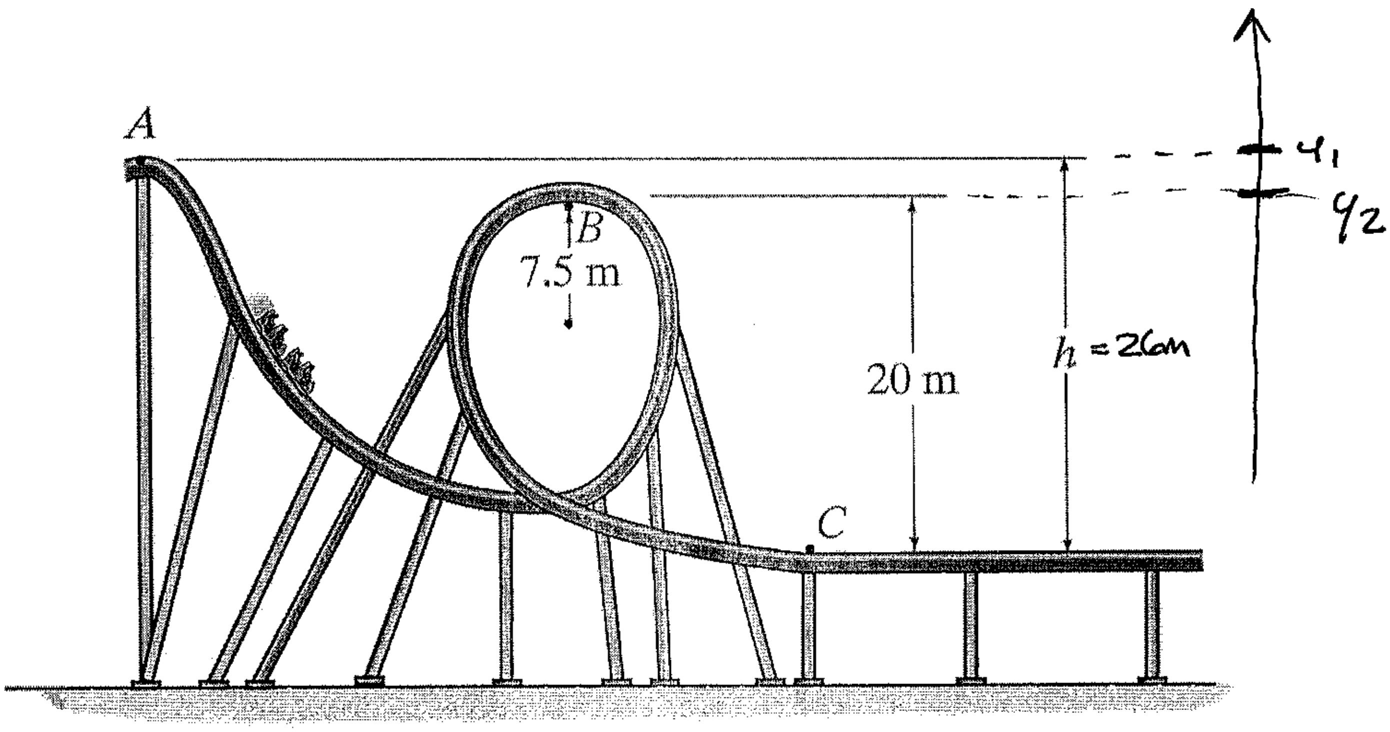


GRAVITY AND SPRING DOING WORK. NO OTHER FORCES

⇒ TOTAL ENERGY CONSERVED ⇒ CONSTANT TOTAL ENERGY

4. A roller coaster starts from rest at point A where the height $h = 26\,m$. It slides along the track without friction. What is the normal force acting on a $75\,kg$ rider of the roller coaster at the top of the loop-to-loop (point B)? As shown, the radius of the loop-to-loop's circle is $7.5\,m$.



(a) 1911 N	(b) 441 N	(c) 0 N	(d) The roller coaster doesn't make it to B

AT TOP [4]

$mg \downarrow \; \downarrow n \; \downarrow a_{rad}$

$\sum F_y = Ma_y \Rightarrow n + Mg = Ma_{rad}$

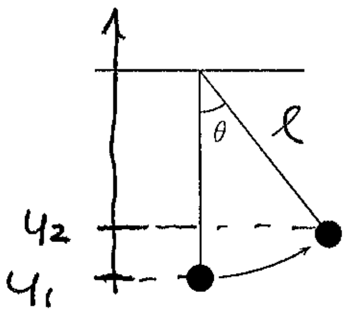
$\Rightarrow n = M(a_{rad} - g) = M\left(\frac{V^2}{r} - g\right)$

FROM A to B, gravity only FORCE DOING WORK: $\frac{1}{2}MV_1^2 + Mgy_1 = \frac{1}{2}MV_2^2 + Mgy_2$

$V_1 = 0,\ y_1 = 6m,\ V_2 = ?,\ y_2 = 0 \Rightarrow (9.8m/s^2)(6m) = \frac{1}{2}V_2^2 \Rightarrow V_2^2 = 117.6m^2/s^2$

$\Rightarrow n = 75Kg\left(\frac{117.6m^2/s^2}{7.5m} - 9.8m/s^2\right) = 75Kg\,(15.68m/s^2 - 9.8m/s^2) = 441N$

5. A 0.9-m long pendulum is started from the vertical position by giving it a speed of $1.65\,m/s$. To what maximum angle θ (from the vertical) will the pendulum go before turning around?



(a) 81.1°	(b) 57.7°
(c) 32.3°	(d) 8.88°

TENSION DOES NO WORK $\Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$

$v_1 = 1.65\,m/s$, $v_2 = 0$, $y_1 = 0$, $y_2 = \ell - \ell\cos\theta = \ell(1-\cos\theta)$

$\Rightarrow \frac{1}{2}(1.65\,m/s)^2 = 9.8\,m/s^2(.9\,m)(1-\cos\theta) \Rightarrow 1.36125\,m^2/s^2 = 8.82\,m^2/s^2(1-\cos\theta)$

$\Rightarrow 1-\cos\theta = .1543 \Rightarrow \cos\theta = .8457 \Rightarrow \theta = \cos^{-1}(.8457) = 32.3°$

6. A $5.00\,kg$ mass with $\vec{v}_{A1} = 3.00\,m/s$ at 45.0° has an elastic collision with a $12.5\,kg$ mass with $\vec{v}_{B1} = 6.00\,m/s$ at 135°. If the $5.00\,kg$ mass bounces with $\vec{v}_{A2} = 7.29\,m/s$ at 163°, with what speed does the $12.5\,kg$ mass bounce? All angles are from the positive x-axis.

(a) $7.72\,m/s$	(b) $-6.97\,m/s$	(c) $4.28\,m/s$	(d) $-0.606\,m/s$

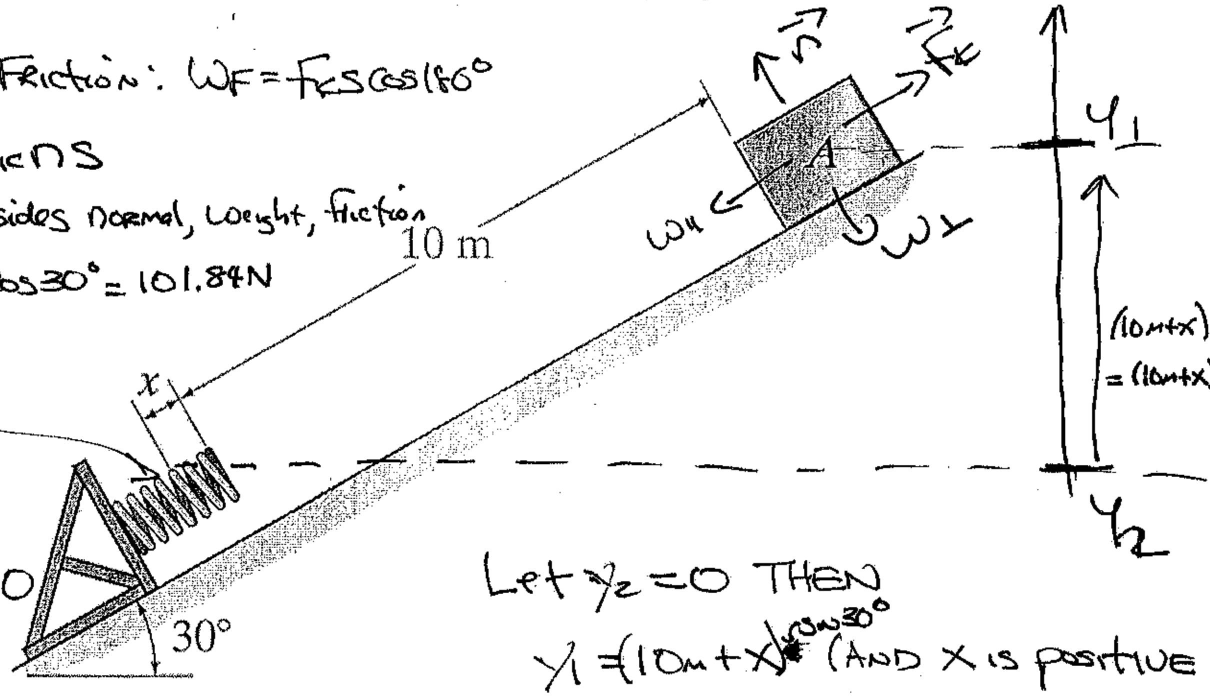
ELASTIC $\Rightarrow \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2$

$\Rightarrow \frac{1}{2}(5\,kg)(3\,m/s)^2 + \frac{1}{2}(12.5\,kg)(6\,m/s)^2 = \frac{1}{2}(5\,kg)(7.29\,m/s)^2 + \frac{1}{2}(12.5\,kg)v_{B2}^2$

$\Rightarrow 247.5\,J = 132.86\,J + \frac{1}{2}(12.5\,kg)v_{B2}^2$

$\Rightarrow v_{B2} = \sqrt{\frac{2(114.64\,J)}{12.5\,kg}} = 4.28\,m/s$

7. A 12.0 kg mass slides 10.0 m down a 30° incline starting with a speed of 3.65 m/s before hitting a 624 N/m spring. If the coefficient of kinetic friction between the mass and the incline is $\mu_k = 0.450$, what *additional* distance, x, does the mass travel before stopping?



Work done by friction: $W_F = f_k S \cos 180° = -f_k S = -\mu_k n S$

No other forces besides normal, weight, friction

$\Rightarrow n = W_\perp = Mg\cos 30° = 101.84\,N$

$S = (10m + x)$

Spring initially uncompressed $\Rightarrow S_1 = 0$

Final: $S_2 = X$

$(10m+x)\sin 30° = (10m+x)\cdot\frac{1}{2}$

Let $y_2 = 0$ THEN $y_1 = (10m + x)\sin 30°$ (AND x is positive)

(a) 0.841 m	(b) 1.56 m	(c) 10 m	(d) 0.919 m

Gravity, Spring, and Friction do work:

$$\frac{1}{2}mV_1^2 + mgy_1 + \frac{1}{2}kS_1^2 + W_F = \frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}kS_2^2$$

$$\frac{1}{2}(12kg)(3.65m/s)^2 + 12kg(9.8m/s^2)(10m+x)\cdot\frac{1}{2} - .45(101.84N)(10m+x) = \frac{1}{2}(624N/m)X^2$$

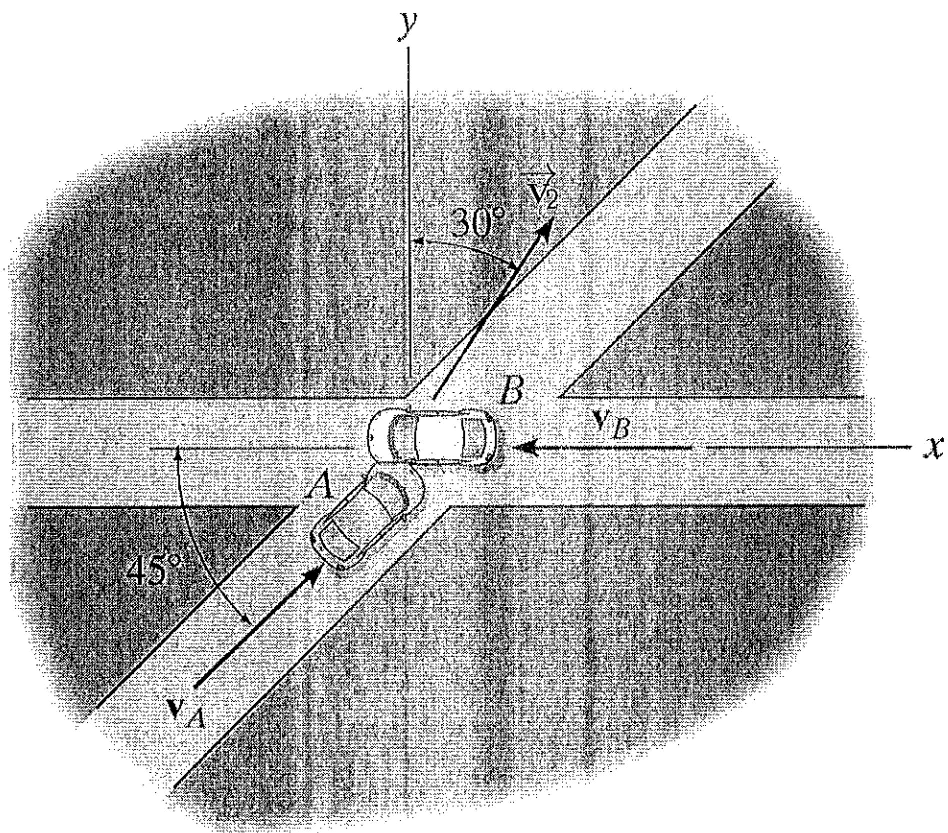
$$\Rightarrow 79.9\,J + 588\,J + 58.8N\,X - 458.3\,J - 45.83N\,X = (312N/m)X^2$$

$$\Rightarrow 209.6J + 12.97N\,X = (312N/m)X^2 \Rightarrow (312N/m)X^2 - (12.97N)X - 209.6J = 0$$

$$\Rightarrow X = \frac{12.97N \pm 511.6N}{2(312N/m)} = \frac{12.97N \pm 511.6N}{624N/m} = .841m \text{ OR } -.799m$$

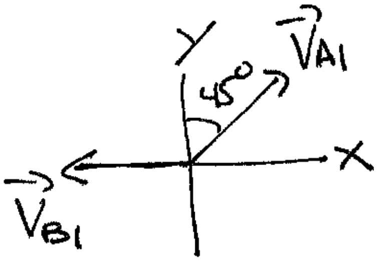
Use positive

8. One day while driving your $M_A = 1500\,kg$ car while going Northeast you, very embarrassingly, smash into your instructor's $M_B = 2000\,kg$ car which was going due West. If the cars have a completely inelastic collision and just after the collision are going $12\,m/s$ at 30° East of North, how fast was each car going before the collision?



Completely inelastic ⇒ $M_A V_{A1,x} + M_B V_{B1,x} = (M_A + M_B) V_{2,x}$

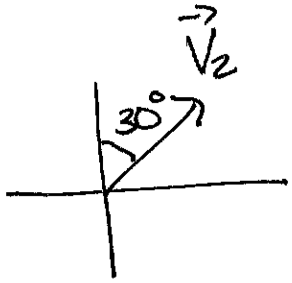
$$M_A V_{A1,y} + M_B V_{B1,y} = (M_A + M_B) V_{2,y}$$



$$V_{A1,x} = V_{A1} \sin 45° = .7071\, V_{A1}$$

$$V_{A1,y} = V_{A1} \cos 45° = .7071\, V_{A1}$$

$$V_{B1,x} = -V_{B1}, \quad V_{B1,y} = 0$$



$$V_{2,x} = V_2 \sin 30^\circ$$
$$= 12\,\text{m/s} \sin 30^\circ = 6\,\text{m/s}$$

$$V_{2,y} = V_2 \cos 30^\circ = 12\,\text{m/s} \cos 30^\circ$$
$$= 10.3923\,\text{m/s}$$

START WITH Y-COMPONENT: $M_A V_{A1,y} + M_B V_{B1,y} = (M_A + M_B) V_{2,y}$

$$\Rightarrow 1500\,\text{kg}\,(.7071\,V_{A1}) + 0 = (1500\,\text{kg} + 2000\,\text{kg})(10.3923\,\text{m/s})$$

$$\Rightarrow 1500\,\text{kg}\,(.7071\,V_{A1}) = (3500\,\text{kg})(10.3923\,\text{m/s})$$

$$\Rightarrow V_{A1} = \frac{3500\,\text{kg}\,(10.3923\,\text{m/s})}{1500\,\text{kg}\,(.7071)} \Rightarrow \boxed{V_{A1} = 34.3\,\text{m/s}}$$

X-COMPONENT: $M_A V_{A1,x} + M_B V_{B1,x} = (M_A + M_B) V_{2,x}$

$$\Rightarrow 1500\,\text{kg}\,(.7071)(34.3\,\text{m/s}) + 2000\,\text{kg}\,(-V_{B1}) = (3500\,\text{kg})(6\,\text{m/s})$$

$$\Rightarrow 36373\,\text{kg}\cdot\text{m/s} - 2000\,\text{kg}\,V_{B1} = 21000\,\text{kg}\cdot\text{m/s}$$

$$\Rightarrow V_{B1} = \frac{-15373\,\text{kg}\cdot\text{m/s}}{-2000\,\text{kg}} \Rightarrow \boxed{V_{B1} = -7.69\,\text{m/s}}$$

I THINK YOU WERE SPEEDING!