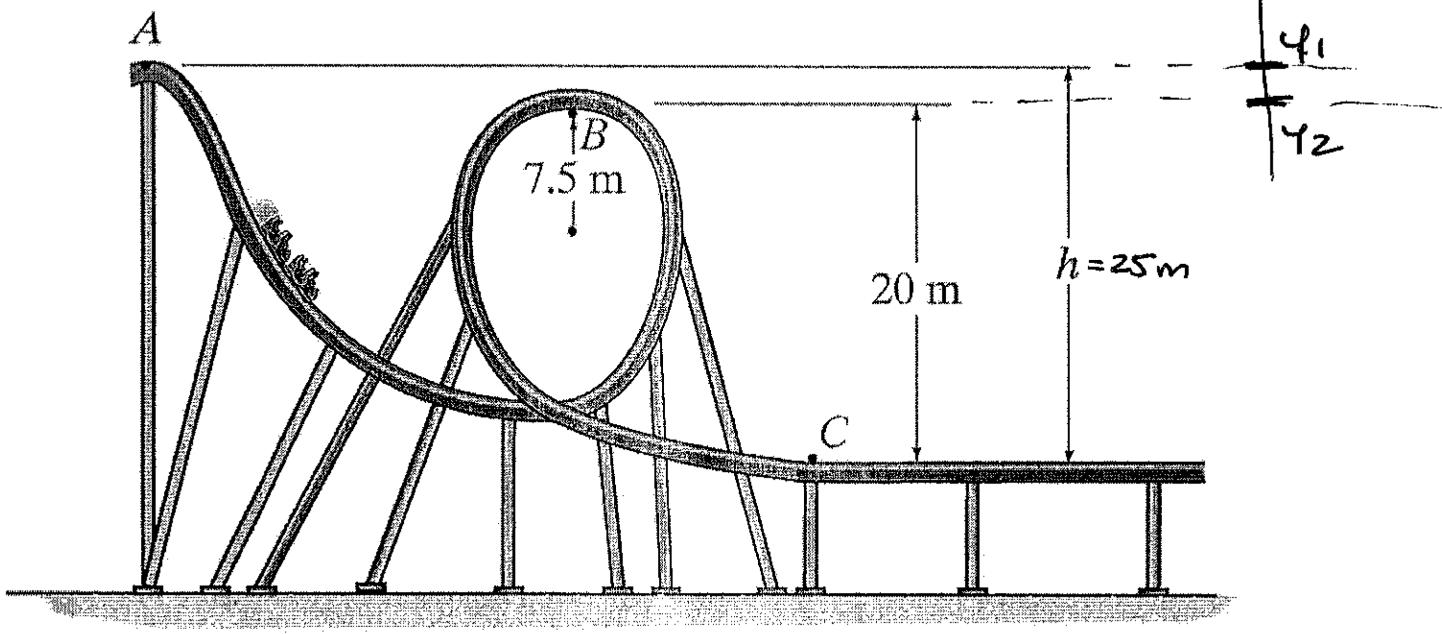
GREEN

1. A roller coaster starts from rest at point A where the height $h = 25\,m$. It slides along the track without friction. What is the normal force acting on a $75\,kg$ rider of the roller coaster at the top of the loop-to-loop (point B)? As shown, the radius of the loop-to-loop's circle is $7.5\,m$.



(a) $0\,N$	(b) $1715\,N$	(c) $245\,N$	(d) The roller coaster doesn't make it to B

at Top: $\quad \Sigma F_y = Ma_y \Rightarrow n + Mg = Ma_{rad} = \frac{MV^2}{r} \Rightarrow n = \frac{MV^2}{r} - Mg$

$\Rightarrow n = M\left(\frac{V^2}{r} - g\right)$. TO FIND V, USE FACT THAT FROM A TO B, GRAVITY ONLY FORCE DOING WORK

$\therefore \frac{1}{2}\cancel{M}V_1^2 + \cancel{M}gy_1 = \frac{1}{2}\cancel{M}V_2^2 + \cancel{Mgy_2}$. $V_1 = 0$, $y_1 = 25m - 20m = 5m$, $V_2 = ?$, $y_2 = 0$

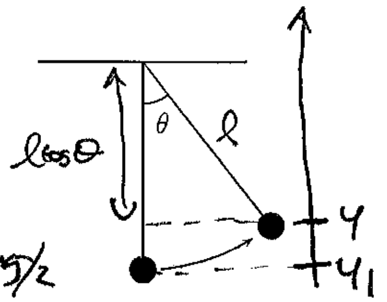
so $V_2^2 = 2gy_1 = 2(9.8m/s^2)(5m) = 98m^2/s^2$, $n = 75kg\left(\frac{98m^2/s^2}{7.5m} - 9.8m/s^2\right)$

$\Rightarrow n = 75kg(13.07m/s^2 - 9.8m/s^2) = 245N$

2. A 1.65-m long pendulum is started from the vertical position by giving it a speed of 0.900 m/s. To what maximum angle θ (from the vertical) will the pendulum go before turning around?

TENSION DOES NO WORK $\Rightarrow$

$\frac{1}{2}\cancel{m}v_1^2 + \cancel{m}g\cancel{y_1} = \frac{1}{2}\cancel{m}v_2^2 + \cancel{m}gy_2$



(a) 88.6°	(b) 77.2°
(c) 34.3°	(d) 12.9°

$V_1 = .9\,m/s,\ V_2 = 0,\ y_1 = 0,\ y_2 = \ell - \ell\cos\theta = \ell(1-\cos\theta)$

$\therefore\ \frac{1}{2}(.9\,m/s)^2 = (9.8\,m/s^2)(1.65\,m)(1-\cos\theta) \Rightarrow .405\,m^2/s^2 = 16.17\,m^2/s^2(1-\cos\theta)$

$\Rightarrow 1-\cos\theta = .025 \Rightarrow \cos\theta = .975 \Rightarrow \theta = \cos^{-1}(.975) = 12.85°$

3. A 5.00 kg mass with $\vec{V}_{A1} = 3.00\,m/s$ at 25.0° has an elastic collision with a 12.5 kg mass with $\vec{V}_{B1} = 6.00\,m/s$ at 165°. If the 5.00 kg mass bounces with $\vec{V}_{A2} = 9.53\,m/s$ at 172°, with what speed does the 12.5 kg mass bounce? All angles are from the positive x-axis.

(a) 1.81 m/s	(b) 3.34 m/s	(c) −8.61 m/s	(d) −0.93 m/s

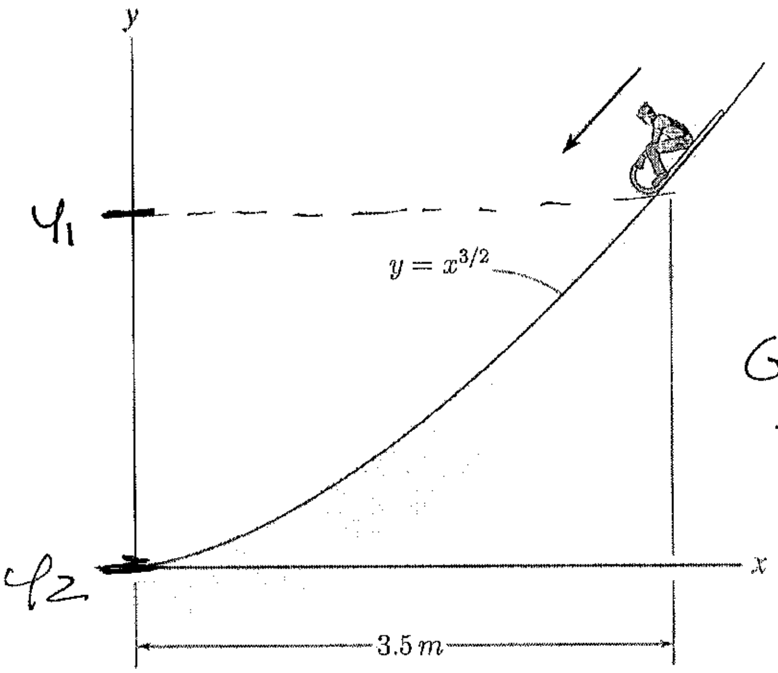
ELASTIC $\Rightarrow \frac{1}{2}M_AV_{A1}^2 + \frac{1}{2}M_BV_{B1}^2 = \frac{1}{2}M_AV_{A2}^2 + \frac{1}{2}M_BV_{B2}^2$

$\frac{1}{2}(5kg)(3m/s)^2 + \frac{1}{2}(12.5kg)(6m/s)^2 = \frac{1}{2}(5kg)(9.53m/s)^2 + \frac{1}{2}(12.5kg)V_{B2}^2$

$22.5J + 225J = 227J + \frac{1}{2}(12.5kg)V_{B2}^2$

$\therefore\ \frac{1}{2}(12.5kg)V_{B2}^2 = 20.5J \Rightarrow V_{B2} = \sqrt{\frac{2(20.5J)}{12.5kg}} \approx 1.81\,m/s$

4. A boy rides a sled down an icy (and therefore frictionless) hill whose height above the ground is given by the equation $y = x^{3/2}$, where y is in meters when x is in meters. If he starts from rest at $x = 3.5\,m$, how fast will he be going at the bottom?



(a) $0\,m/s$	(b) $8.28\,m/s$
(c) $5.42\,m/s$	$\boxed{\text{(d) } 11.3\,m/s}$

GRAVITY ONLY FORCE DOING WORK

$$\tfrac{1}{2}\cancel{m}V_1^2 + \cancel{m}gY_1 = \tfrac{1}{2}\cancel{m}V_2^2 + \cancel{mgY_2}$$

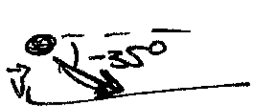
$$V_1 = 0,\ Y_1 = (3.5)^{3/2} = 6.55\,m$$

$$V_2 = ?,\ Y_2 = 0$$

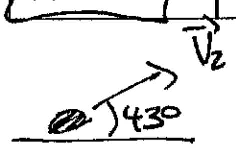
$$V_2 = \sqrt{2gY_1} = \sqrt{2(9.8\,m/s^2)(6.55\,m)} = 11.3\,m/s$$

5. A 5.0-kg ball going $6.0\,m/s$ at $-35°$ hits the ground and bounces at $5.3\,m/s$ at $+43°$ (both angles are from the positive x-axis). If during its bounce, the average force on the ball has a y-component $F_{av,y} = 2350\,N$, how long was the bounce time?

$\boxed{\text{(a) } 0.015\,s}$	(b) $0.0187\,s$	(c) $0.024\,s$	(d) $0.00034\,s$



BEFORE



After

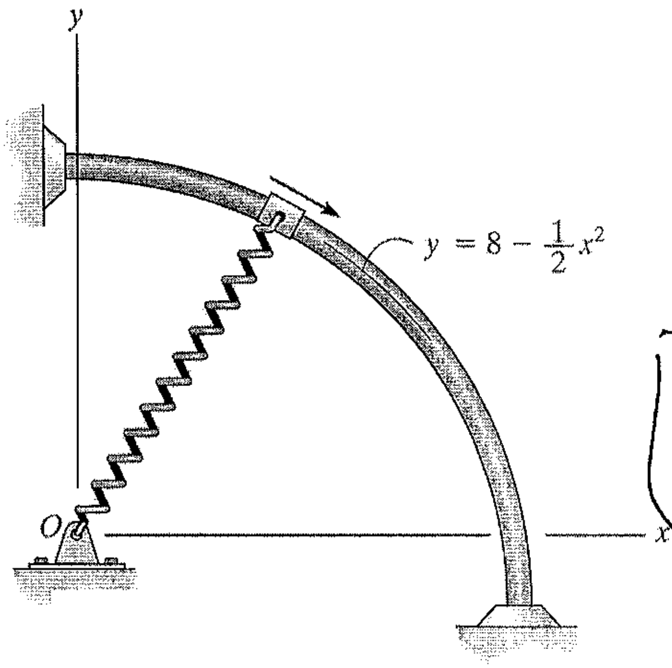
$$\vec{J} = \Delta\vec{P} \Rightarrow J_y = \Delta P_y = MV_{2,y} - MV_{1,y}$$

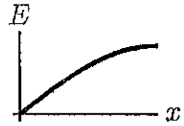
$$J_y = F_{Av,y}\,\Delta t \quad \therefore \quad F_{Av,y}\,\Delta t = M(V_{2,y} - V_{1,y})$$

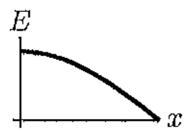
$$\text{so } \Delta t = \frac{5\,kg\,(5.3\,m/s \sin 43° - 6\,m/s \sin -35°)}{2350\,N} = \frac{5\,kg\,[3.61\,m/s - (-3.44\,m/s)]}{2350\,N}$$

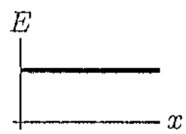
$$\Delta t = \frac{5\,kg\,[3.61\,m/s + 3.44\,m/s]}{2350\,N} = .015\,s$$

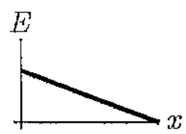
6. A $6\,kg$ collar is allowed to slide over a frictionless pole whose height above the ground obeys the parabolic equation $y = 8-(1/2)\,x^2$, where y is in meters when x is in meters. Attached to the collar is a $k = 30\,N/m$ spring. The spring, unstretched length $1\,m$, is connected such that as the collar moves, the spring is always oriented along the line connecting the point O and the collar. If the collar is started from rest at $x = 0$, which of the following graphs correctly displays the collar's total energy, E, versus position, x as it slides down the pole?



(a)


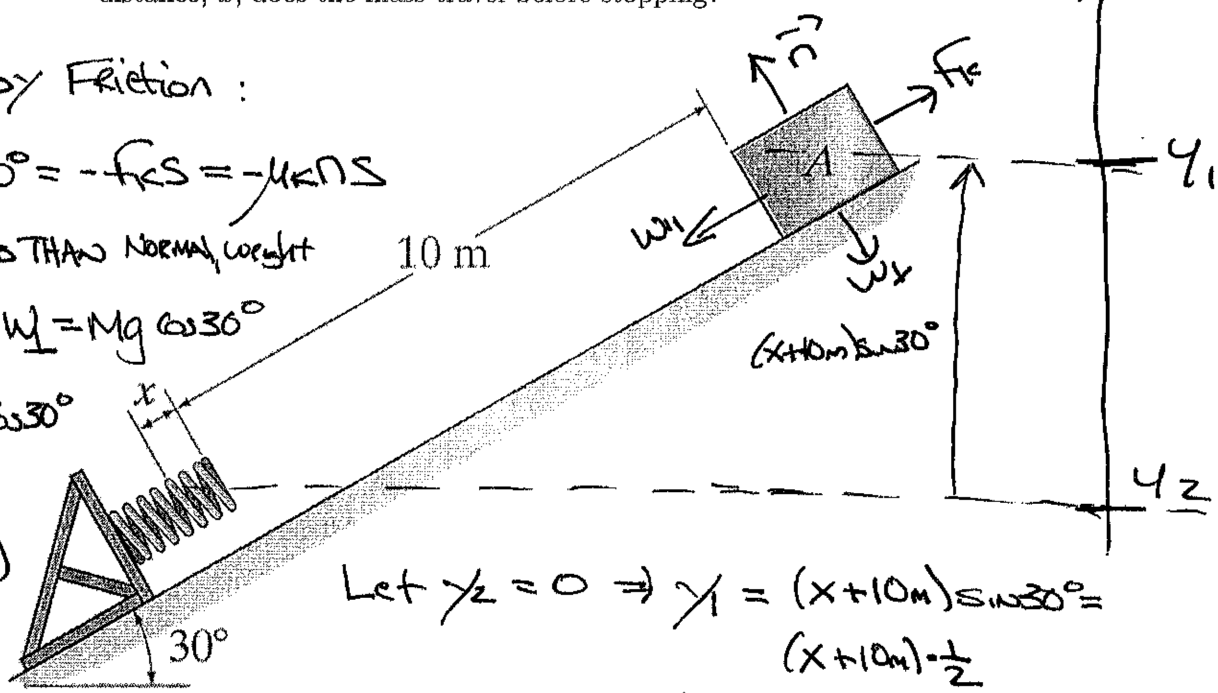
(b)


(c)


(d)


NO FRICTION SO ONLY GRAVITY AND SPRING DOING WORK
TWO CONSERVATIVE FORCES, SO TOTAL ENERGY WILL BE
CONSERVED ⇒ TOTAL ENERGY CONSTANT

7. A 12.0 kg mass slides 10.0 m down a 30° incline starting with a speed of 5.85 m/s before hitting a 424 N/m spring. If the coefficient of kinetic friction between the mass and the incline is $\mu_k = 0.350$, what *additional* distance, x, does the mass travel before stopping?



WORK DONE by Friction:

$W_f = f_k s \cos 180° = -f_k s = -\mu_k n s$

NO OTHER FORCES THAN NORMAL, weight friction $\Rightarrow n = W_\perp = Mg\cos 30°$

$n = (12kg)(9.8 m/s^2)\cos 30°$
$= 101.84N$

$\mu_k n = .35(101.84N)$
$= 35.6N$

$S = (10m + x)$

Let $y_2 = 0 \Rightarrow y_1 = (x+10m)\sin 30° = (x+10m)\cdot\frac{1}{2}$

Also, x is positive

(a) 2.08 m	(b) 1.49 m	(c) 10.0 m	(d) 1.6 m

gravity, spring, friction DO WORK $\Rightarrow \frac{1}{2}MV_1^2 + Mgy_1 + \frac{1}{2}kS_1^2 + W_f = \frac{1}{2}MV_2^2 + Mgy_2 + \frac{1}{2}kS_2^2$

$V_1 = 5.85 m/s,\ V_2 = 0,\ S_1 = 0$ (Not Compressed), $S_2 = -x$

$\therefore \frac{1}{2}(12kg)(5.85 m/s)^2 + (12kg)(9.8 m/s^2)(10m+x)\cdot\frac{1}{2} - 35.6N(10m+x) = \frac{1}{2}(424 N/m)x^2$

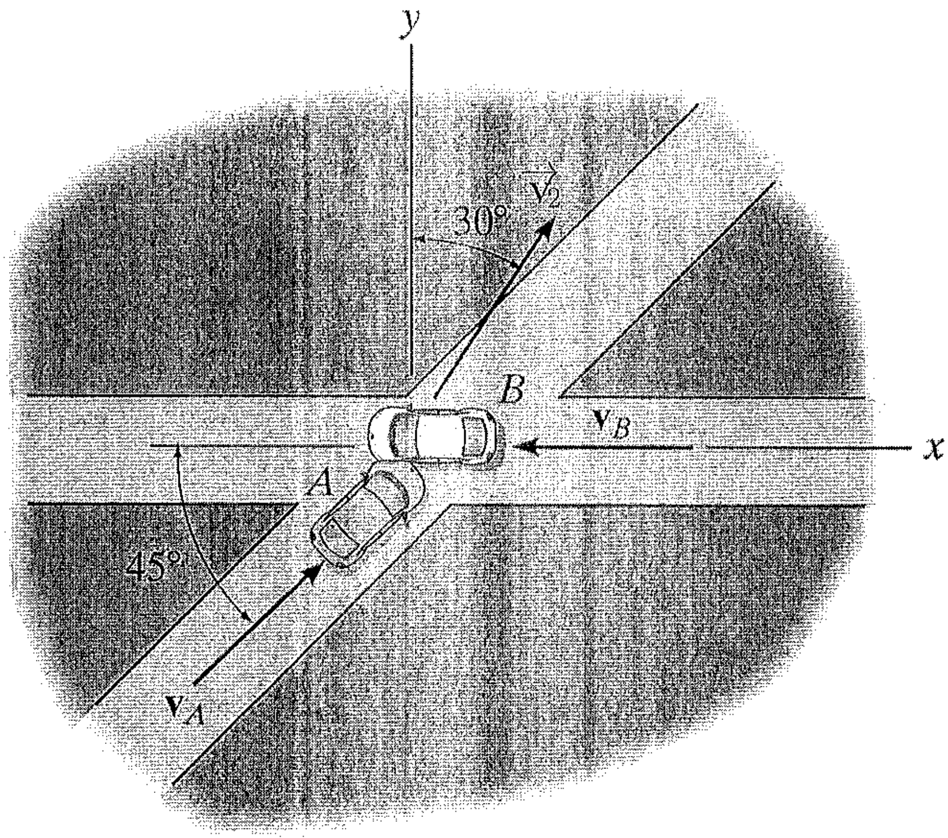
$\Rightarrow 205.3J + 588J + 58.8N(x) - 356J - 35.6N(x) = (212 N/m)x^2$

$\Rightarrow 437.3J + 23.2N(x) = (212 N/m)x^2$

$\Rightarrow (212 N/m)x^2 - 23.2N(x) - 437.3J = 0 \Rightarrow X = \dfrac{23.2N \pm 609.4N}{424 N/m}$

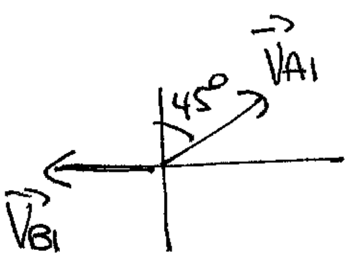
$\Rightarrow X = 1.49m$ OR $-1.38m$ USE positive ANSWER

8. One day while driving your $M_A = 2000\,kg$ car while going Northeast you, very embarrassingly, smash into your instructor's $M_B = 1500\,kg$ car which was going due West. If the cars have a completely inelastic collision and just after the collision are going $12\,m/s$ at 30° East of North, how fast was each car going before the collision?



COMPLETELY INELASTIC ⟹ $M_A V_{A1,x} + M_B V_{B1,x} = (M_A + M_B) V_{2,x}$

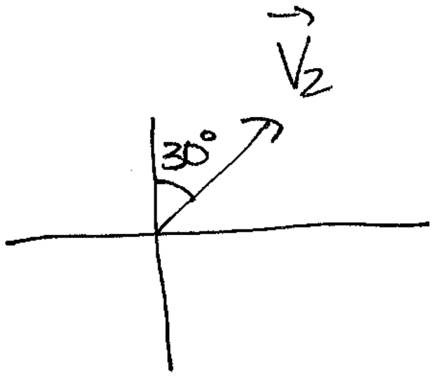
$$M_A V_{A1,y} + M_B V_{B1,y} = (M_A + M_B) V_{2,y}$$



$$V_{A1,x} = V_{A1} \sin 45° = .7071\, V_{A1}$$

$$V_{A1,y} = V_{A1} \cos 45° = .7071\, V_{A1}$$

$$V_{B1,x} = -V_{B1}, \quad V_{B1,y} = 0$$



$$V_{2,x} = V_2 \sin 30^\circ = 12\,m/s \sin 30^\circ$$
$$= 6\,m/s$$

$$V_{2,y} = V_2 \cos 30^\circ = 12\,m/s \cos 30^\circ$$
$$= 10.3923\,m/s$$

START WITH y-COMPONENTS: $M_A V_{A1,y} + M_B V_{B1,y} = (M_A + M_B) V_{2,y}$

$$\Rightarrow 2000\,kg\,(.7071\,V_{A1}) + 0 = (2000\,kg + 1500\,kg)(10.3923\,m/s)$$

$$\Rightarrow 2000\,kg\,(.7071)\,V_{A1} = 3500\,kg\,(10.3923\,m/s)$$

$$\Rightarrow V_{A1} = \left(\frac{3500}{2000}\right)\left(\frac{10.3923\,m/s}{.7071}\right) \Rightarrow \boxed{V_{A1} = 25.7\,m/s}$$

X-COMPONENTS: $M_A V_{A1,x} + M_B V_{B1,x} = (M_A + M_B) V_{2,x}$

$$\Rightarrow 2000\,kg\,(.7071)(25.7\,m/s) + 1500\,kg\,(-V_{B1}) = 3500\,kg\,(6\,m/s)$$

$$\Rightarrow 36373\,kg \cdot m/s - 1500\,kg\,V_{B1} = 21000\,kg \cdot m/s$$

$$\Rightarrow V_{B1} = -\frac{15373\,kg \cdot m/s}{1500\,kg} \Rightarrow \boxed{V_{B1} = +10.2\,m/s}$$