Five Easy Pieces

1. A $M_A = 5\,kg$ mass with $\vec{v}_{A1} = 25\,m/s$ at 32° (relative to the $+x$-axis) collides with a stationary $M_B = 15\,kg$ mass. If M_B bounces to the right with a speed of $8\,m/s$, with what speed does M_A bounce?

$M_A\vec{V}_{A1} + M_B\vec{V}_{B1} = M_A\vec{V}_{A2} + M_B\vec{V}_{B2}$

$\Rightarrow 5kg(25\cos 32°) + 0$

$= 5kg\,V_{A2,x} + 15kg(8m/s)$

$\Rightarrow V_{A2,x} = -2.8\,m/s$

(a) $13.54\,m/s$	(b) $20.8\,m/s$	(c) $25\,m/s$	(d) $8\,m/s$

$5kg(25\sin 32°) + 0 = 5kg\,V_{A2,y} + 0$

$\Rightarrow V_{A2,y} = 13.25\,m/s$ $\quad V_{A2} = \sqrt{(2.8)^2 + (13.25)^2} = 13.54\,m/s$

2. Using the information from the previous problem, at what angle does M_A bounce? Note: All answers are given relative to the positive x-axis

$\theta = \tan^{-1}\left(\frac{13.25}{-2.8}\right) + 180°$ ⟸ 2ND Quadrant

$= 102°$

(a) 180°	(b) ~~168°~~ 102°	(c) ~~−12°~~ −78°	(d) Cannot be determined.

3. A $M_A = 5\,kg$ mass with $v_{A1} = 6\,m/s$ and a $M_B = 7\,kg$ mass with $v_{B1} = 2\,m/s$ have an elastic collision. If $v_{A2} = 3\,m/s$, v_{B2} has what value?

(a) $1\,m/s$	(b) $4\,m/s$	(c) $4.83\,m/s$	(d) $4.14\,m/s$

Elastic $\Rightarrow \frac{1}{2}M_AV_{A1}^2 + \frac{1}{2}M_BV_{B1}^2 = \frac{1}{2}M_AV_{A2}^2 + \frac{1}{2}M_BV_{B2}^2$

$\frac{1}{2}(5kg)(6m/s)^2 + \frac{1}{2}(7kg)(2m/s)^2 = \frac{1}{2}(5kg)(3m/s)^2 + \frac{1}{2}(7kg)V_{B2}^2$

$\Rightarrow 104\,J = 22.5 + \frac{1}{2}(7kg)V_{B2}^2$

$\Rightarrow V_{B2} = \sqrt{\frac{2(81.5J)}{7kg}} = 4.83\,m/s$

4. A $M_A = 5\,kg$ mass with $\vec{v}_{A1} = 6\,m/s$ at 32° and a $M_B = 3\,kg$ with $\vec{v}_{B1} = 2\,m/s$ at 175° have a two-dimensional elastic collision. What $\vec{v}_{A2}$ and $\vec{v}_{B2}$ make the collision elastic?

(a) $\vec{v}_{A2} = 4\,m/s$ at 122°, $\vec{v}_{B2} = 6.11\,m/s$ at 32°
(b) $\vec{v}_{A2} = 4\,m/s$ at 122°, $\vec{v}_{B2} = 6.11\,m/s$ at 212°
(c) $\vec{v}_{A2} = 4\,m/s$ at 32°, $\vec{v}_{B2} = 6.11\,m/s$ at 122°
(d) There is not enough information to determine.

There are infinitely ways to have a 2D Elastic Collision

5. A $2\,kg$, $2\,m$-radius solid cylinder (moment of inertia, $I = 1/2MR^2$) is rolling without slipping with an angular speed of $2.5\,rad/s$. How much kinetic energy does it have?

(a) $25\,J$	(b) $9.375\,J$	(c) $37.5\,J$	(d) $6.25\,J$

$$K = \frac{1}{2}mv^2\left(1+\frac{I}{MR^2}\right) = \frac{1}{2}mv^2\left(1+\frac{\frac{1}{2}MR^2}{MR^2}\right)$$

$$= \frac{1}{2}mv^2\left(1+\frac{1}{2}\right) = \frac{1}{2}mv^2\left(\frac{3}{2}\right) = \frac{3}{4}mv^2$$

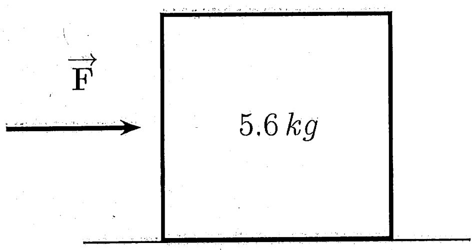
Rolls, No slipping $\Rightarrow V = \omega R \Rightarrow V = (2.5\,rad/s)(2\,m) = 5\,m/s$

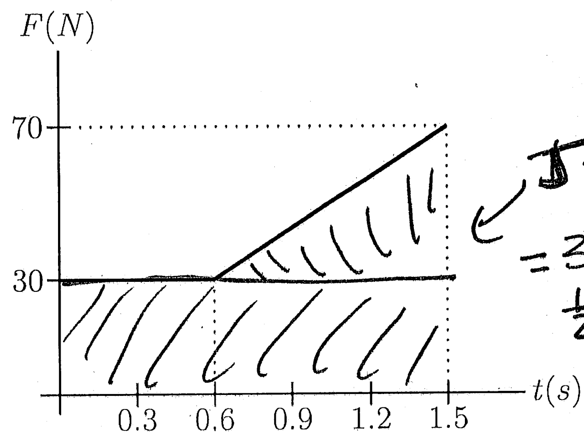
$$K = \frac{3}{4}(2\,kg)(5\,m/s)^2 = 37.5\,J$$

Sliding

Questions 6 through 10 refer to the following setup.

A 5.6 kg box is sitting stationary on a frictionless surface when your instructor steps up to it and applies, in the positive x direction, a non-constant force, F.





$J = \text{Area}$
$= 30(1.5) + \frac{1}{2}(40)(.9)$
$= 63 \, N \cdot s$

6. What impulse was imparted to the mass by your instructor's force?

(a) 45 $N \cdot s$	(b) 105 $N \cdot s$	(c) 18 $N \cdot s$	(d) 63 $N \cdot s$

7. What was the average force exerted on the mass?

(a) 30 N	(b) 42 N	(c) 50 N	(d) 70 N

$J = F_{AV} \Delta t$

$\Rightarrow 63 = F_{AV}(1.5)$

$F_{AV} = \frac{63}{1.5} = 42 N$

8. How fast was the 5.6 kg, initially at rest, mass going after 1.5 s?

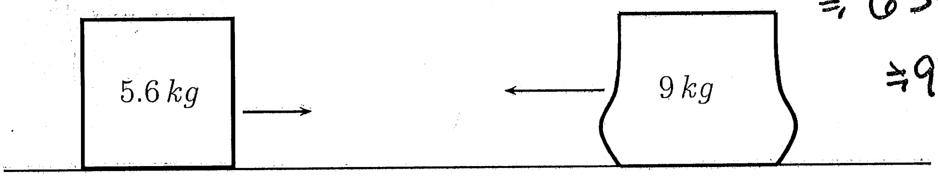
(a) 11.25 m/s	(b) 353 m/s	(c) 4.74 m/s	(d) 22.5 m/s

$J = \Delta P$
$= mV_2 - mV_1$
$V_1 = 0,\ V_2 = ?$
$63 N\cdot s = (5.6 kg)V_2$
$\Rightarrow V_2 = 11.25 m/s$

9. After being pushed, the 5.6 kg box slides across the frictionless floor and collides with a 9.0 kg lump of clay that is traveling in the opposite direction. If the box and the clay stick to each other and have a post-collision velocity of 0.5 m/s to the right, how fast was the clay going the instant before the collision?

(a) 7.3 m/s	(b) 9 m/s	(c) 6.2 m/s	(d) 56 m/s

$M_A V_{A1} + M_B V_{B1} = (M_A + M_B)V_2$
$M_A = 5.6 kg,\ V_{A1} = 11.25 m/s$
$M_B = 9 kg,\ V_{B1} = ?$
$V_2 = .5 m/s$
$\Rightarrow 63 + 9V_{B1} = (14.6)(.5)$
$\Rightarrow 9V_{B1} = 7.3 - 63$
$V_{B1} = -6.188...$
$= -6.2 m/s$



10. How much heat was produced during this collision?

(a) 354 J	(b) 1.825 J	(c) 172 J	(d) 525 J

"Missing" Kinetic Energy Converted into Heat

$K_2 = \frac{1}{2}(M_A + M_B)V_2^2 = \frac{1}{2}(14.6 kg)(.5 m/s)^2 = 1.825 J$

$K_1 = \frac{1}{2}M_A V_{A1}^2 + \frac{1}{2}M_B V_{B1}^2 = \frac{1}{2}(5.6 kg)(11.25 m/s)^2 + \frac{1}{2}(9 kg)(6.188 m/s)^2$

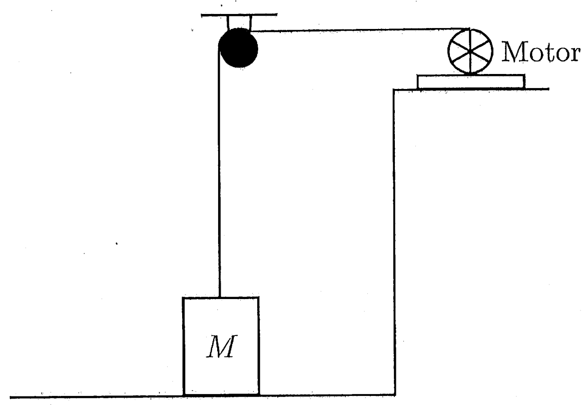
$= 526.7 J$

$K_1 - K_2 = 525 J$

Lifting

Questions 11 through 15 refer to the following setup and figure.

One day finds your instructor needing to lift a 10.0 kg box. As usual, he has completely over-complicated the procedure by using a massless pulley and a motor as shown below. Your instructor observes that when the motor has lifted the box 4.0 m above the floor, it has a speed of 5.0 m/s.



11. How much gravitational potential energy does the box have? (Assume the on the floor, the box had zero potential energy.)

(a) 98 J	(b) 4 J	(c) 392 J	(d) 400 J

$U = mgy$
$= (10kg)(9.8m/s^2)(4m)$
$= 392J$

12. How much kinetic energy does the box have?

(a) 250 J	(b) 392 J	(c) 517 J	(d) 125 J

$K = \frac{1}{2}mv^2$
$= \frac{1}{2}(10kg)(5m/s)^2$
$= 125J$

13. Assuming the box started from rest, how much work was done by the motor lifting the box to $4\,m$?

$U_1 + K_1 + W_{other} = U_2 + K_2$

(a) $267\,J$	(b) $392\,J$	(c) $517\,J$	(d) $125\,J$

$U_1 = 0,\ K_1 = 0$

$W_{other} = ?,\ U_2 = 392\,J$

$K_2 = 125\,J \Rightarrow W_{other} = 517\,J$

14. If at the instant the box is $4\,m$ above the ground the motor is supplying $640\,Watt$ of power, what is the box's acceleration? **HINT**: Use the equation $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$ where $\vec{\mathbf{F}}$ is the force the motor exerts on the rope and $\vec{\mathbf{v}}$ is the velocity of the rope entering the motor.

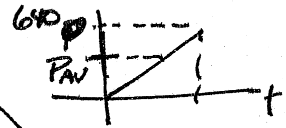
(a) $9.8\,m/s^2$	(b) $0\,m/s^2$	(c) $128\,m/s^2$	(d) $3\,m/s^2$

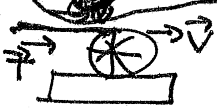
15. Assuming constant acceleration, which of the following is a true statement about the average power, P_{av} supplied by the motor while the box was rising $4\,m$? → MASS & Rope start from rest, Constant Acceleration

$\Rightarrow V = v_0 + at = (3\,m/s^2)t \Rightarrow P = TV = (128N)(3\,m/s^2)t = (384)t$

Linearly increasing Power

(a) $P_{av} = 640\,Watt$	(b) $P_{av} > 640\,Watt$
(c) $P_{av} < 640\,Watt$	(d) There is not enough information to determine.

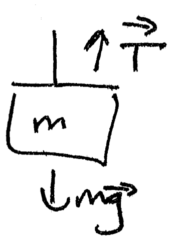


For motor 

$\vec{F} = \vec{T}$ = tension in Rope

$P = \vec{T} \cdot \vec{V} = TV\cos 0° = TV$. Rope has same speed as MASS $\Rightarrow V = 5\,m/s$

$\Rightarrow 640\,Watt = T(5\,m/s) \Rightarrow T = 128N$

FOR MASS 

$\sum F_y = ma_y \Rightarrow T - mg = ma_y$

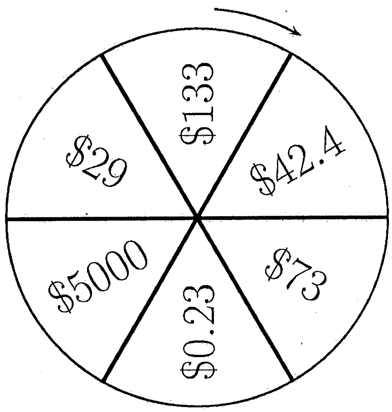
$\Rightarrow 128N - (10\,kg)(9.8\,m/s^2) = (10\,kg)a_y$

$\Rightarrow a_y = \dfrac{30N}{10\,kg} = 3\,m/s^2$

Round and Round

Questions 16 through 20 refer to the following setup and figure.

Through circumstances too bizarre to be detailed here, you find yourself competing in a cheap wheel-of-fortune© knockoff. The possible payouts are shown below.



16. If the wheel is spinning clockwise about its center, as shown, what is the **direction** of its angular velocity?

(a) $\odot$	(b) $\otimes$	(c) *clockwise*	(d) *counter – clockwise*

RIGHT-HAND-RULE $\Rightarrow \vec{\omega} = \otimes$

Note:
Clockwise is not a direction.

17. If the wheel's angular speed is decreasing, what **direction** is the wheel's angular acceleration?

(a) $\odot$	(b) $\otimes$	(c) *clockwise*	(d) *counter – clockwise*

Decreasing $\Rightarrow \vec{\alpha}$ opposite to $\vec{\omega}$

$\Rightarrow \vec{\alpha} = \odot$

18. If the number 5 in the \$5000 payoff is $1.2\,m$ from the center of the wheel, what is its linear velocity at the instant $\omega = 3\,rad/s$ and the wheel is oriented as shown on the previous page? Assume the payoffs are equally spaced around the circle.

(a) $3\,m/s$ at 120°	(b) $3.6\,m/s$ at 210°	(c) $3.6\,m/s$ at 120°	(d) $3.6\,m/s$ at 300°

$V = \omega r = (3\,rad/s)(1.2\,m) = 3.6\,m/s$ at $210° - 90° = 120°$ (SEE Below picture)

19. What is the centripetal acceleration of the 5 in the \$5000 payout?

(a) $3\,m/s^2$ at 30°	(b) $10.8 m/s^2$ at 30°	(c) $3.6\,m/s^2$ at 120°	(d) $10.8 m/s^2$ at 210°

$a_{rad} = \omega^2 r = (3\,rad/s)^2 (1.2\,m) = 10.8\,m/s^2$ at $210° - 180° = 30°$

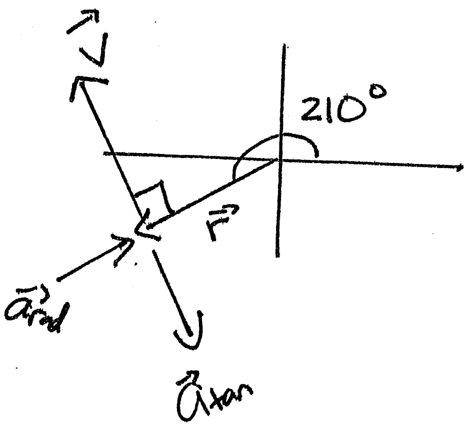
20. What direction is the tangential acceleration of the 5 in the \$5000 payout? Remember that the wheel was decreasing angular speed at this time.

$\vec{a}_{tan}$ opposite to $\vec{V}$ since speed decreasing

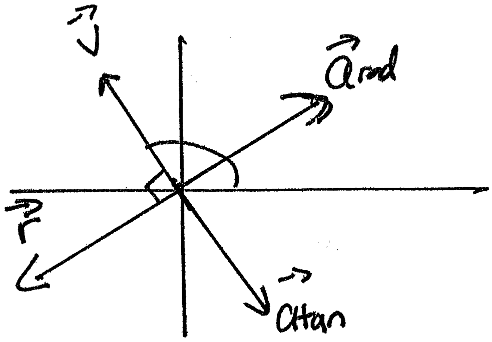
(a) 120°	(b) 210°	(c) 30°	(d) 300°

$\Rightarrow 120° + 180° = 300°$

EQUALLY SPACED, 6 OF THEM $\Rightarrow \frac{360°}{6} = 60°$ Between EACH PAYOUT. AT INSTANT SHOWN \$42.4 at 30° $\Rightarrow$ \$5000 at $30° + 180° = 210°$ is direction of $\vec{r}$!



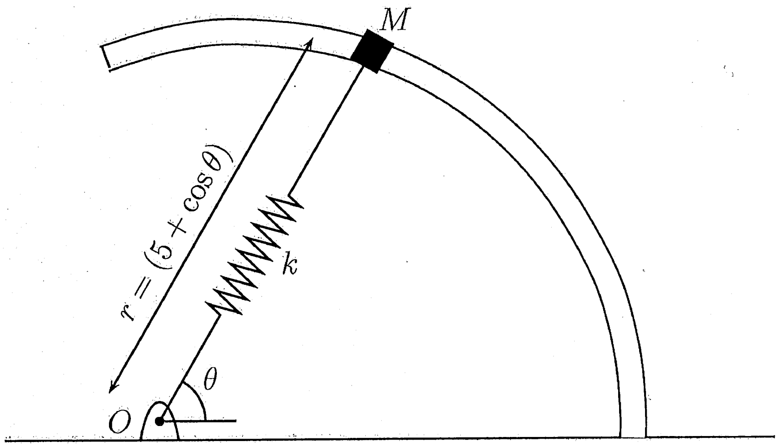
$\Rightarrow$



21. **High Art**

One day whilst on a walking tour of Santa Fe, you come across a most curious piece of kinetic art. It consists of an 18 kg steel collar that slides over a frictionless, fancy-shaped track while attached to a 26 N/m spring. As shown below, the spring, unstretched length 1.1 m, is connected so that it is free to swing around with the mass and is always oriented along the line connecting the collar and the point labeled O.

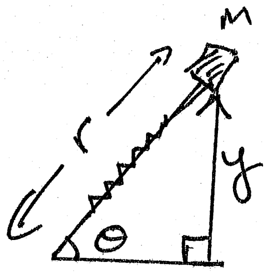
The *artiste* who designed the sculpture proudly tells you that she has carefully designed the track so that it has the famous shape called a limaçon. She's even able to give you the exact equation for the track's limaçon, $r = (5 + \cos\theta)$, where r is the distance (in meters) from O to the collar and θ is the angle shown below.



(a) The *artiste* informs you that her original vision had the 18 kg collar starting from rest at $\theta = 90°$ and gracefully sliding down to $\theta = 0°$. She was "bummed" (to use her phrase) when the collar did no such thing. It had to be started with some minimum speed. Using methods discussed in the bonus, you find that the collar's potential energy is greatest at $\theta = 73.4°$. What minimum speed must be given to the collar at $\theta = 90°$ for it to just reach $\theta = 73.4°$? *(10pts)*

gravity and spring doing work

$$\frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}ks_1^2 = \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}ks_2^2$$



$$y = r\sin\theta = (5+\cos\theta)\sin\theta$$

$$y_1 = y(\theta=90°) = 5m$$

$$y_2 = y(\theta=73.4°) = (5+\cos 73.4°)\sin 73.4° = 5.065m$$

$$s = r - l_0 = (5+\cos\theta) - 1.1m = 3.9m + \cos\theta$$

↳ unstretched

$$s_1 = 3.9m + \cos 90° = 3.9m, \quad s_2 = 3.9m + \cos 73.4° = 4.186m$$

$V_1 = ?$, $V_2 = 0$ (Barely makes it)

$$\Rightarrow \frac{1}{2}(18kg)V_1^2 + (18kg)(9.8m/s^2)(5m) + \frac{1}{2}(26N/m)(3.9m)^2 = \emptyset + (18kg)(9.8m/s^2)(5.065m) + \frac{1}{2}(26N/m)(4.186m)^2$$

$$\Rightarrow \frac{1}{2}(18kg)V_1^2 + 882J + 197.73J = 893.466J + 227.8J$$

$$\Rightarrow \frac{1}{2}(18kg)V_1^2 = 41.536J \Rightarrow \boxed{V_1 = \sqrt{\frac{41.536(2)}{18}} = 2.14828 = 2.15m/s}$$

(b) During one particulary memorable run of the sculpture, the collar went from $\theta = 73.4°$ (where it was momentarily at rest) down to $\theta = 0°$ whilst a big gust of wind was blowing. (It was so impressive that your monocle nearly popped out.) If the collar reached $\theta = 0°$ with a speed of $6.5\,m/s$, how much work was done by that gust of wind? *(10pts)*

Now gravity, Spring, AND WIND DO work

$W_{WIND} = W_{OTHER} = ?$

$$\frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}KS_2^2 + W_{WIND} = \frac{1}{2}mV_3^2 + mgy_3 + \frac{1}{2}KS_3^2$$

$V_2 = 0$, $y_2 = 5.065m$, $S_2 = 4.186m$

$V_3 = 6.5m/s$, $y_3 = y(\theta = 0°) = 0$, $S_3 = 3.9m + \cos 0° = 4.9m$

$$\Rightarrow 0 + 893.466J + 227.8J + W_{WIND} = \frac{1}{2}(18kg)(6.5m/s)^2 + \phi + \frac{1}{2}(26N/m)(4.9m)^2$$

$$\Rightarrow 1121.266J + W_{WIND} = 380.25J + 312.13J$$

$$\Rightarrow \boxed{W_{WIND} = -428.886J = -429J}$$

(c) **BONUS:** Show that the potential energy of the collar has its maximum value at the point where $\theta = 73.4°$. **HINT:** Find the potential energy as a function of theta. Use your new/old-found calculus skills to find the maximum. *(+3pts)*

Total Potential $U = U_g + U_{el} = mgy + \frac{1}{2}ks^2$

$\Rightarrow U = mg(5 + \cos\theta)\sin\theta + \frac{1}{2}k(3.9 + \cos\theta)^2$

FIND MAXIMA $\Rightarrow$ TAKE derivative AND set EQUAL to zero!

i.e. $\frac{dU}{d\theta} = 0$ gives θ. Easier with Numbers, $m = 18\,kg$, $k = 26\,N/m$

$\Rightarrow U = 176.4(5 + \cos\theta)\sin\theta + 13(3.9 + \cos\theta)^2$

$$\frac{dU}{d\theta} = 176.4(-\sin\theta)(\sin\theta) + 176.4(5 + \cos\theta)\cos\theta + 13(2)(3.9 + \cos\theta)'(-\sin\theta)$$

$$= -176.4\sin^2\theta + 882\cos\theta + 176.4\cos^2\theta - 26(3.9 + \cos\theta)\sin\theta$$

$\rightarrow$ THIS IS impossible to solve w/out MATLAB / graphing Calculator which is why THE BONUS IS TO SHOW $\theta = 73.4°$ works

i.e. $-176.4\sin^2 73.4° + 882\cos 73.4° + 176.4\cos^2 73.4° - 26(3.9 + \cos 73.4°)\sin 73.4°$

$= -162 + 251.98 + 14.40 - 104.29 = .09 \approx 0$