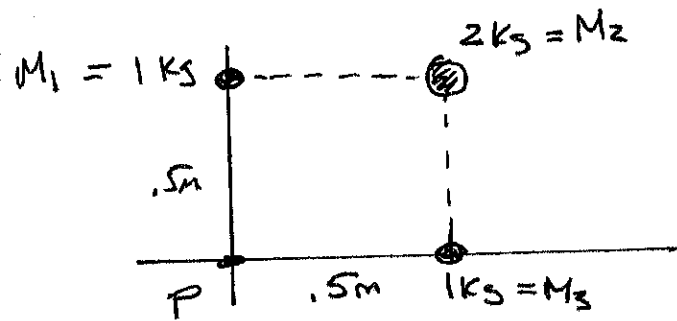


Physic 160, Hw# 11

Mastering Physics: 5 Problems from  
Chapter 13

13.43



A  $M_4 = .015 \text{ kg}$  MASS IS  
PLACED AT ORIGIN,  
WHAT IS NET FORCE ON IT?

LABEL  $M_1, M_2, M_3$  AS SHOWN ABOVE.  $\Rightarrow$  NET FORCE ON  $M_4$  IS

$$\sum \vec{F}_4 = \vec{F}_{41} + \vec{F}_{42} + \vec{F}_{43}$$

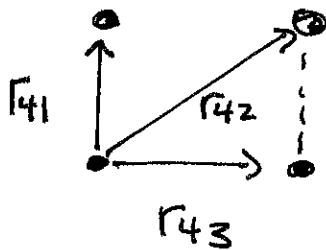
$\uparrow$   
Force on #4 due to #1

$\uparrow$   
Force on #4 due to #2

$\uparrow$   
Force on #4 due to #3

NEWTON'S LAW OF GRAVITY  $\Rightarrow F_{41} = \frac{GM_4M_1}{r_{41}}, F_{42} = \frac{GM_4M_2}{r_{42}}$

$$F_{43} = \frac{GM_4M_3}{r_{43}}$$



$$r_{41} = r_{43} = .5 \text{ m}$$

$$r_{42} = \sqrt{(.5\text{m})^2 + (.5\text{m})^2}$$

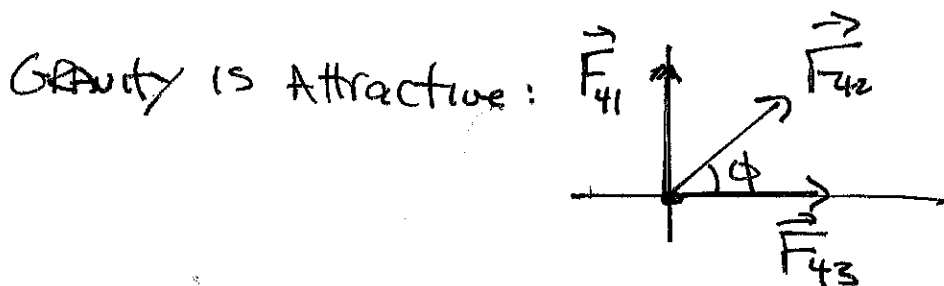
$$= \sqrt{.25\text{m}^2 + .25\text{m}^2} = \sqrt{.5\text{m}^2}$$

$$\textcircled{a} M_1 = M_3 = 1 \text{ kg} \Rightarrow F_{41} = F_{43} = \frac{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(0.015 \text{ kg})(1 \text{ kg})}{(0.5 \text{ m})^2}$$

$$\Rightarrow F_{41} = F_{43} = 4 \times 10^{-12} \text{ N}$$

$$F_{42} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2)(0.015 \text{ kg})(2 \text{ kg})}{(\sqrt{0.5 \text{ m}^2})^2} = \frac{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2})(0.015 \text{ kg})(2 \text{ kg})}{0.5 \text{ m}^2}$$

$$\Rightarrow F_{42} = 4 \times 10^{-12} \text{ N}$$



$$\phi = \tan^{-1}\left(\frac{0.5}{0.5}\right) = 45^\circ$$

$$\sum F_{4,x} = F_{41,x} + F_{42,x} + F_{43,x} = 0 + (4 \times 10^{-12} \text{ N}) \cos 45^\circ + 4 \times 10^{-12} \text{ N}$$

$$\Rightarrow \sum F_{4,x} = 6.828 \times 10^{-12} \text{ N}$$

$$\sum F_{4,y} = F_{41,y} + F_{42,y} + F_{43,y} = 4 \times 10^{-12} \text{ N} + (4 \times 10^{-12} \text{ N}) \sin 45^\circ + 0 = 6.828 \times 10^{-12} \text{ N}$$

$$\sum F_4 = \sqrt{\sum F_{4,x}^2 + \sum F_{4,y}^2} = \sqrt{(6.828 \times 10^{-12} \text{ N})^2 + (6.828 \times 10^{-12} \text{ N})^2} = 9.66 \times 10^{-12} \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\sum F_{4,y}}{\sum F_{4,x}}\right) = \tan^{-1}(1) = 45^\circ$$

Part C: IF .015 kg particle is released from rest 300m from others, how fast will it be going at origin?

gravity only force doing work  $\Rightarrow \frac{1}{2} M_4 V_1^2 + U_{g,1} = \frac{1}{2} M_4 V_2^2 + U_{g,2}$

Compared to .5m, 300m  $\approx$  INFINITY  $\Rightarrow U_{g,1} = 0, V_1 = 0$

Particle has potential energy due to  $M_1, M_2,$  AND  $M_3$ .

$U_g = -\frac{GM_4 M_b}{r}$ , Energy A SCALAR  $\Rightarrow U_{g,2} = -\frac{GM_4 M_1}{r_{41}} - \frac{GM_4 M_2}{r_{42}} - \frac{GM_4 M_3}{r_{43}}$

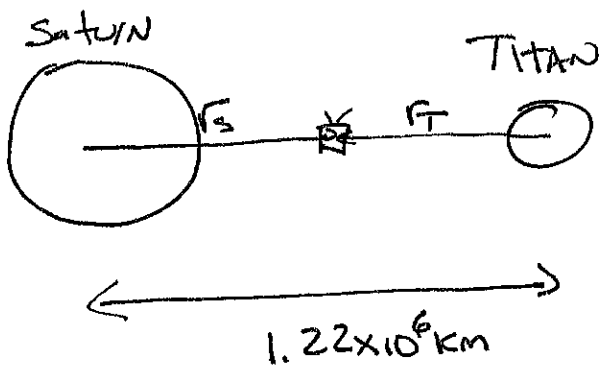
$\therefore 0 = \frac{1}{2} M_4 V_2^2 - \frac{GM_4 M_1}{r_{41}} - \frac{GM_4 M_2}{r_{42}} - \frac{GM_4 M_3}{r_{43}}$

$\Rightarrow \frac{1}{2} V_2^2 = \frac{GM_1}{r_{41}} + \frac{GM_2}{r_{42}} + \frac{GM_3}{r_{43}} \Rightarrow V_2^2 = 2G \left( \frac{M_1}{r_{41}} + \frac{M_2}{r_{42}} + \frac{M_3}{r_{43}} \right)$

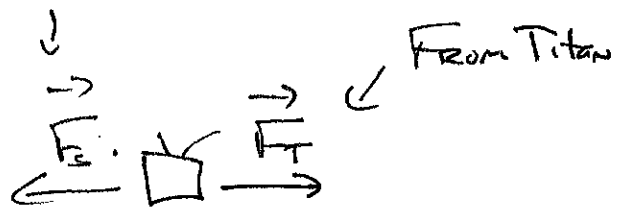
$= 2(6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}) \left( \frac{1 kg}{.5m} + \frac{2 kg}{1.5m^2} + \frac{1 kg}{.5m} \right) = 2(6.67 \times 10^{-11})(6.8284)$

$\Rightarrow V_2^2 = 9.109 \times 10^{-10} m^2/s^2 \Rightarrow V_2 = \sqrt{9.109 \times 10^{-10}} = 3.02 \times 10^{-5} m/s$

13.46



From Saturn



$$\sum \vec{F} = 0$$

$$\Rightarrow \vec{F}_T = -\vec{F}_S$$

$$\Rightarrow F_T = F_S$$

Let Huygens Probe have MASS  $M$ : Newton's LAW  $\Rightarrow$

$$\frac{GM_T}{r_T^2} = \frac{GM_S}{r_S^2} \Rightarrow \frac{M_T}{r_T^2} = \frac{M_S}{r_S^2}$$

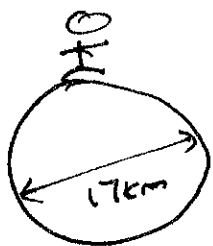
$$\Rightarrow \frac{r_T^2}{r_S^2} = \left( \frac{M_T}{M_S} \right) \Rightarrow r_T^2 = r_S^2 \left( \frac{1.35 \times 10^{25} \text{ kg}}{5.68 \times 10^{26} \text{ kg}} \right) = r_S^2 (2.37 \times 10^{-4})$$

↑  
Appendix F

$$\Rightarrow r_T = r_S \sqrt{2.37 \times 10^{-4}} = r_S (0.0154)$$

$$r_T + r_S = 1.22 \times 10^6 \text{ km} \Rightarrow r_S (0.0154) + r_S = 1.22 \times 10^6 \text{ km}$$

$$\Rightarrow r_S = \frac{1.22 \times 10^6 \text{ km}}{1.0154} = 1.2 \times 10^6 \text{ km} \quad \therefore r_T = 0.0185 \times 10^6 \text{ km} = 18500 \text{ km}$$



$$W_{\text{weight on Earth}} = 6065 \text{ N}$$

$$W = Mg \text{ NEAR SURFACE}$$

$$\Rightarrow M = \frac{W}{g} = \frac{6065 \text{ N}}{9.8 \text{ m/s}^2} = 67.857 \text{ kg}$$

$$\text{ON "SURFACE" OF NEUTRON STAR } g = \frac{GM}{R^2}$$

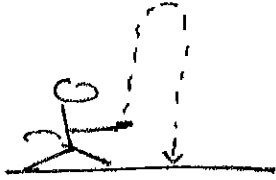
$$M = \text{MASS OF NEUTRON STAR} = 1.99 \times 10^{30} \text{ kg}$$

$$R = \text{RADIUS} = \frac{17 \text{ km}}{2} = 8.5 \text{ km} = 8500 \text{ m}$$

$$\Rightarrow g = \frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30} \text{ kg})}{(8500 \text{ m})^2} = 1.837 \times 10^{12} \text{ m/s}^2$$

$$\therefore W = (67.857 \text{ kg})(1.837 \times 10^{12} \text{ m/s}^2) = 1.24 \times 10^{14} \text{ N}$$

13.58



$M = 2.5 \text{ kg}$ ,  $V_0 = 12 \text{ m/s}$ , RETURNS IN 6s  
NO ATMOSPHERE ON MONGO.

$$\Rightarrow y = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$y = y_0 = 0, V_0 = 12 \text{ m/s}, t = 6 \text{ s}$$

$$\Rightarrow 0 = 12 \text{ m/s}(6 \text{ s}) - \frac{1}{2} g (6 \text{ s})^2 \Rightarrow g = 4 \text{ m/s}^2$$

OR  $V = V_0 + g t$ ,  $V = 0$ ,  $V_0 = 12 \text{ m/s}$ ,  $t = \frac{6 \text{ s}}{2} = 3 \text{ s} \Rightarrow g = 4 \text{ m/s}^2$

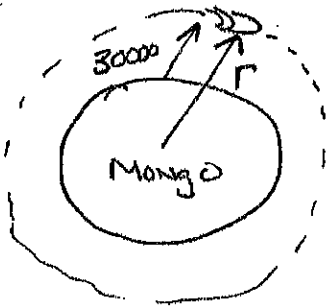
$$g = \frac{G M_M}{R_M^2}. \quad M_M = \text{MONGO'S MASS} \quad R_M = \text{MONGO'S RADIUS}$$

$$\text{CIRCUMFERENCE} = 2 \times 10^5 \text{ km} = 2 \times 10^8 \text{ m}$$

$$C = 2\pi R_M \Rightarrow R_M = \frac{2 \times 10^8 \text{ m}}{2\pi} = 3.18 \times 10^7 \text{ m}$$

$$M_M = \frac{g R_M^2}{G} = \frac{(4 \text{ m/s}^2)(3.18 \times 10^7 \text{ m})^2}{6.67 \times 10^{-11} \text{ N} \cdot \text{kg}^2/\text{m}^2} = \underline{\underline{6.08 \times 10^{25} \text{ kg} = M_M}}$$

b) How many hours for circular orbit  
30000 km above surface?



$$r = 30000 \text{ km} + R_M$$

$$= 30 \times 10^4 \times 1000 \text{ m} + 3.18 \times 10^7 \text{ m}$$

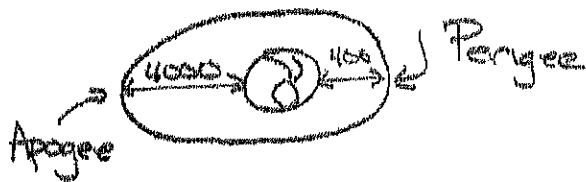
$$= 3 \times 10^7 \text{ m} + 3.18 \times 10^7 \text{ m} = 6.18 \times 10^7 \text{ m}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{GM_M}} = \frac{2\pi (6.18 \times 10^7 \text{ m})^{3/2}}{\sqrt{(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}) (6.0 \times 10^{25} \text{ kg})}}$$

$$\Rightarrow T = 47996 \text{ s} \times \frac{1 \text{ h}}{3600 \text{ s}} = \underline{\underline{13.3 \text{ hour}}}$$



13.77



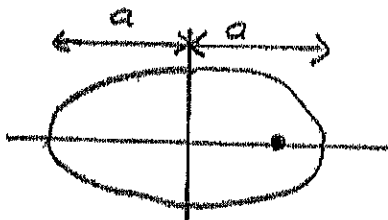
NOT TO SCALE obviously

$$400 \text{ km} = 4 \times 10^5 \text{ m}$$

$$4000 \text{ km} = 4 \times 10^6 \text{ m}$$

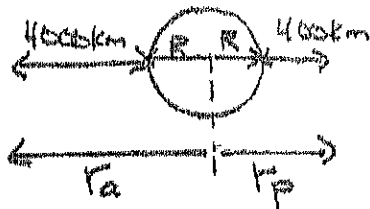
a) WHAT IS PERIOD?  $T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}}$

EARTH IS AT ONE FOCUS:



TOTAL DISTANCE IS  $2a$ .

HAVE TO INCLUDE EARTH'S RADIUS



$$r_p = R + 400 \text{ km}$$

$$= 6.38 \times 10^6 \text{ m} + 4 \times 10^5 \text{ m} = 6.78 \times 10^6 \text{ m}$$

$$r_a = R + 4000 \text{ km}$$

$$= 6.38 \times 10^6 \text{ m} + 4 \times 10^6 \text{ m} = 10.38 \times 10^6 \text{ m}$$

$$r_a + r_p = 2a \Rightarrow a = \frac{10.38 \times 10^6 \text{ m} + 6.78 \times 10^6 \text{ m}}{2} = 8.58 \times 10^6 \text{ m}$$

$$\Rightarrow T = \frac{2\pi (8.58 \times 10^6 \text{ m})^{3/2}}{[(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})]^{1/2}} \Rightarrow \boxed{T = 7913 \text{ s} = 2.2 \text{ h}}$$

b)  $L = MvR$  IS CONSERVED  $\Rightarrow Mv_a r_a = Mv_p r_p$

$$\Rightarrow \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{10.38 \times 10^6}{6.78 \times 10^6} \Rightarrow \boxed{\frac{v_p}{v_a} = 1.53}$$

c)  $E = \frac{1}{2}MV^2 - \frac{GM_E M}{r}$  IS CONSERVED

$$\Rightarrow \frac{1}{2}MV_a^2 - \frac{GM_E M}{r_a} = \frac{1}{2}MV_p^2 - \frac{GM_E M}{r_p}$$

$$\Rightarrow \frac{1}{2}V_a^2 - \frac{1}{2}V_p^2 = \frac{GM_E}{r_a} - \frac{GM_E}{r_p} = GM_E \left( \frac{1}{r_a} - \frac{1}{r_p} \right)$$

$$V_p = 1.53V_a \Rightarrow \frac{1}{2}V_a^2 - \frac{1}{2}(1.53V_a)^2 = GM_E \left( \frac{1}{r_a} - \frac{1}{r_p} \right)$$

$$\Rightarrow \frac{1}{2}(1-1.53^2)V_a^2 = GM_E \left( \frac{1}{r_a} - \frac{1}{r_p} \right) \Rightarrow V_a^2 = \frac{2GM_E}{(1-1.53^2)} \left( \frac{1}{r_a} - \frac{1}{r_p} \right)$$

$$\Rightarrow V_a^2 = \frac{2(6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2)(5.97 \times 10^{24} \text{kg})}{(1-1.53^2)} \left( \frac{1}{10.38 \times 10^6 \text{m}} - \frac{1}{6.75 \times 10^6 \text{m}} \right)$$

$$= (-5.94 \times 10^{14})(-5.12 \times 10^{-8}) \text{m}^2/\text{s}^2 = 3.04 \times 10^7 \text{m}^2/\text{s}^2$$

$$\Rightarrow \boxed{V_a = 5512 \text{m/s}} \quad V_p = 1.53(5512 \text{m/s}) \Rightarrow \boxed{V_p = 8434 \text{m/s}}$$

d) TO ESCAPE EARTH.  $V_{es} = \sqrt{\frac{2GM_E}{r}}$

AT PERIGEE,  $V_{es} = \sqrt{\frac{2GM_E}{r_p}} = 10838 \text{m/s}$  (I THINK YOU KNOW THE NUMBERS BY NOW!)

IT ALREADY HAS  $V_p = 8434 \text{m/s} \Rightarrow \Delta V_p = V_{es} - V_p \Rightarrow \boxed{\Delta V_p = 2404 \text{m/s}}$

AT APOGEE,  $V_{es} = \sqrt{\frac{2GM_E}{r_a}} = 8759 \text{m/s}$ ,  $V_a = 5512 \text{m/s}$

$$\Delta V_a = V_{es} - V_a \Rightarrow \boxed{\Delta V_a = 3247 \text{m/s}}$$

EASIER TO DO AT PERIGEE.