Physics 160, HW #7:

Mastering Physics: 6 problems from Chapter 7

Written: 7.60

Kinetic and Potential Energy Graphing

Ball caught at $t = 5s$

Sketch K vs. t

No air resistance $\Rightarrow V = V_0 - gt$

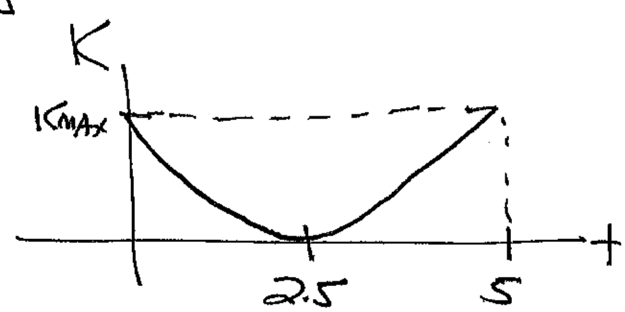
$\Rightarrow K = \frac{1}{2}MV^2 = \frac{1}{2}M(V_0 - gt)^2$

So $K \propto t^2 \Rightarrow$ parabola

At top, $V = 0 \Rightarrow K = 0$, ball caught at $t = 5s$, caught and thrown at same height, so $K = 0$ at $t = 2.5s$

at $t = 0$ and $t = 5s$, $|V| = V_0 \Rightarrow V^2 = V_0^2 \Rightarrow K = K_{MAX} = \frac{1}{2}MV_0^2$

K_{MAX} since at $t = 0$ and $t = 5s$, potential energy is zero so K = maximum. Putting this all together:



PART B: SKETCH POTENTIAL ENERGY.

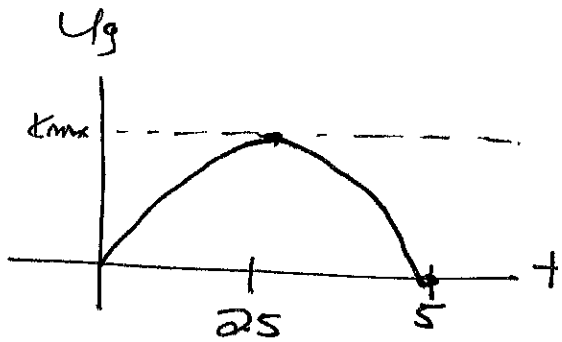
GRAVITY ONLY FORCE DOING WORK $\Rightarrow$ TOTAL ENERGY CONSERVED.

$K + U_g = \text{Constant}$ At $t = 0$, $t = 5s$, $U_g = 0$

$\therefore$ Constant $= K_{MAX}$ $\Rightarrow$ $K + U_g = K_{MAX}$

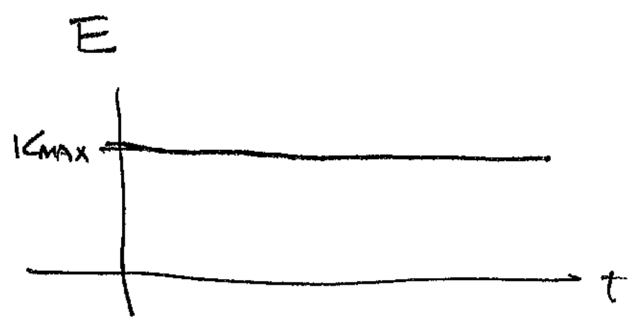
$\therefore$ at ~~t=0, t=5s~~ $t = 2.5s$ $U_g = K_{MAX}$

$\therefore$ U_g Also PARABOLA

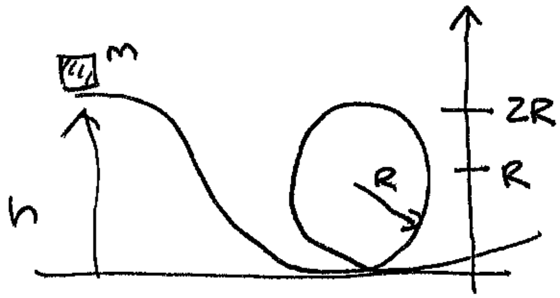


Part C:

As stated ABOVE $E = K + U_g$ IS Constant



Loop-THE-Loop



Starts From Rest

SLIDES Along Frictionless TRACK

Part A: FIND EXPRESSION FOR KINETIC ENERGY at top OF Loop

No FRICTION, SO GRAVITY ONLY FORCE DOING WORK

$$\Rightarrow \frac{1}{2}\cancel{MV_1^2} + Mgy_1 = \frac{1}{2}MV_2^2 + Mgy_2$$

$$V_1 = 0, \quad y_1 = h, \quad \frac{1}{2}MV_2^2 = K_2 = ?, \quad y_2 = 2R$$

$$\Rightarrow Mgh = K_2 + Mg2R \quad \Rightarrow K_2 = Mgh - Mg2R$$

$$\Rightarrow \boxed{K_2 = Mg(h - 2R)}$$

PART B: FIND MINIMUM h (IN TERMS OF R) to stay IN CONTACT.

AT TOP:

$$\sum F_y = M a_y$$

$$\Rightarrow mg + n = m a_{rad}$$

$$\Rightarrow mg + n = \frac{m v^2}{R}$$

SO AS V DECREASES SO DOES n. IF MASS BARELY MAKES IT, $n = 0 \Rightarrow mg = \frac{m V_{MIN}^2}{R}$

$$\Rightarrow V_{MIN}^2 = Rg$$

SO MINIMUM h OCCURS WHEN $V_2 = V_{MIN}$

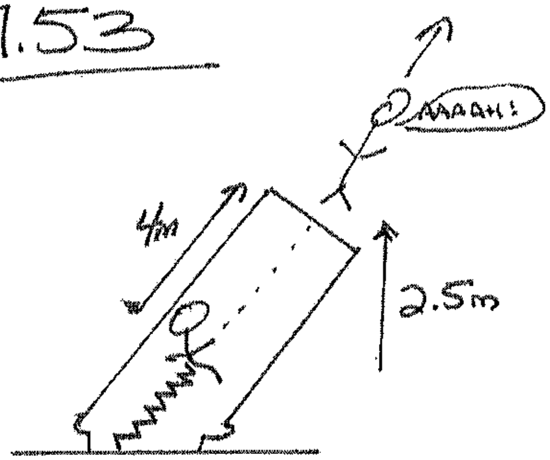
$$\frac{1}{2}\cancel{m V_1^2} + mgy_1 = \frac{1}{2} m v_2^2 + mgy_2$$

$$V_1 = 0,\ y_1 = h_{min} = ?,\ V_2 = V_{MIN},\ y_2 = 2R$$

$$\therefore \cancel{m} g h_{MIN} = \frac{1}{2}\cancel{m} V_{MIN}^2 + \cancel{m} g (2R) \Rightarrow g h_{MIN} = \frac{1}{2}(Rg) + g(2R)$$

$$\Rightarrow g h_{MIN} = g\left(\frac{1}{2}R + 2R\right) \Rightarrow \boxed{h_{MIN} = (2 + 1/2)R = 2.5R = 5/2 R}$$

7.53



$m = 60\,kg$

Spring: $k = 1100\,N/m$

Compressed with 4400N force

Friction: 40N during 4m

During launch, gravity, spring, and friction doing work

$$\Rightarrow \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}kS_1^2 + W_f = \frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}kS_2^2$$

$V_1 = 0$ (starts from rest), $V_2 = ?$

$y_1 = 0$, $y_2 = 2.5m$

$S_1 \rightarrow$ found from $F_s = kS \Rightarrow S_1 = \frac{F_s}{k} = \frac{4400N}{1100N/m} = 4m$

$S_2 = 0$ since no longer attached to spring

W_f = work done by friction, $f = 40N$ constant $\Rightarrow W_f = fS\cos\phi$.

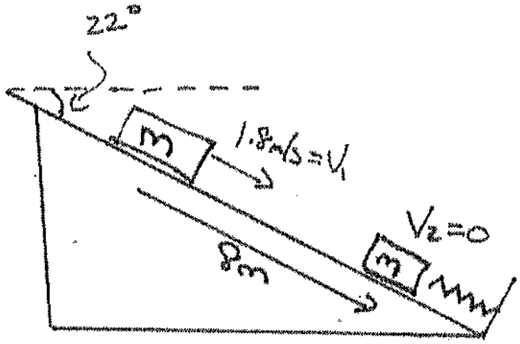
$\phi = 180°$ for friction $\Rightarrow W_f = -40N(4m) = -160J$

$\therefore \quad 0 + 0 + \frac{1}{2}(1100N/m)(4m)^2 - 160J = \frac{1}{2}(60kg)V_2^2 + (60kg)(9.8m/s^2)(2.5m) +$

$\Rightarrow 8800J - 160J = \frac{1}{2}(60kg)V_2^2 + 1470J$

$\Rightarrow V_2 = \sqrt{\frac{2(7170J)}{60kg}} = \sqrt{239m^2/s^2} = 15.459m/s = \underline{\underline{15.5m/s}}$

7.54



$mg = 1470N$

550N kinetic friction force

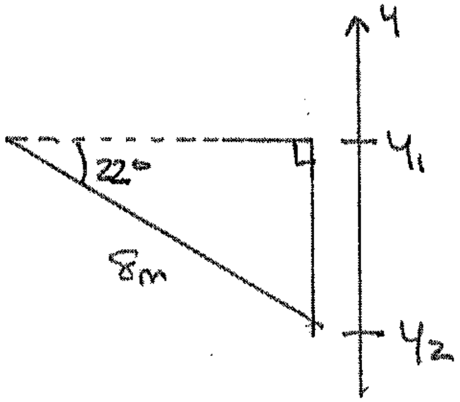
$f_{s,max} = 550N$

Crate does not rebound, what spring constant?

gravity, spring, and friction do work on mass

$$\Rightarrow \frac{1}{2}mV_1^2 + mgy_1 + \frac{1}{2}KS_1^2 + W_f = \frac{1}{2}mV_2^2 + mgy_2 + \frac{1}{2}KS_2^2$$

$V_1 = 1.8m/s$, $V_2 = 0$, $m = \frac{1470N}{9.8m/s^2} = 150kg$

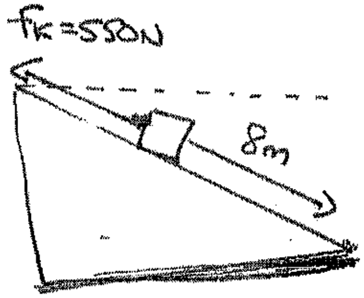


Let $y_2 = 0$ $\Rightarrow \sin 22° = \frac{y_1}{8m}$

$\Rightarrow y_1 = 8m \sin 22° = 2.997m$

$S_1 = 0$ (not touching spring initially, so it must be unstretch

$S_2 = ?$



Constant friction

$$\Rightarrow W_f = f_k s \cos\phi = f_k s \cos 180^\circ$$
$$= -f_k s = -550N(8m) = -4400J$$

$$\frac{1}{2}mv_1^2 + mgy_1 + \cancel{\frac{1}{2}ks_1^2} + W_f = \cancel{\frac{1}{2}mv_2^2} + \cancel{mgy_2} + \frac{1}{2}ks_2^2$$

$$\Rightarrow \frac{1}{2}(150kg)(1.8m/s)^2 + (1470N)(2.997m) - 4400J = \frac{1}{2}ks_2^2$$

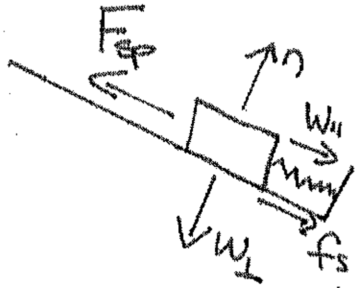
$$\Rightarrow 243J + 4405.6J - 4400J = \frac{1}{2}ks_2^2$$

$$\Rightarrow \frac{1}{2}ks_2^2 = 248.6J \Rightarrow ks_2^2 = 497.2J$$ → NEED ANOTHER EQUATION. Use Forces

To stay in place f_s must be at its max value $\Rightarrow f_s = 550N$

at resting, touching spring

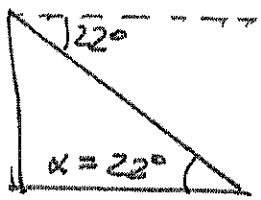
spring force is up since spring is compressed



static friction trying to prevent rebound.

$\sum F_{\parallel} = ma_{\parallel}$. No motion $\Rightarrow a_{\parallel} = 0$

$\therefore F_{sp} - W_{\parallel} - f_s = 0 \Rightarrow F_{sp} = W_{\parallel} + f_s$



$\Rightarrow W_{\parallel} = W \sin 22° = 1470 \sin 22° = 550.67N$

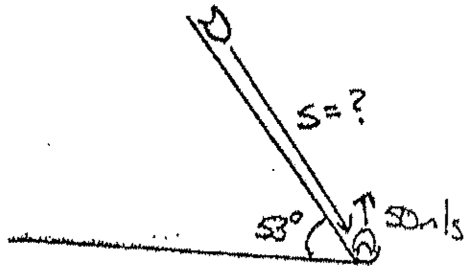
$F_{sp} = 550.67N + 550N = 1100.67N$

$F_{sp} = KS \Rightarrow 1100.67N = KS_2$. Previously found that $KS_2^2 = 497.2J \Rightarrow KS_2(S_2) = 497.2J$

$\Rightarrow (1100.67N)S_2 = 497.2J \Rightarrow S_2 = \frac{497.2J}{1100.67N} = .4517m$

$K = \frac{1100.67N}{S_2} = \frac{1100.67N}{.4517m} = 2436.597 N/m = \underline{\underline{2440 N/m}}$

7.56



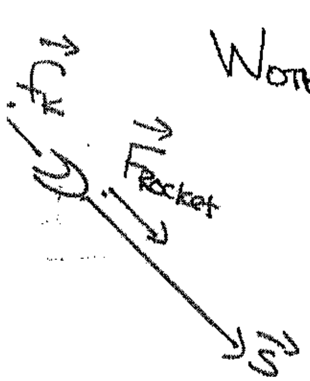
$M = 1500\,kg$

$F_{rocket} = 2000\,N$

$f_k = 500\,N$

Gravity, Rocket, and friction doing Work $\Rightarrow$

$$\frac{1}{2}MV_1^2 + Mgy_1 + W_{OTHER} = \frac{1}{2}MV_2^2 + Mgy_2.$$



$$W_{OTHER} = W_{rocket} + W_f.$$

F_{rocket}, f_k constant $\Rightarrow W_{rocket} = F_{rocket}\, s \cos 0°$

$W_f = f_k s \cos 180°$

$$\Rightarrow \frac{1}{2}MV_1^2 + Mgy_1 + F_{rocket}\, s - f_k s = \frac{1}{2}MV_2^2 + Mgy_2$$

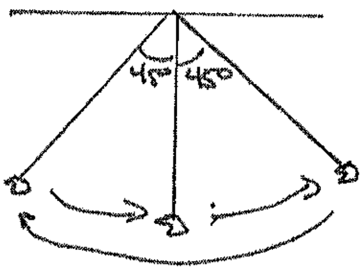
$V_1 = 0,\ y_1 = s \sin 53°,\ V_2 = 50\,m/s,\ y_2 = 0$

$$\Rightarrow 1500\,kg(9.8\,m/s^2)\, s \sin 53° + 2000\,N(s) - 500\,N(s) = \frac{1}{2}(1500\,kg)(50\,m/s)^2$$

$$\Rightarrow s(14700\,N \sin 53° + 2000\,N - 500\,N) = 1875000\,J$$

$$\Rightarrow \boxed{s = 141.6\,m = 142\,m}$$

7.82



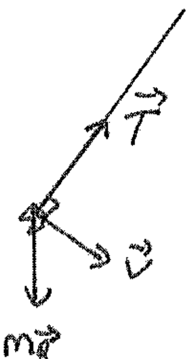
$M = .12\,Kg$

String is .8m long

Maximum Angle = 45°

a) Speed as passes through vertical ⇒ 0°

Forces on mass: tension and gravity



$\vec{T}$ along string ⇒ Along radius of circular path. For circle, $\vec{v}$ is 90° to radius ⇒ $\vec{T}$ at 90° to $\vec{v}$. $\vec{v}$ is always in same direction as displacement ⇒ $\vec{T}$ at 90° to displacement ⇒ ~~Does~~ $\vec{T}$ does no work.

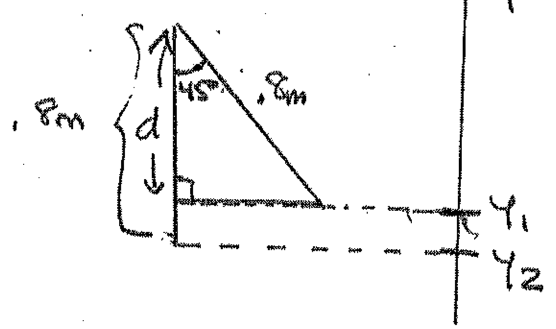
(Note, here is a place you would have to use a $d\vec{s}$ vector but $\vec{F} \cdot d\vec{s} = 0$ at every point)

⇒ We can use $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2$

AT MAXIMUM Angle 45°, MASS changes DIRECTION

$\Rightarrow V = 0$

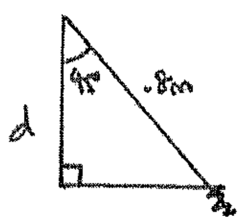
$\therefore V_1 = 0,\ y_1 = y \text{ at } 45°,\ V_2 = ?,\ y_2 = y \text{ at } 0°$



Set $y_2 = 0$ then

$y_1 = .8m - d = .8m - .8m\cos 45°$

$= .8m(1 - \cos 45°) = .2343\,m$



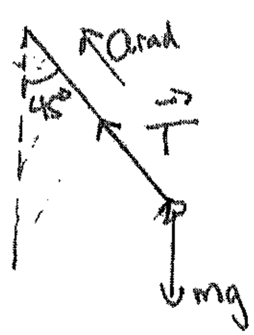
$\cos 45° = \frac{d}{.8m}$

$\Rightarrow d = .8m\cos 45°$

$\therefore\ 0 + m(9.8m/s^2)(.2343m) = \frac{1}{2} m V_2^2 + 0$

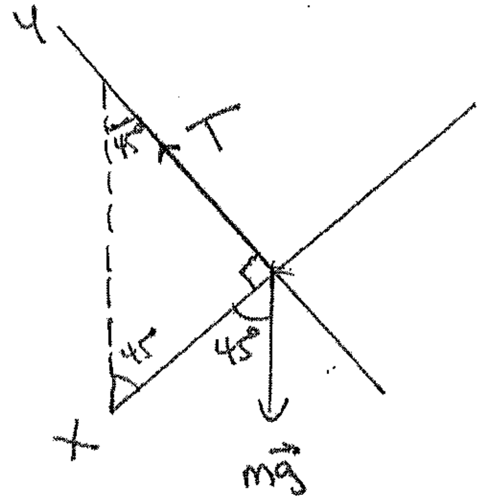
$\Rightarrow V_2 = \sqrt{2(9.8m/s^2)(.2343m)} = \underline{\underline{2.143m/s}} = \underline{2.14m/s}$

b) WHAT is tension at 45°?



$a_{rad} = \frac{V^2}{r} \Rightarrow$ at 45°, $a_{rad} = 0$

$\Rightarrow$ USE Coordinates Along radius



$\Rightarrow \vec{mg}$ at 45° to string

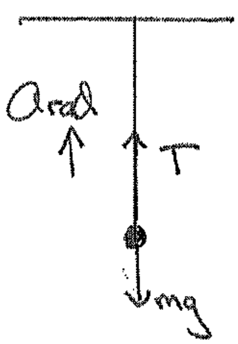
$$\sum F_y = T - mg\sin 45° = ma_y$$

$$a_y = a_{rad} = 0$$

$$\Rightarrow T = mg\sin 45° = (.12\,kg)(9.8\,m/s^2)\sin 45°$$

$$\Rightarrow \underline{T = .83156\,N = .832\,N}$$

c) Tension at vertical?



at Bottom: $a_{rad} = \dfrac{V^2}{r} = \dfrac{(2.143\,m/s)^2}{.8\,m} = 5.74\,m/s^2$

$a_y = a_{rad}$ since center is up

$$\sum F_y = ma_y \Rightarrow T - mg = ma_{rad}$$

$$\Rightarrow T = \underbrace{mg + ma_{rad}}_{m(g+a_{rad})} = .12\,kg\,(9.8\,m/s^2 + 5.74\,m/s^2)$$

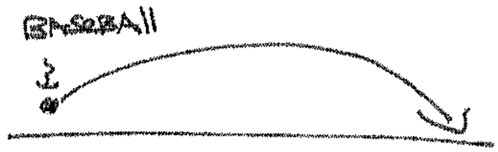
$$\Rightarrow \underline{T = 1.86\,N}$$

7.60

Data:

t	x	y	V_x	V_y
0	0	0	30 m/s	40 m/s
3.05 s	70.2 m	53.6 m	18.6 m/s	0
6.59 s	124.4 m	0	11.9 m/s	−28.7 m/s

$m = .145\,\text{Kg}$



AIR RESISTANCE.

a). HOW MUCH WORK DONE by air as baseball went from INITIAL to max height?

GRAVITY AND AIR RESISTANCE DOING WORK ⇒

$$\frac{1}{2}mV_1^2 + mgy_1 + W_{air} = \frac{1}{2}mV_2^2 + mgy_2$$

IN KINETIC ENERGY $V^2 = \text{SPEED}^2 = V_x^2 + V_y^2$

$$\Rightarrow V_1^2 = (30\,\text{m/s})^2 + (40\,\text{m/s})^2 = 2500\,\text{m}^2/\text{s}^2$$

$y_1 = 0$, $W_{air} = ?$, MAX height where $V_y = 0$

$$\Rightarrow V_2^2 = (18.6\,\text{m/s})^2, \quad y_2 = 53.6\,\text{m}$$

$$\Rightarrow \frac{1}{2}(.145\,kg)(2500\,m^2/s^2) + W_{air} = \frac{1}{2}(.145kg)(18.6m/s)^2 + (.145kg)(9.8m/s^2)(53.6m$$

$$\Rightarrow 181.25\,J + W_{air} = 25.0821\,J + 76.16565$$

$$\Rightarrow W_{air} = -80.0023\,J = \underline{\underline{-80J}}$$

b) $W_{air} = ?$ for motion FROM MAX height BACK DOWN

$$\Rightarrow \frac{1}{2}mV_2^2 + mgy_2 + W_{air} = \frac{1}{2}mV_3^2 + mgy_3$$

$V_2 = 18.6m/s$, $y_2 = 53.6m$, $W_{air} = ?$, $V_3^2 = (11.9m/s)^2 + (28.7m/s)^2 = 965.3m^2/$

$y_3 = 0$

$$\Rightarrow \frac{1}{2}(.145\,kg)(18.6m/s)^2 + (.145kg)(9.8m/s^2)(53.6m) + W_{air} = \frac{1}{2}(.145kg)(965.3m^2/$$

$$\Rightarrow 25.0821J + 76.1656J + W_{air} = 69.98425J$$

$$\Rightarrow W_{air} = -31.26345\,J = \underline{\underline{-31.3J}}$$

c) W_{air} is smaller in part b Because the speed is less on way BACK DOWN (Due to AIR Resistance on way up). AIR Resistance depends on speed. So LOWER speeds $\Rightarrow$ LESS AIR RESISTANCE $\Rightarrow$ less work.