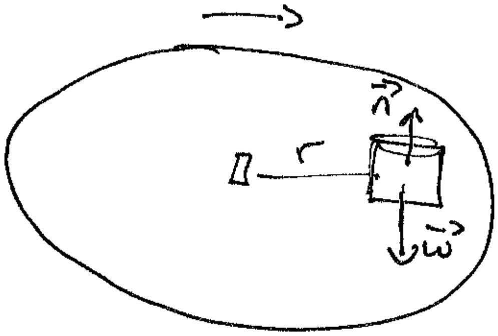
PHYSICS 160, HW#6

Mastering Physics: 9 Problems from chapters 5 & 6

Written: NONE

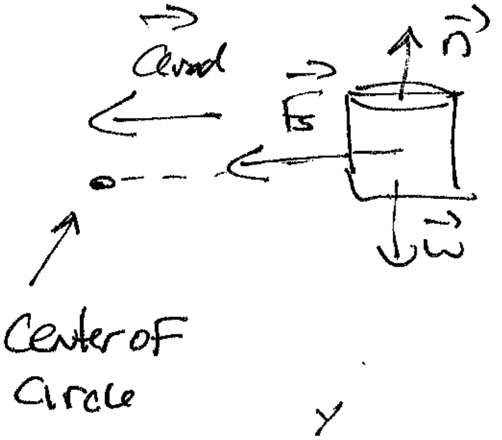
MASS ON A TURNTABLE

$M = .2Kg$, $\mu_s = .080$, $r = .15m$

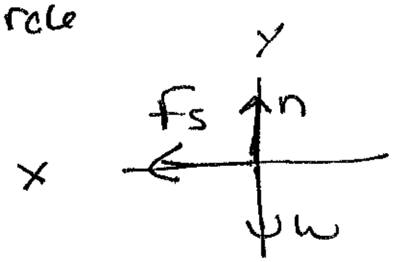


WHAT IS MAXIMUM speed of CYLINDER WITHOUT slipping?

FORCES ON MASS: NORMAL, WEIGHT, AND Static FRICTION



Static Friction Must point towards Center OR Circular motion isn't possible



$$\sum F_x = Ma_x, \quad \sum F_y = Ma_y$$

$$a_x = a_{rad}, \quad a_y = 0$$

$$\sum F_y = 0 \Rightarrow n - w = 0 \Rightarrow n = W = Mg$$

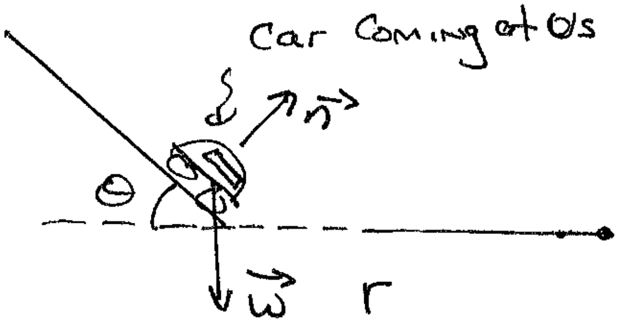
$$\sum F_x = Ma_x \Rightarrow f_s = Ma_{rad} = \frac{Mv^2}{r}$$

At MAXIMUM SPEED $f_s = f_{s,MAX} = \mu_s n$

$\Rightarrow \mu_s n = \frac{M v_{max}^2}{r}$ $\Rightarrow \mu_s \cancel{M} g = \frac{M v_{max}^2}{r}$

$\Rightarrow V_{max} = \sqrt{\mu_s r g} = \sqrt{.08(.15m)(9.8m/s^2)} = .343 m/s$

BANKED CURVE :



$M = 1500\,kg$

$V = \frac{50\,Km}{h} \times \frac{1000\,m}{Km} \times \frac{h}{3600\,s}$

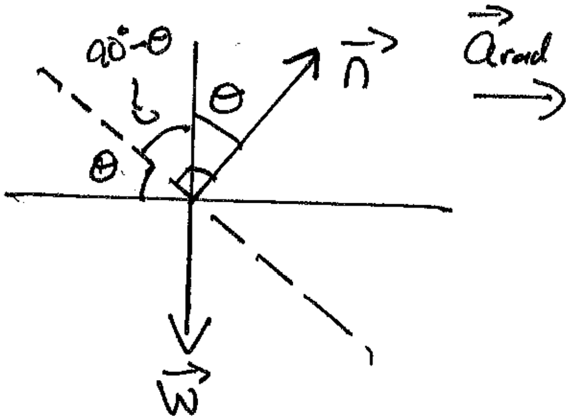
$= 13.88\bar{8}..\ m/s$

NO FRICTION

PART A: $\theta = 20°$ what is r?

FORCES ON CAR : $\vec{n}$ at 90° to SURFACE, AND $\vec{w}$ DOWN

to go AROUND CIRCLE, CAR MUST HAVE $\vec{a}_{rad}$ TOWARDS CENTER $\Rightarrow$ $\vec{a}_{rad}$ to RIGHT IN DRAWING.



NON-STANDARD ANGLE:

$\sin\theta = \frac{n_x}{n} \Rightarrow n_x = n\sin\theta$

$\cos\theta = \frac{n_y}{n} \Rightarrow n_y = n\cos\theta$

$$\sum F_x = Ma_x, \quad \sum F_y = Ma_y$$

$\vec{a}_{rad}$ to RIGHT $\Rightarrow a_x = a_{rad},\ a_y = 0$

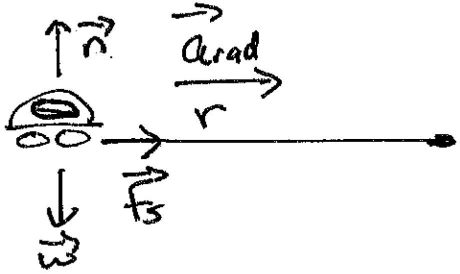
$$\therefore \sum F_y = 0, \quad \sum F_x = Ma_{rad} = \frac{Mv^2}{r}$$

$$\sum F_y = 0 \Rightarrow n\cos\theta - w = 0 \Rightarrow n = \frac{w}{\cos\theta} = \frac{Mg}{\cos\theta}$$

$$\sum F_x = \frac{Mv^2}{r} \Rightarrow n\sin\theta = \frac{Mv^2}{r} \Rightarrow \left(\frac{Mg}{\cos\theta}\right)\sin\theta = \frac{Mv^2}{r}$$

$$\Rightarrow g\tan\theta = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{g\tan\theta} = \frac{(13.888\,\text{m/s})^2}{9.8\,\text{m/s}^2\tan 20^\circ}$$ = ~~[illegible]~~ 54.1m

PART B :

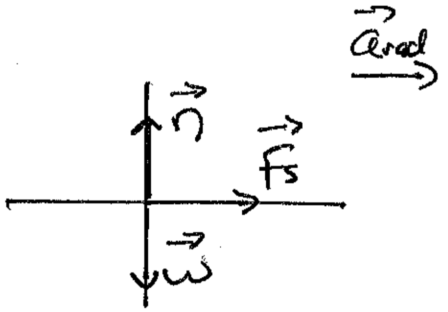


WHAT IS MINIMUM Coefficient
For Car to have V =13.888m/s
AND r=54.1m

Flat SURFACE SO $\vec{n}$ up, $\vec{w}$ DOWN,

Static FRiction, $\vec{f_s}$

$\vec{a}_{rad}$ still to RIGHT $\Rightarrow$ $\vec{f_s}$ must be to RIGHT Also OTHERWISE NO FORCE would be creating a_{rad}.



$$\sum F_x = Ma_x, \quad \sum F_y = Ma_y$$

$$a_x = a_{rad} = \frac{v^2}{r}, \quad a_y = 0$$

$$\sum F_y = 0 \Rightarrow n - w = 0 \Rightarrow n = w = Mg$$

$$\sum F_x = Ma_x \Rightarrow f_s = \frac{Mv^2}{r}.$$

MINIMUM COEFFICIENT $\Rightarrow f_s = f_{s,MAX} = \mu_s n$

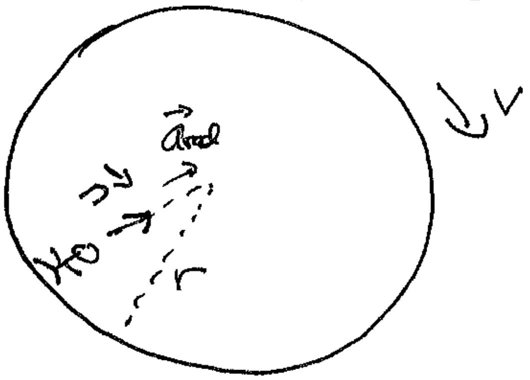
$$\therefore \mu_s n = \frac{Mv^2}{r} \Rightarrow \mu_s Mg = \frac{Mv^2}{r} \Rightarrow \mu_s = \frac{v^2}{rg} = \frac{(13.88\overline{8}\,m/s)^2}{(54.1m)(9.8m/s^2)}$$

$$\Rightarrow \mu_s = .364$$

5.49

"space-station"

with Diameter 800m ⇒ $r = 400m$



Only Force is Normal force $\vec{n}$

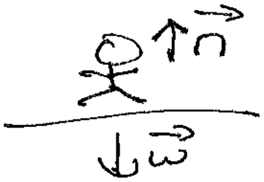
$\vec{n}$ towards Center

$$\sum \vec{F} = M\vec{a} \Rightarrow \sum F = Ma$$

so $n = Ma_{rad} \Rightarrow n = \frac{Mv^2}{r}$

Like Always n = Apparent weight.

On Earth (or Mars)



$\sum F_y = Ma_y$. Normally $a_y = 0$

$\Rightarrow n - w = 0 \Rightarrow n = w = Mg$

So to make spacestation feel Normal $n = Mg$

So $Mg = \frac{Mv^2}{r} \Rightarrow \frac{v^2}{r} = g$ (As you might have guessed $a_{rad} = g$)

$\Rightarrow V = \sqrt{rg}$, FOR EARTH gravity

$V = \sqrt{400m(9.8m/s^2)} = 62.6m/s$ ← speed of outer edge

TO FIND F = REV per Minute.

Remember 1 rev = 1 CIRCUMFERENCE = $2\pi r$ meters

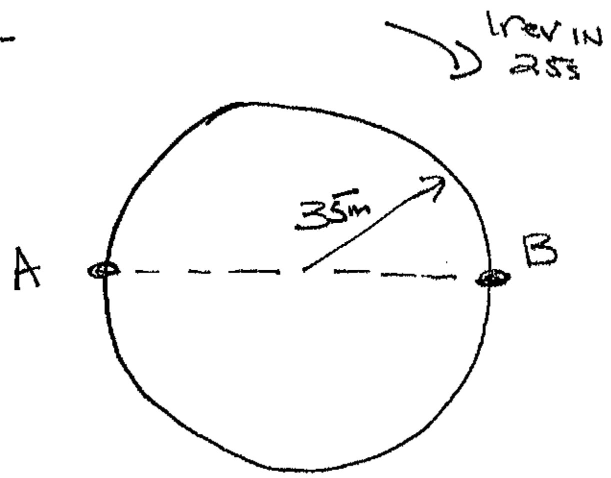
$f = \frac{62.6m}{s} \times \frac{1 rev}{2\pi(400)m} \times \frac{60s}{MIN} = 1.4947 rev/MIN \approx 1.5 rev/MIN$

FOR MARS gravity, $g = 3.7m/s^2$

$V = \sqrt{400m(3.7m/s^2)} = 38.47m/s$

$38.47m/s \times \frac{1rev}{2\pi(400)m} \times \frac{60s}{MIN} = .918 rev/MIN$

5.116



Ferris wheel Rider

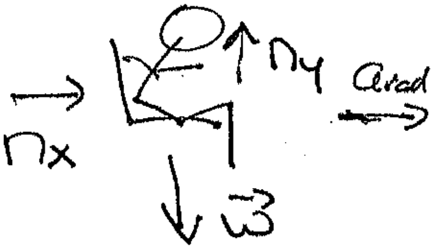
~~R~~ with M = 85kg

Find Magnitude and Direction of net Force exerted by seat on passenger at points A and B

At A: Center is to right ⇒ $a_x = +a_{rad} = \frac{v^2}{r}$, $a_y = 0$

We know there's gravity so seat has to push up on person to make $a_y = 0$ and push to right to make $a_x \neq 0$.

⇒



n_x, n_y are the components of $\vec{n}$

$$\sum F_x = M a_x \Rightarrow n_x = \frac{Mv^2}{r}$$

$$\sum F_y = M a_y \Rightarrow n_y - w = 0 \Rightarrow n_y = w = M$$

1rev = once around = 1 circumference = $2\pi r$

$$\Rightarrow V = \frac{2\pi r}{25s} = \frac{2\pi(35m)}{25s} = (2.8m/s)\pi$$

∴

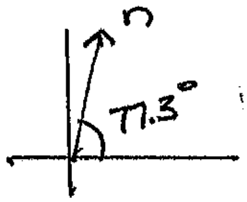
$$n_x = \frac{mv^2}{r} = \frac{(85kg)(2.8m/s)^2\pi^2}{35m} \cong (85kg)(2.21m/s^2)$$

$$= 187.92N$$

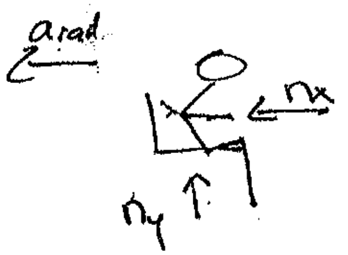
$$n_y = Mg = (85kg)(9.8m/s^2) = 833N$$

$$n = \sqrt{n_x^2+n_y^2} = \sqrt{(187.92N)^2+(833N)^2} = 8\underline{54}N$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}\left(\frac{833}{187.92}\right) \cong 77.3°$$



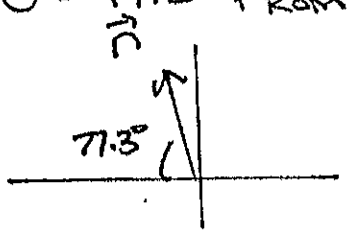
AT B, Center is to LEFT ⇒ n_x must be to Left



Here, IN REALITY n_x would be provided BY FRICTION, OR IN THE WORST CASE SCENARIO ~~THE~~ A LAP Belt OR BAR.

Same V AND r ⇒ $n_x = -187.92N$, $n_y = 833N$

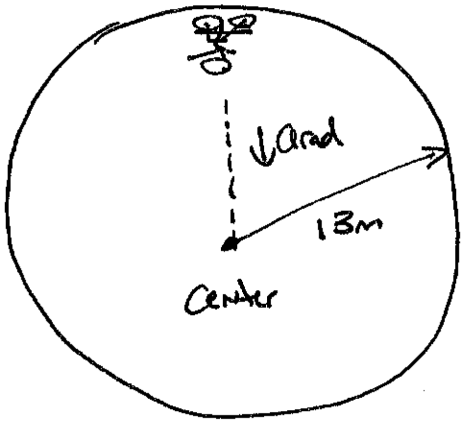
⇒ $n = 854N$ $\theta = 77.3°$ FROM −x AXIS



Standard Angle: 180° − 77.3° =

$\underline{102.7°}$

5.118

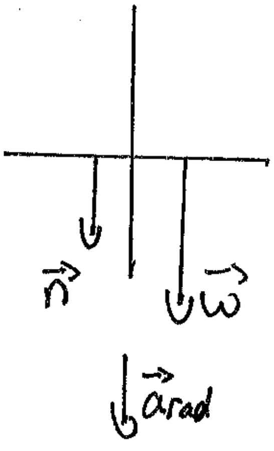


$M_{\text{physics major}} = 70\text{Kg}$

$M_{\text{motorcycle}} = 40\text{Kg}$

a) WHAT IS MINIMUM speed to make it over the top?

AT TOP, forces are $\vec{w}$ DOWN AND $\vec{n}$ DOWN.



$\sum F_y = Ma_y$

Make Down positive

$\Rightarrow n + w = Ma_{rad}$

$$\underline{\underline{\Rightarrow n + Mg = \frac{Mv^2}{r}}}$$

SURFACES CAN ONLY PUSH THEY CANNOT pull. Motorcycle is Below the sphere, So NORMAL Force is DOWN.

Mg is constant, so as V decreases so does NORMAL.

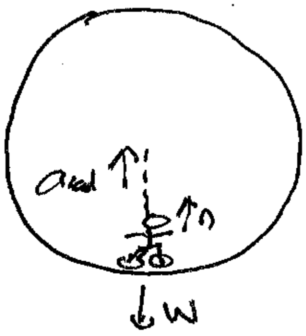
AT MINIMUM SPEED NORMAL BECOMES ZERO → Motorcycle loses contact with SPHERE.

$$n = 0 \Rightarrow \cancel{M}g = \frac{\cancel{M}v_{MIN}^2}{r}$$

$$\Rightarrow v_{MIN}^2 = rg \Rightarrow v_{MIN} = \sqrt{13m(9.8m/s^2)} = \underline{\underline{11.3m/s}}$$

b) At bottom, $V = 2V_{MIN} = 22.6m/s$

WHAT is NORMAL force on motorcycle?



$\sum F_y = Ma_y$. Make up positive

$\Rightarrow a_y = a_{rad}$

$\therefore n - w = Ma_{rad} \Rightarrow n = w + Ma_{rad}$

$$\Rightarrow n = Mg + Ma_{rad} = M(g + a_{rad}) = M\left(g + \frac{v^2}{r}\right)$$

Bottom must hold up both

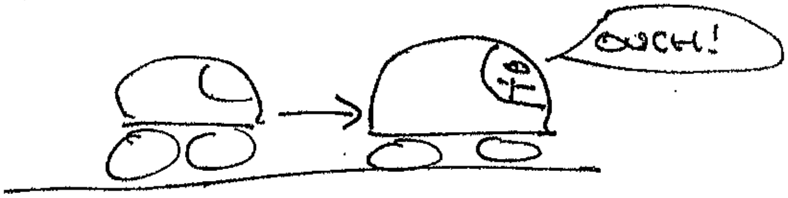
Motorcycle AND MAJOR $\Rightarrow M = M_{total} = 70kg + 40kg = 110kg$

$$\therefore n = 110kg\left(9.8m/s^2 + \frac{(22.6m/s)^2}{13m}\right) = 110kg(9.8m/s^2 + 39.3m/s^2)$$

$$= 110kg(49.1m/s^2) = 5399.8N$$

$$= 5\underline{400}N$$

6.69



Neck Bones can withstand 8J, 10ms Collision

$10ms = 10\times10^{-3}s = .01s$

$M = 5kg$ ← Head only

a) Greatest speed during Collision? If $W_{TOTAL} = 8J$ bones break. $W_{TOTAL} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$. $V_1 = 0$ since initially at rest $\Rightarrow \frac{1}{2}mV_{max}^2 = W_{TOTAL}$

$$\Rightarrow \frac{1}{2}(5kg)V_{MAX}^2 = 8J \Rightarrow V_{max} = \sqrt{\frac{2(8J)}{5kg}} = \sqrt{3.2m^2/s^2} = \underline{\underline{1.79m/s}}$$

$$\text{Unit: } \frac{J}{kg} = \frac{kg\cdot m^2/s^2}{kg} = m^2/s^2$$

$$1.79m/s = 1.79m/s \times \frac{1mi/h}{.447m/s} = \underline{\underline{4mi/h}}$$

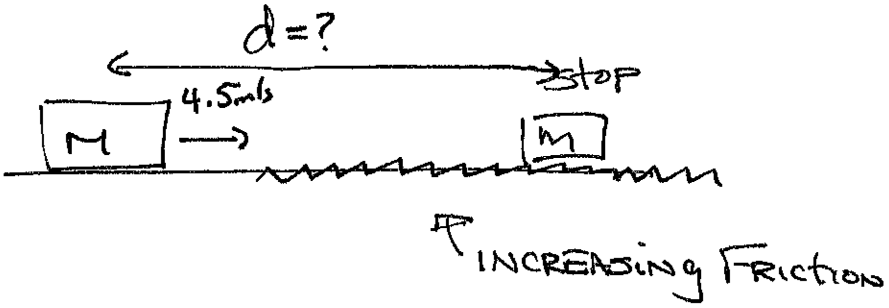
b. What is Acceleration? → Assumed Constant (of course)

$$V = \overset{0}{\cancel{V_0}} + at \Rightarrow a = \frac{V}{t} = \frac{1.79m/s}{.01s} = \underline{\underline{179m/s^2}} \quad \frac{179m/s^2}{9.8m/s^2} = \underline{\underline{18.3g's}}$$

How large is Force ?

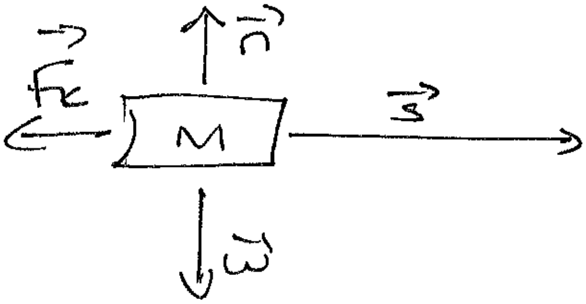
$$F = Ma = (5\,\mathrm{kg})(179\,\mathrm{m/s^2}) = 8\underline{95\,\mathrm{N}}$$

6.73



μ_K INCreases From .1 to .6 over A distance of 12.5m

Forces ON BOX: $\vec{n}$ up, $\vec{\omega}$ DOWN, $\vec{f}_K$ to left



For displacement to right $\vec{n}, \vec{\omega}$ DO NO WORK

$\Rightarrow \vec{f}_K$ only Force Doing work

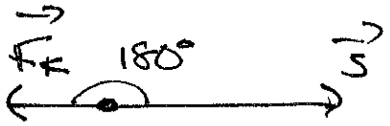
$\Rightarrow$ Work done by Friction, $W_F = W_{total}$

WORK-ENERGY $\Rightarrow$ $W_F = \Delta K = \frac{1}{2}MV_2^2 - \frac{1}{2}MV_1^2$

$V_2 = 0$, $V_1 = 4.5m/s$ $\therefore$ $W_F = -\frac{1}{2}MV_1^2$

FRICTION IS A VARIABLE FORCE $\Rightarrow$ AREA UNDER CURVE (OR INTEGRATION IF you PREFER): $W_F = \int_0^d f_K \cos\phi \, dx$

d = distance traveled = ?

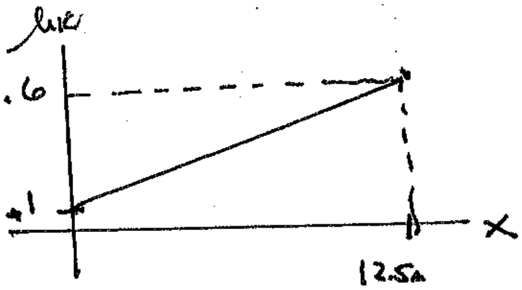
For Friction 

$\phi = 180^\circ$ (Always)

$\cos 180^\circ = -1 \Rightarrow W_F = -\int_0^d f_K \, dx$

$f_K = \mu_K n$. $\sum F_y = Ma_y$. $a_y = 0$ since no motion in y-direction $\Rightarrow n - w = 0$
$\Rightarrow n = w = Mg$

$\Rightarrow f_K = \mu_K Mg$. So need to find equation for μ_K to find f_K.



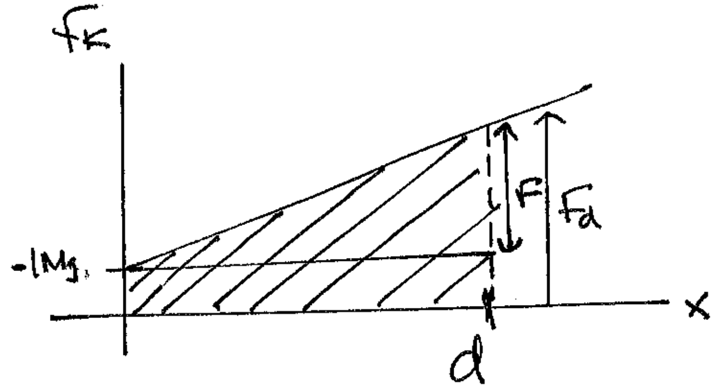
Straight line: $\mu_K = mx + b$

$m = \text{slope} = \frac{(.6 - .1)}{(12.5m - 0)} = \frac{.5}{12.5m} = .04/m$

b = y-intercept = .1

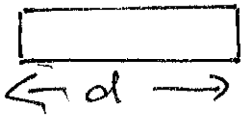
$\therefore \mu_K = (.04/m)x + .1$

$$f_K = \mu_K Mg = (.04/m)x\,Mg + .1Mg$$

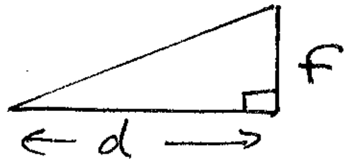


$-\text{Area} = W_F$

Area = rectangle + Triangle



$\Rightarrow A_r = (.1mg)d$



$A_t = \frac{1}{2}df$

at $x=d$,

$f_K = (.04/m)dMg + .1Mg$

$f = f_d - .1Mg = (.04/m)Mgd$

$\Rightarrow A_t = \frac{1}{2}d(.04/m)Mgd$

$= (.02/m)Mgd^2$

So $A = (.1mgd) + (.02/m)Mgd^2$

FOR CALCULUS LOVERS:

$$A = \int_0^d f_K\,dx = \int_0^d \left((.04/m)Mg\,x + .1Mg\right)dx$$

$$= \int_0^d (.04/m)Mg\,x\,dx + \int_0^d .1Mg\,dx$$

$$= (.04/m)Mg\int_0^d x\,dx + .1Mg\int_0^d dx$$

$$= (.04/m)Mg\left(\tfrac{1}{2}x^2\right)\Big|_0^d + .1Mg\,x\Big|_0^d$$

$$= (.02/m)Mg(d^2-0) + .1Mg(d-0)$$

$$\Rightarrow A = (.02/m)Mgd^2 + .1Mgd \longleftarrow \text{SAME RESULT (OF COURSE)}$$

$$W_F = -A = -\left[.1Mgd + (.02/m)Mgd^2\right] = -Mg\left[.1d + (.02)d^2\right]$$

BACK TO WORK-ENERGY: $W_F = -\frac{1}{2}MV_1^2$

$$\Rightarrow +Mg\left[.1d + (.02/m)d^2\right] = +\frac{1}{2}MV_1^2$$

$$\Rightarrow (.02/m)d^2 + .1d = \frac{V_1^2}{2g}$$

$$\Rightarrow (.02/m)d^2 + .1d - \frac{V_1^2}{2g} = 0$$

$$\frac{V_1^2}{2g} = \frac{(4.5m/s)^2}{2(9.8m/s^2)} = 1.033m$$

$$\therefore (.02/m)d^2 + .1d - 1.033m = 0 \leftarrow \text{QUADRATIC}$$

$$d = \frac{-.1 \pm \sqrt{(.1)^2 - 4(.02/m)(-1.033m)}}{2(.02/m)} = \frac{-.1 \pm \sqrt{.09264}}{.04/m}$$

$\Rightarrow d = 5.1092m$ OR ~~$-10.1m$~~

so $\boxed{d = 5.11m}$

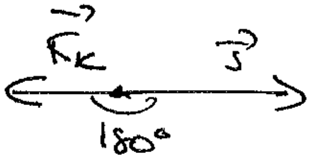
WHAT IS μ_K at d? $\mu_K = (.04/m)(5.1092m) + .1 = \underline{\underline{.304}}$

How far would box go if $\mu_K = .1$ (constant)?

IF $\mu_K = .1$, $f_K = .1Mg \Rightarrow$ Constant force

So $W_F = \vec{f_K} \cdot \vec{s} = f_K s \cos 180°$

↑ Always for friction



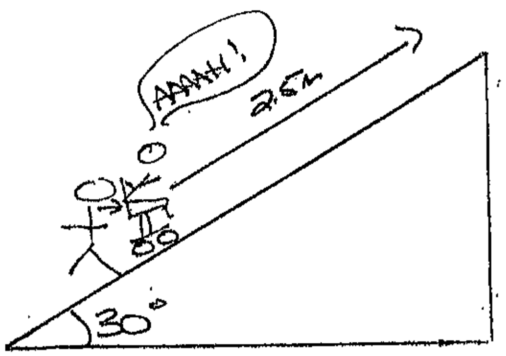
Let $s = d = ?$ $\Rightarrow W_F = -f_K d = -.1Mgd$

$W_F = W_{total} \Rightarrow -.1Mgd = \frac{1}{2}M\overset{0}{v_2^2} - \frac{1}{2}Mv_1^2$

$\Rightarrow +.1Mgd = +\frac{1}{2}Mv_1^2 \Rightarrow d = \frac{v_1^2}{2(.1)g} = \frac{(14.5\,m/s)^2}{0.2(9.8\,m/s^2)}$ ~~≈ [illegible]~~

$$\boxed{d = 10.3m}$$

6.84



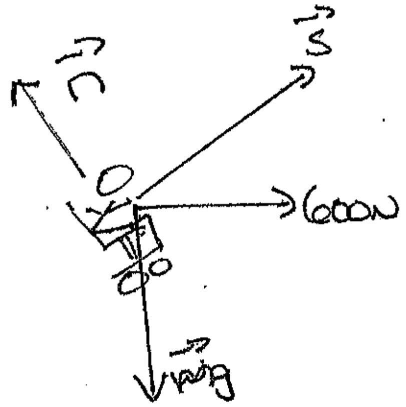
M = 85 Kg

HORIZONTAL, 600N FORCE

$V_1 = 2 m/s$

FIND speed at top.

FORCES ON Chair/professor: NORMAL, Weight, 600N



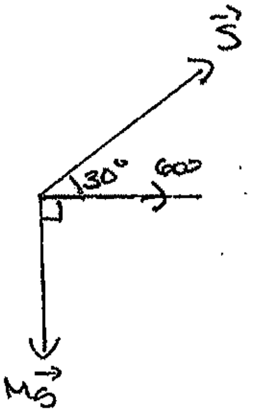
$$W_{TOTAL} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

$$W_{TOTAL} = W_n + W_g + W_{600}$$

AS ALWAYS, NORMAL DOES NO WORK

$$\Rightarrow W_g + W_{600} = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$$

gravity AND 600N both Constant $\Rightarrow W_g = M\vec{g}\cdot\vec{s}$, $W_{600} = \vec{F}_{600}\cdot\vec{s}$



$$\Rightarrow W_g = Mgs\cos 120° = (85kg)(9.8m/s^2)(2.5m)\cos 120°$$

$$= -1041.25J$$

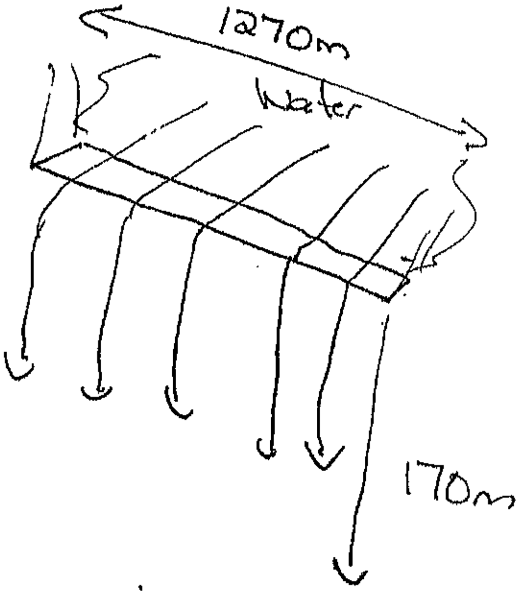
$$W_{600} = (600N)(2.5m)\cos 30° = 1299 J$$

$$\therefore\ -1041.25\,\text{J} + 1299\,\text{J} = \tfrac{1}{2}(85\,\text{kg})V_2^2 - \tfrac{1}{2}(85\,\text{kg})(2\,\text{m/s})^2$$

$$\Rightarrow 257.75\,\text{J} = \tfrac{1}{2}(85\,\text{kg})V_2^2 - 170\,\text{J}$$

$$\Rightarrow V_2 = \sqrt{\frac{2(427.75\,\text{J})}{85\,\text{kg}}} = \underline{\underline{3.17\,\text{m/s}}}$$

6.94



GRAND COULEE DAM
generates 2000MW
↑
Mega-Watt

92% OF Work Done by Gravity Converted to Electri
Energy.

1m³ of Water has 1000kg of MA

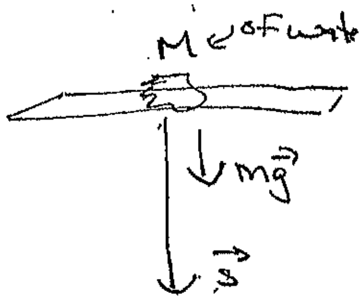
How MANY Cubic Meters of water per second?

$2000\text{MWatt} = 2000\times10^6\text{ Watt} = 2\times10^9\text{ Watt} = 2\times10^9\text{ J/s}$

$\therefore\ .92\,W_g = 2\times10^9\text{ J}$ EVERY SECOND. $\Rightarrow W_g = \frac{2\times10^9\text{ J}}{.92} = 2.173$

↓
Work done by gravity

So Gravity Needs to do $2.1739\times10^9\text{ J}$ of Work every second.



Gravity is Constant Force $\Rightarrow W_g = m\vec{g}\cdot\vec{s} = mgs$

$s = 170\text{m} \Rightarrow 2.1739\times10^9\text{J} = m(9.8\text{m/s}^2)(170\text{m})$

$\Rightarrow m = 1.3049\times10^6\text{ Kg}$ of water every second

1m^3 of water $= 1000\text{kg}$ $\Rightarrow$ Volume $= 1.3049\times10^6\text{kg}\times\frac{1\text{m}^3}{1000\text{kg}} = 1364.9\text{ m}^3$

$= \underline{\underline{1305\text{m}^3}}$