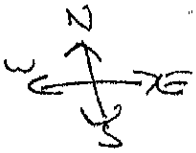
PHYS 160, HW #4

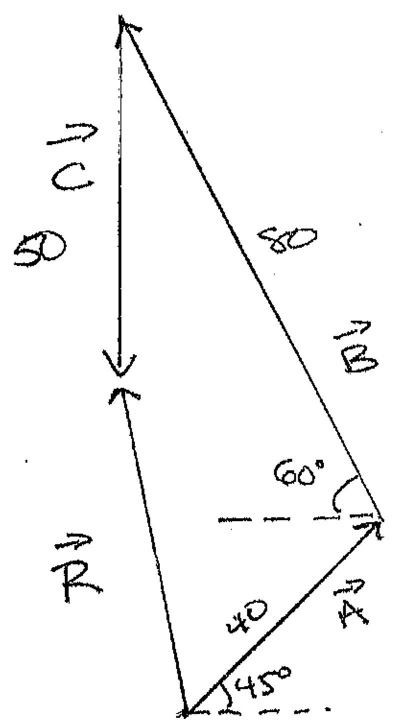
MASTERING: 9 PROBLEMS FROM Chapters 1 & 3

Written: 3.56

1.76 40 steps at NE, 80 steps at 60° N of W, 50 steps Due South.

USE traditional 

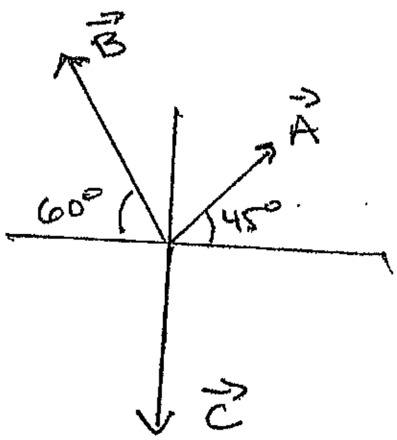
NE at 45°



$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

POINTS FROM HUT TO HIS FINAL LOCATION

$\Rightarrow -\vec{R}$ will BRING HIM BACK



$A_x = A\cos 45° = 40\cos 45° = 28.28$

$A_y = A\sin 45° = 40\sin 45° = 28.28$

USE STANDARD ANGLE $\Rightarrow \vec{B}$ at $180° - 60° = 120°$

$\Rightarrow B_x = B\cos 120° = 80\cos 120° = -40$

$B_y = B\sin 120° = 80\sin 120° = 69.282$

$\vec{C}$ STRAIGHT DOWN $\Rightarrow C_x = 0 \quad C_y = -C = -50$

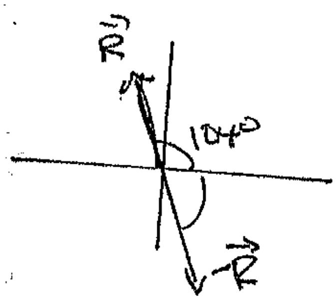
$R_x = A_x + B_x + C_x = 28.28 - 40 + 0 = -11.72$

$R_y = A_y + B_y + C_y = 28.28 + 69.282 - 50 = 47.562$

$\Rightarrow R = \sqrt{R_x^2 + R_y^2} = 48.98 \approx 49$ steps

~~Now refer.~~ $R_x < 0,\ R_y > 0 \Rightarrow 2^{ND}$ QUADRANT

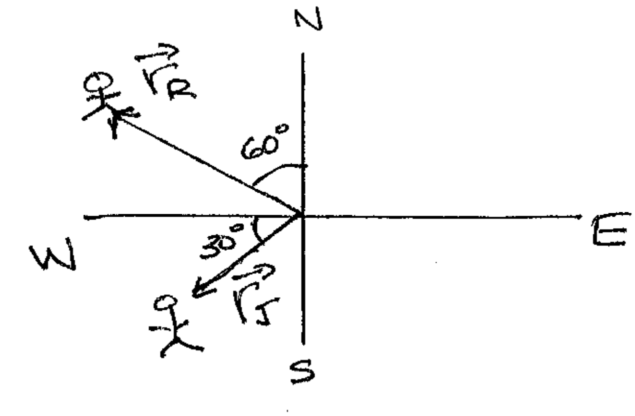
$\Rightarrow \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) + 180° = \tan^{-1}\left(\frac{47.562}{-11.72}\right) + 180° = 103.8 = 104°$



$\Rightarrow$ TO RETURN $180° - 104° = 76°$

$\Rightarrow 49$ steps, 76° SOUTH OF EAST

Problem 1.84



Ricardo: 26m, 60° W of North

$\Rightarrow$ 60° From North towards West

Jane: 16m, 30° South of West

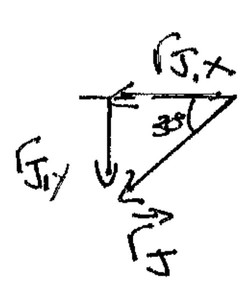
$\Rightarrow$ 30° From West toward South

a) What Distance? Find Displacement Vector From Ricardo to Jane $\Rightarrow$ $\vec{r}_R = \vec{r}_1$ = Initial.

$\vec{r}_J = \vec{r}_2$ = Final

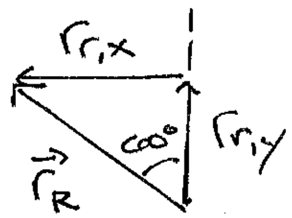
$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{r}_J - \vec{r}_R$$

Practice with non-standard Angles:



$$r_{J,x} = -r_J \cos 30° = -16\text{m} \cos 30° \doteq 13.8564\text{m}$$

$$r_{J,y} = -r_J \sin 30° = -16\text{m} \sin 30° = -8\text{m}$$



$$r_{r,x} = -r_r \sin 60° = -26\,m \sin 60° = -22.51666\,m$$

$$r_{r,y} = +r_r \cos 60° = +26\,m \cos 60° = 13\,m$$

$$\Delta\vec{r} = \vec{r}_J - \vec{r}_r \Rightarrow \Delta r_x = \Delta X = r_{J,x} - r_{r,x}$$

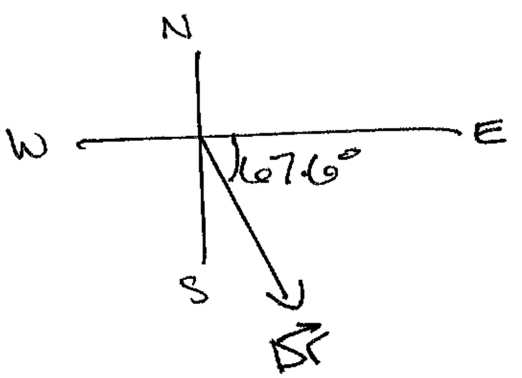
$$\Rightarrow \Delta X = -13.8564\,m - (-22.51666\,m) = 8.66026\,m$$

$$\Delta r_y = \Delta y = r_{J,y} - r_{r,y} = -8\,m - (13\,m) = -21\,m$$

$$\Delta r = \sqrt{\Delta X^2 + \Delta y^2} = \sqrt{(8.66026\,m)^2 + (21\,m)^2} = \sqrt{516}\,m = \underline{\underline{22.7\,m}}$$

b) WHAT Angle? $\Delta X > 0,\ \Delta y < 0 \Rightarrow$ 4th QUADRANT

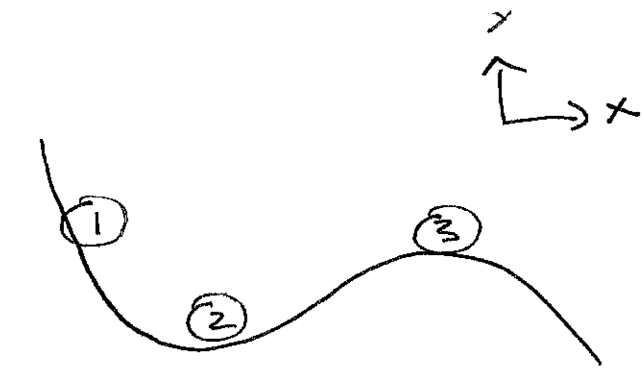
$\Rightarrow$ Calculator OK, $\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta X}\right) = \tan^{-1}\left(\frac{-21}{8.66026}\right) = \underline{\underline{-67.6°}}$



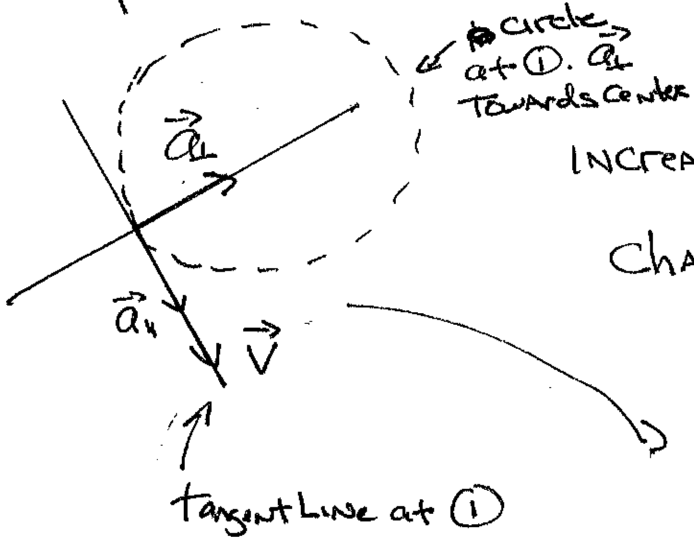
WE Could Also SAY: $67.6°$ S OF E

OR EVEN $90° - 67.6° = 22.4°$ E OF S

ACCELERATING ON A RAMP:

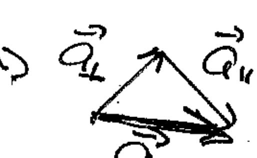


At ① BALL IS going DOWN HILL ⇒ INCREASING SPEED AND CHANGING DIRECTION



INCREASING speed ⇒ $\vec{a}_\parallel$ along $\vec{v}$

Changing DIRECTION ⇒ $\vec{a}_\perp$ 90° to $\vec{v}$



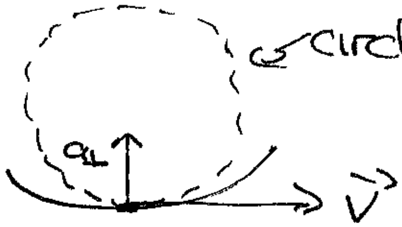
$$\vec{a} = \vec{a}_\perp + \vec{a}_\parallel$$

VECTOR ADDITION ⇒ $\vec{a}$ IS DOWN AND TO RIGHT

At ② AT BOTTOM OF HILL

THIS THE TURNING POINT Between going DOWN HILL ⇒ INCREASING SPEED AND UP the Hill ⇒ decreasing speed. ∴ NO CHANGE IN speed only direction.

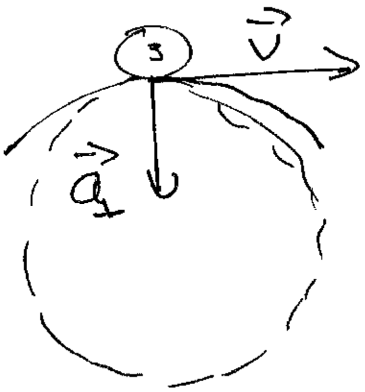
AT ②:



No $a_{\parallel}$ since no change in speed

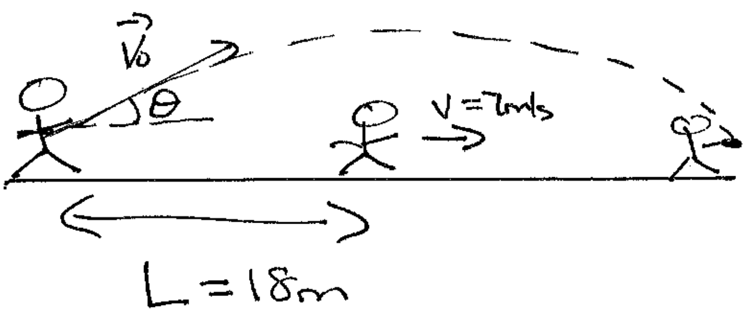
$\Rightarrow \vec{a} = \vec{a}_\perp \Rightarrow$ points upward.

At ③: Again a turning point (From up to down hill) $\Rightarrow a_{\parallel} = 0$



$\vec{a} = \vec{a}_\perp \Rightarrow$ points downward

SPEED OF A SOFTBALL



CATCHES 2s AFTER BEING HIT

a, b FIND LAUNCH SPEED V_0 AND ANGLE θ

~~CATCHER~~ BASEMAN RUNS WITH CONSTANT SPEED IN STRAIGHT LINE

$\Rightarrow X_{BASEMAN} = X_0 + V_0 t + \frac{1}{2} a t^2$

$X_0 = L = 18m, \; V_0 = 7m/s, \; a = 0$ (CONSTANT SPEED)

$\Rightarrow X_{BASEMAN} = 18m + 7m/s(2s) = 18m + 14m = 32m$

HE CATCHES BALL $\Rightarrow$ $X_0 = 0$, $X = 32m$ FOR BALL, AT SAME HEIGHT $\Rightarrow$ $y_0 = y_\bullet = 0$ WHEN CAUGHT,

SO FOR BALL: $X_0 = 0, \; y_0 = 0, \; X = 32m, \; y = 0, \; t = 2s$

$V_{0x} = ?, \; V_{0y} = ?,$

SO SOLVE FOR V_{ox}, V_{oy} AND USE $V_0 = \sqrt{V_{ox}^2 + V_{oy}^2}$, $\theta = \tan^{-1}\left(\frac{V_{oy}}{V_{ox}}\right)$

$$X = \cancel{X_0}^{0} + V_{ox}t \Rightarrow V_{ox} = \frac{X}{t} = \frac{32m}{2s} = 16m/s$$

$$\cancel{Y}^{0} = \cancel{Y_0}^{0} + V_{oy}t - \frac{1}{2}gt^2 \Rightarrow V_{oy} = \frac{\frac{1}{2}gt^2}{t} = \frac{1}{2}gt = \frac{1}{2}(9.8m/s^2)(2s)$$

$$= 9.8m/s$$

$$V_0 = \sqrt{(16m/s)^2 + (9.8m/s)^2} = 18.8m/s$$

$$\theta = \tan^{-1}\left(\frac{9.8}{16}\right) = 31.5°$$

c) FIND V_x, V_y .1s BEFORE CAUGHT. CAUGHT AT 2s

$$\Rightarrow t = 2s - .1s = 1.9s$$

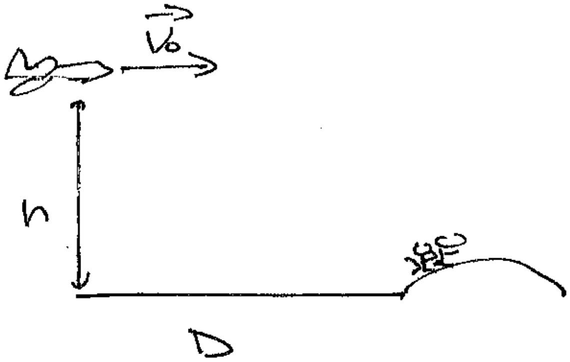
$$V_x = V_{0,x} = 16m/s, \quad V_y = V_{oy} - gt = 9.8m/s - (9.8m/s^2)(1.9s) = -8.82m/s$$

d) FIND x, y .1s BEFORE CAUGHT

$$X = \cancel{X_0}^{0} + V_{ox}t \Rightarrow X = 16m/s(1.9s) = 30.4m$$

$$Y = \cancel{Y_0} + V_{oy}t - \frac{1}{2}gt^2 = 0 + (9.8m/s)(1.9s) - 4.9m/s^2(1.9s)^2 = .931m$$

Delivering a Package by Air:



$V_0 = 200 mph$, $h = 1000m$

$\vec{V_0}$ Horizontal,

a) How long to reach ground?

$\vec{V_0}$ Horizontal $\Rightarrow V_{0x} = V_0,\ V_{0y} = 0 \Rightarrow y = y_0 - \frac{1}{2}gt^2$

$y = 0,\ y_0 = h = 1000m \Rightarrow 0 = h - \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

$$t = \sqrt{\frac{2(1000m)}{9.8m/s^2}} = 14.3s$$

b) $D = ?$ in meters

$X = X_0 + V_{0,x}t$; $X = D$, $X_0 = ?$, $V_{0,x} = V_0 = 200mph$, $t = 14.3s$

$\Rightarrow D = V_{0x}t$, but first have to convert mph to m/s

Using textbook: $1mph = .4470 m/s$ $\therefore$ $200mph \times \frac{.4470 m/s}{mph} = 89.4 m/s$

$$\therefore D = (89.4 m/s)(14.3s) = 1278.42m = 1280m$$

c) WHAT IS SPEED IN mph when package hits ground?

SPEED $\Rightarrow V = ?$ $\quad V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_{0x} = 200\text{mph} = 89.4\text{m/s}$

$V_y = V_{0y} - gt = 0 - 9.8\text{m/s}^2(14.3\text{s}) = -140.14\text{m/s}$

$\therefore \quad V = \sqrt{(89.4\text{m/s})^2 + (140.14\text{m/s})^2} = 166.227\text{m/s}$

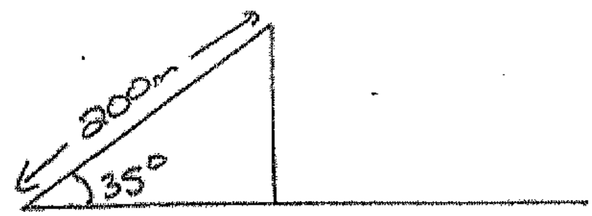
$166.227\text{m/s} \times \frac{\text{mph}}{.4470\text{m/s}} = 371.8735\ldots\text{mph} = 372\text{mph}$

d) WHAT WOULD make V SMALLER?

DECREASE HEIGHT BECAUSE t WOULD BE SMALLER AND SO $|V_y|$ WOULD BE SMALLER

DECREASE SPEED BECAUSE V_x WOULD BE SMALLER.

3.47



ROCKET ACCELERATED AT $1.25\,m/s^2$ for 200m. THEN MOVES UNDER force of gravity only.

ON INCLINE ROCKET MOVING IN A STRAIGHT LINE, SO WE CAN USE $V^2 = V_0^2 + 2a(r - r_0)$ TO FIND SPEED WITH WHICH ROCKET LEAVES INCLINE.

$V = ?,\ V_0 = 0,\ a = 1.25\,m/s^2,\ r = 200m,\ r_0 = 0$

$\Rightarrow V^2 = 0 + 2(1.25\,m/s^2)(200m) \Rightarrow V = \sqrt{500\,m^2/s^2} = 22.36\,m/s$

AFTER LEAVING INCLINE ROCKET BECOMES A PROJECTILE WHOSE INITIAL VELOCITY IS $\vec{V}_0 = 22.36\,m/s$ at $35°$. ITS INITIAL POSITION IS $X_0 = 200m\cos 35°,\ y_0 = 200m\sin 35°$

a) FIND $y = ?$ WHEN $V_y = 0$. $y = y_0 + V_{0y}t - \frac{1}{2}gt^2 \rightarrow$ NEED t.

$V_y = V_{0y} - gt \Rightarrow 0 = 22.36\,m/s \sin 35° - 9.8\,m/s^2\, t$

$\Rightarrow t = 1.309s \Rightarrow y = 200m\sin 35° + 22.36\,m/s \sin 35°(1.309s) - \frac{1}{2}g(1.3$

$\Rightarrow \boxed{y = 123.1m = 123m}$

b) $X = ?$ WHEN $y = 0$. $X = X_0 + V_{0x}t \rightarrow$ NEED t

$y = y_0 + V_{0y}t - \frac{1}{2}gt^2 \Rightarrow 0 = 200m\sin 35° + (22.36\,m/s \sin 35°)t - \frac{1}{2}(9.8\,m/s^2$

$\Rightarrow 0 = 114.7m + 12.825\,m/s\, t - 4.9\,m/s^2\, t^2$

$$t = \frac{-12.825\,m/s \pm \sqrt{(12.825\,m/s)^2 - 4(-4.9\,m/s^2)(114.7\,m)}}{2(-4.9\,m/s^2)}$$

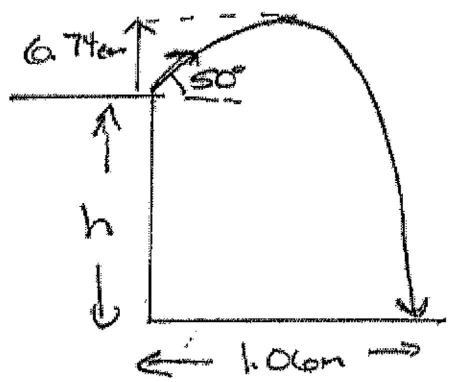
$$= \frac{-12.825\,m/s \pm \sqrt{2412.6\,m^2/s^2}}{-9.8\,m/s^2}$$

$$\Rightarrow t = -3.7s,\ \underline{\underline{6.32s}} \quad \text{use } t>0$$

$$\Rightarrow X = 200m\cos 35° + 22.36\,m/s\cos 35°\,(6.32s)$$

$$\Rightarrow \boxed{X = 279.6m = 280m}$$

3.63



FIND INITIAL SPEED AND height.

SET ORIGIN AT LAUNCH POINT $\Rightarrow X_0=0, y_0=0$

$6.74cm = .0674m \Rightarrow$ max height $\Rightarrow V_y = 0$ when $y = .067$

$V_y = V_{0,y} - gt \Rightarrow 0 = V_0 \sin 50° - 9.8m/s^2 t$

$y = y_0 + V_{0,y}t - \frac{1}{2}gt^2 \Rightarrow .0674m = 0 + V_0 \sin 50° t - \frac{1}{2}(9.8m/s^2)t^2$

$\Rightarrow .0674m = V_0 \sin 50° t - 4.9m/s^2 t^2$

1st EQN $\Rightarrow V_0 \sin 50° = 9.8m/s^2 t$

$\Rightarrow .0674m = (9.8m/s^2 \cdot t)t - 4.9m/s^2 t^2$

$\Rightarrow .0674m = (9.8m/s^2 - 4.9m/s^2)t^2 = 4.9m/s^2 t^2$

$\Rightarrow t = \sqrt{\frac{.0674m}{4.9m/s^2}} = .117s$

$\rightarrow$ UNIT: $m \times s^2/m = s^2$, $\sqrt{s^2} = s$

$$V_0 \sin 50° = 9.8\,m/s^2\, t$$

$$\Rightarrow V_0 = \frac{9.8\,m/s^2 (.117s)}{\sin 50°} \Rightarrow \underline{\underline{V_0 = 1.5\,m/s}}$$

FIND HEIGHT: $y = -h$ when $X = 1.06m$

$$y = y_0 + V_{0,y}t - \tfrac{1}{2}gt^2. \quad V_{0y} = V_0 \sin 50° = 1.5\,m/s \sin 50°$$
$$= 1.15\,m/s$$
$$V_{0,x} = V_0 \cos 50° = 1.5\,m/s \cos 50° = .964\,m/s$$

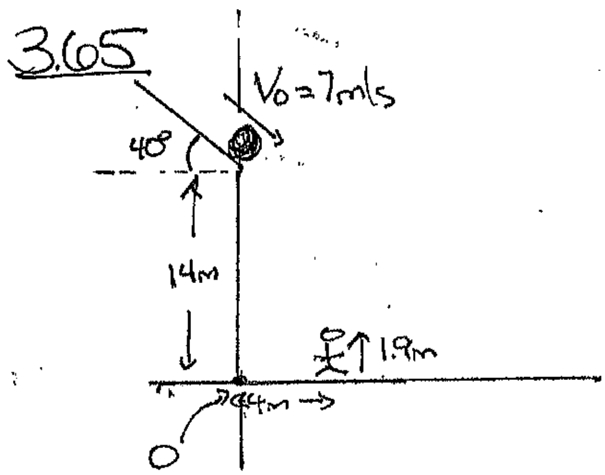
$-h = 0 + 1.15\,m/s\, t - \tfrac{1}{2}(9.8\,m/s^2)t^2$; FIND t FROM $X = X_0 + V_{0x}t$

$$\Rightarrow 1.06m = 0 + .964\,m/s\, t \Rightarrow t = \frac{1.06m}{.964\,m/s} = 1.0996s$$

$$\Rightarrow -h = 1.15\,m/s\,(1.0996s) - 4.9\,m/s^2 (1.0996s)^2$$

$$\Rightarrow -h = -4.66m \Rightarrow \underline{\underline{h = 4.66m}}$$

3.65



Put origin as shown

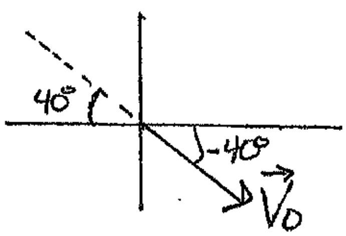
⇒ From picture

$x_0 = 0$, $y_0 = 14m$

⇒ up is positive ⇒

$a_x = 0$, $a_y = -g$

a) How far from barn does snowball land?



$V_{0,x} = 7m/s \cos(-40°) = 5.362 m/s$

$V_{0,y} = 7m/s \sin(-40°) = -4.4995 m/s$

How far ⇒ $x = ?$ when $y = 0$

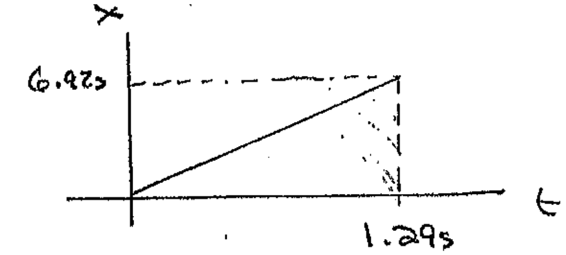
$x = x_0 + V_{0,x} t$ ← Need t

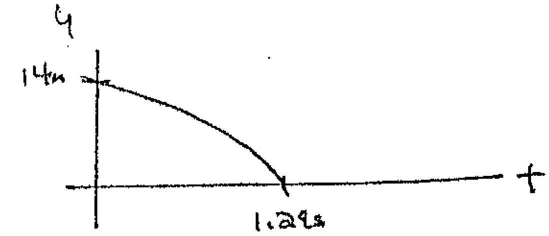
Use $y = y_0 + V_{0,y} t - \frac{1}{2} g t^2$ to find

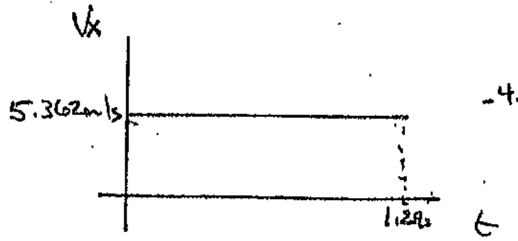
$\therefore\ 0 = 14m - 4.4995 m/s\ t - \frac{1}{2}(9.8 m/s^2) t^2$

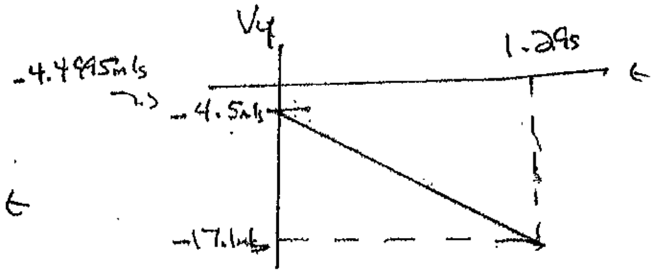
$\Rightarrow 0 = 14m - 4.4995 m/s\ t - 4.9 m/s^2\ t^2$

$\Rightarrow +4.9 m/s^2\ t^2 + 4.4995 m/s\ t - 14m = 0$









c) WILL MAN BE HIT? $\Rightarrow$ WHEN SNOWBALL'S $x = 4m$

IS ITS y BETWEEN 0 AND 1.9m

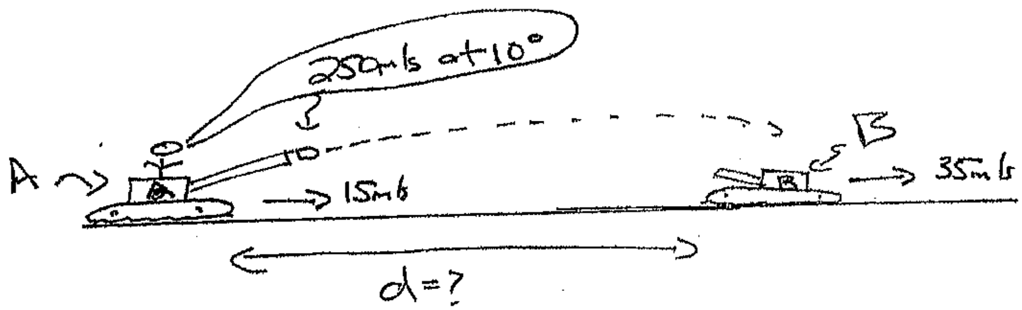
$y = y_0 + V_{0,y}t - \frac{1}{2}gt^2$ $\Leftarrow$ NEED t when $x = 4m$

$\Rightarrow$ $X = X_0 + V_{0,x}t$ to FIND $4m = 0 + (5.362m/s)t$

$\Rightarrow t = \frac{4m}{5.362m/s} = .746s \Rightarrow y = 14m - 4.4995m/s(.746s) - \frac{1}{2}(9.8m/s^2)(.746)$

$\Rightarrow y = 7.92m$ $\Leftarrow$ NO!

3.73



OTHER TANK HIT. HOW FAR APART INITIALLY AND WHEN HIT?

IF YOU ARE ON THE GROUND, THE SHELL IS ALREADY GOING WITH ~~THE~~ SAME VELOCITY AS THE TANK

$\Rightarrow V_{0,x} = 250\,m/s \cos 10° + 15\,m/s = 261.2\,m/s$

$V_{0,y} = 250\,m/s \sin 10° = 43.412\,m/s$

LET FIRST TANK BE A AND SECOND BE B.

$\therefore X_{0A} = 0, \quad X_{0B} = d = ?$

SHELL LAUNCHED FROM A $\Rightarrow X_{0S} = 0$.

SHELL HITS B $\Rightarrow X_S = X_B$.

SHELL PROJECTILE, B NOT ACCELERATING

$X_{0,S} + V_{0,xS}t = X_{0,B} + V_{0,xB}t \Rightarrow 0 + 261.2\,m/s\,t = d + 35\,m/s\,t$

$\Rightarrow 261.2\,m/s\,t - 35\,m/s\,t = d \Rightarrow 226.2\,m/s\,t = d$

TO FIND t USE FACT THAT SHELL AT SAME HEIGHT WHEN IT HITS B

$\Rightarrow y_s = y_{0,s} = 0$

PROJECTILE $\Rightarrow y = y_0 + V_{0,y}t - \frac{1}{2}gt^2$

$\Rightarrow 0 = 0 + 43.412\text{m/s}\ t - \frac{1}{2}(9.8\text{m/s}^2)t^2$

$\Rightarrow 0 = t[43.412\text{m/s} - 4.9\text{m/s}^2 t]$

$\Rightarrow t = 0$ OR $t = \frac{43.412\text{m/s}}{4.9\text{m/s}^2} = 8.86\text{s}$

$\therefore\ \underline{d = 226.2\text{m/s}(8.86\text{s})} = \underline{2004\text{m}} = 2000\text{m}$

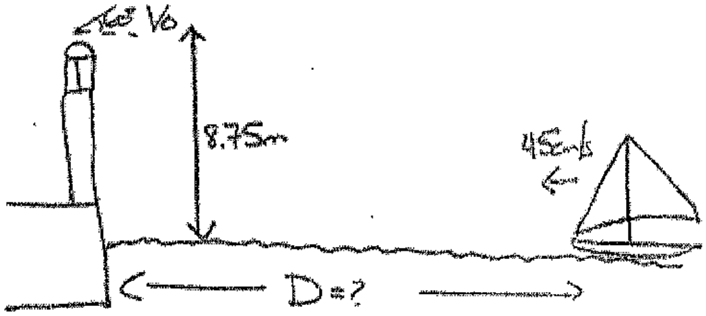
HOW FAR APART AT $t = 8.86$s?

A ALSO NOT ACCELERATING $\Rightarrow X_A = X_{0,A} + 15\text{m/s}\ t$

$\Rightarrow X_A = 0 + 15\text{m/s}(8.86\text{s}) = 132.9\text{m}$

$X_B = d + 35\text{m/s}\ t = 2004\text{m} + 35\text{m/s}(8.86\text{s}) = 2314.1\text{m}$

$\Rightarrow X_B - X_A = 2314.1\text{m} - 132.9\text{m} = \underline{2181.2\text{m}} = \underline{2180\text{m}}$



$\vec{V}_o = 15\,m/s$ at $60°$

PACKAGE THROWN TO SAILBOAT

D=? SUCH THAT $X_{package} = X_{sailboat}$

FOR PACKAGE: $X_o = 0$, $y_o = 8.75m$, $y = 0$, $V_{ox} = 15\,m/s \cos 60° = 7.5\,m/s$

$V_{oy} = 15\,m/s \sin 60° = 7.5(\sqrt{3})\,m/s$

$a_x = 0$, $a_y = -g$

For SAILBOAT: $X_o = D$, $y_o = y = 0$, $V_{ox} = -45\,cm/s = -.45\,m/s$, $V_{oy} = 0$

$a_x = a_y = 0$

$X = X_o + V_{ox}t + \frac{1}{2}a_x t^2$ & $X_{package} = X_{sailboat} \Rightarrow 0 + 7.5\,m/s\,t + 0 = D - .45\,m/s\,t + 0$

$\Rightarrow 7.5\,m/s\,t = D - .45\,m/s\,t \Rightarrow D = (7.95\,m/s)t \rightarrow$ NEED t.

USE $y = y_o + V_{oy}t - \frac{1}{2}gt^2$ OF PACKAGE TO FIND t

$0 = 8.75m + (7.5)\sqrt{3}\,m/s\,t - \frac{1}{2}(9.8\,m/s^2)t^2 \Rightarrow 0 = -4.9\,m/s^2\,t^2 + (7.5\sqrt{3})\,m/s\,t + 8.75m$

$\Rightarrow 4.9\,m/s^2\,t^2 - 7.5(\sqrt{3})\,m/s\,t - 8.75m = 0$

$$\Rightarrow t = \frac{7.5\sqrt{3}\,m/s \pm \sqrt{(7.5)^2 3\,m^2/s^2 - 4(4.9\,m/s^2)(-8.75m)}}{2(4.9\,m/s^2)} = \frac{7.5(\sqrt{3})\,m/s \pm \sqrt{340.25\,m^2/s^2}}{9.8\,m/s^2}$$

$\Rightarrow t = 3.21s$ OR $\cancel{-.5560s}$ $\Rightarrow D = (7.95\,m/s)(3.21s) \Rightarrow \boxed{D = 25.5m}$