

# May 2, Week 15

Today: Chapter 14, Periodic Motion

Homework #11 due Monday, May 7 at 11:59pm  
Mastering Physics: 5 problems from chapter 13  
Written: none.

Final Exam, Wednesday, May 9, 10:00-12:00 AM.

Review Sessions, Monday and Tuesday afternoon. Time and Place: TBD.

# Survey

Which of the following time slots works best for a review session?

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(a) 1:00-2:00

(b) 2:00-3:00

(c) 3:00-4:00

(d) 4:00-5:00

# Periodic Motion

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$$f = \frac{1}{T}$$

Unit:  $\frac{1}{s} = Hz$  (Hertz)

# Simple Harmonic Motion

Simple Harmonic Motion (SHM) - The simplest type of periodic motion. Occurs when a mass is connected to a spring with no friction.

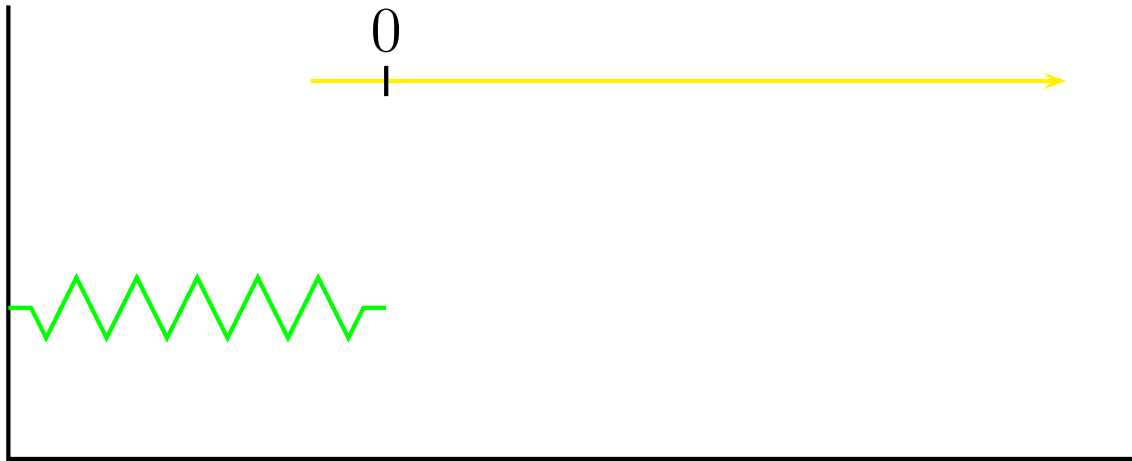
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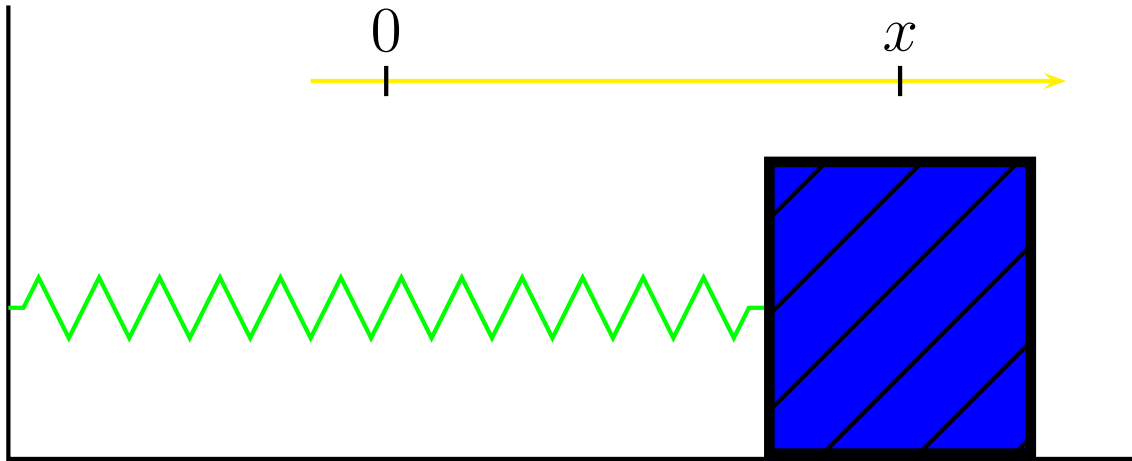
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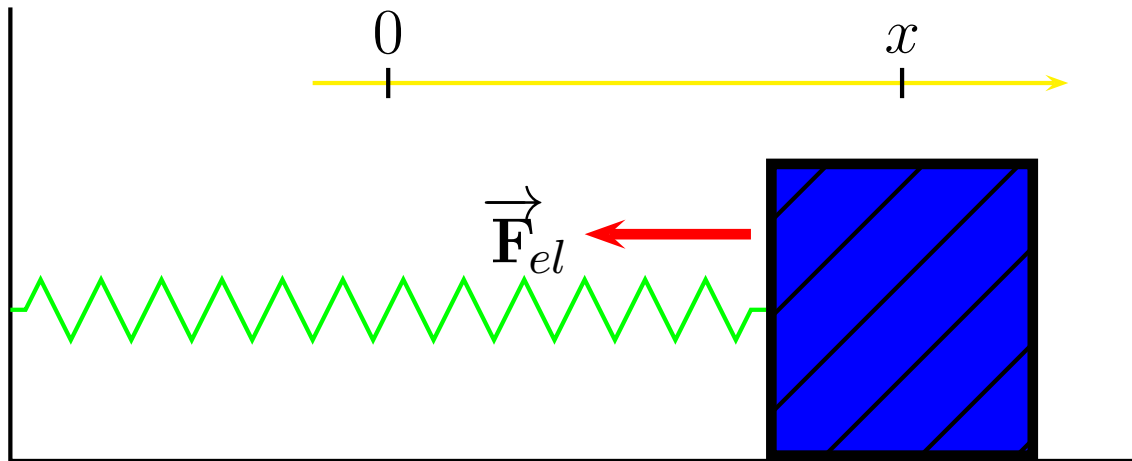
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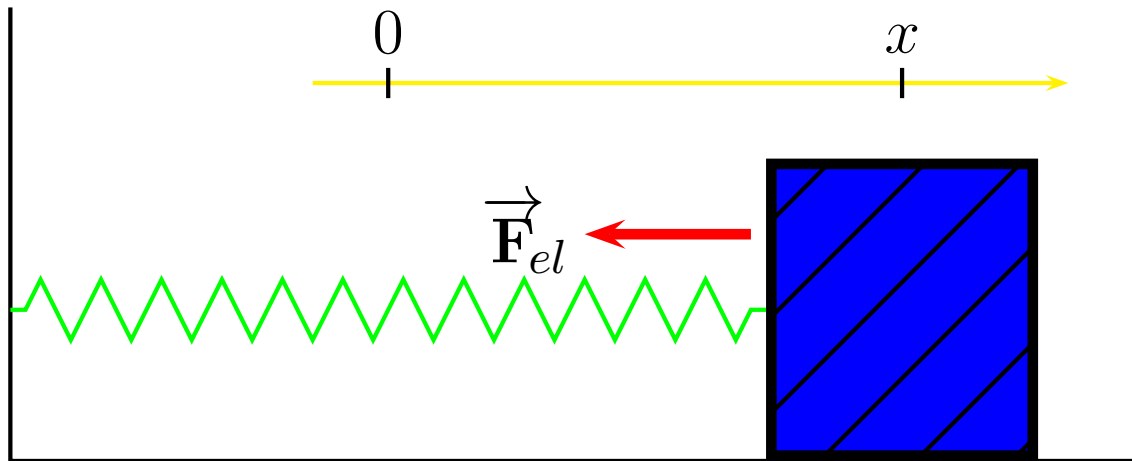




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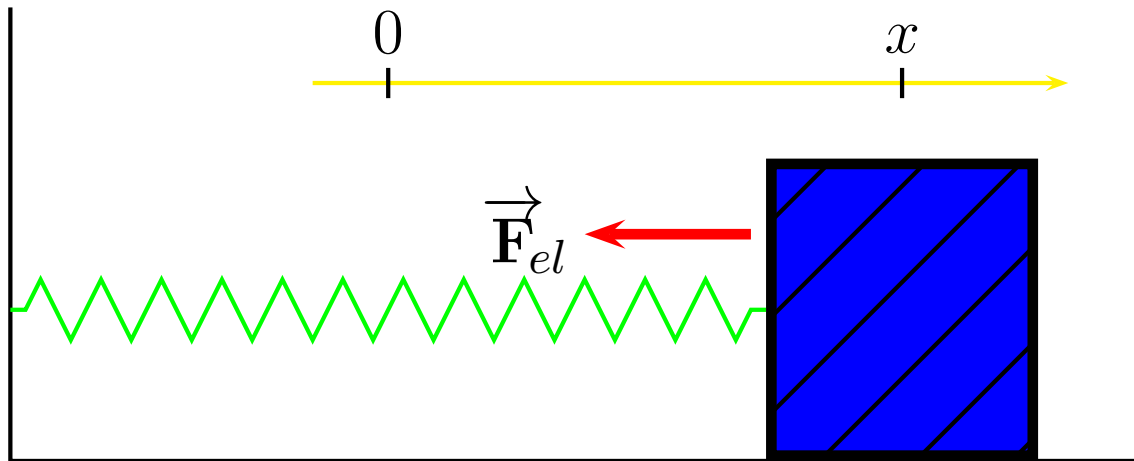
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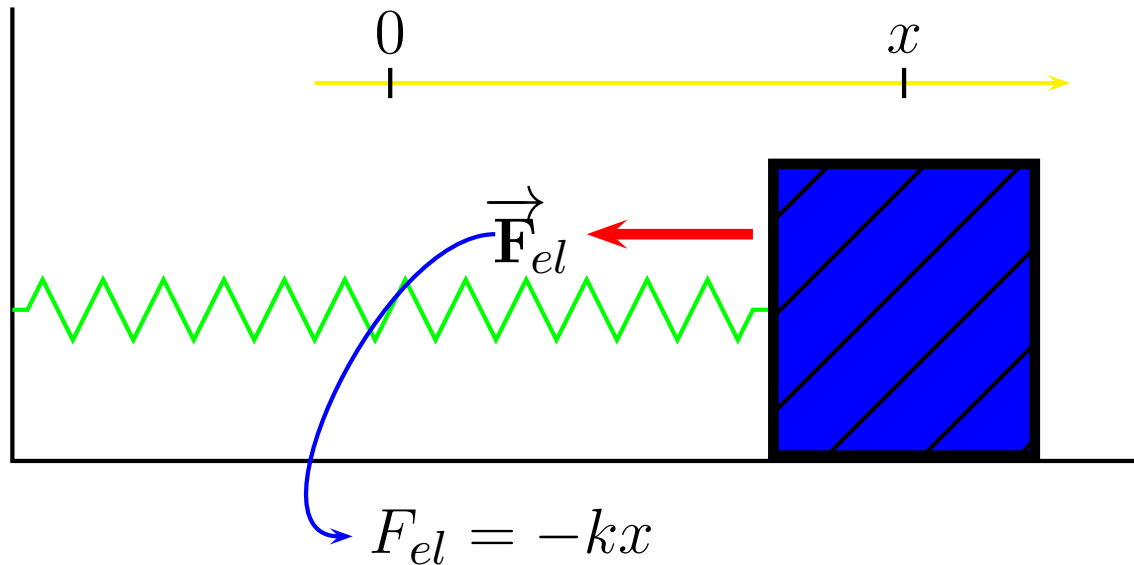


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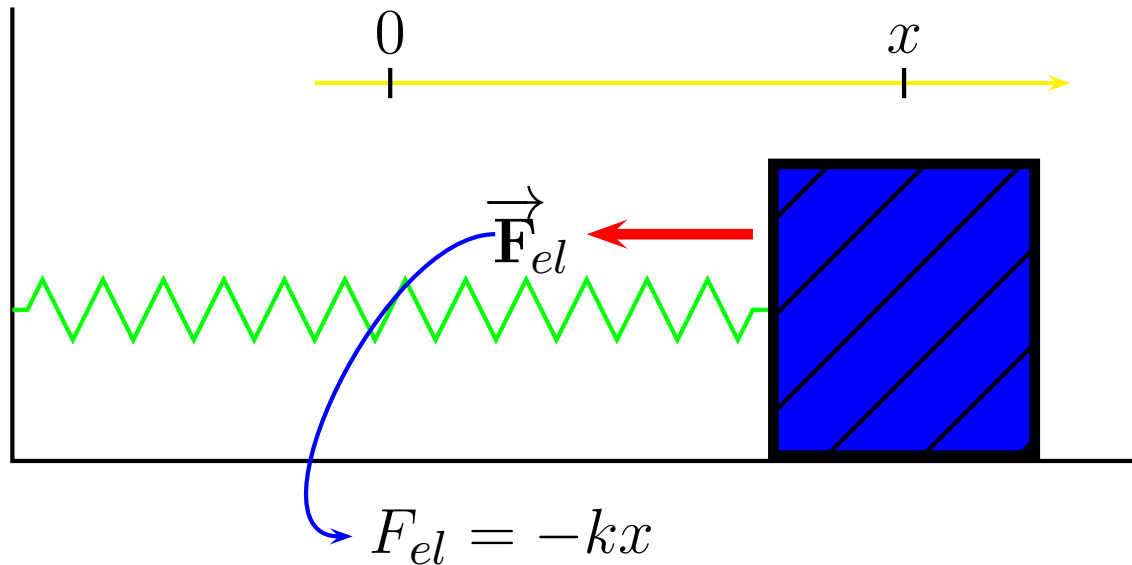


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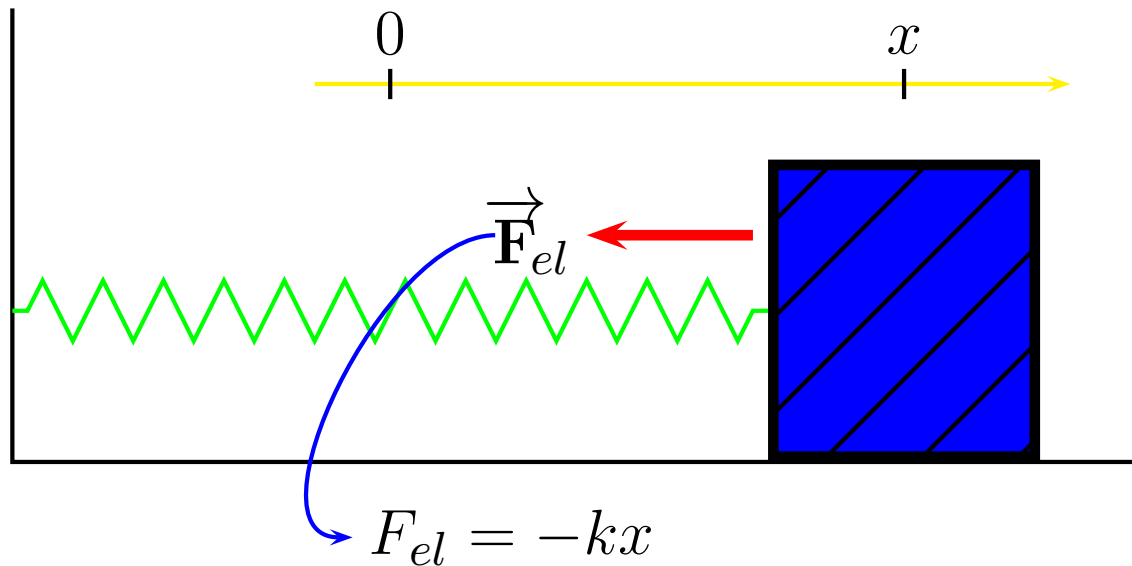
$$-F_{el} = M a_x$$

$$-kx = M a_x$$

# Simple Harmonic Motion II

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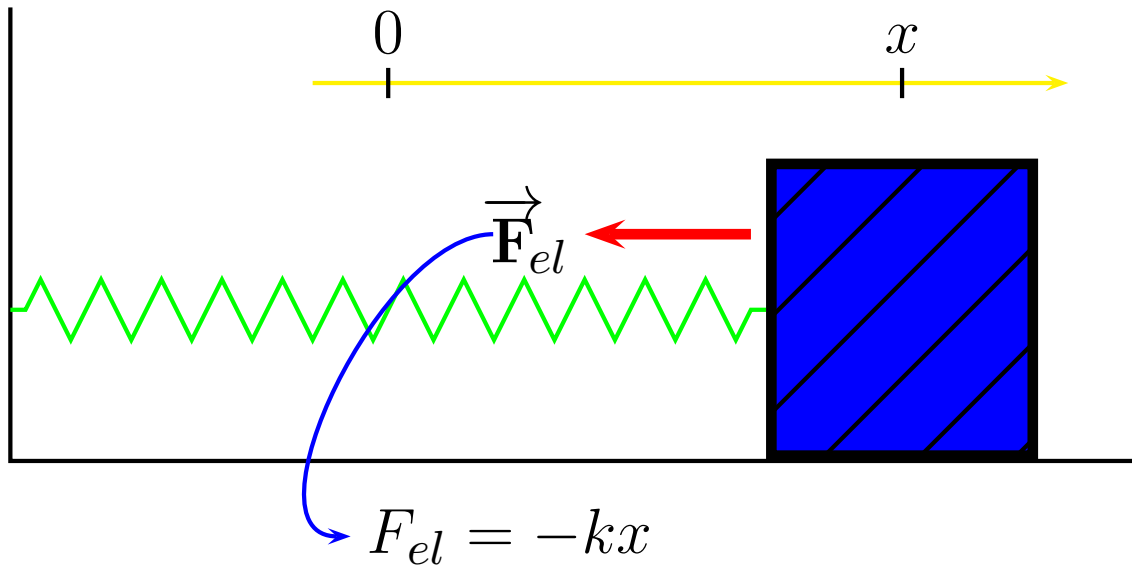
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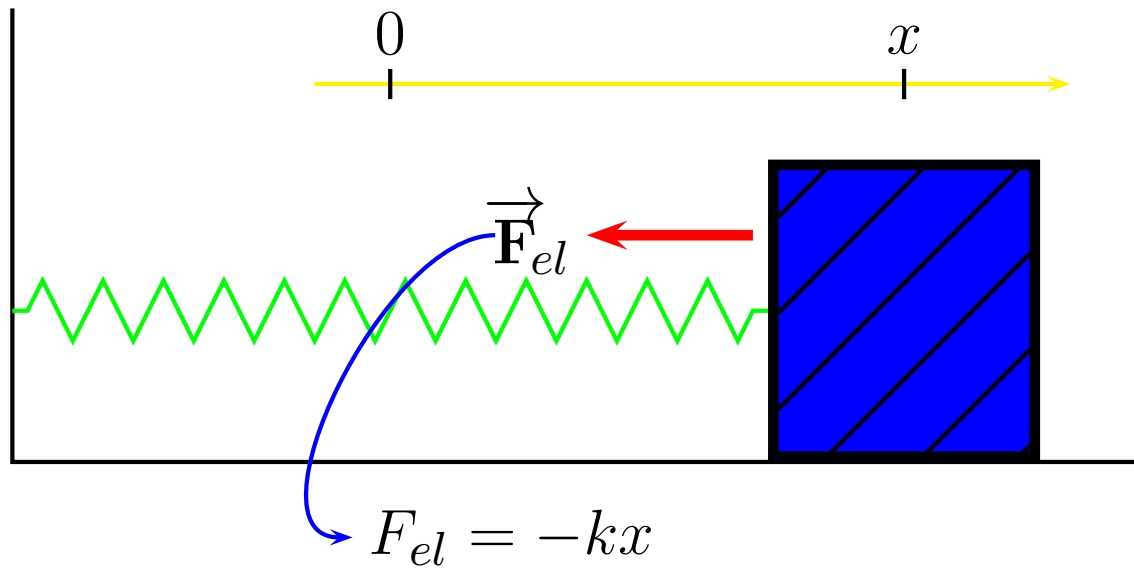


$$v_x = \frac{dx}{dt}$$

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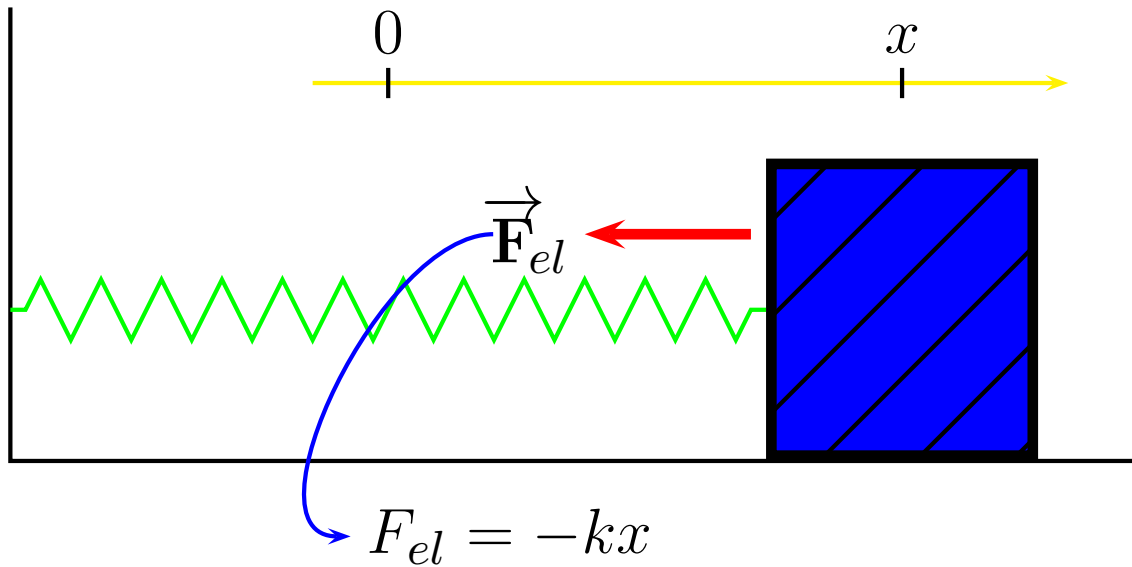


$$a_x = \frac{dv_x}{dt}$$

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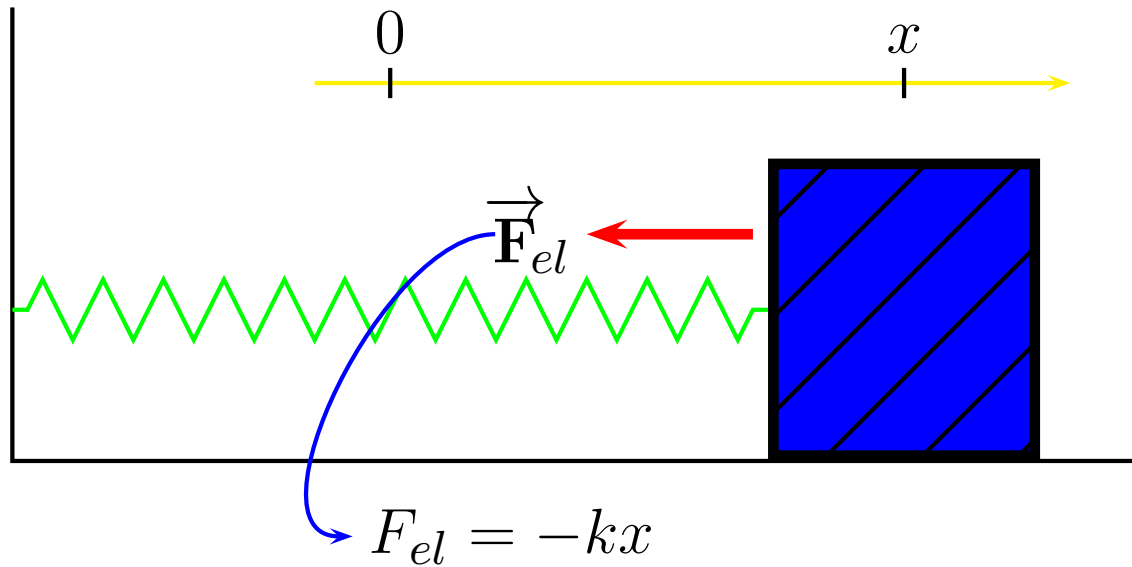
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# Simple Harmonic Motion III

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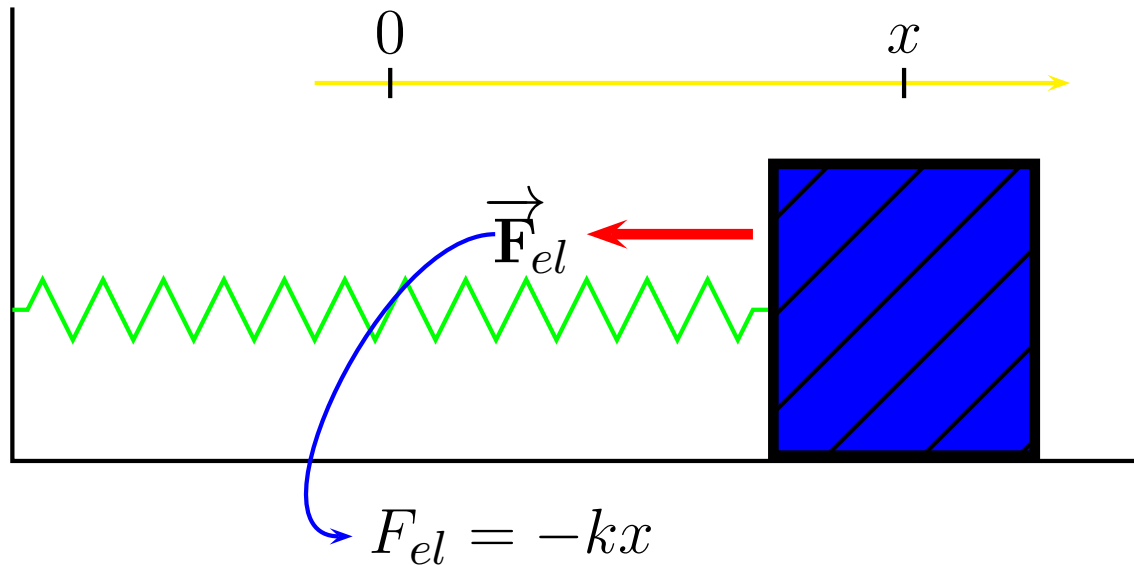
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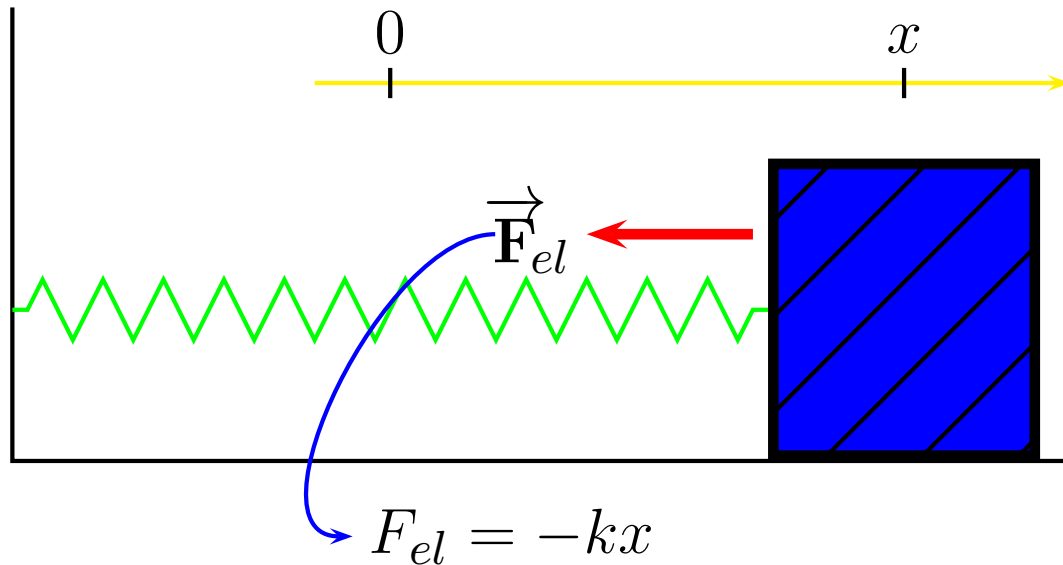
$$\frac{d^2x}{dt^2} = - \left( \frac{k}{M} \right) x$$

Differential Equation  
for SHM

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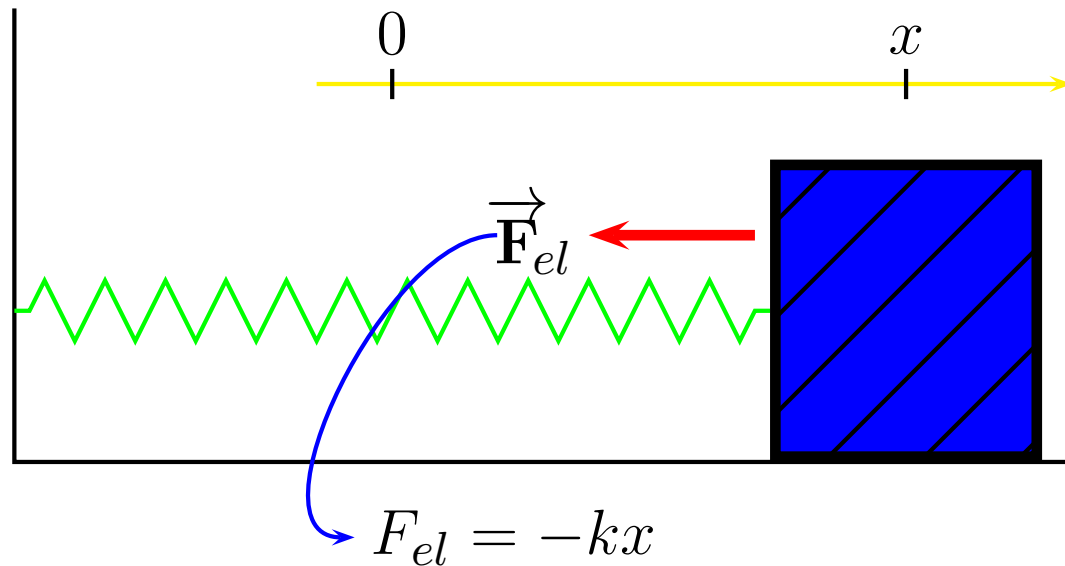
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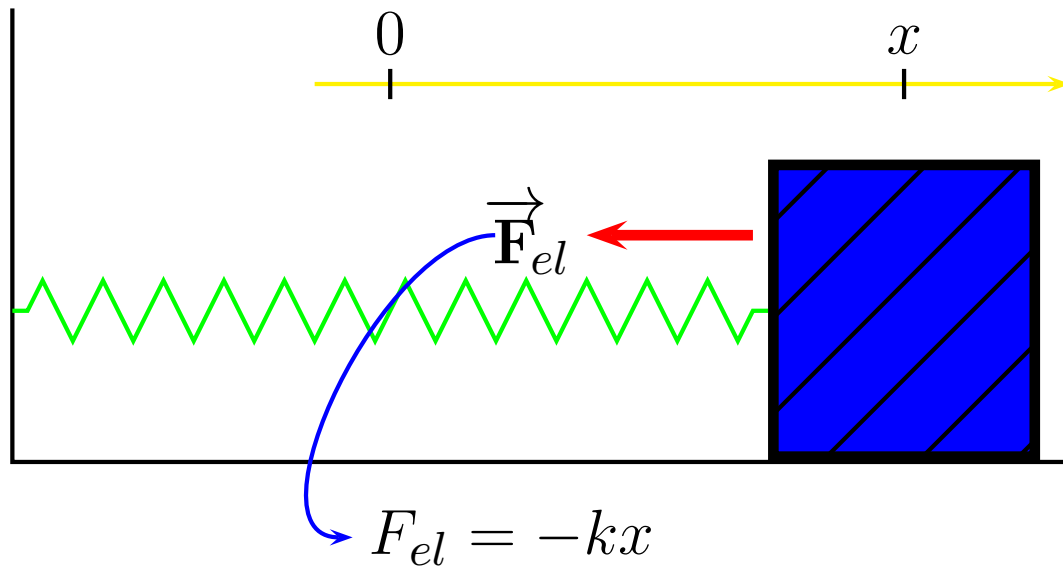


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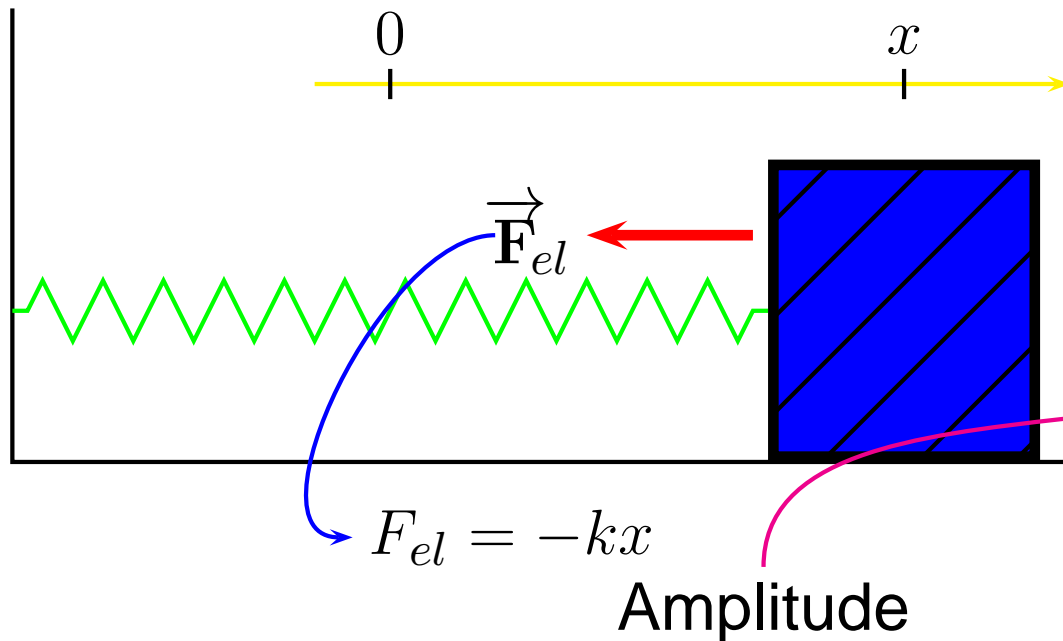
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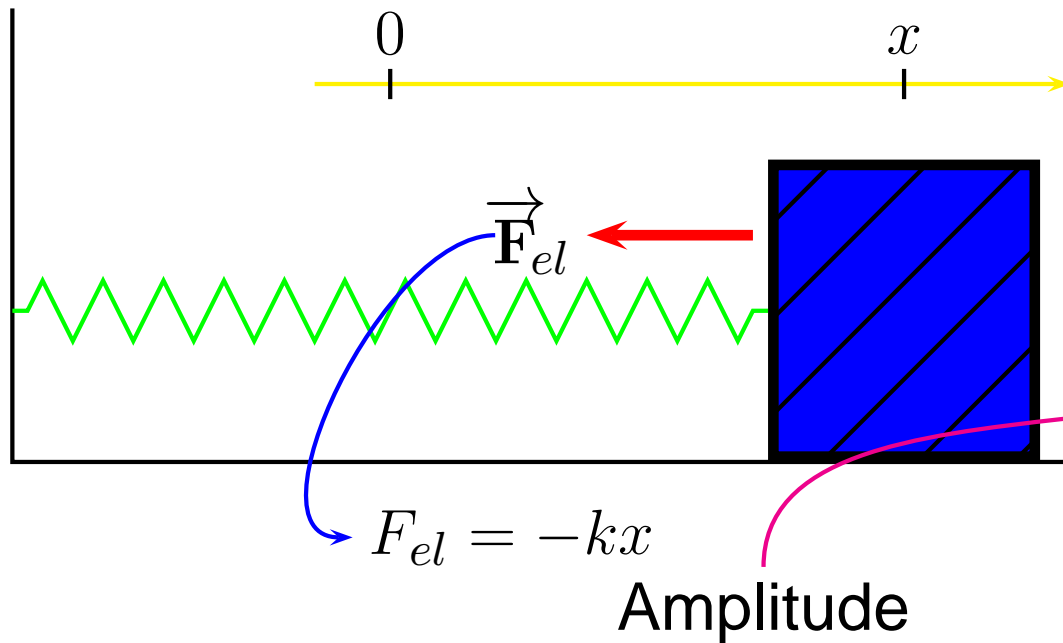
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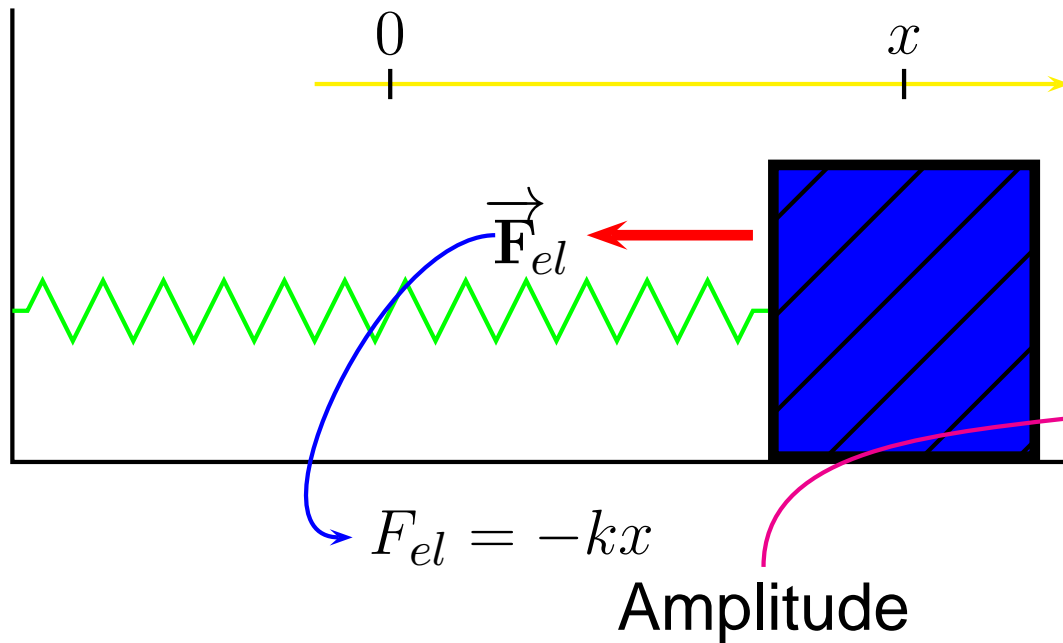
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Phase Angle

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Angular frequency,

$$\omega = 2\pi f = \frac{2\pi}{T}$$



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Amplitude - Maximum distance from zero.

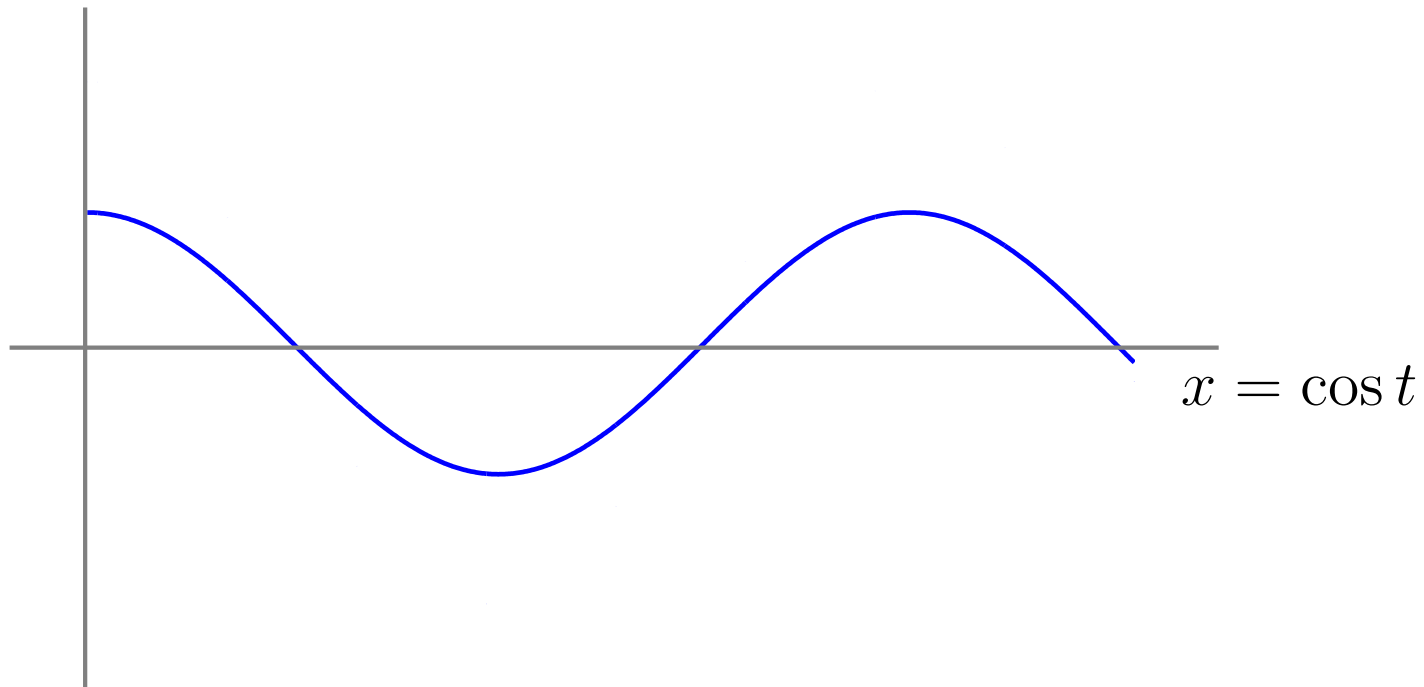
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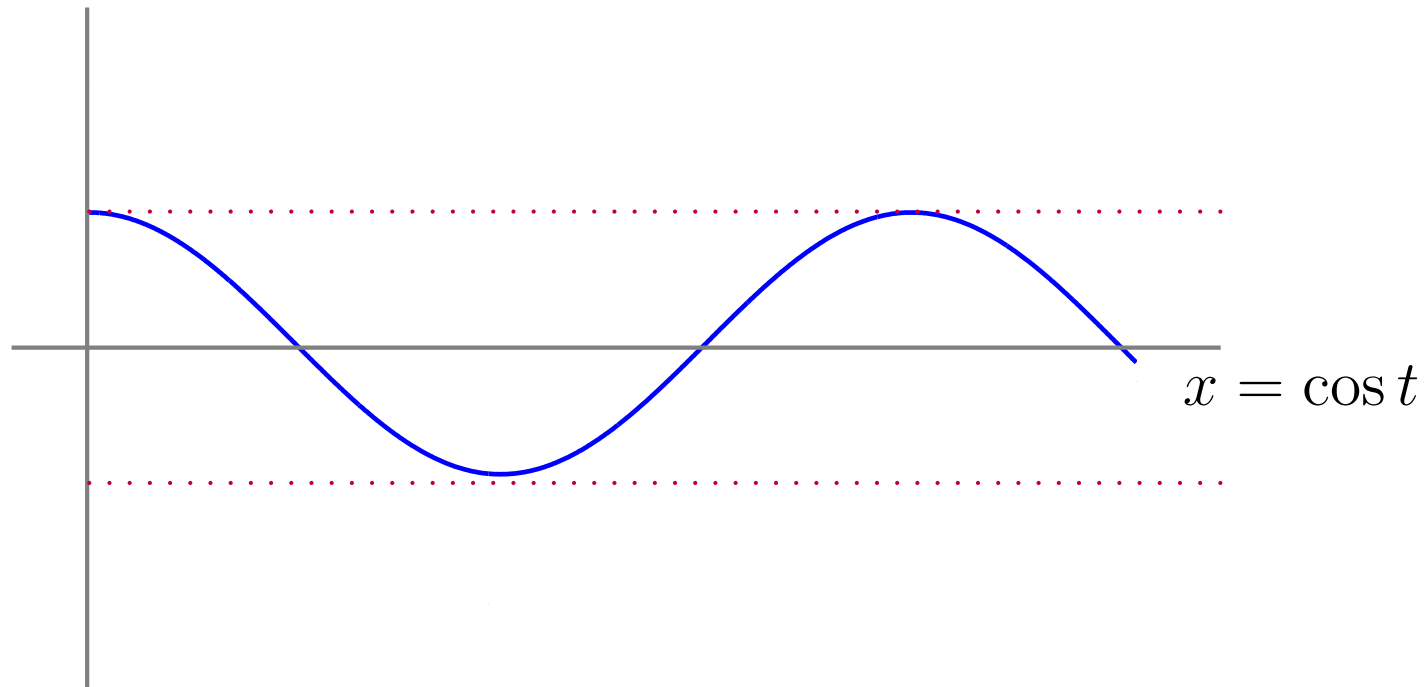
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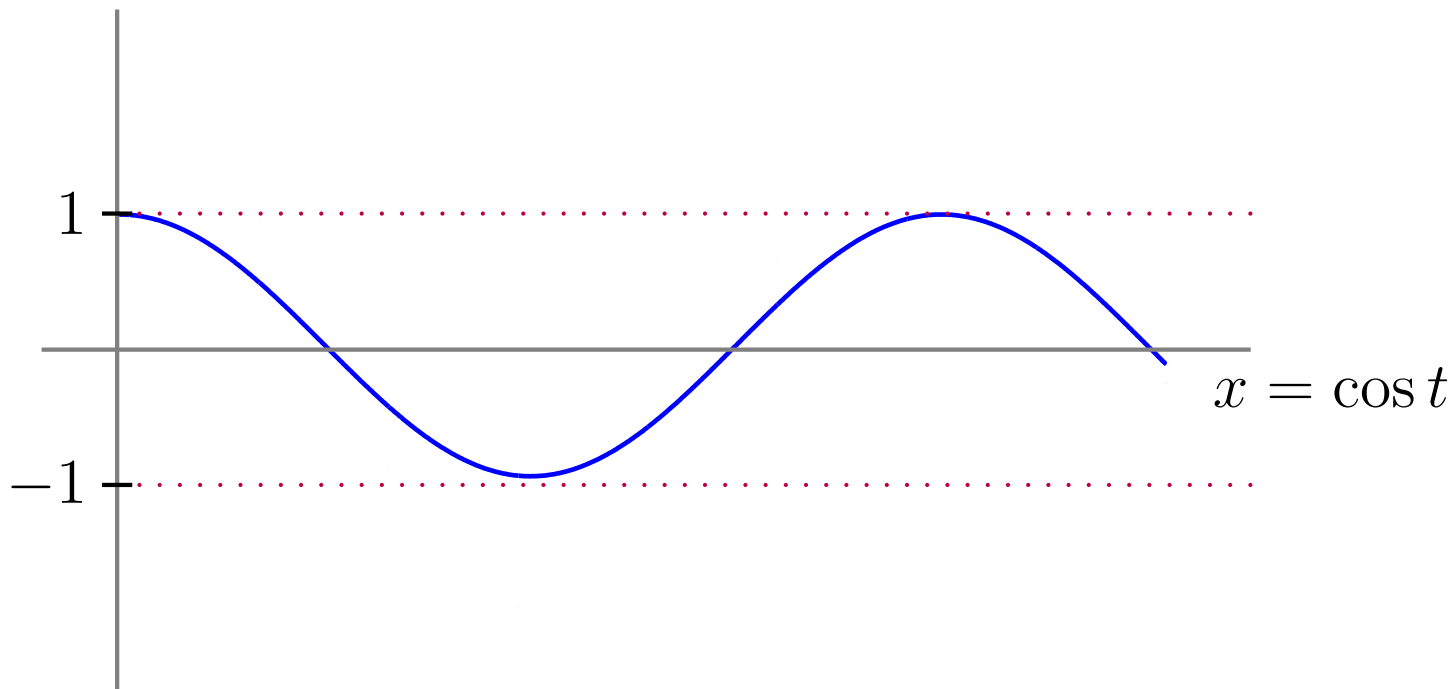
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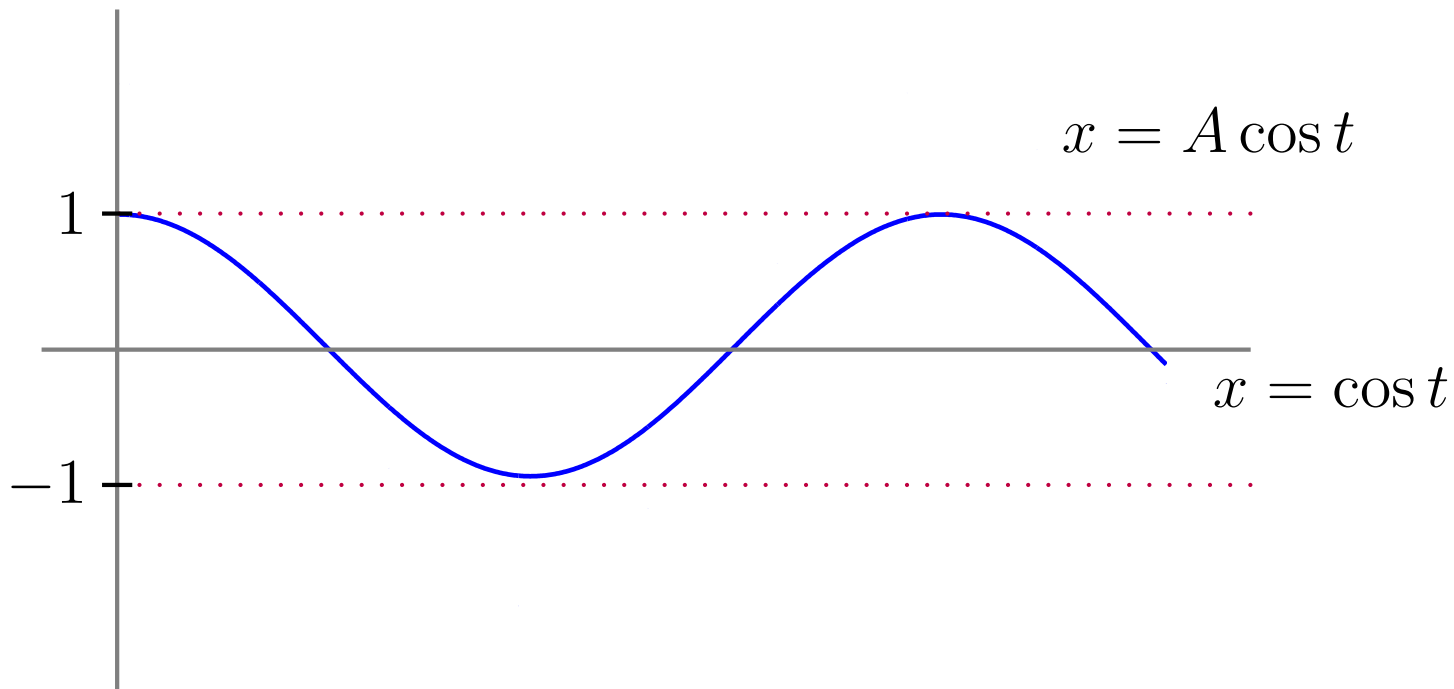
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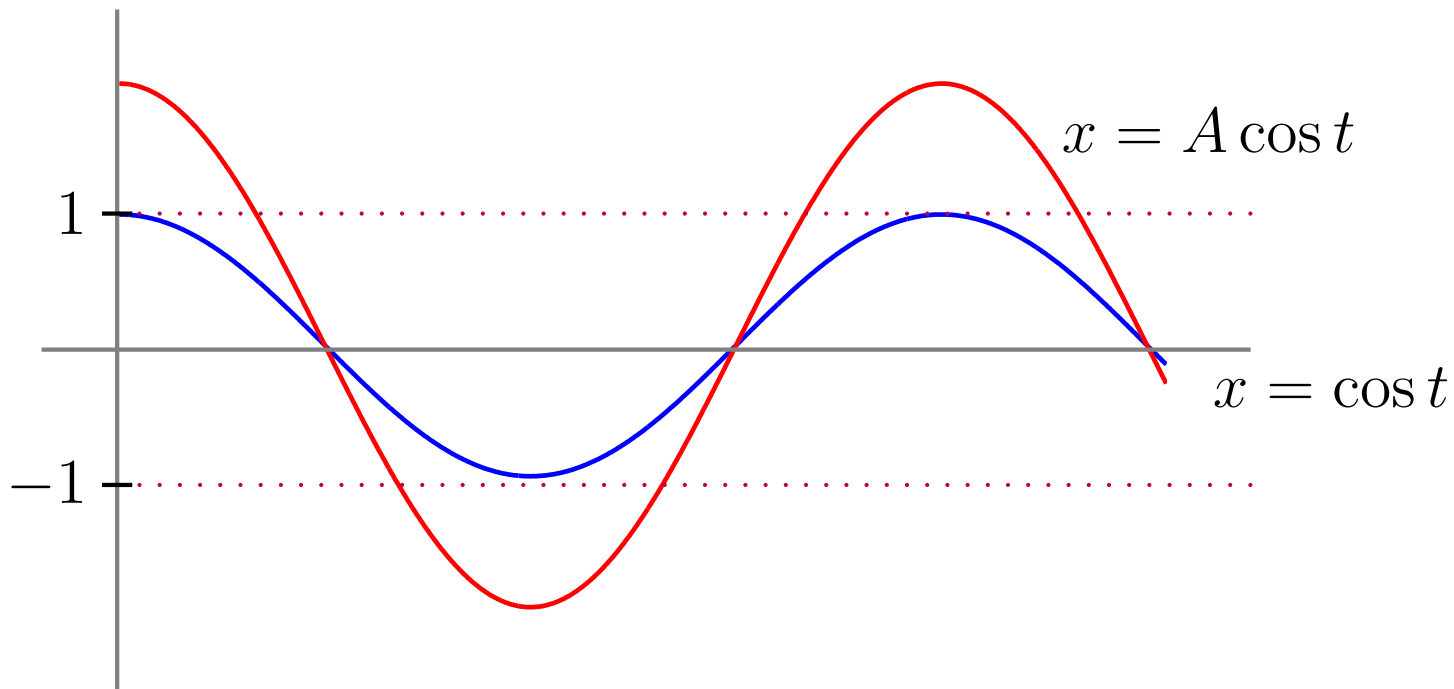
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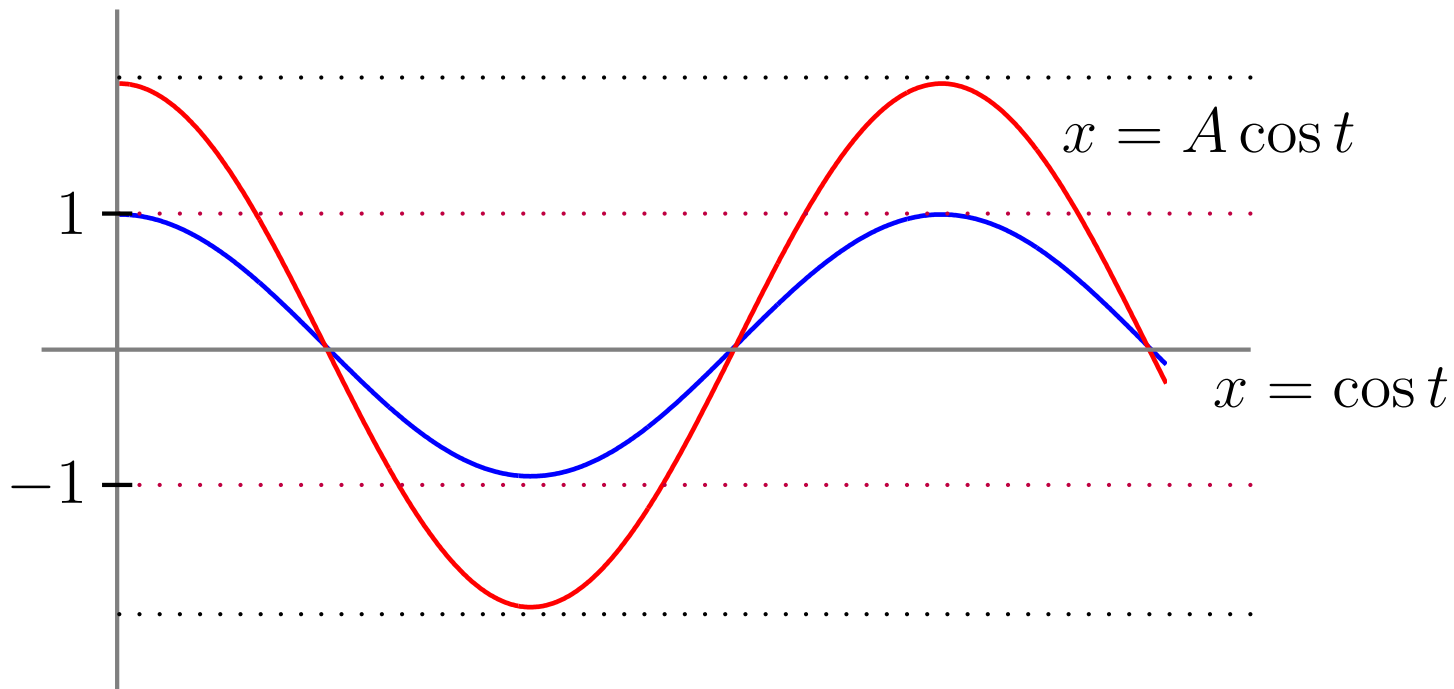
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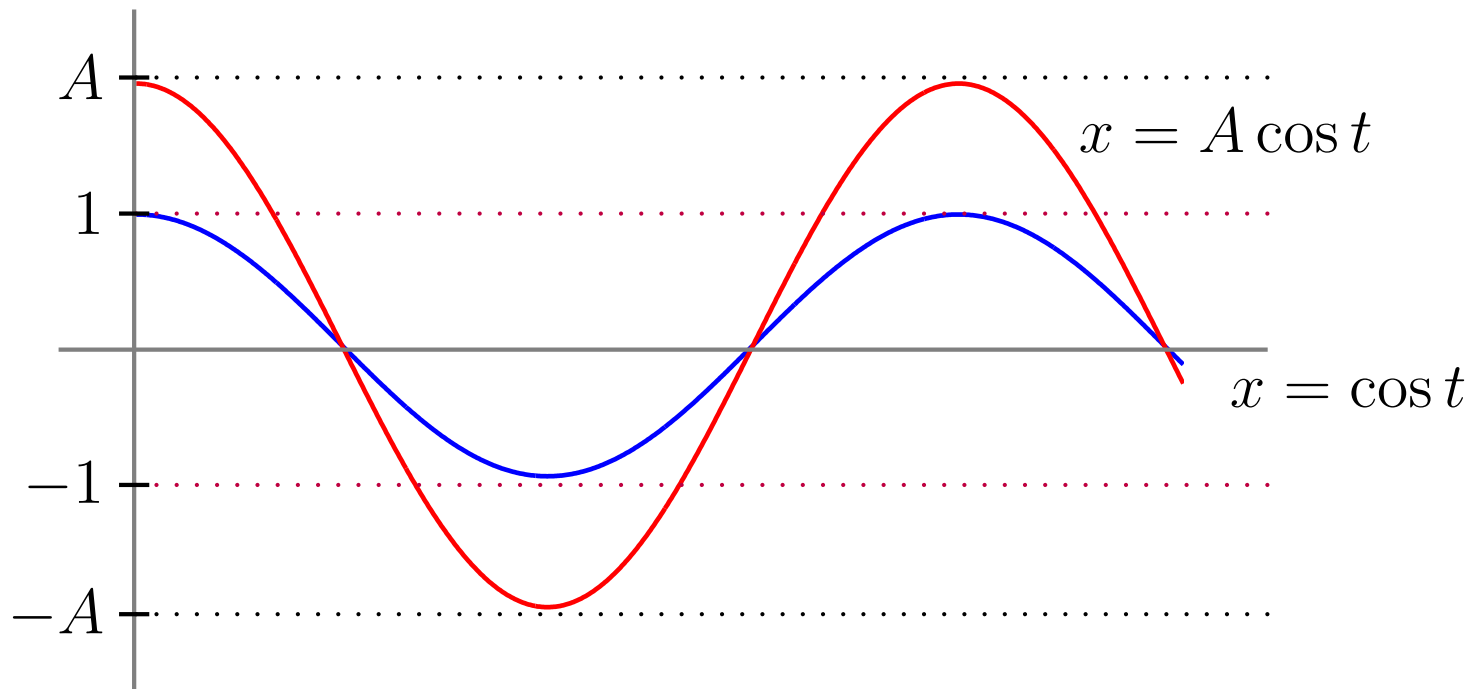
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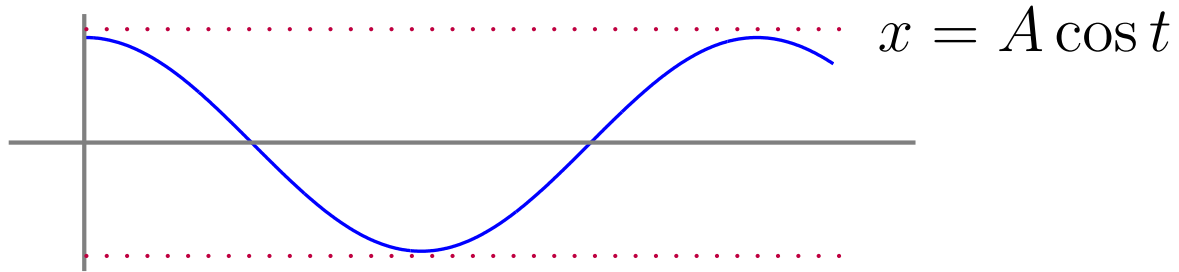


# Phase Angle

Phase Angle -  $\phi$ , Units: *rad*. Shifts the Cosine to start wherever needed.

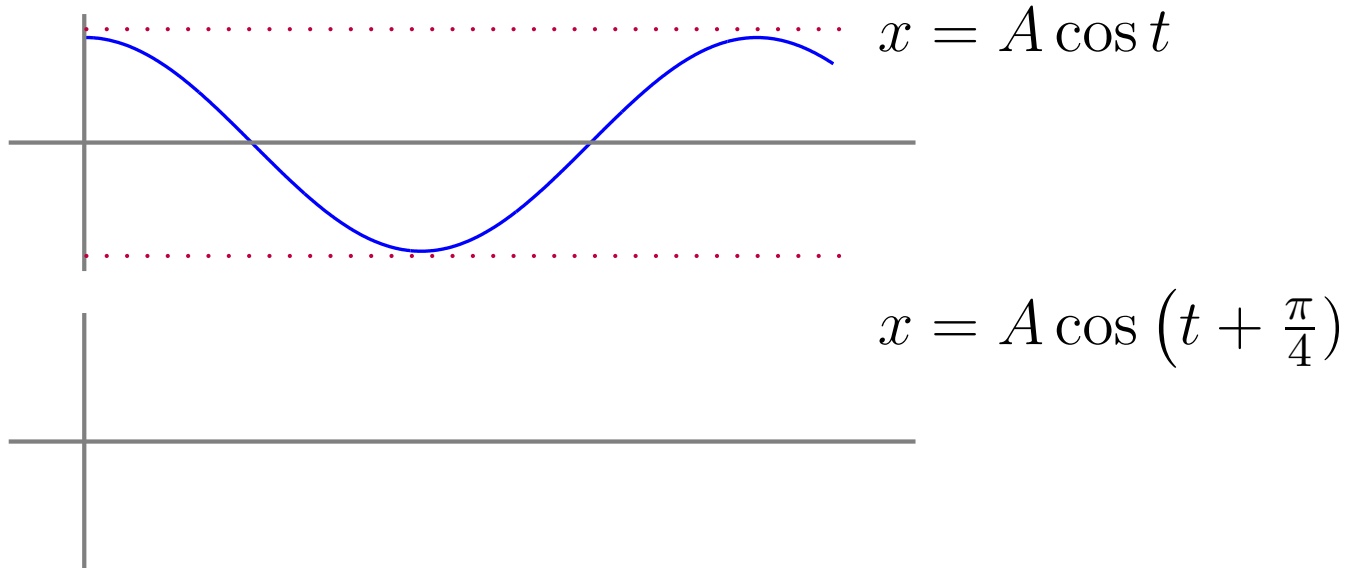
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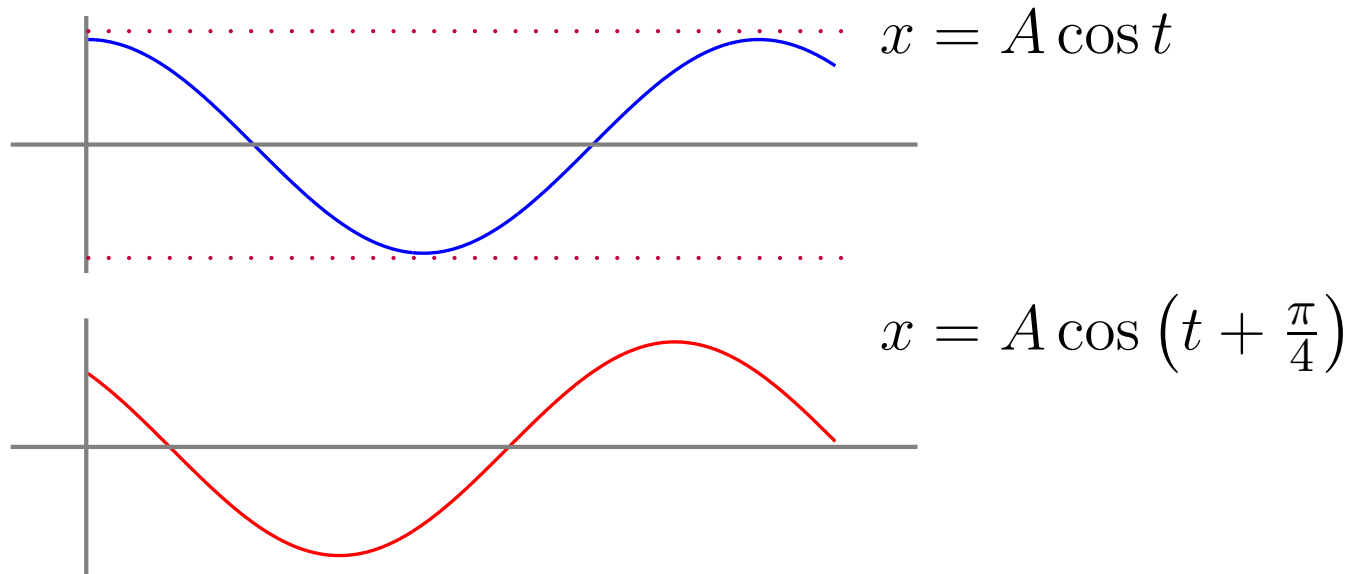
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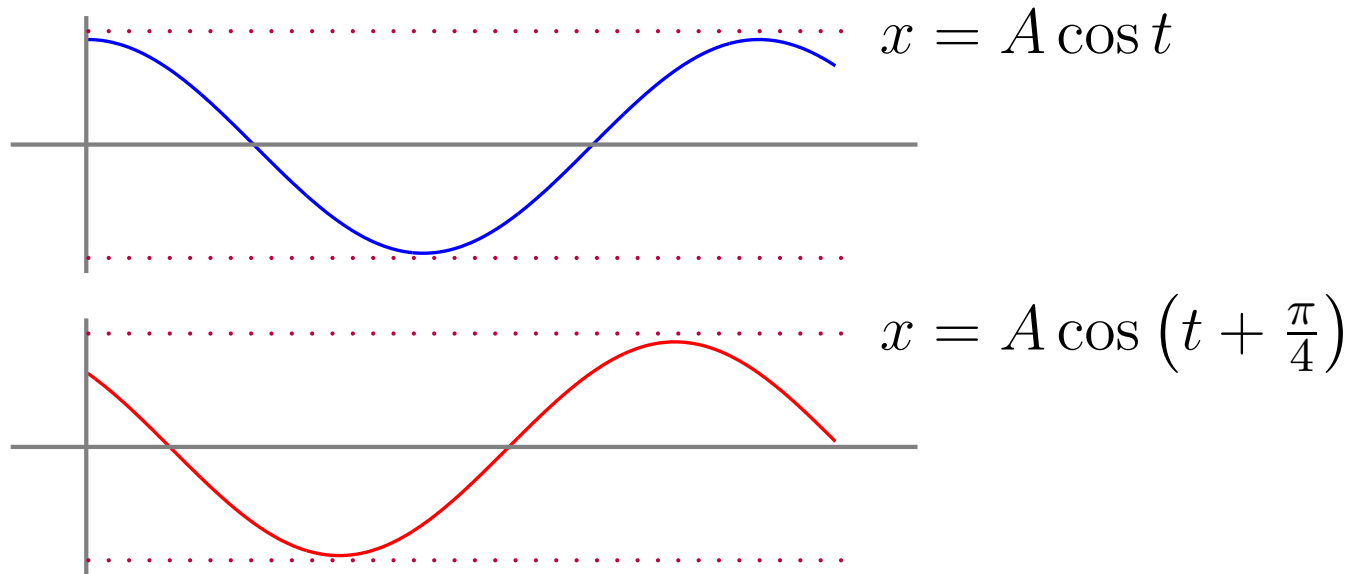
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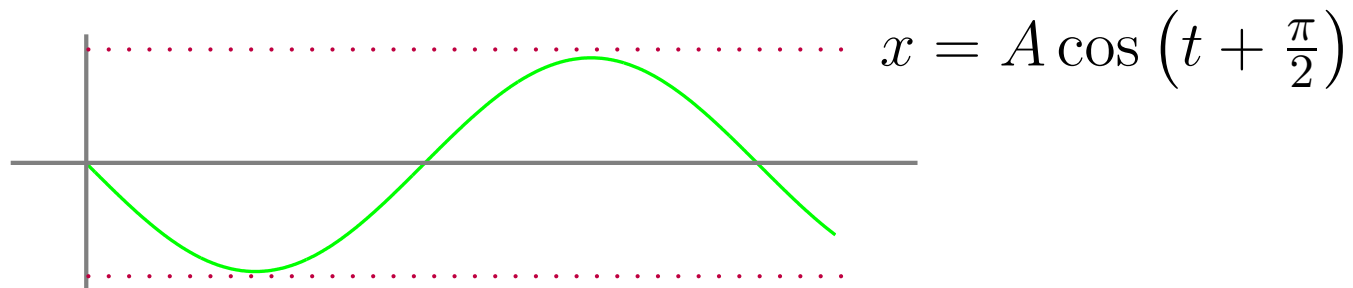
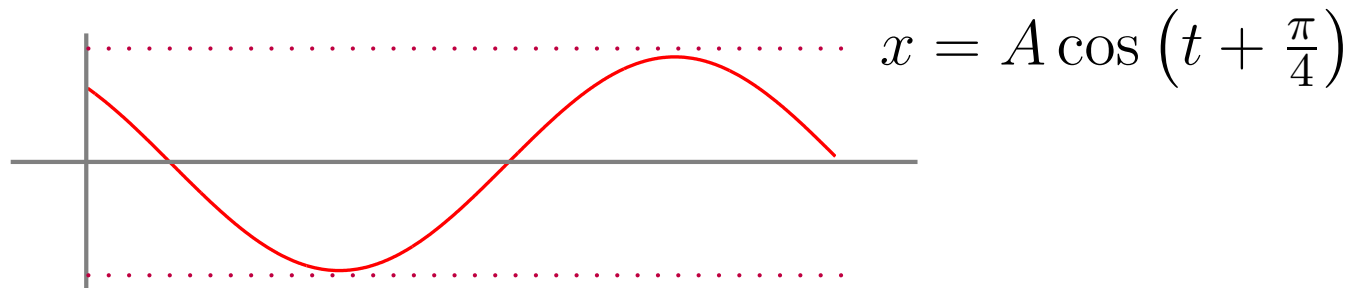
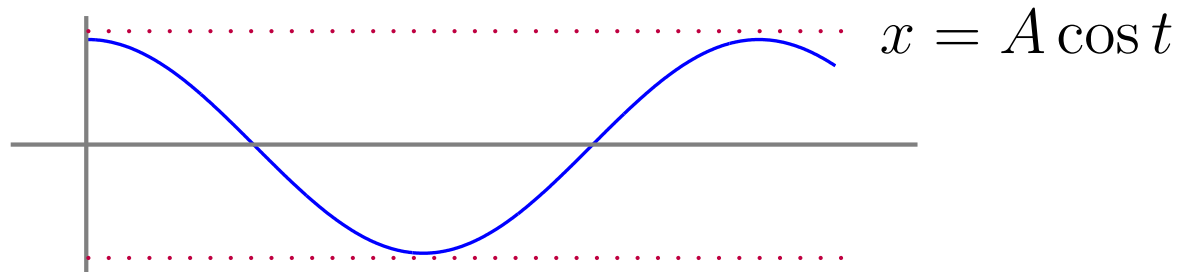
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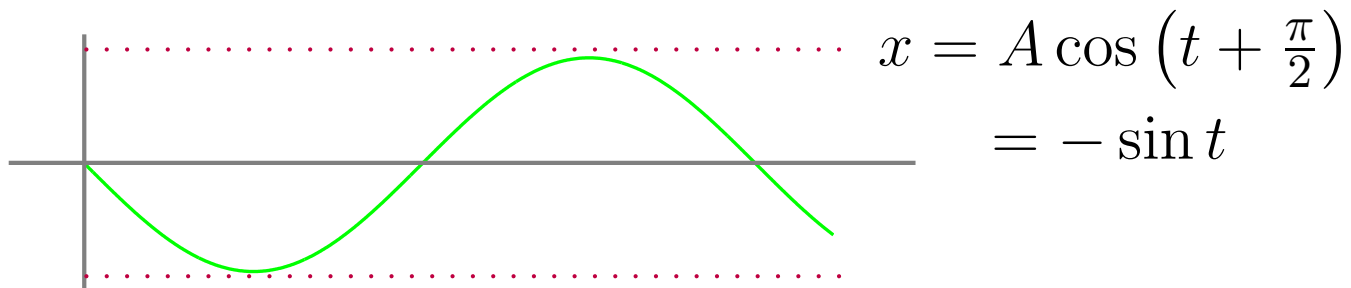
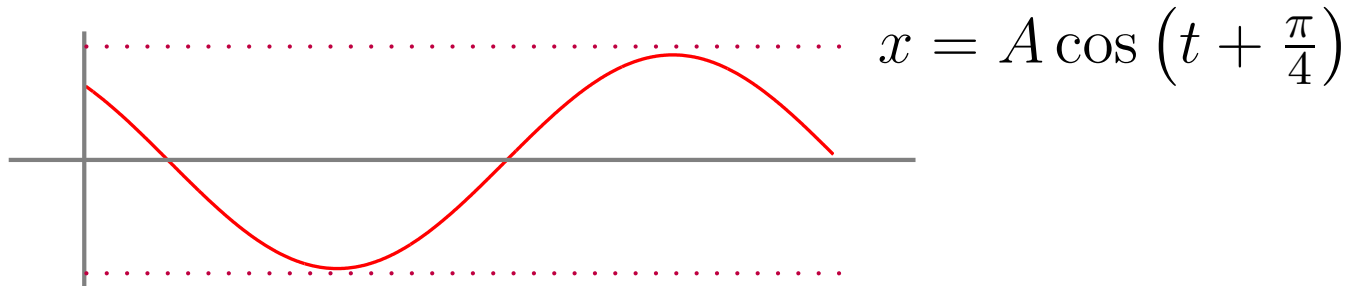
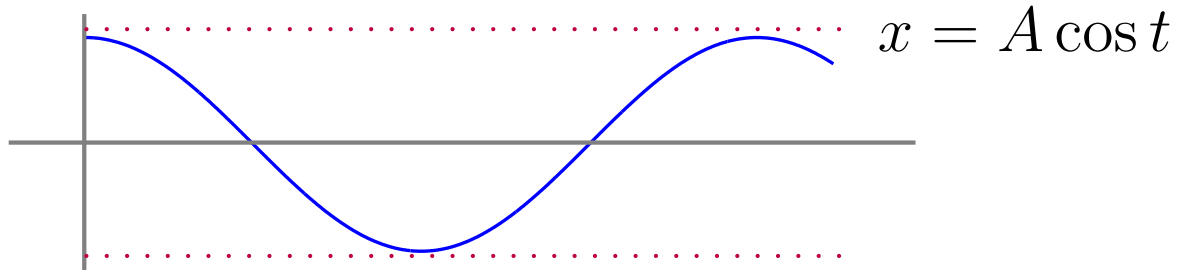
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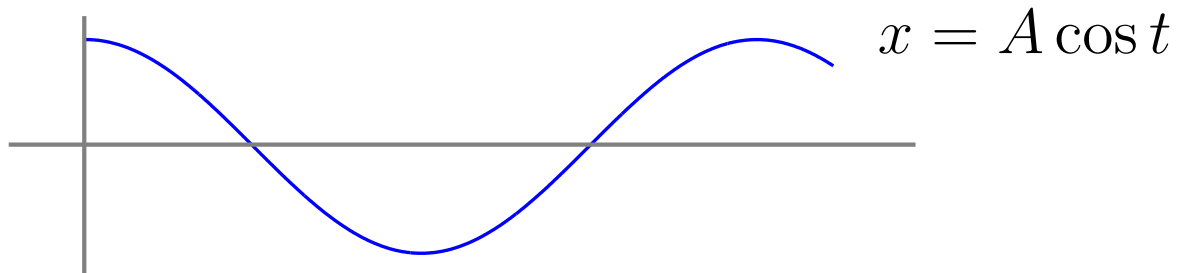


# Angular Frequency

Angular Frequency -  $\omega = 2\pi f = \frac{2\pi}{T}$     Units: *rad/s*

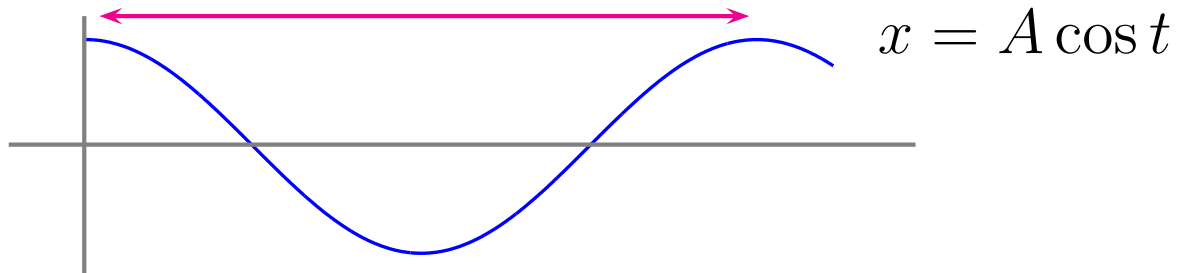
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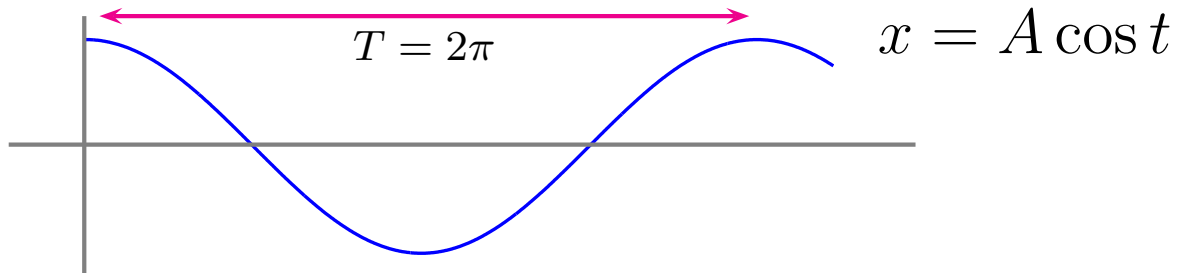
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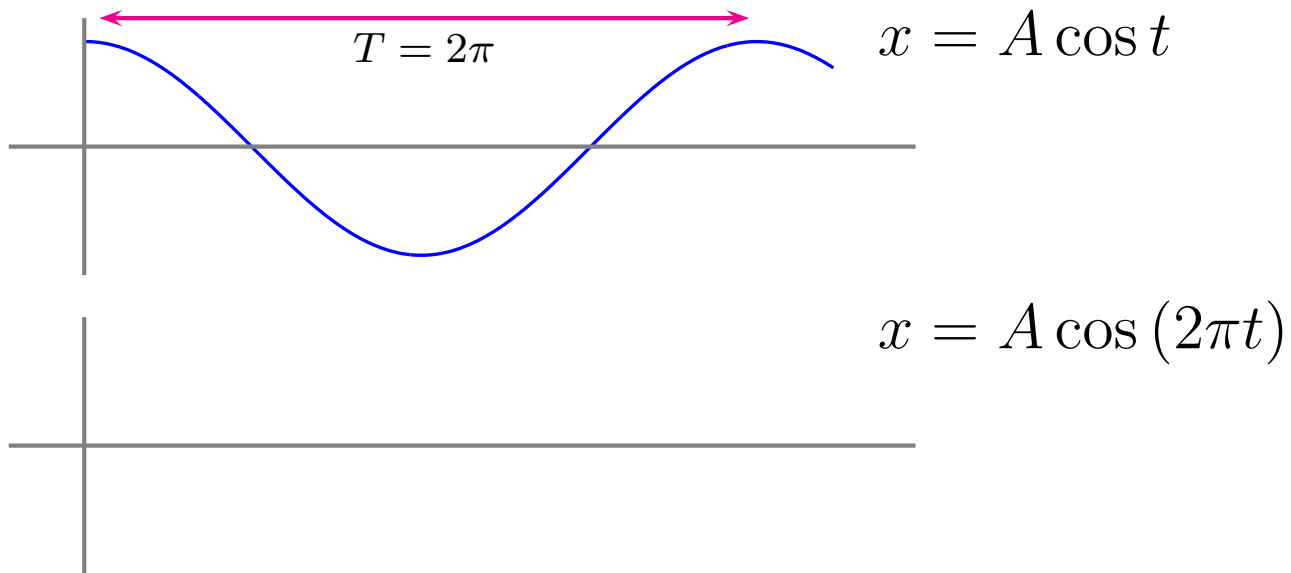
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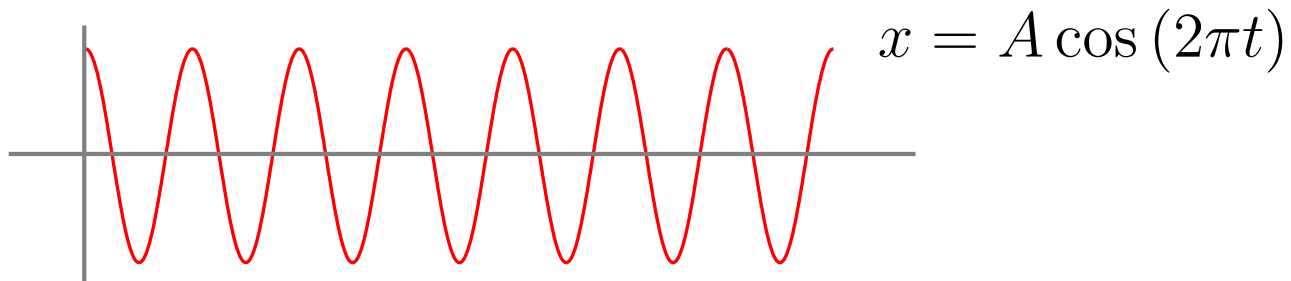
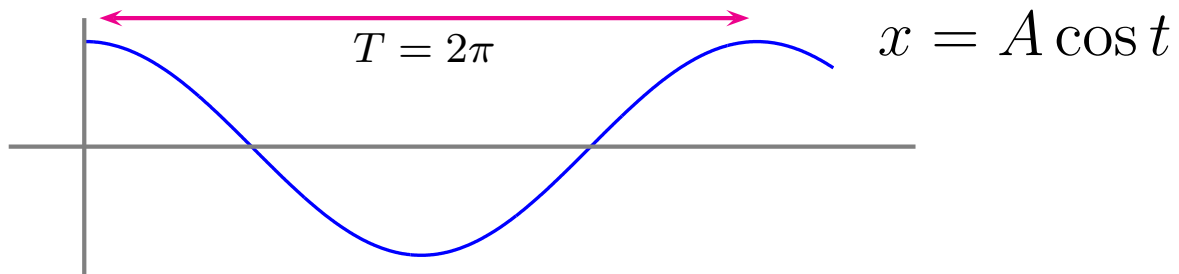
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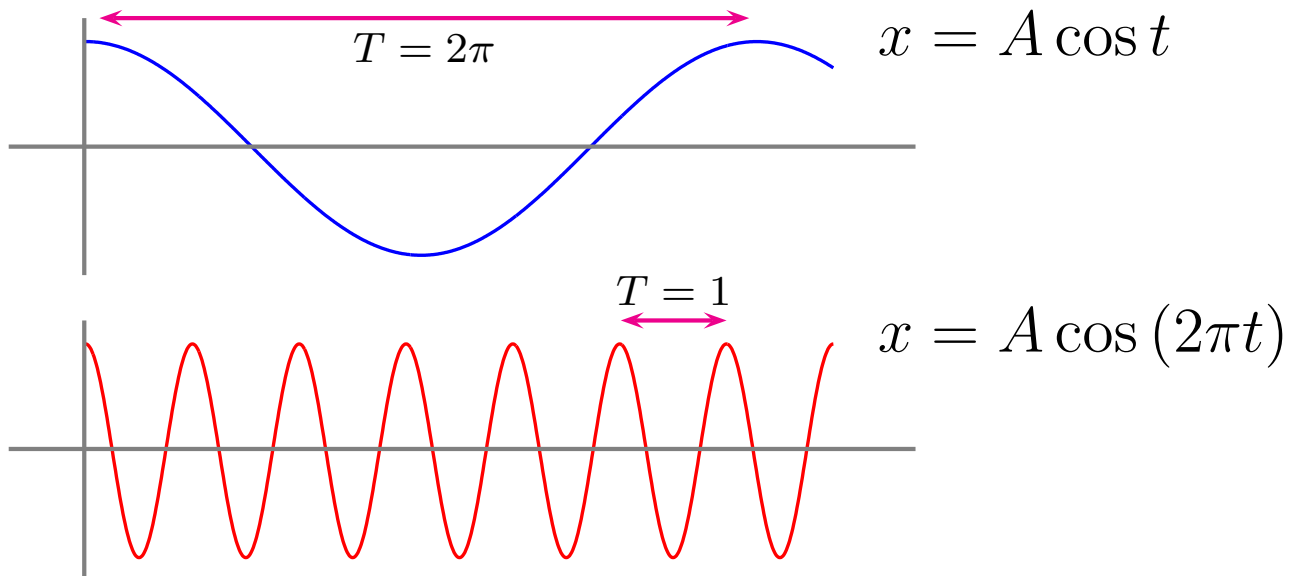
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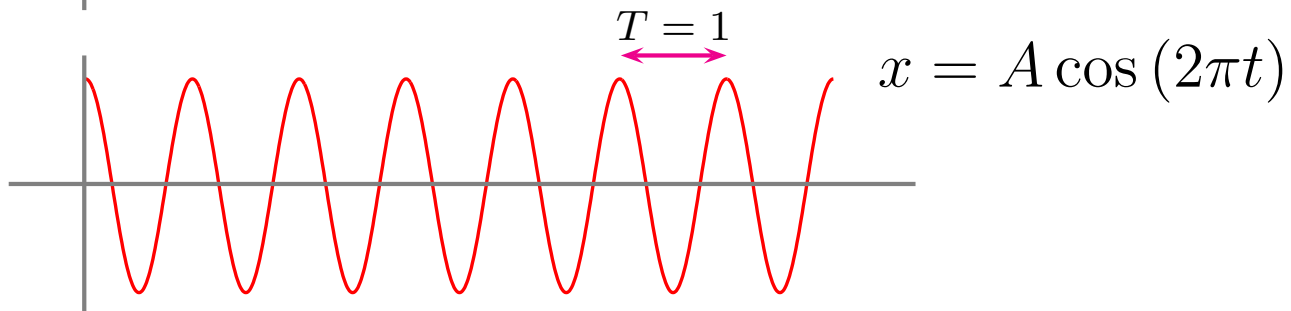
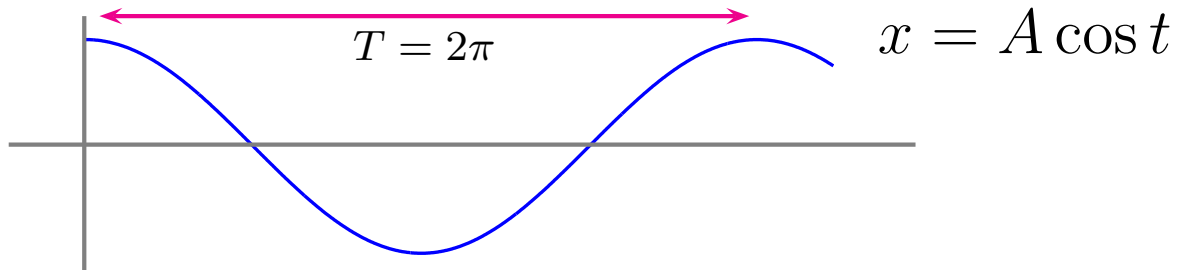
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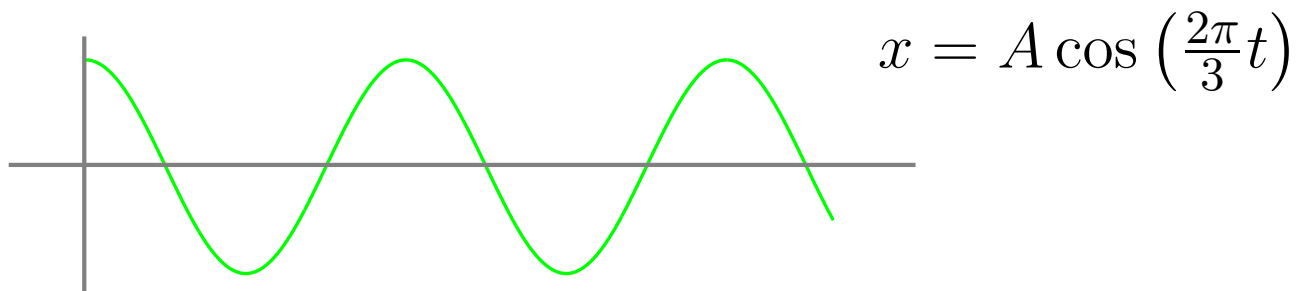
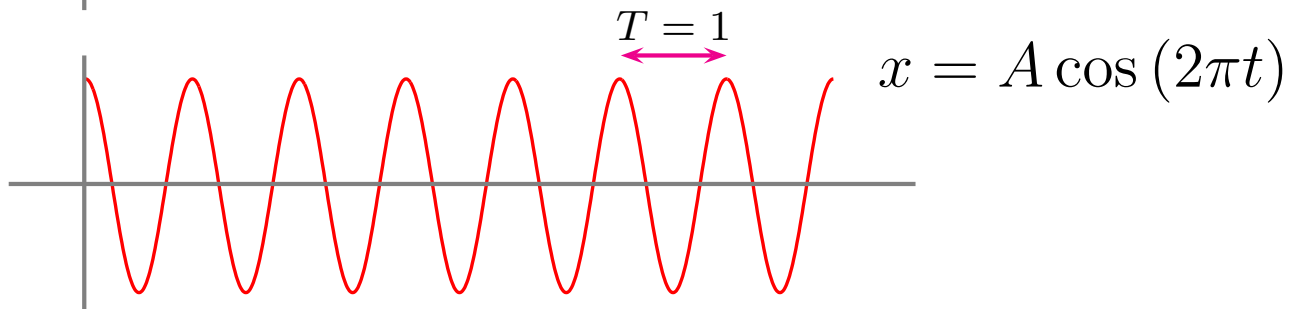
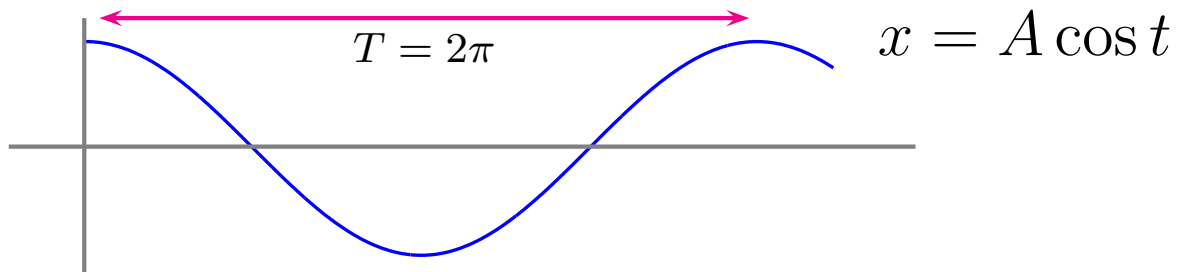
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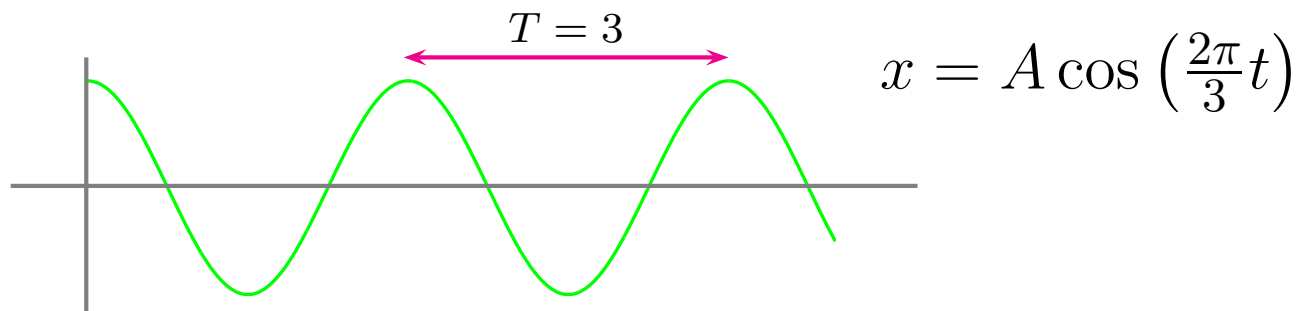
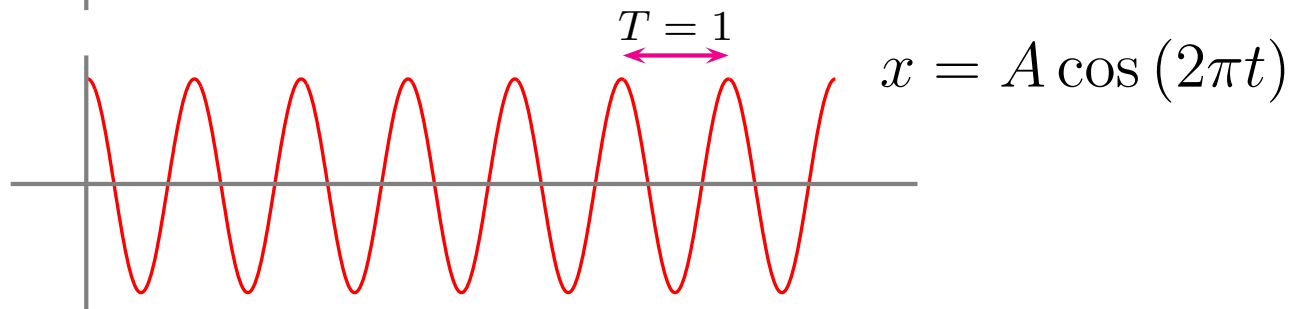
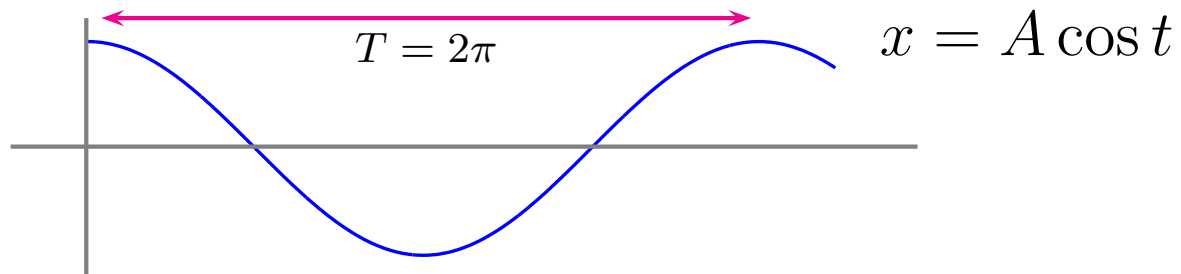
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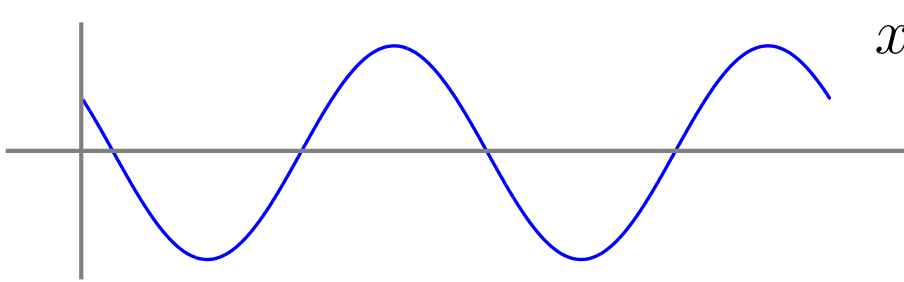


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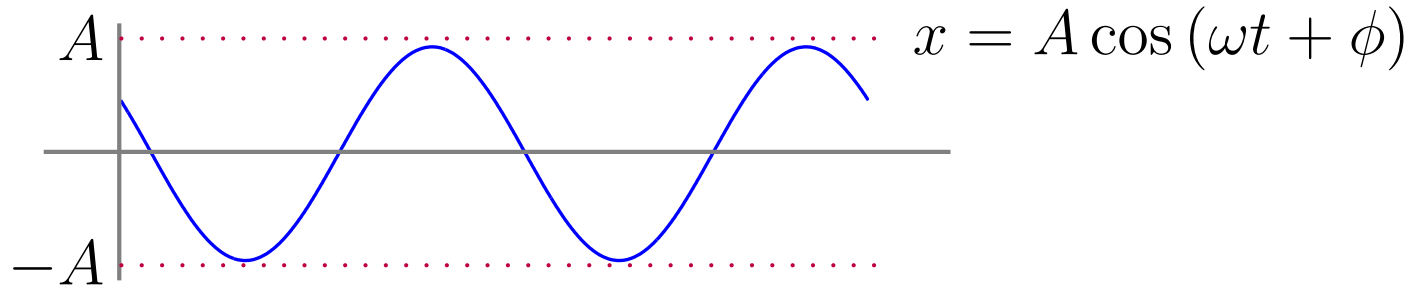
$$x = A \cos(\omega t + \phi)$$


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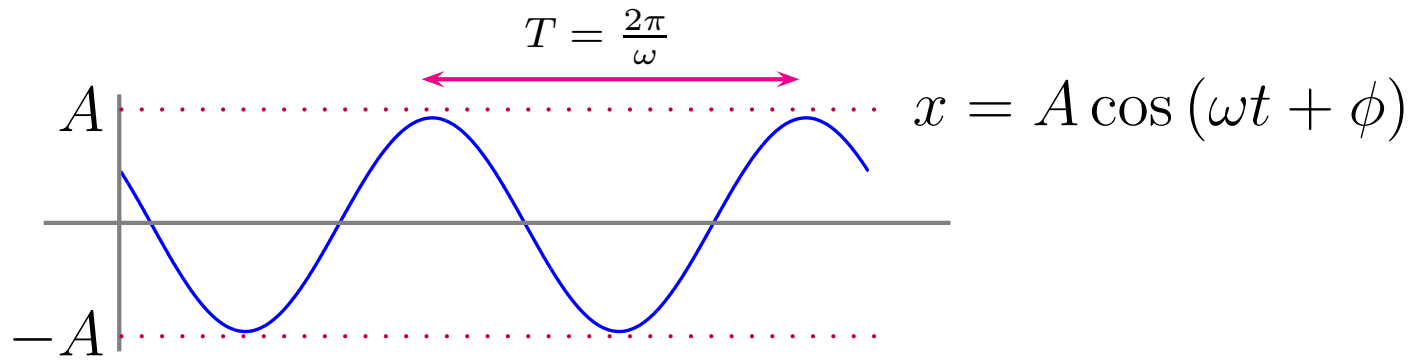


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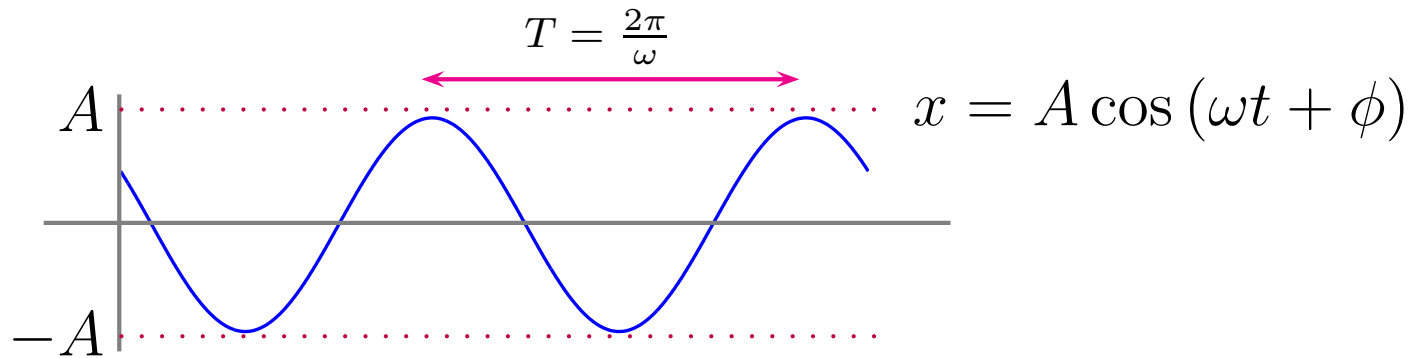
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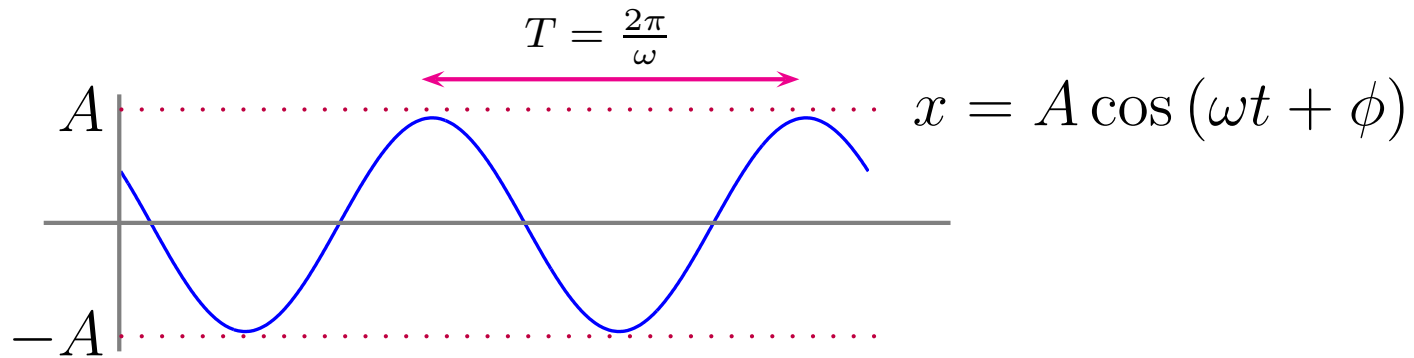
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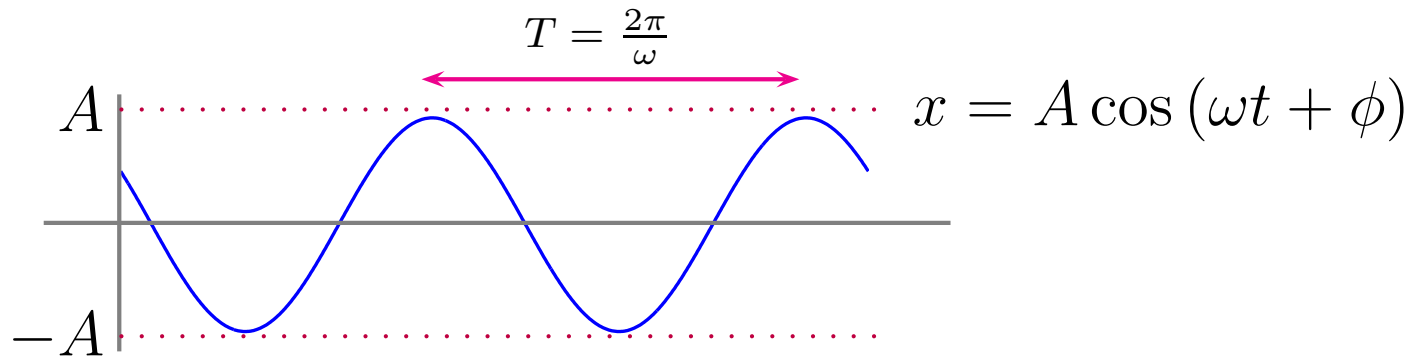
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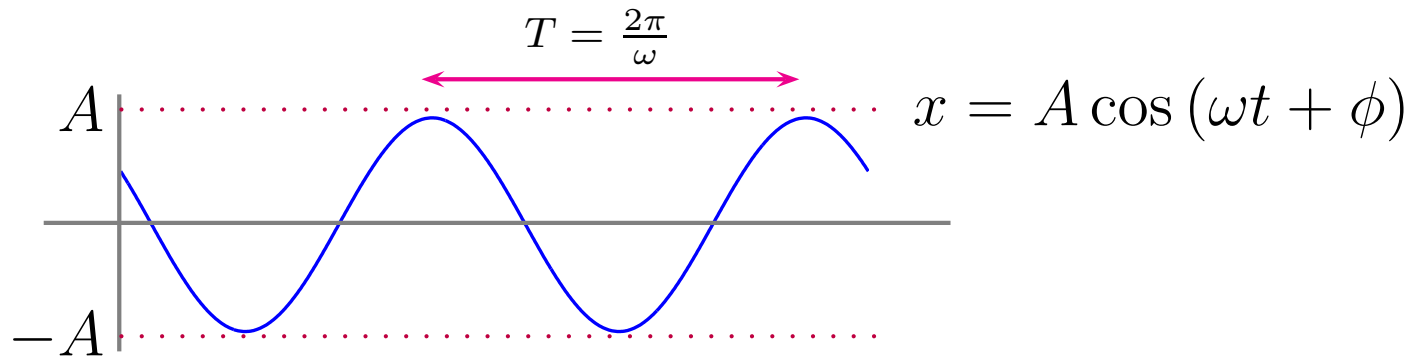


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# General Solution

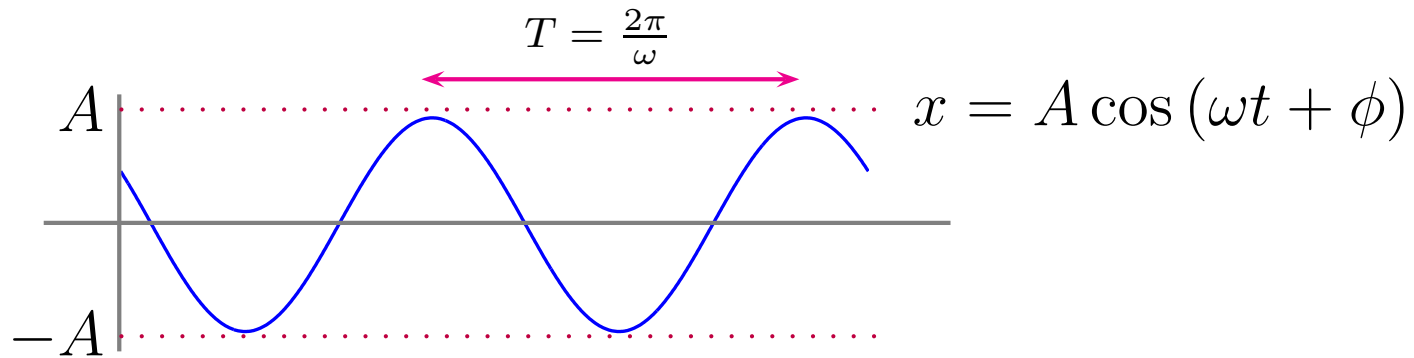


Differential Equation for SHM:  $\frac{d^2x}{dt^2} = -\left(\frac{k}{M}\right)x$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) = -\omega^2 x$$

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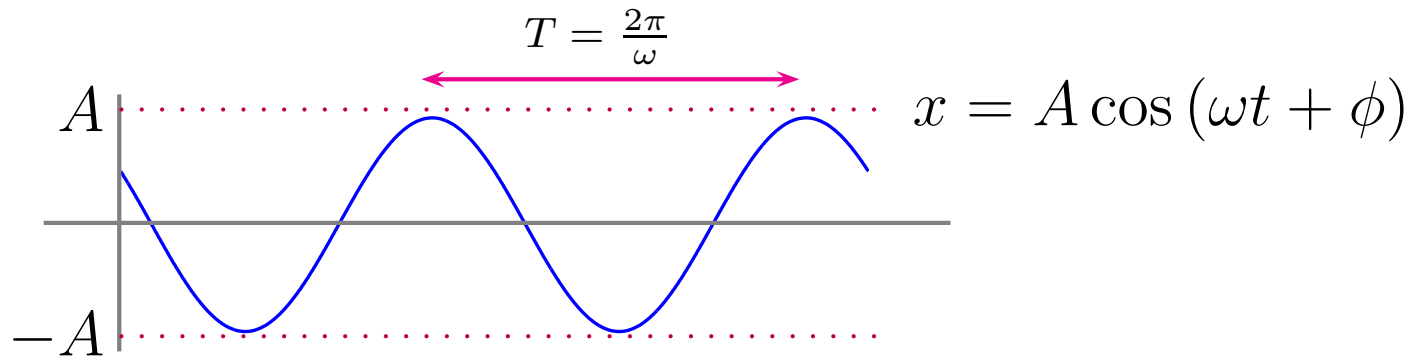


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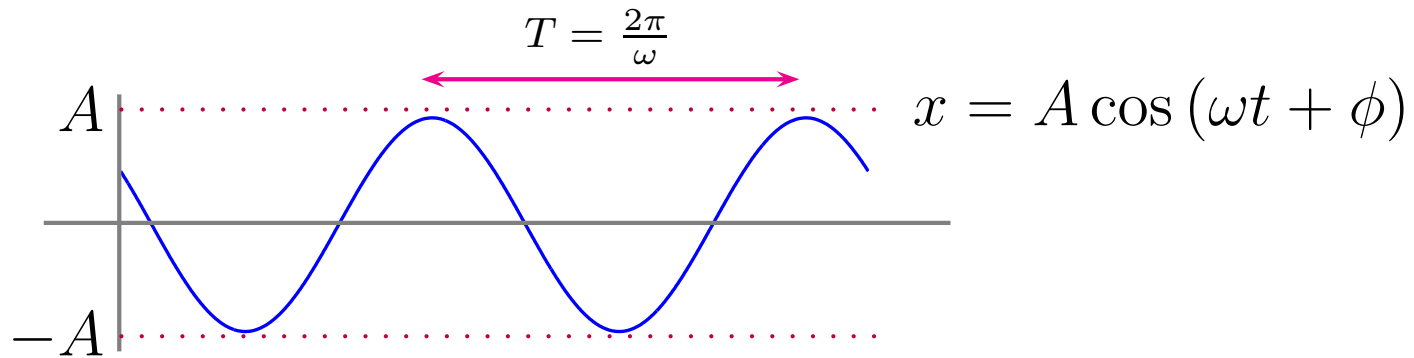
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A red arrow points from the  $\left(\frac{k}{M}\right)$  term in the differential equation above to the  $\omega^2$  term in this equation.

$$\omega^2 = \left(\frac{k}{M}\right)$$

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$$\omega^2 = \left(\frac{k}{M}\right)$$

$$\omega = \sqrt{\frac{k}{M}}$$

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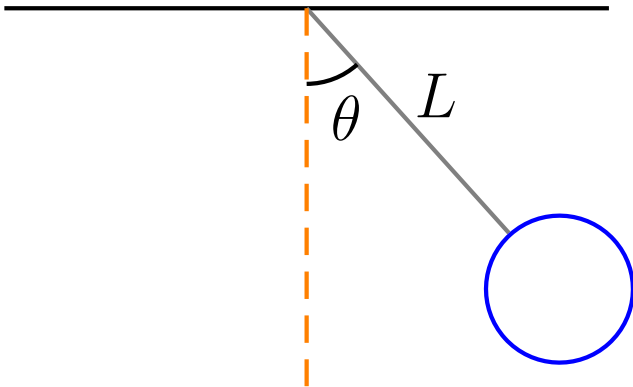
Simple Pendulum - Massless connector

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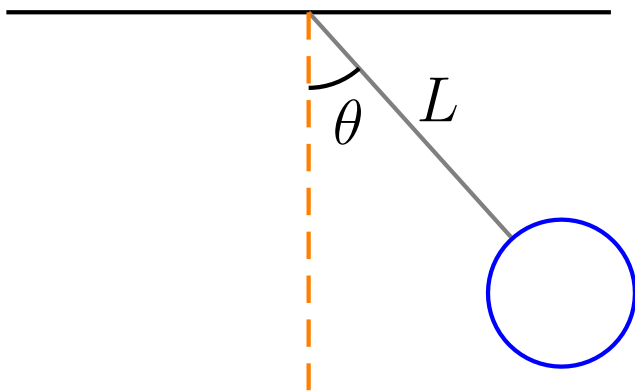


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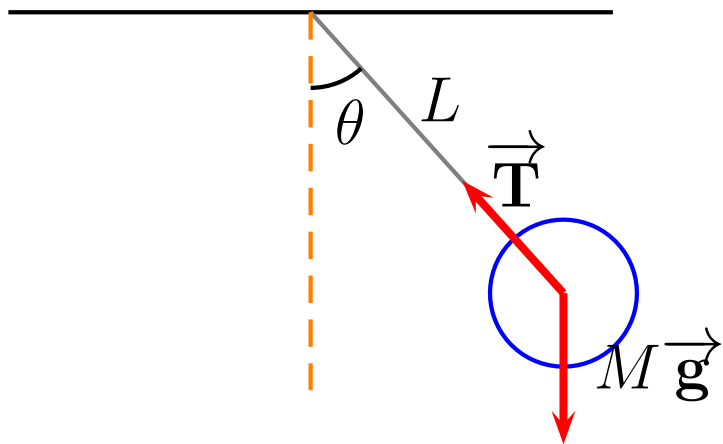
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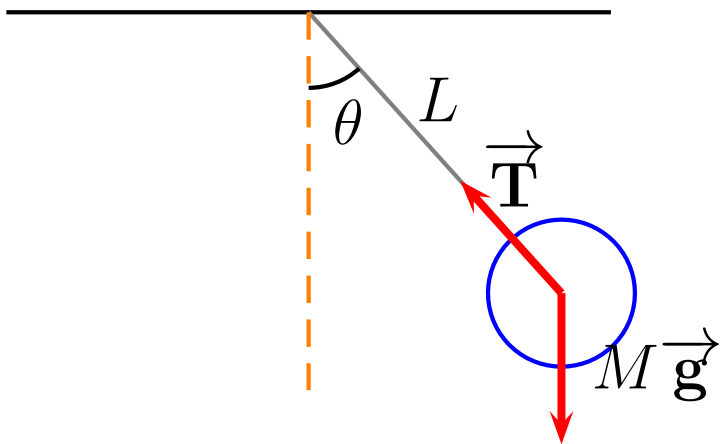
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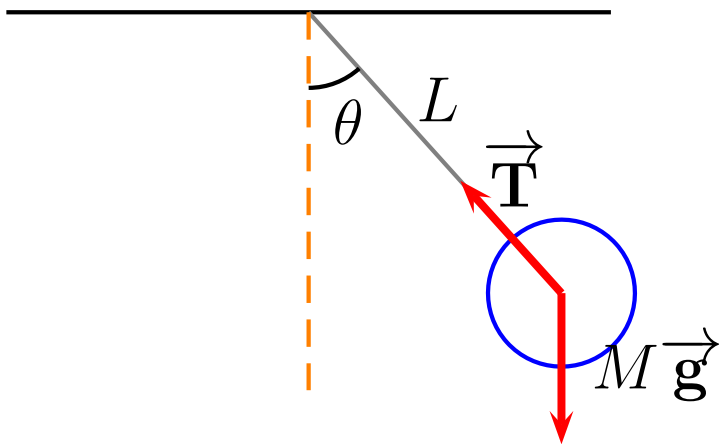
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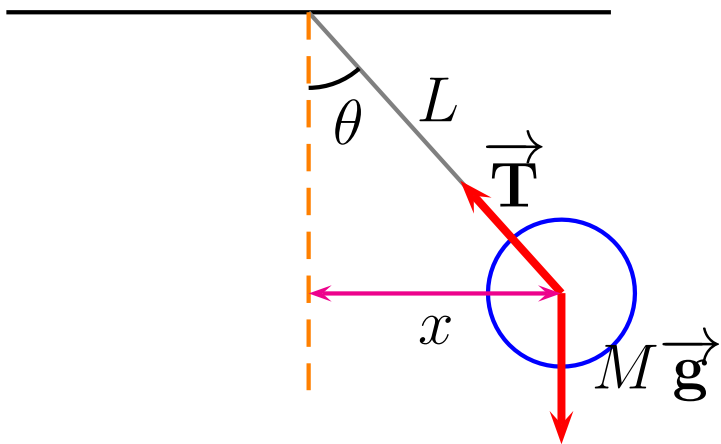
$$\Rightarrow \sum \tau = \tau_g = -xMg$$

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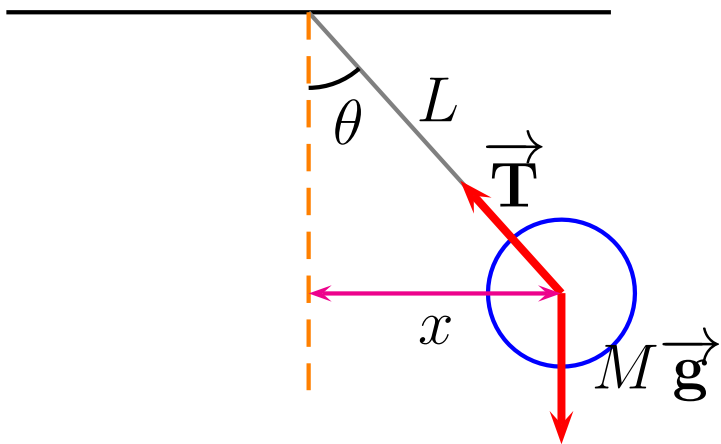
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$$x = L \sin \theta$$

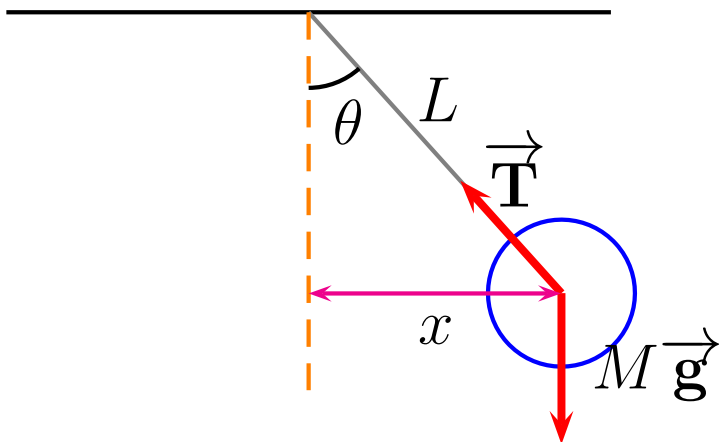


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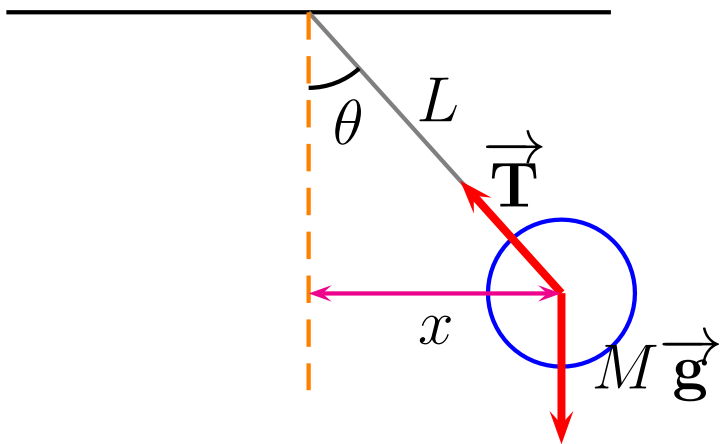
$$x = L \sin \theta \quad I = ML^2$$

# Pendulum II

Under the right conditions, a pendulum will undergo simple harmonic motion.

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Simple Pendulum - Massless connector



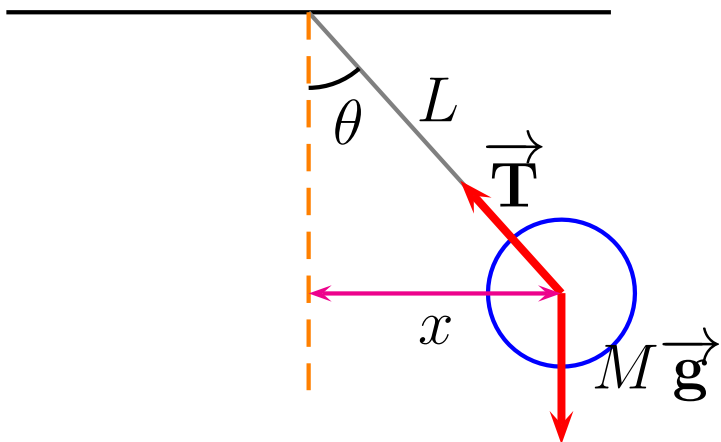
$$L (\sin \theta) M g = -M L^2 \alpha$$

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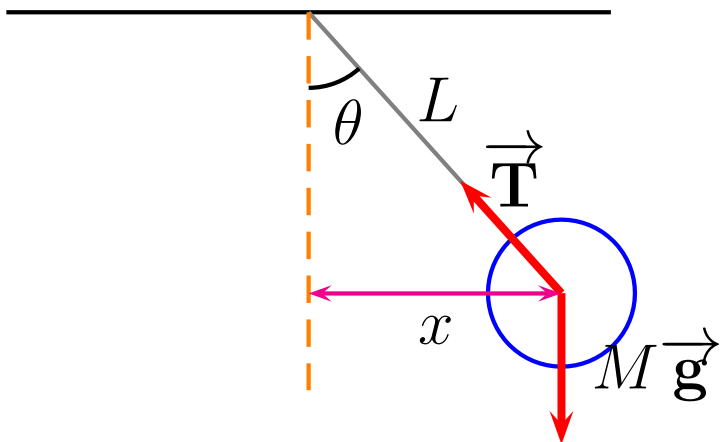
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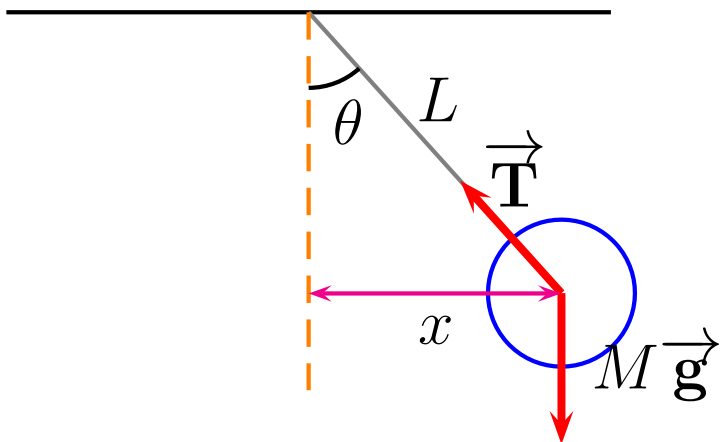
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True equation  
for pendulum