April 30, Week 15

Today: Chapter 13, Newton's Law of Gravity

Homework #11 due Monday, May 7 at 11:59pm
Mastering Physics: 5 problems from chapter 13
Written: none.

Kepler's Laws

Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

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Kepler's Laws:

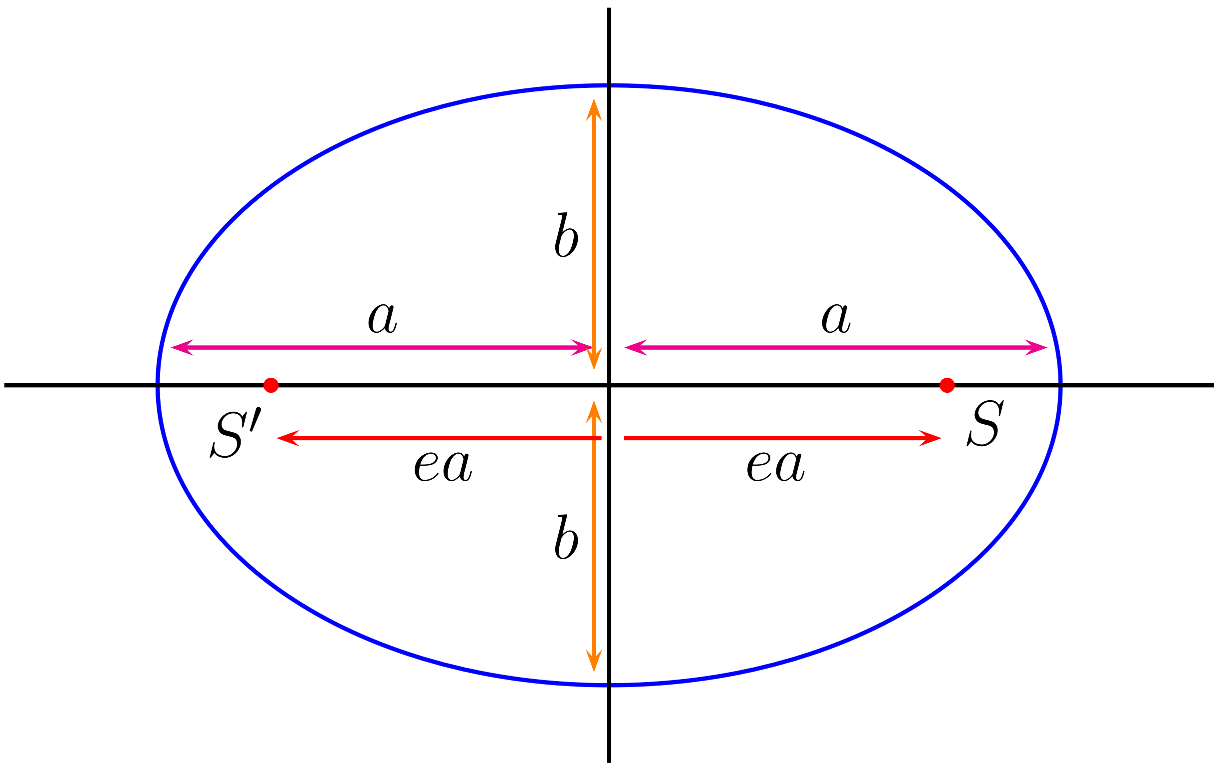
1: Each planet's orbit traces out the shape of an ellipse with the sun located at one focus.

2: The imaginary line from the sun to a planet sweeps out equal areas in equal times.

3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.

Ellipses - The Algebraic Approach

Ellipse - ovals.



a: semi-major axis

b: semi-minor axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

e: eccentricity

Kepler's First Law

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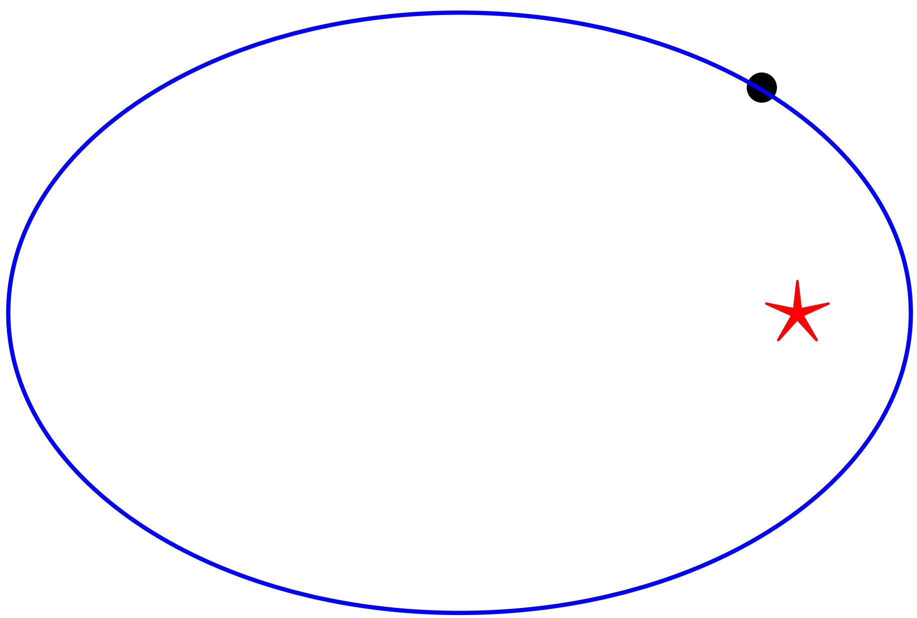

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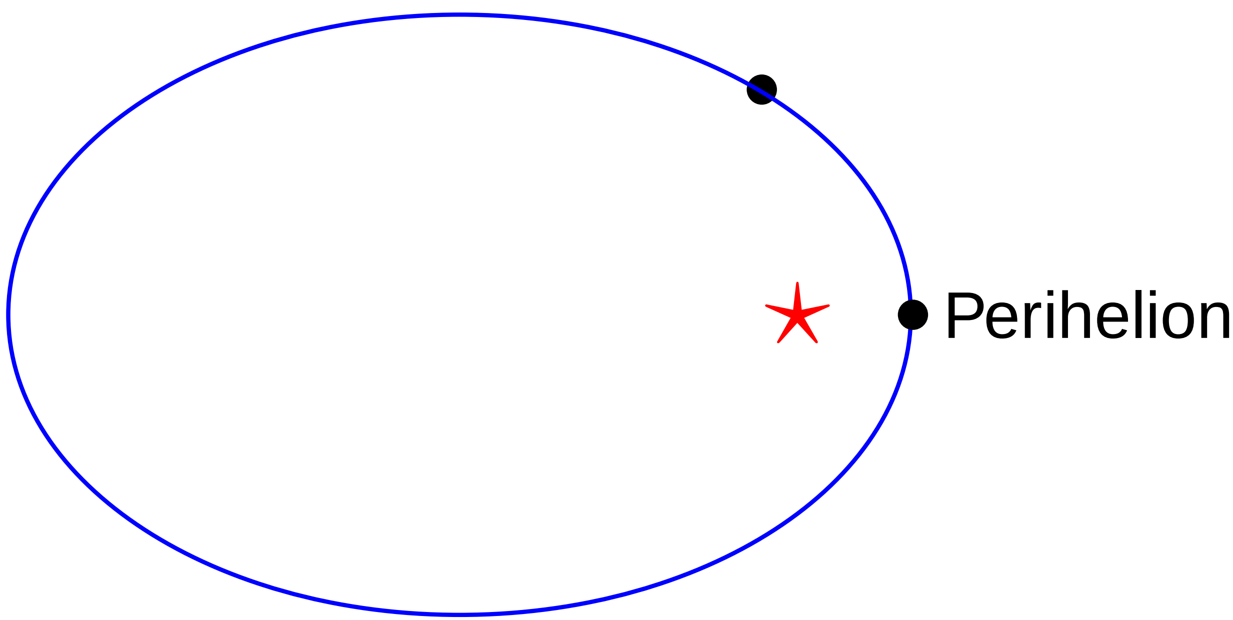

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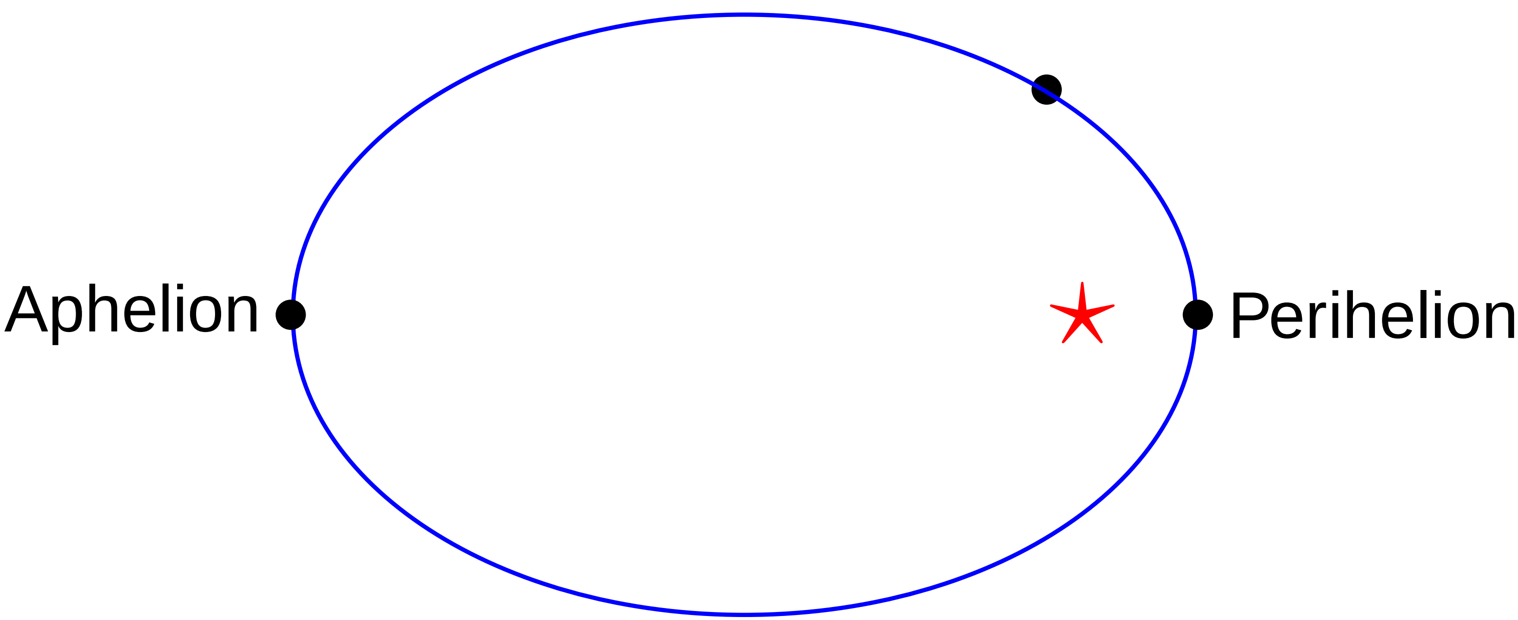

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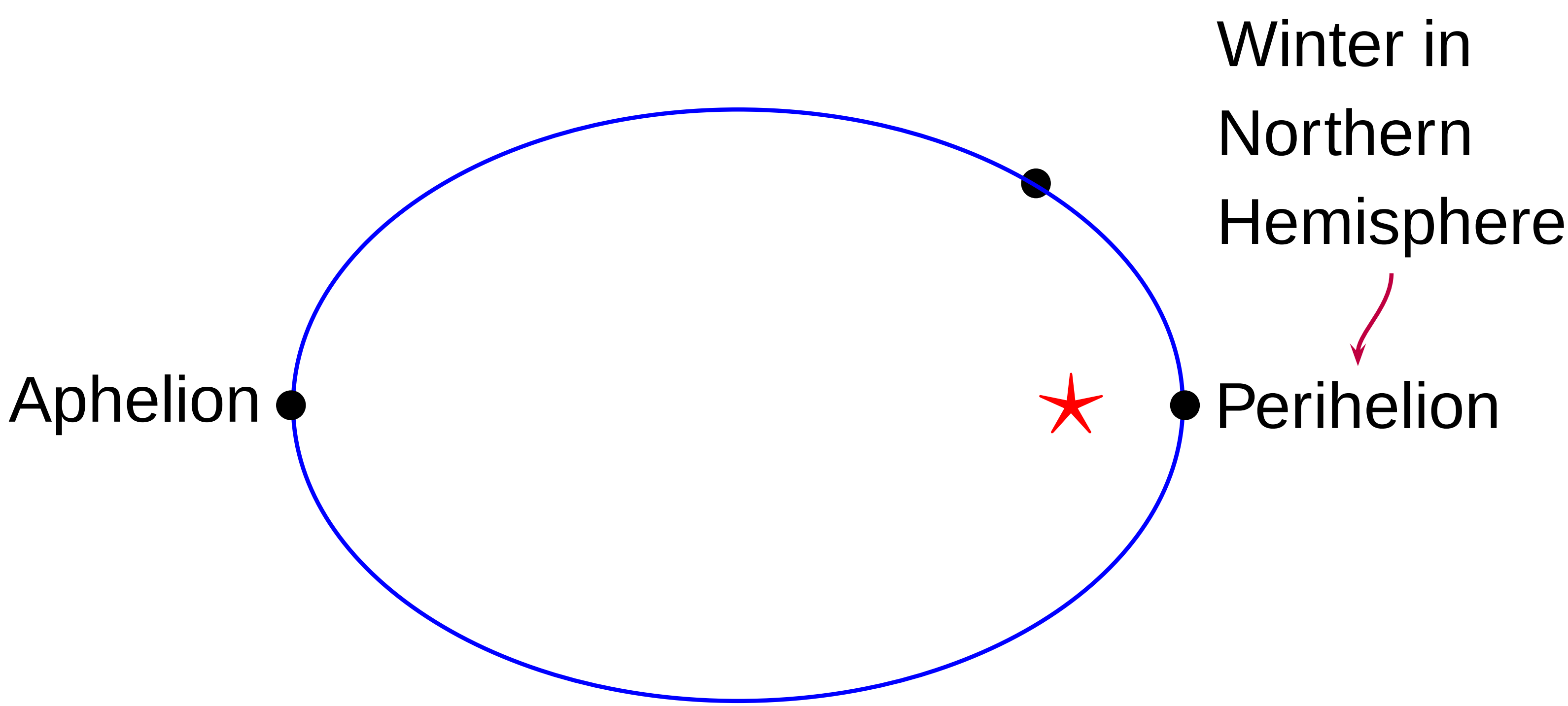

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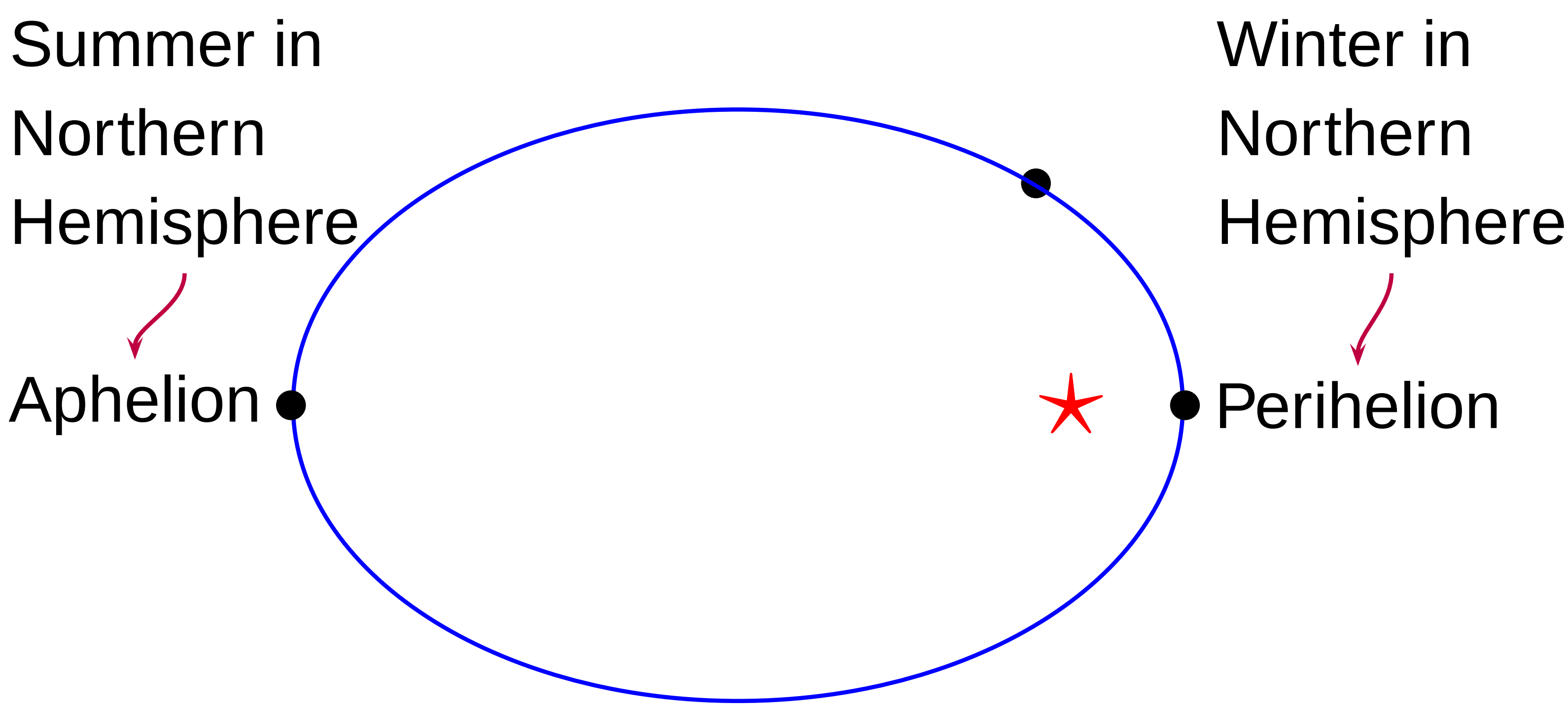

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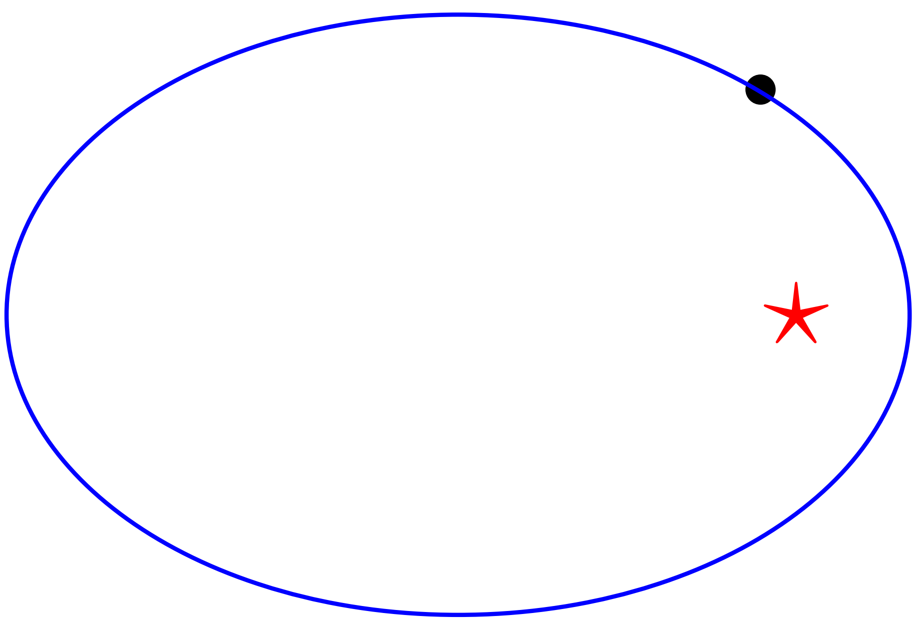

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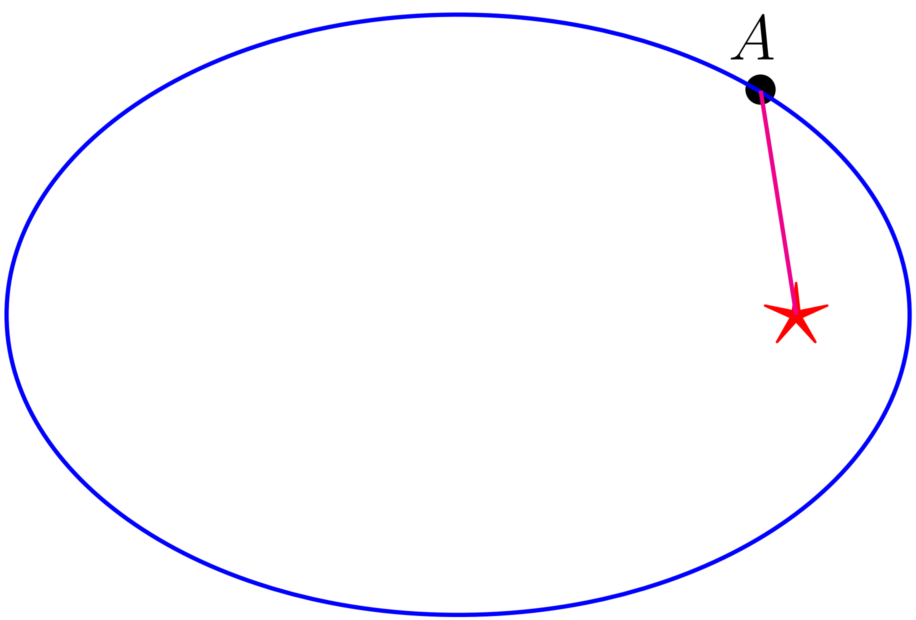

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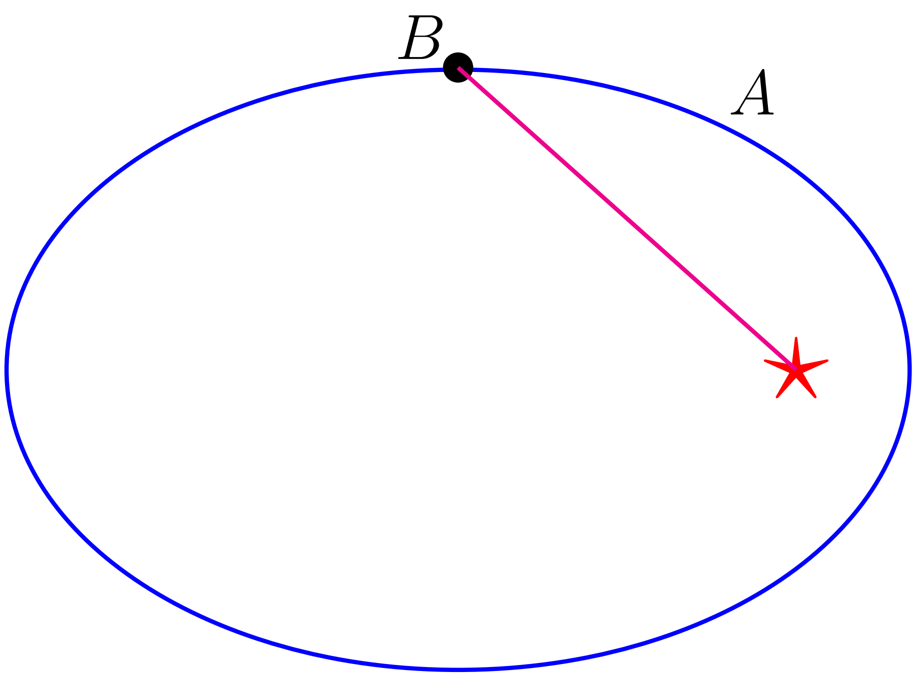

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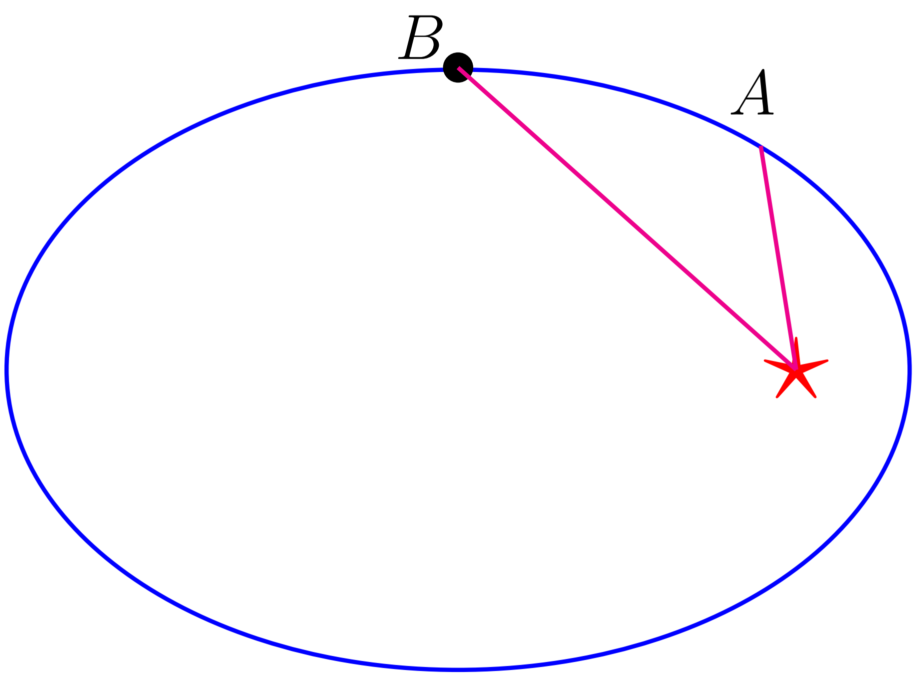

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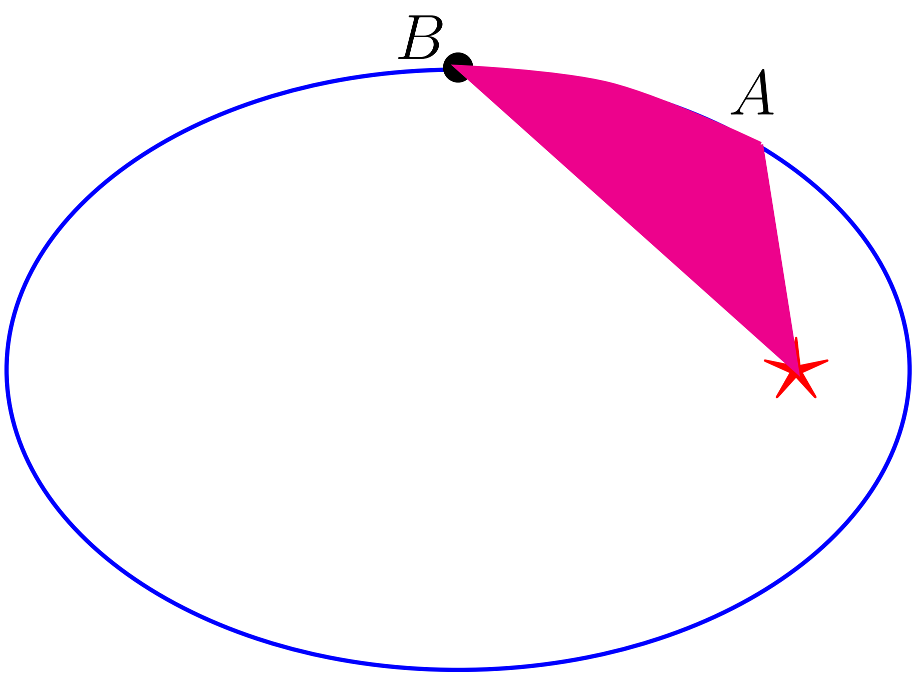

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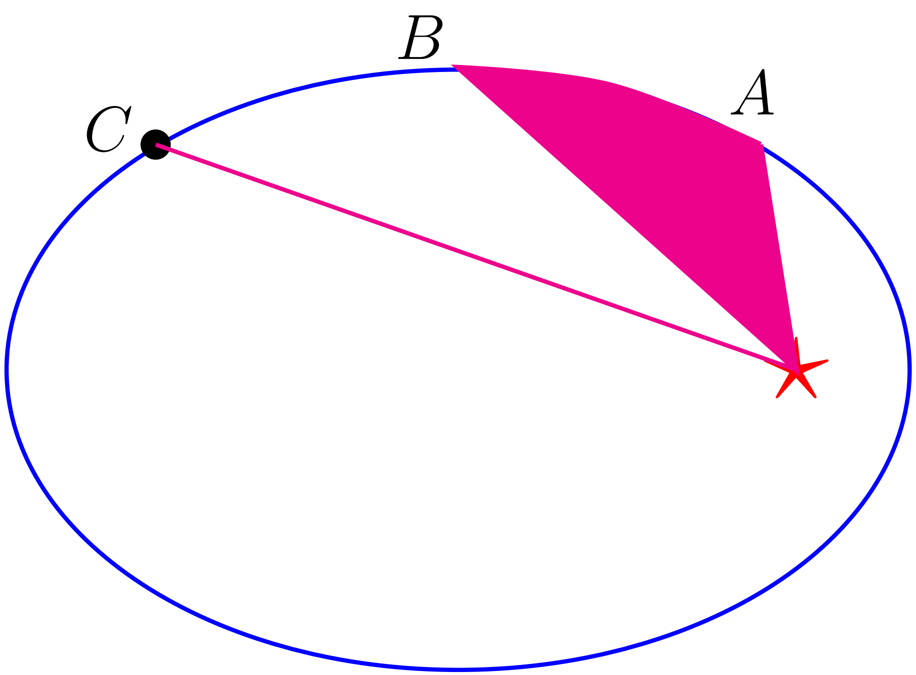

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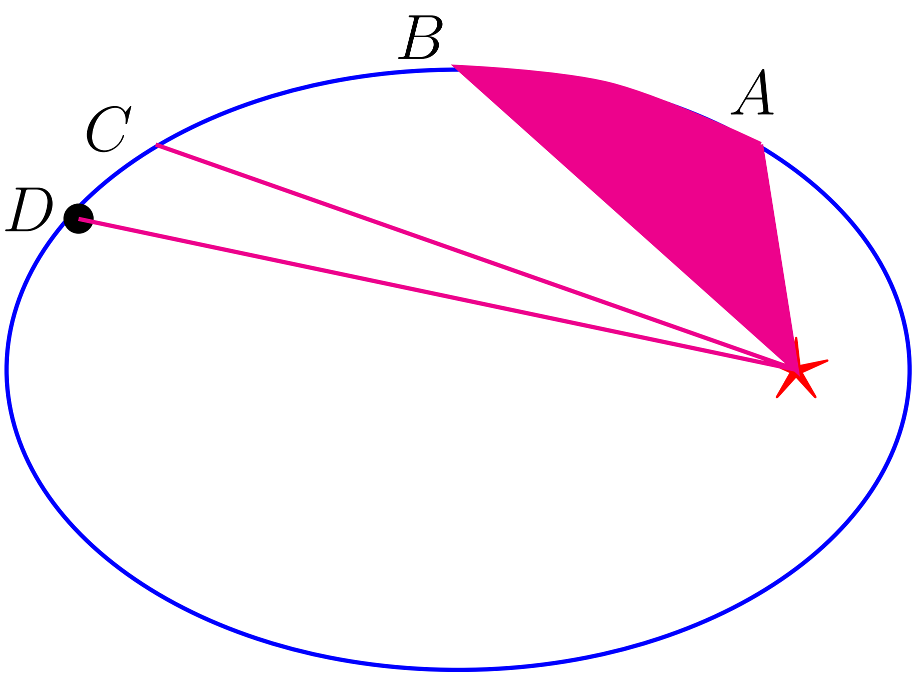

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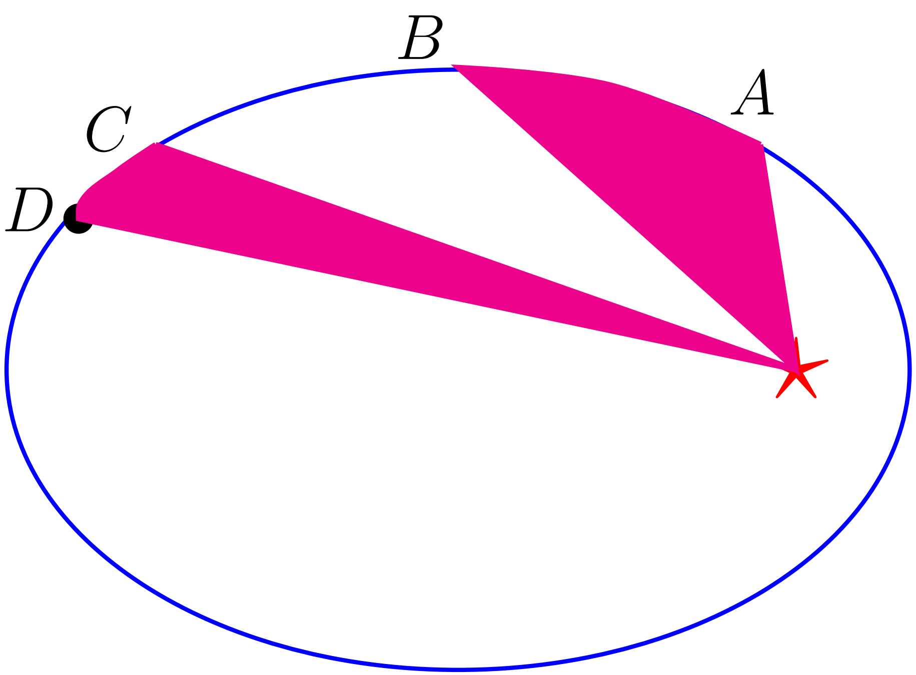

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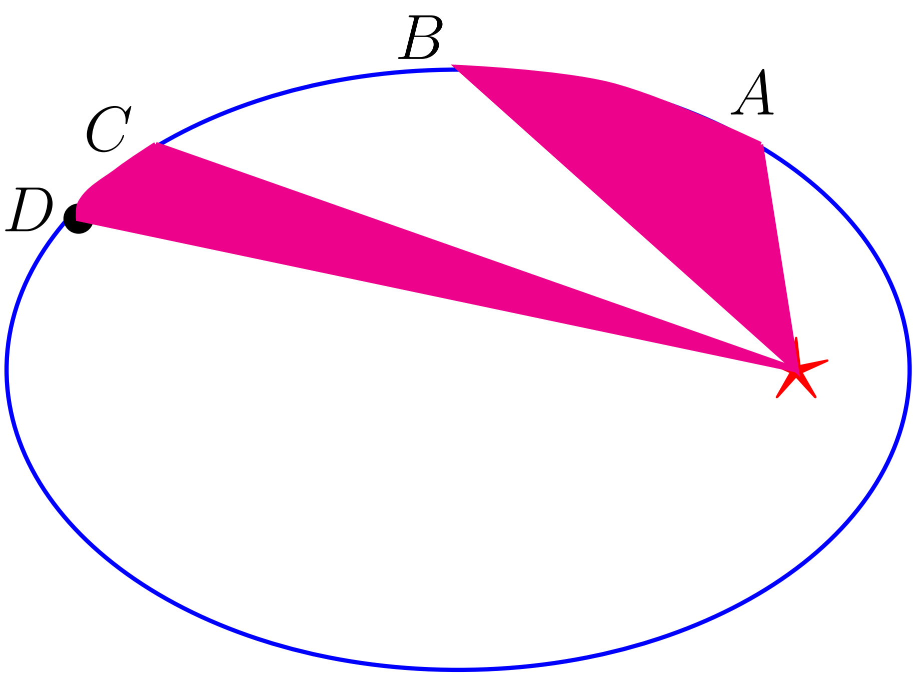

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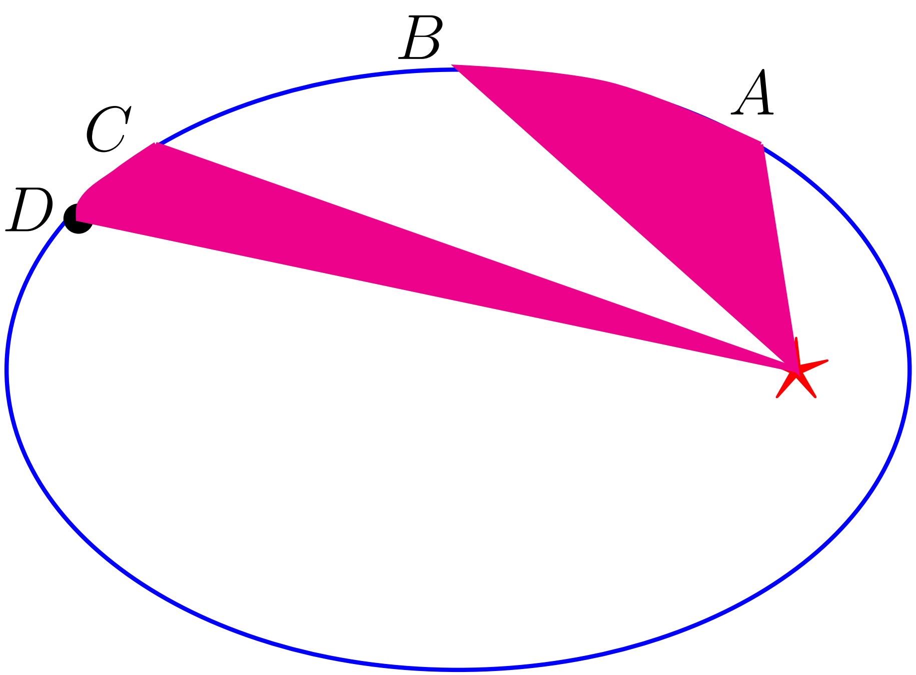

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The speed of *any* object in an elliptical orbit is NOT constant

Kepler's Second Law II

Proof:

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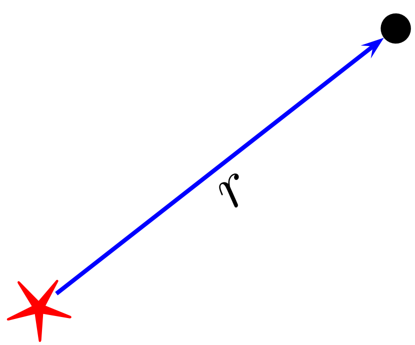

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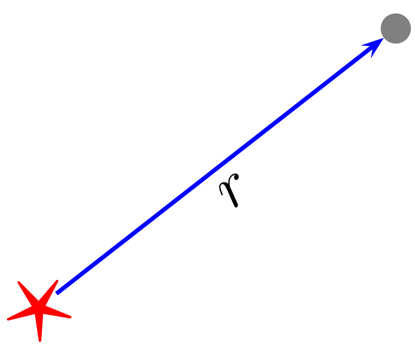

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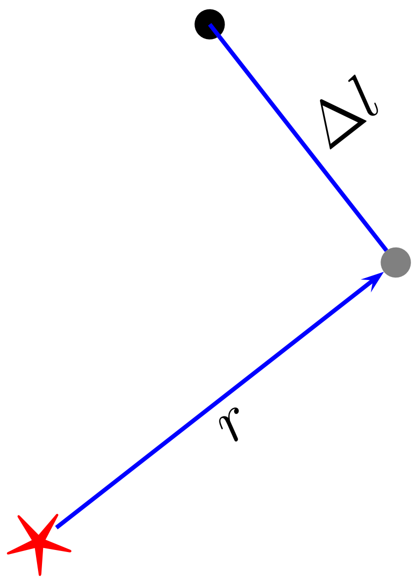

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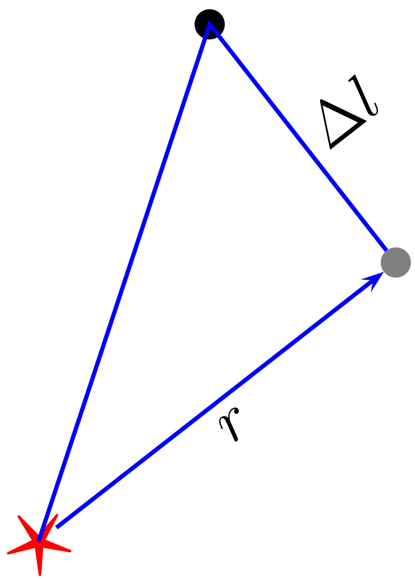

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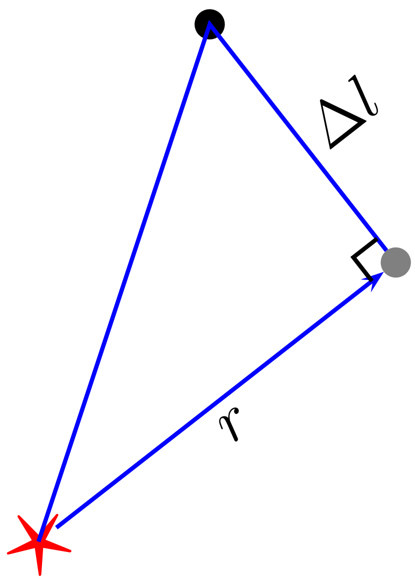

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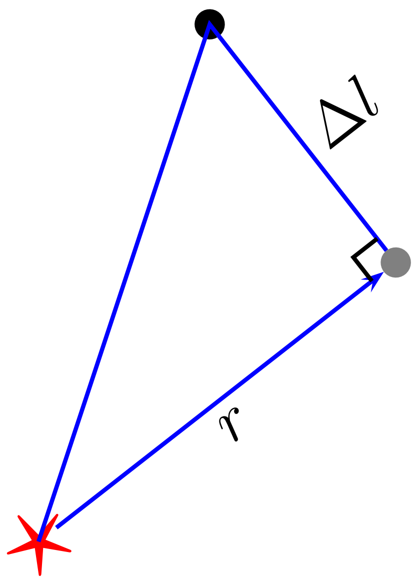

$$A \approx \frac{1}{2}(r)(\Delta l)$$

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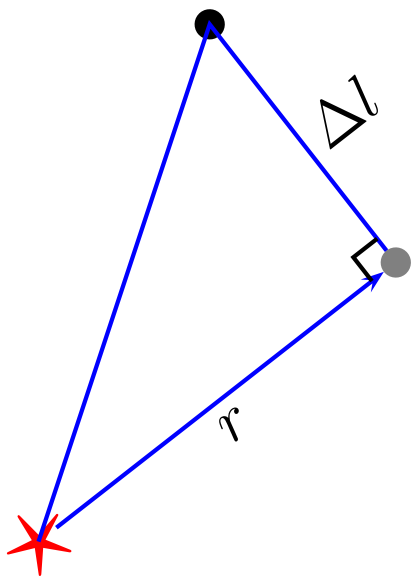

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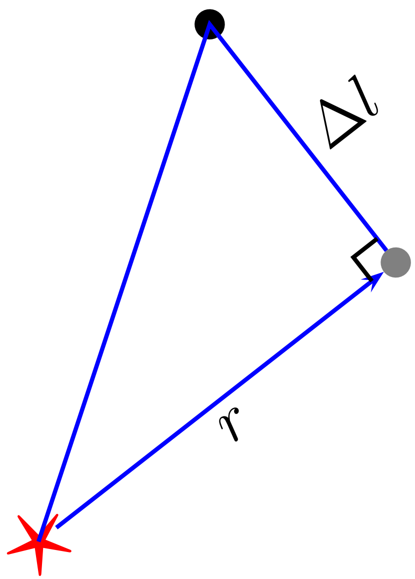

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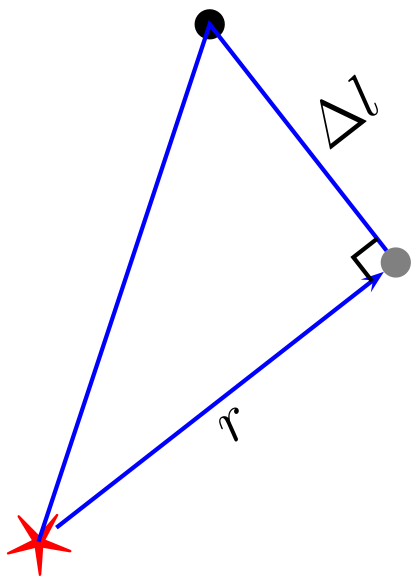

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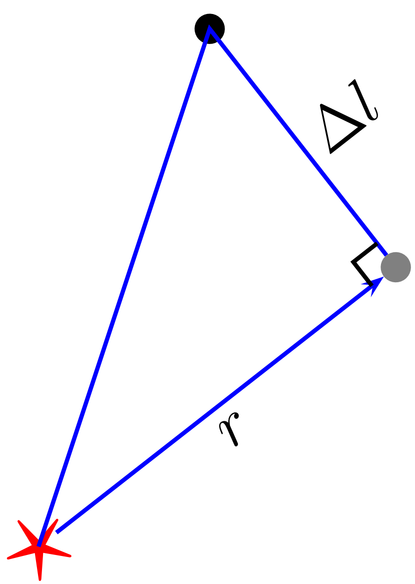

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Kepler's Second Law III

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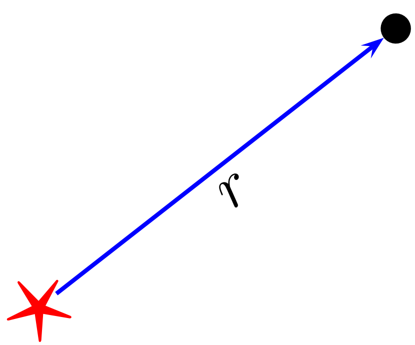

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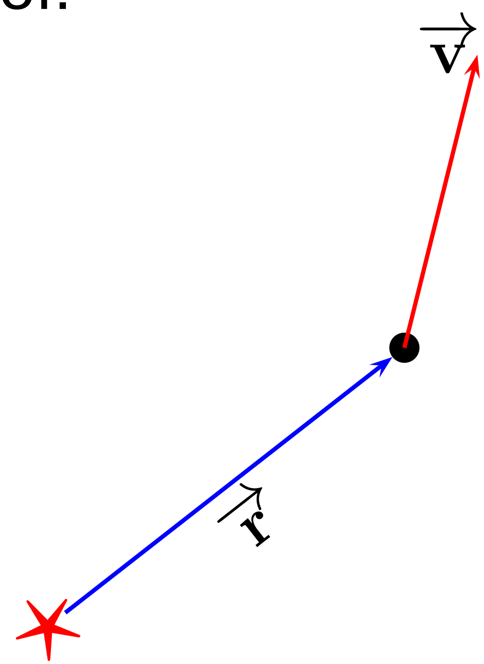

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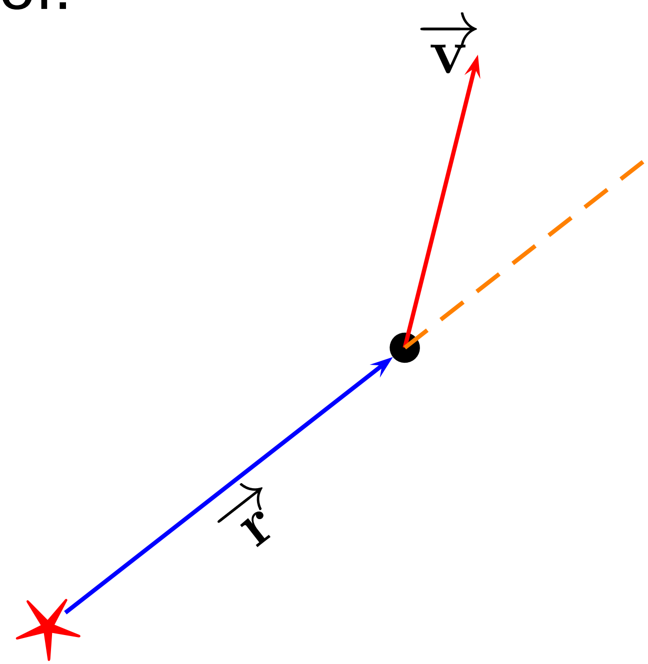

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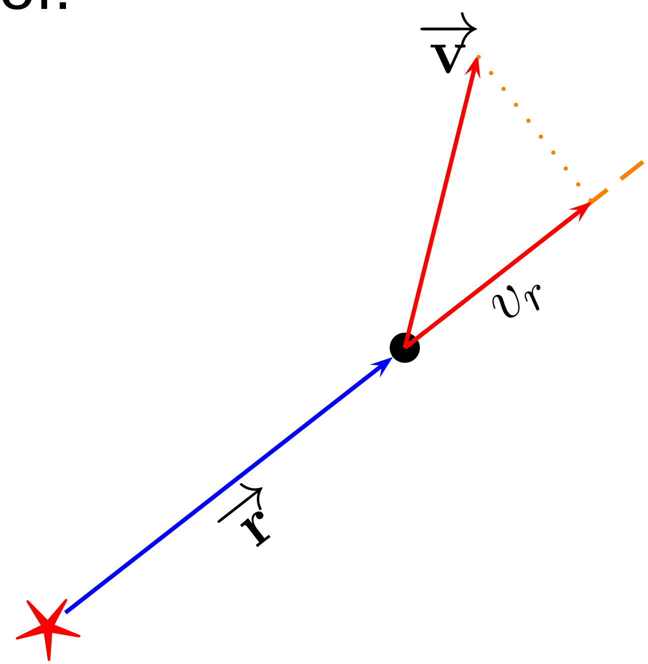

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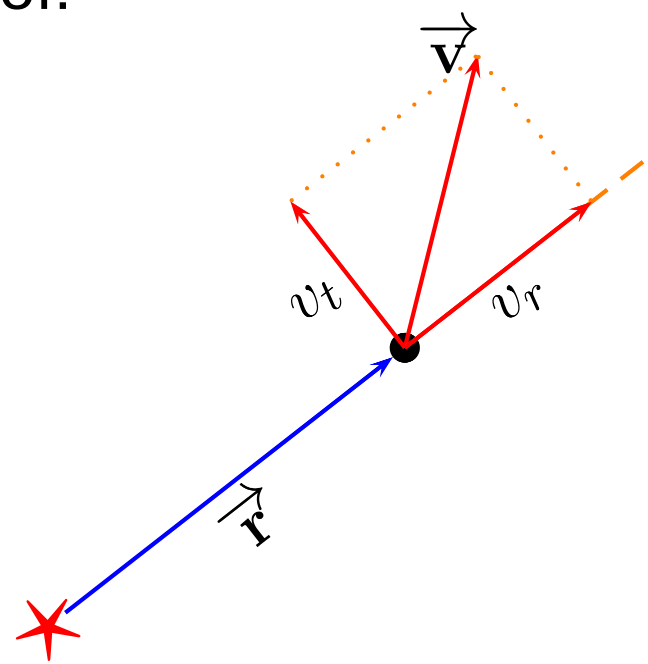

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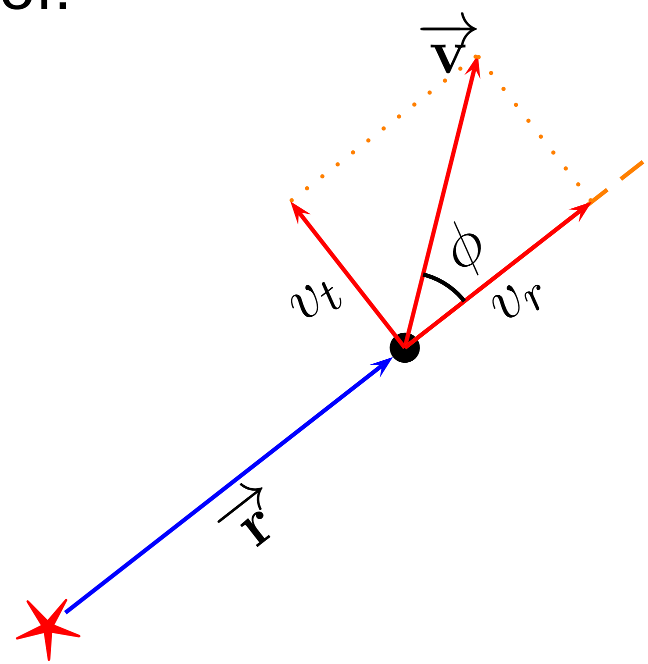

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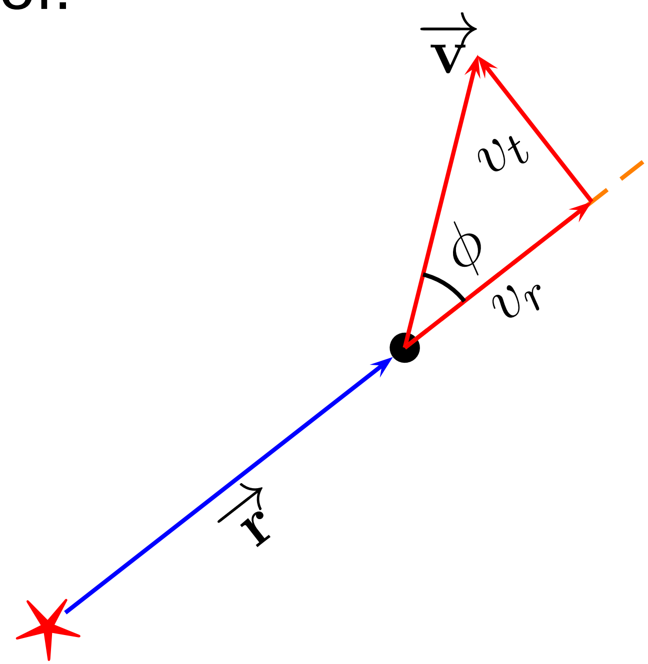

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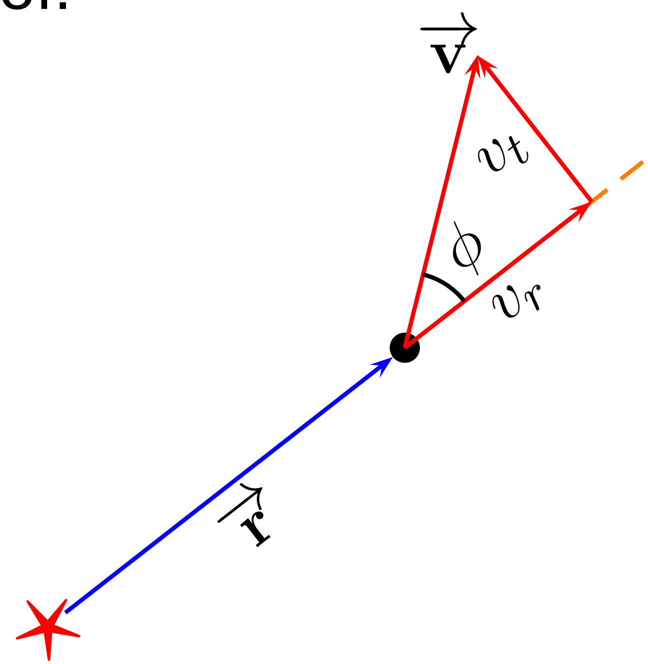

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$$v_t = v \sin\phi$$

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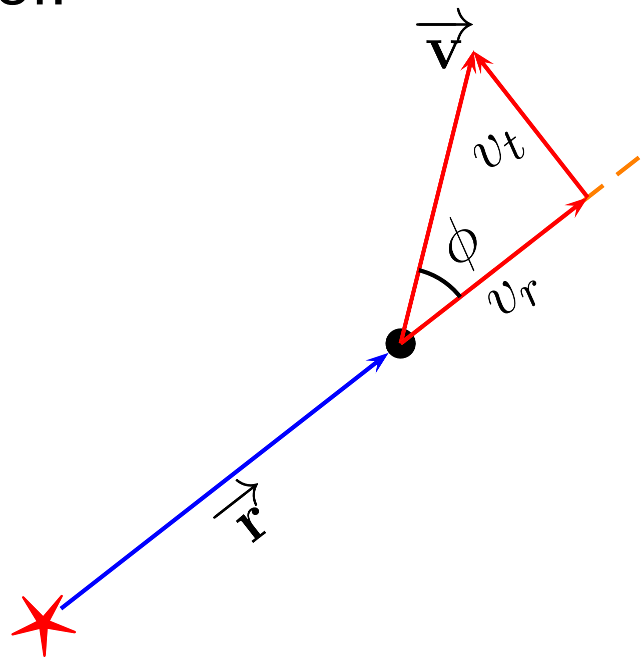

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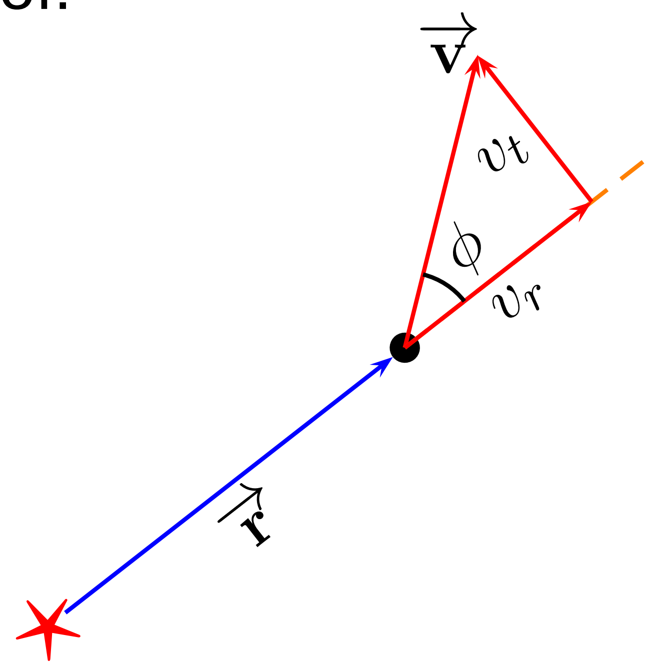

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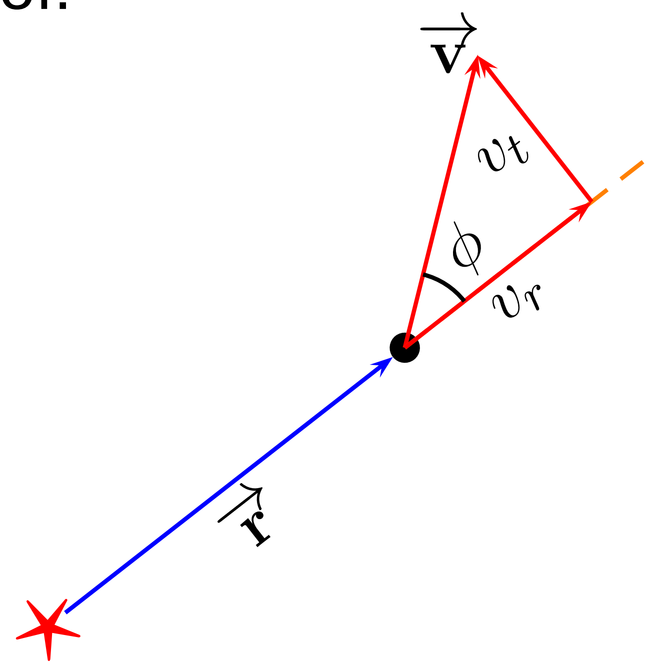
Angular Momentum of a Satellite: $\boxed{L = Mvr\sin\phi}$

Kepler's Second Law III

Proof:



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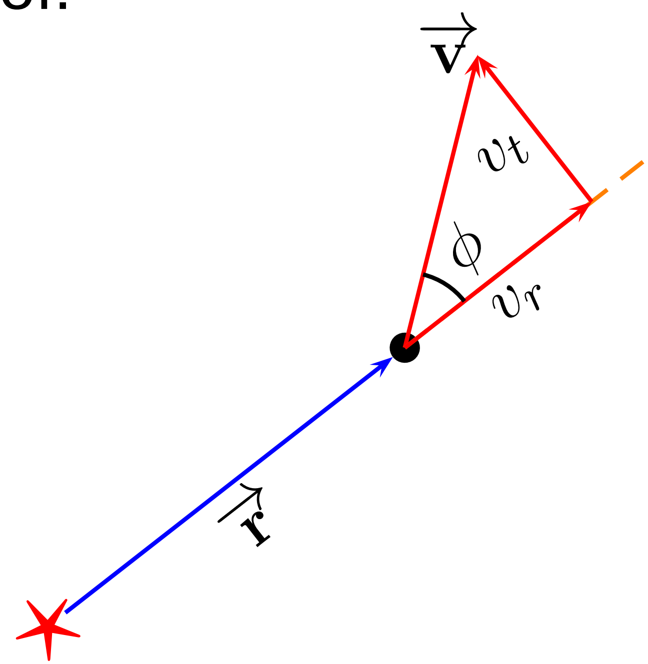

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$$\boxed{\frac{dA}{dt} = \frac{L}{2M}}$$

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Kepler's Second Law IV

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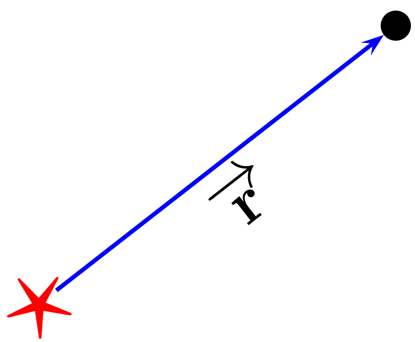
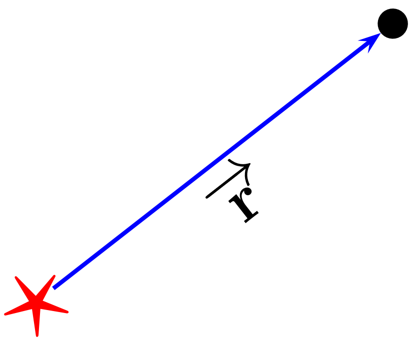


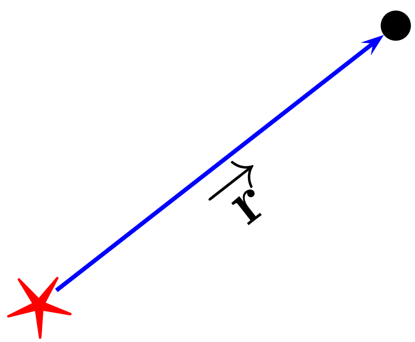
Kepler's Second Law IV



$$\frac{dA}{dt} = \frac{L}{2M}$$

$$\overrightarrow{\boldsymbol{\tau}} = \frac{d\overrightarrow{\mathbf{L}}}{dt}$$

Kepler's Second Law IV



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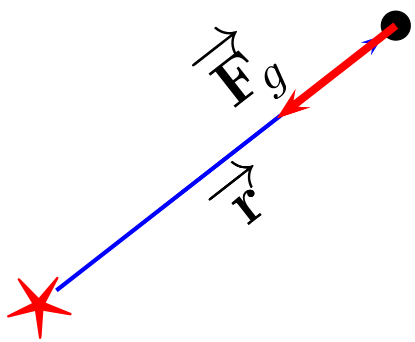
On satellites, gravity causes no torque

Kepler's Second Law IV



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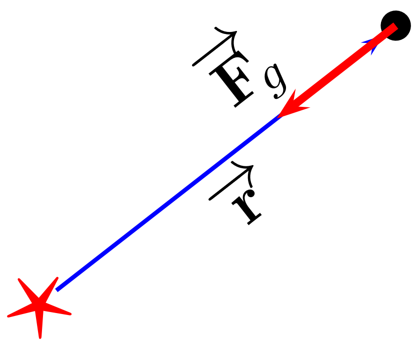

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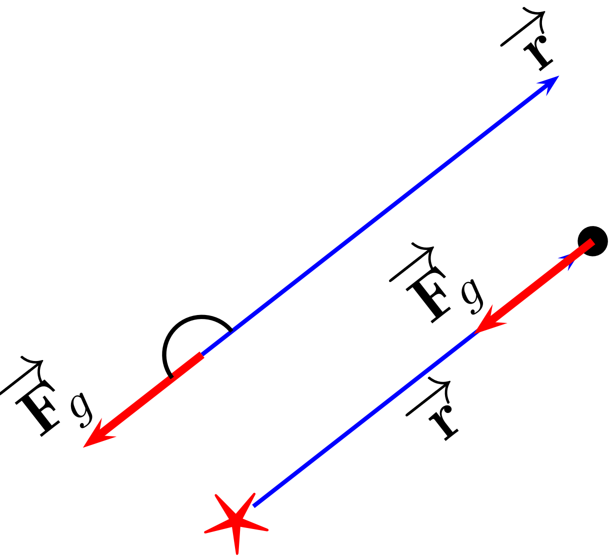

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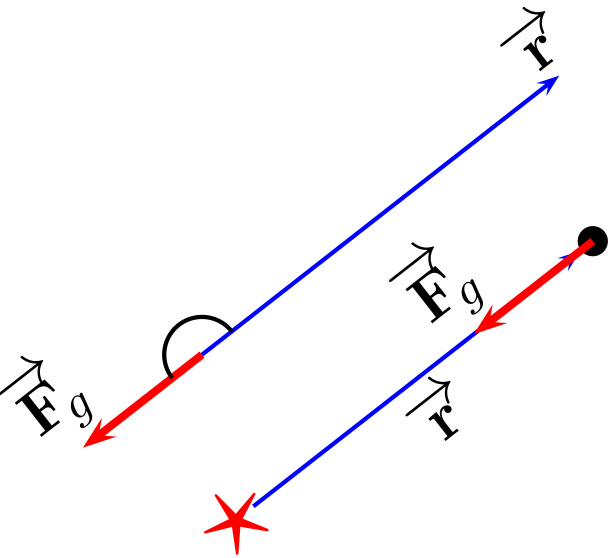

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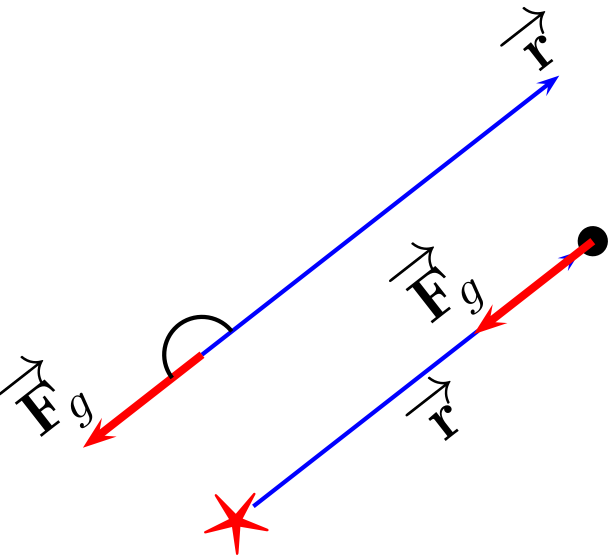
$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}_g \Rightarrow \tau = rF_g \sin 180^\circ$$

Kepler's Second Law IV



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$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}_g \Rightarrow \tau = rF_g \sin 180^\circ = 0$$

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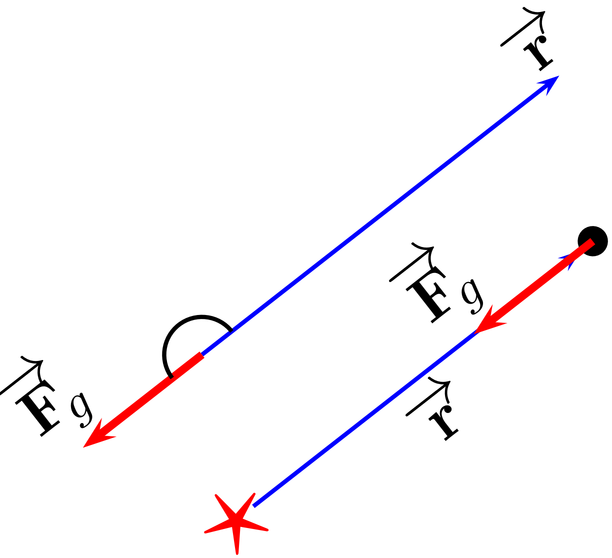

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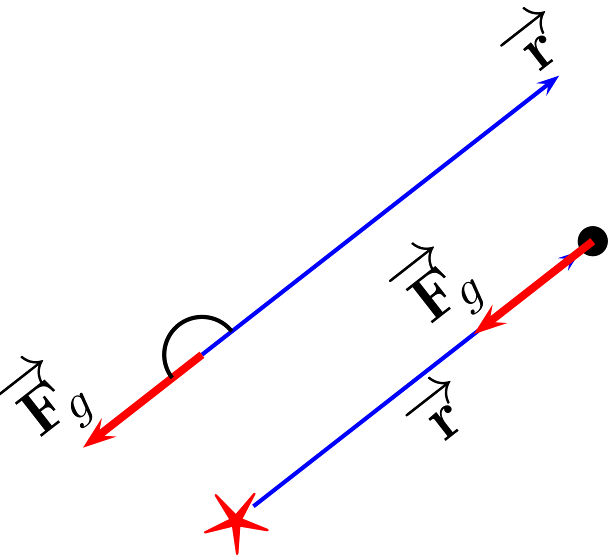
The angular momentum of a satellite is constant

Kepler's Second Law IV



$$\frac{dA}{dt} = \frac{L}{2M}$$

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equal ΔA for equal Δt

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At perihelion and aphelion the angle ϕ for satellite motion is 90°.

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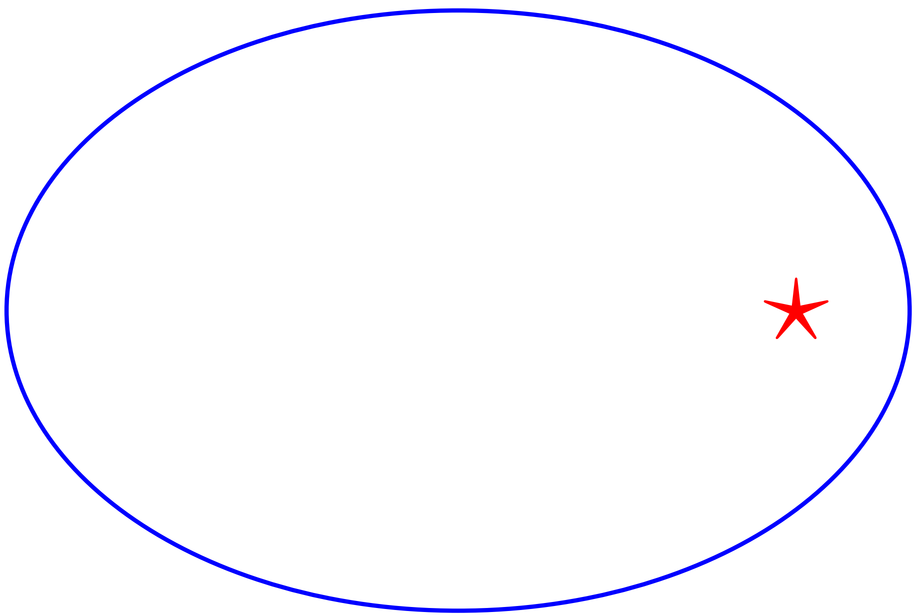

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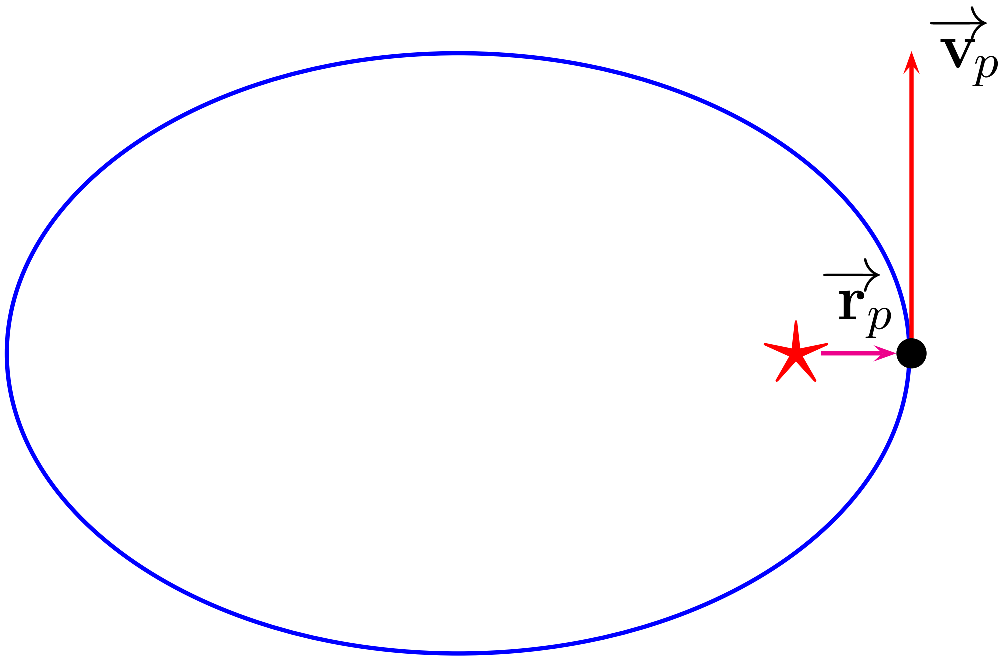

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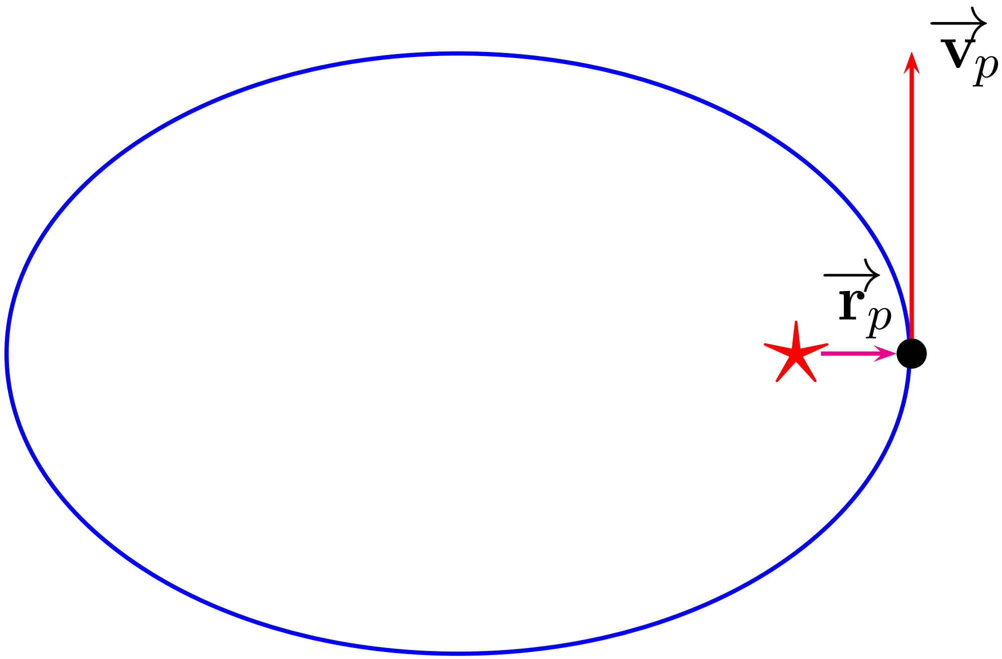

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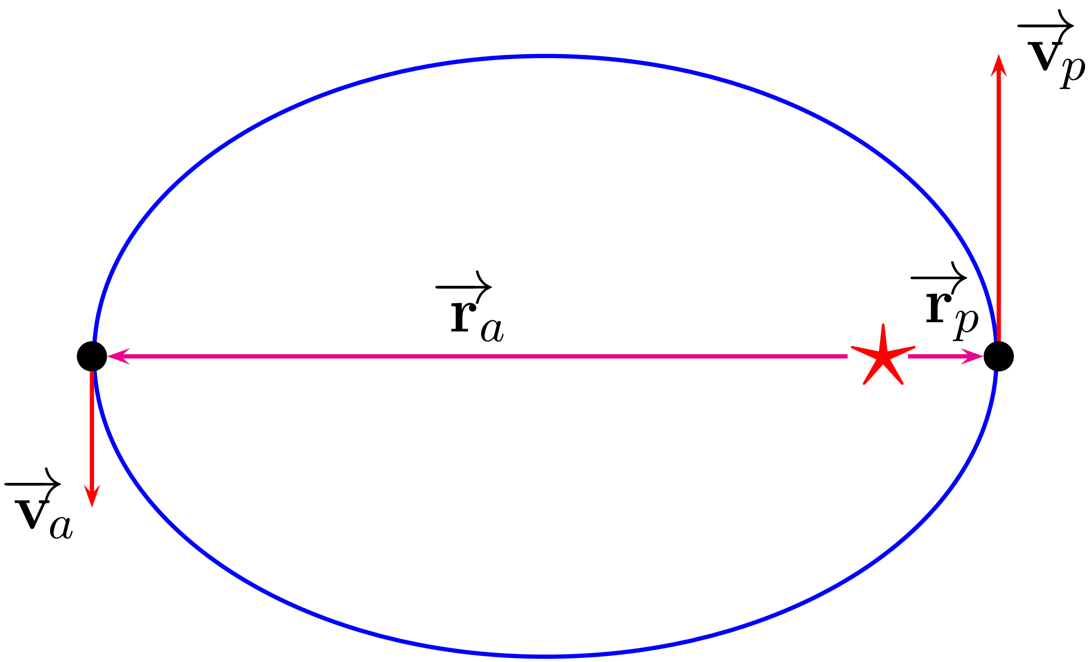

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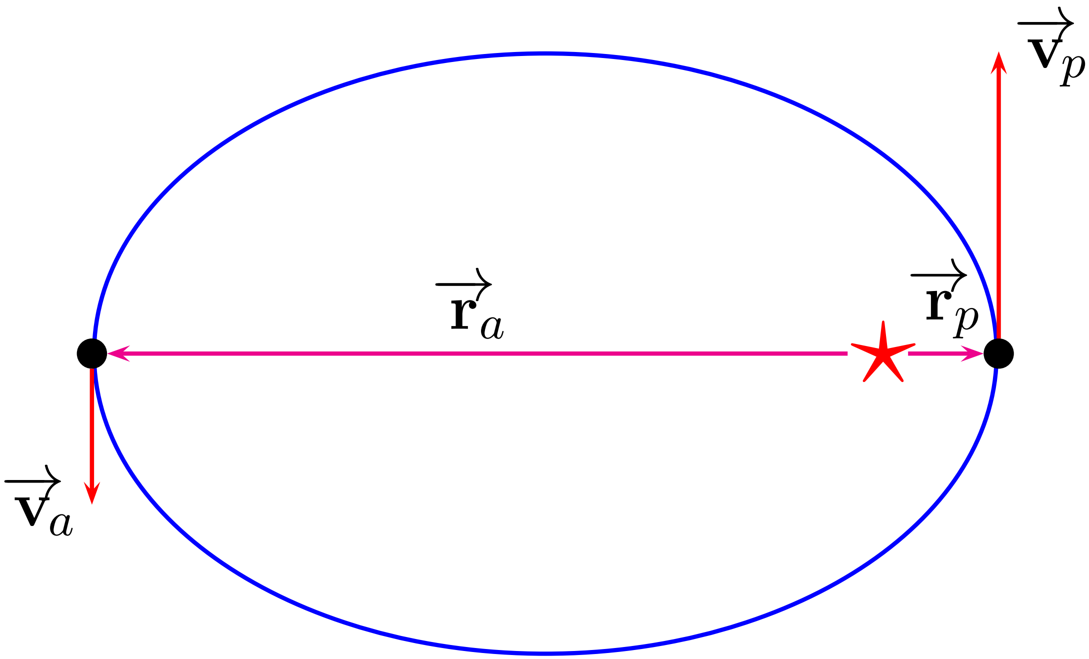

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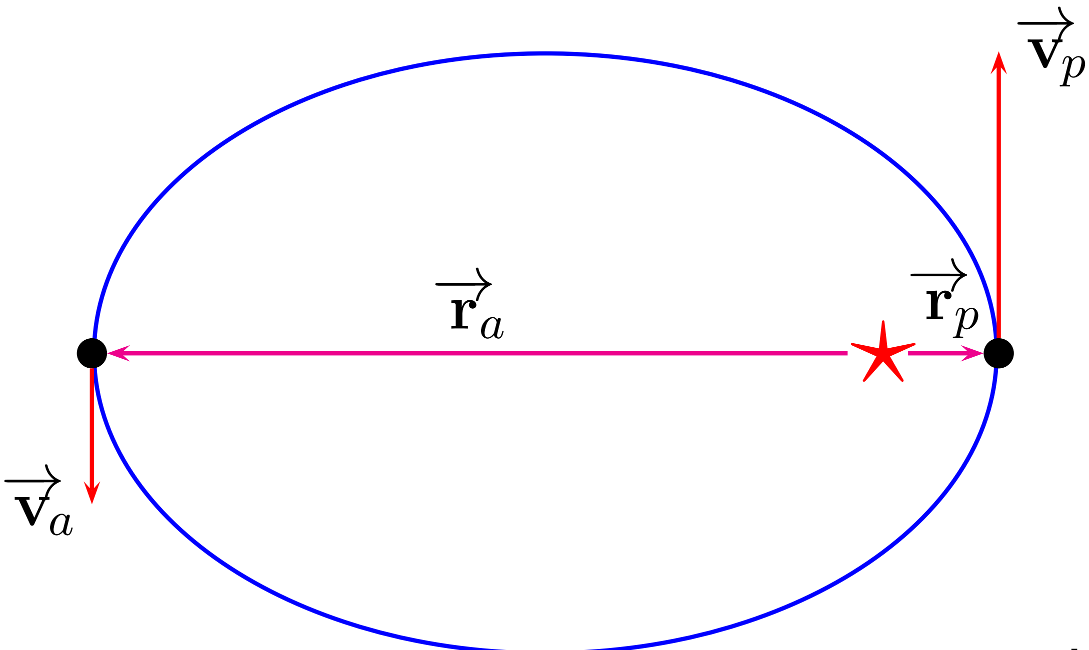

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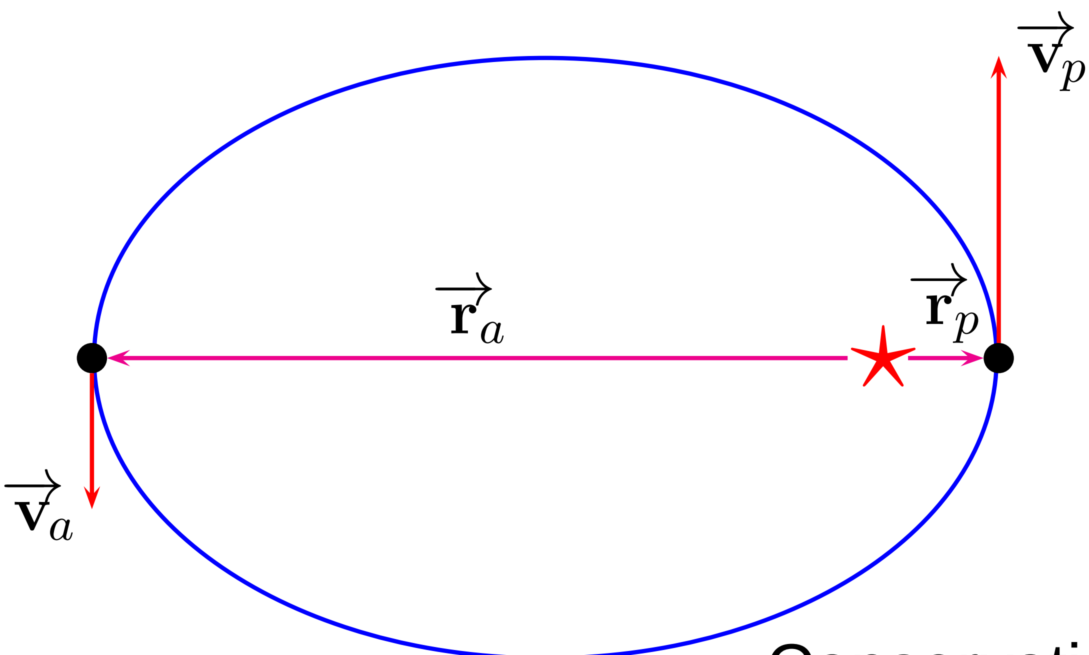

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3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.

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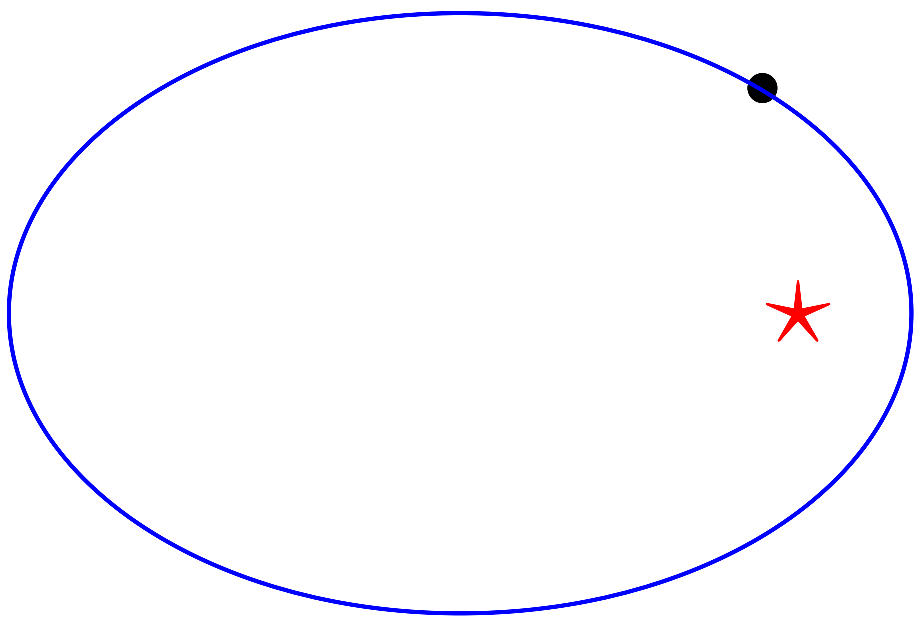

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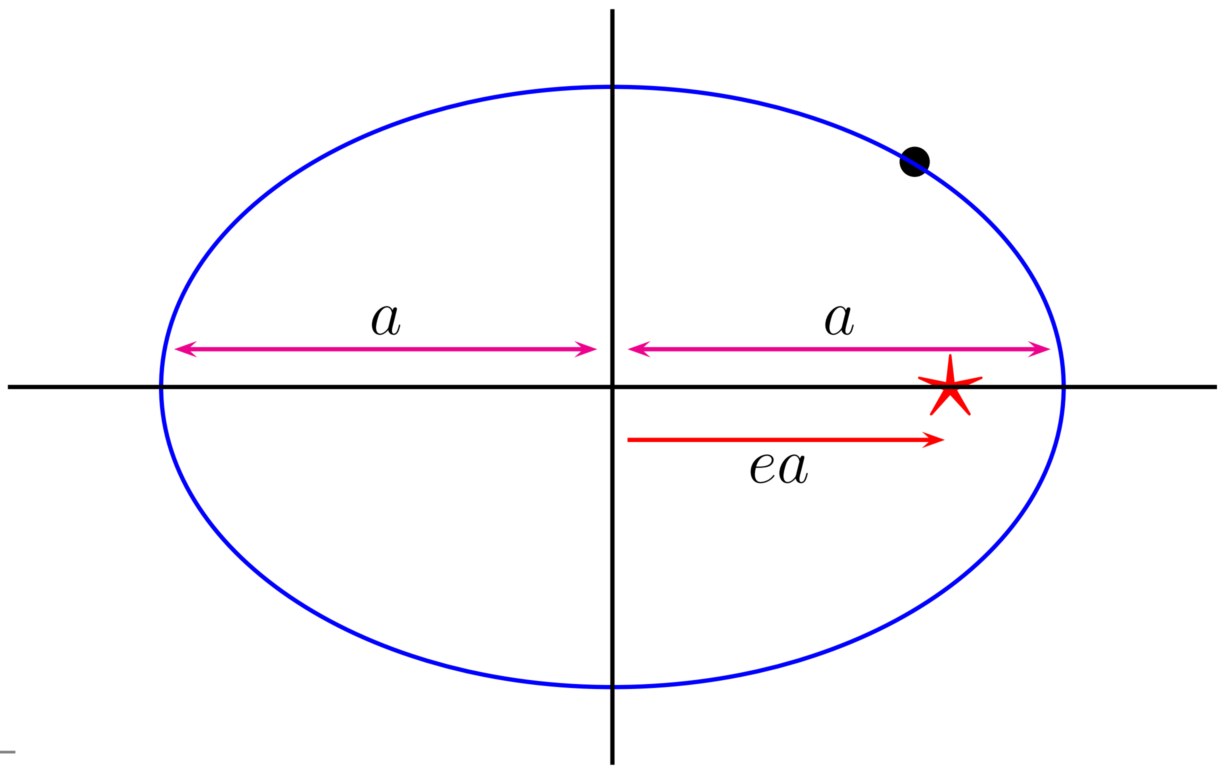

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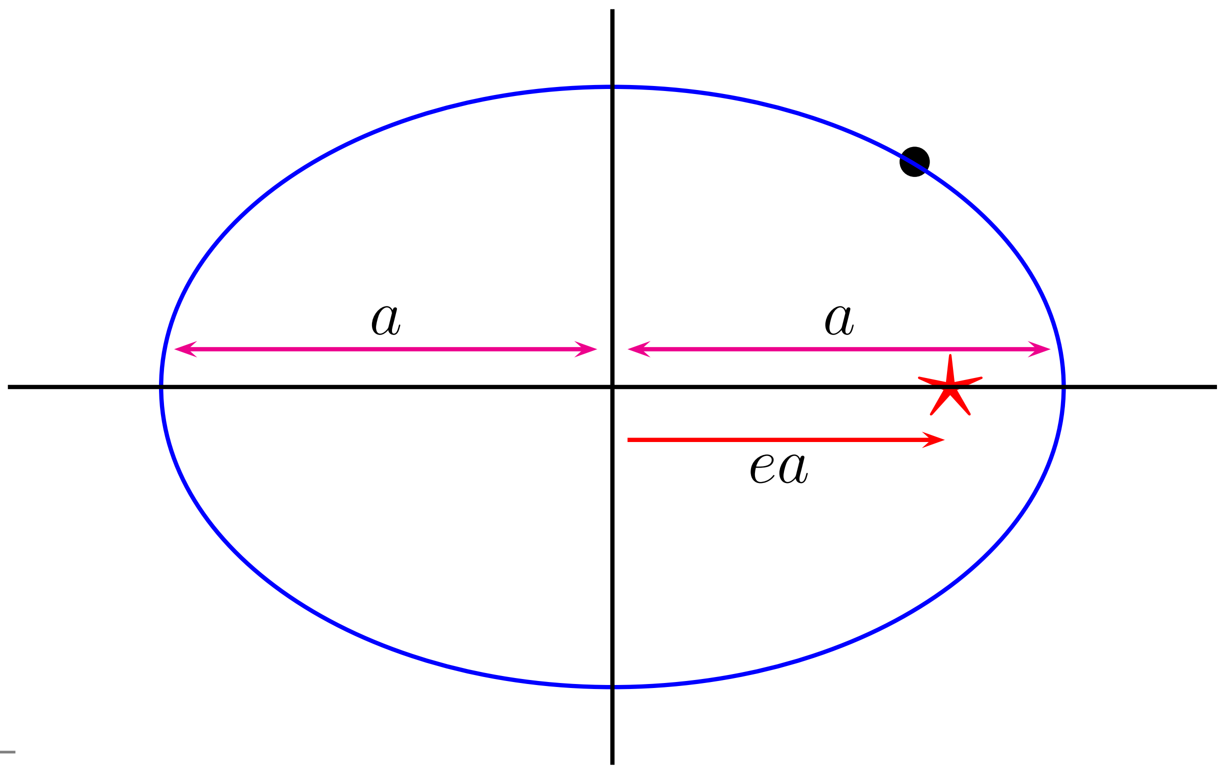

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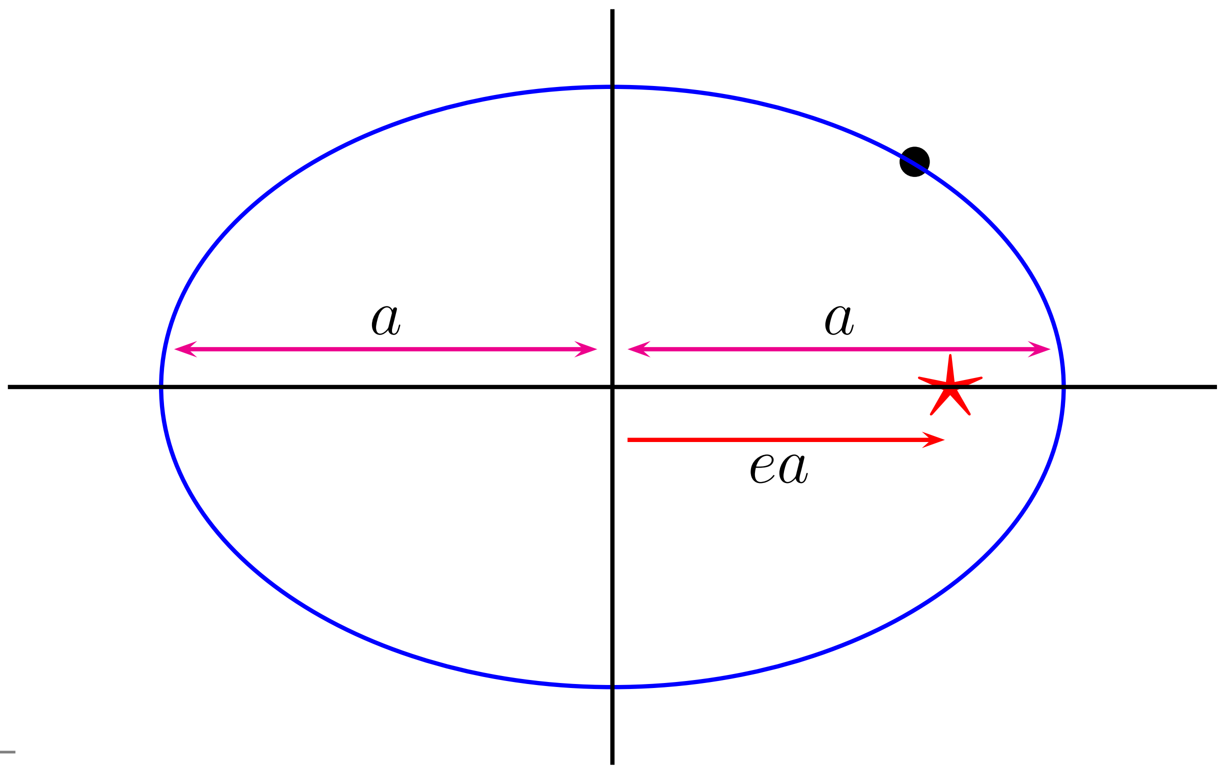

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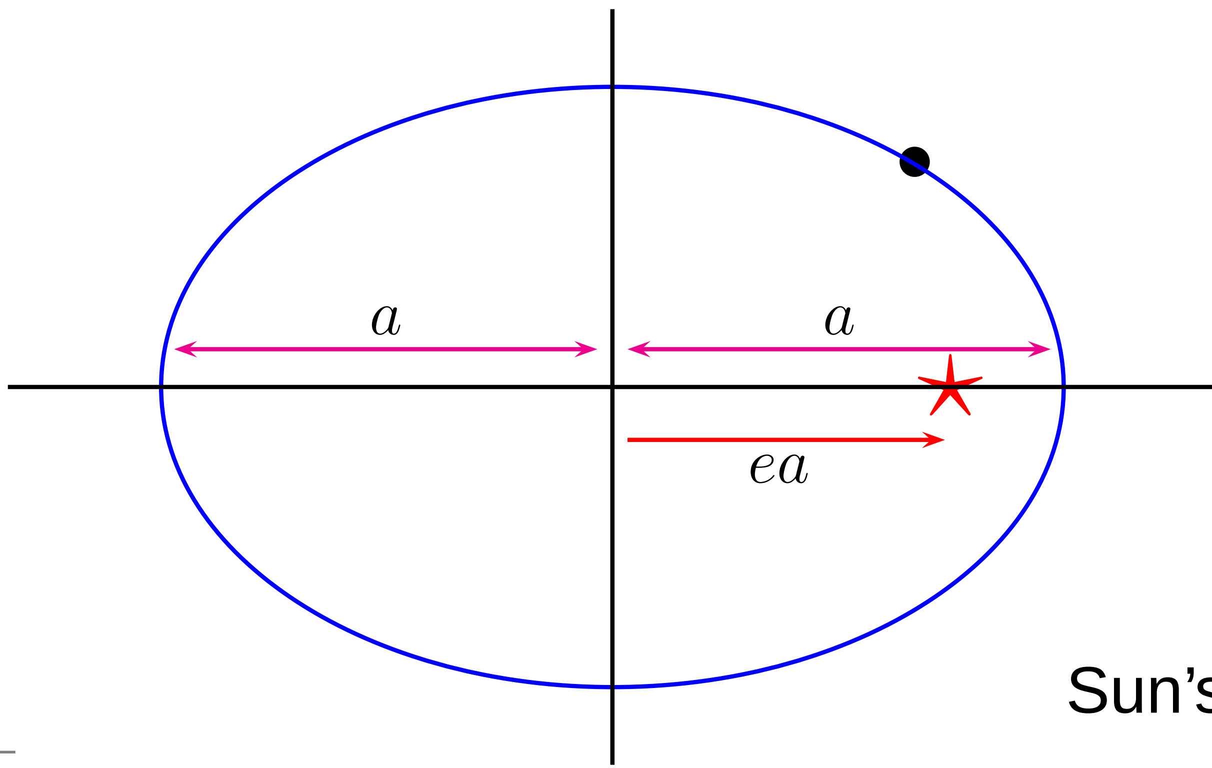

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Sun's Mass