April 25, Week 14

Today: Chapter 13, Newton's Law of Gravity

Exam #5, Friday, April 27
On Chapters 9 and 10

Review Session: Thursday, April 26, 7:30PM, Room 114 of Regener Hall.

Practice Exam on Website.

Practice Problems on Mastering.

Satellites

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<u>Example:</u> A projectile is fired, $1\,m$ above the ground, horizontally with a speed of $8000\,m/s$. How far above the ground is it after $1\,s$?

Satellites

The earth is not flat! It has a curvature of roughly $8000\,m$ to $5\,m$ (horizontal to vertical).

Example: A projectile is fired, $1\,m$ above the ground, horizontally with a speed of $8000\,m/s$. How far above the ground is it after $1\,s$?

Satellite - Any projectile with sufficient horizontal velocity to “miss" the ground.

Orbits

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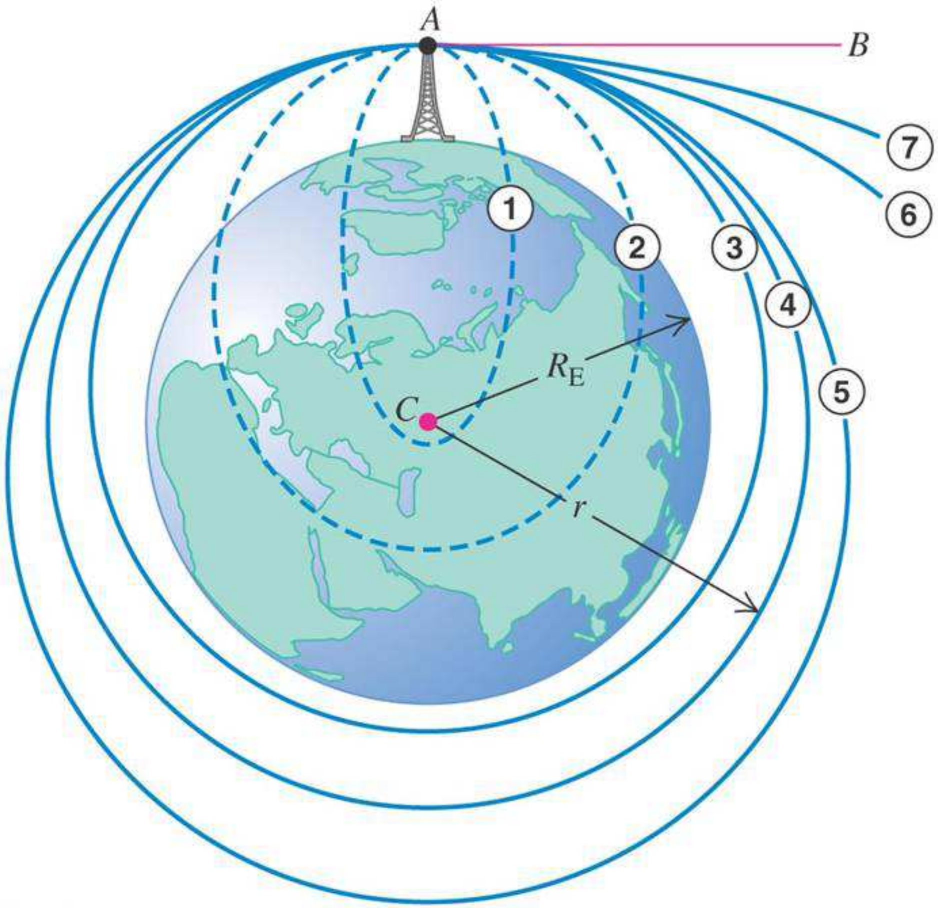
Newton showed that when gravity is the only force doing work, the only allowed closed orbits are circular or elliptical in shape. While the only open orbits are parabolic or hyperbolic.

Orbits II

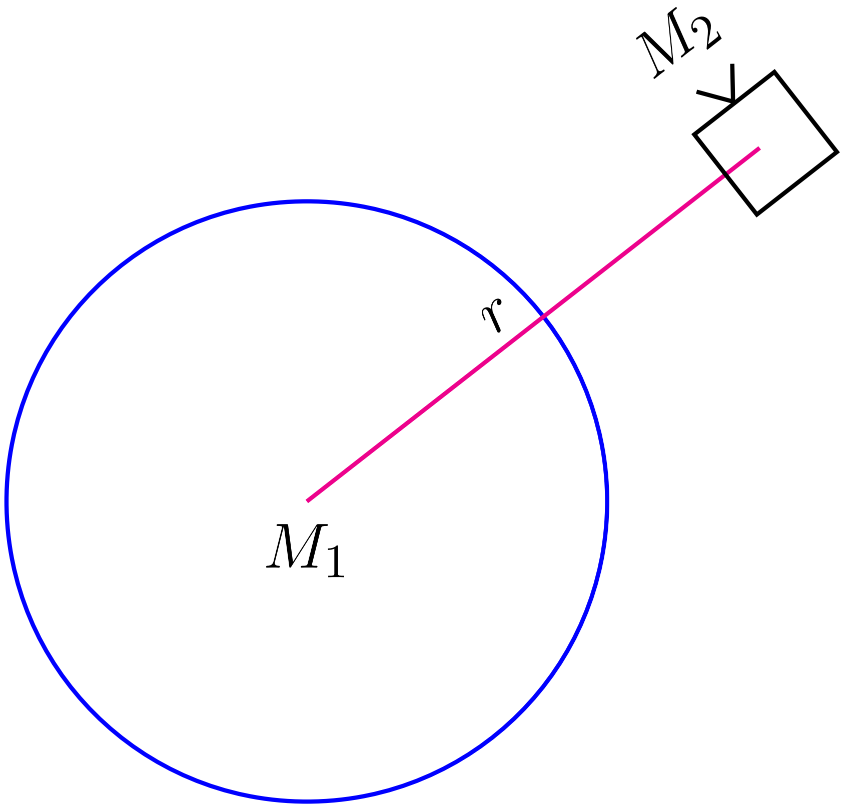
The initial velocity of the satellite determines whether the orbit is open or closed.



A projectile is launched from *A* toward *B*. Trajectories (1) through (7) show the effect of increasing initial speed.

Circular Orbits

In circular orbit, gravity creates the centripetal acceleration.



$$\sum \overrightarrow{\mathbf{F}}_2 = M_2 \overrightarrow{\mathbf{a}}$$

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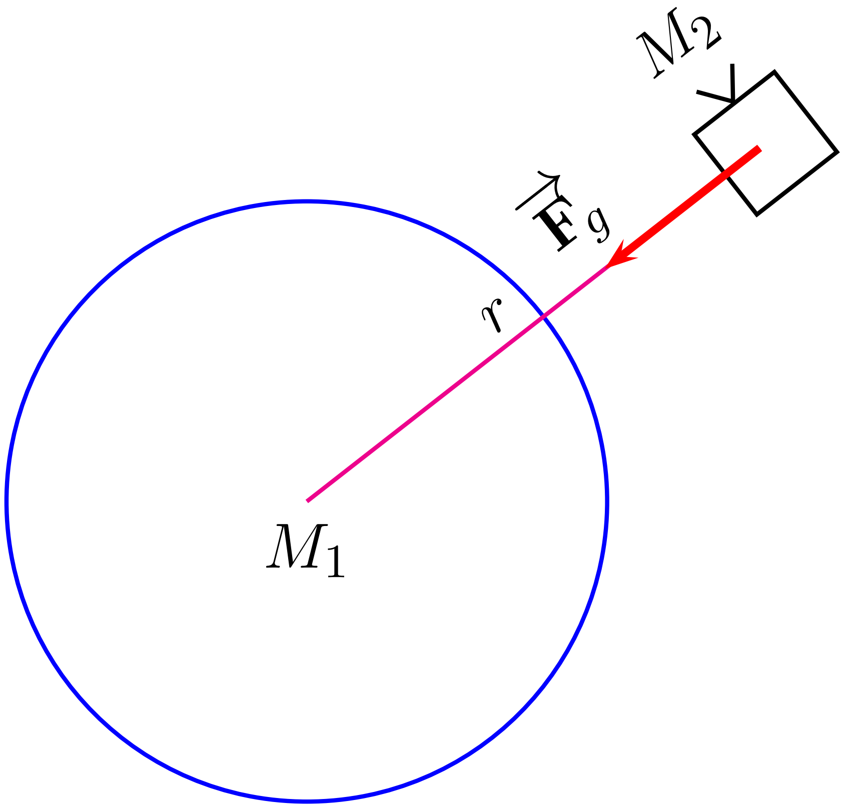

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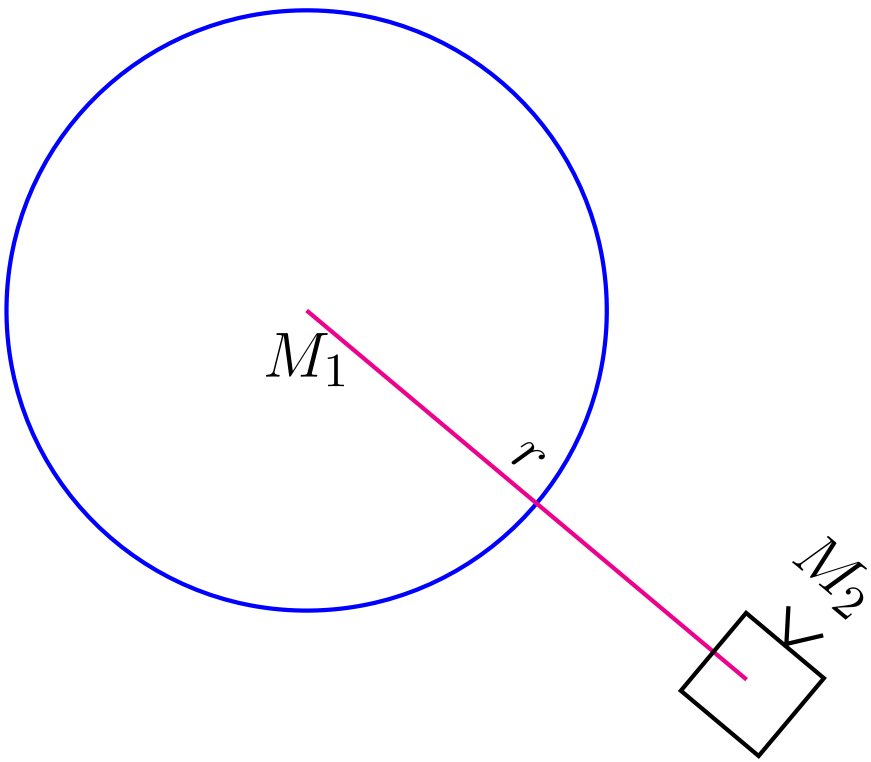

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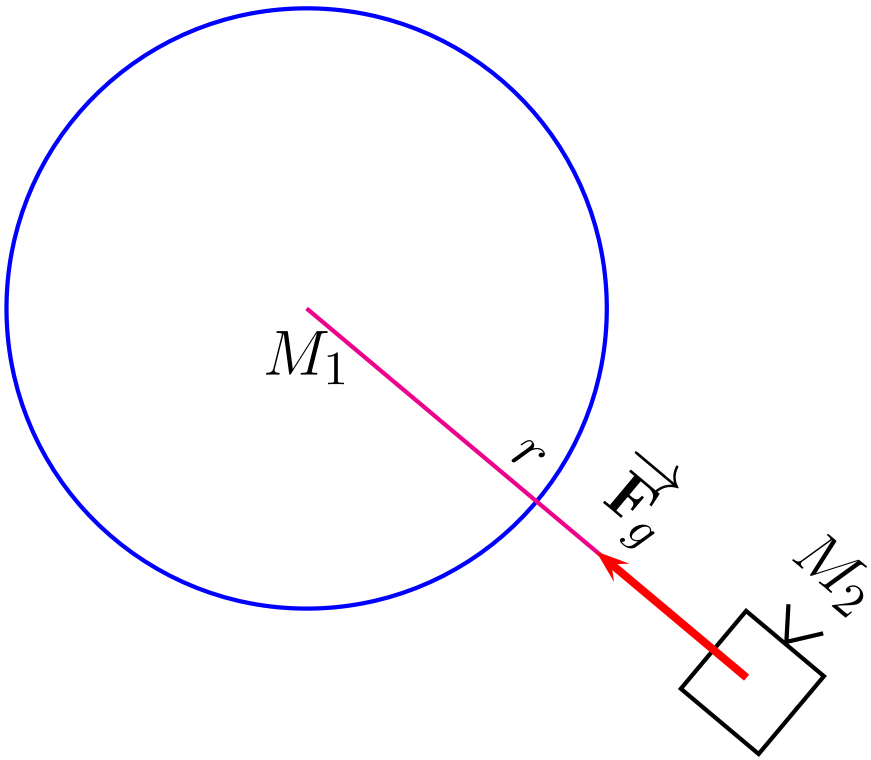

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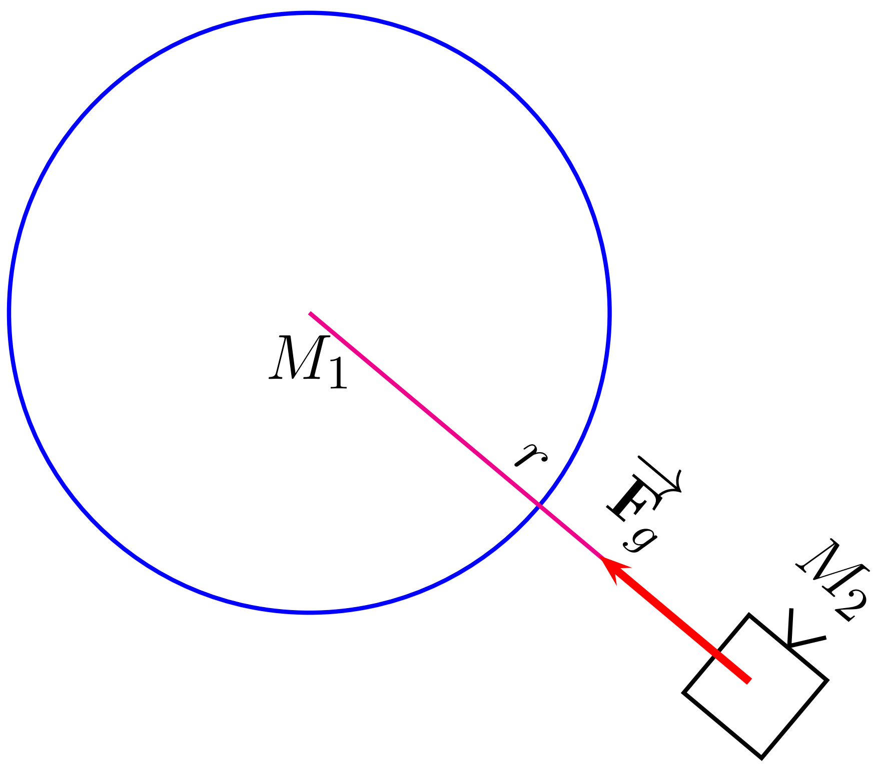

$$\sum \vec{\mathbf{F}}_2 = M_2 \vec{\mathbf{a}} \Rightarrow F_g = M_2 a_{rad}$$

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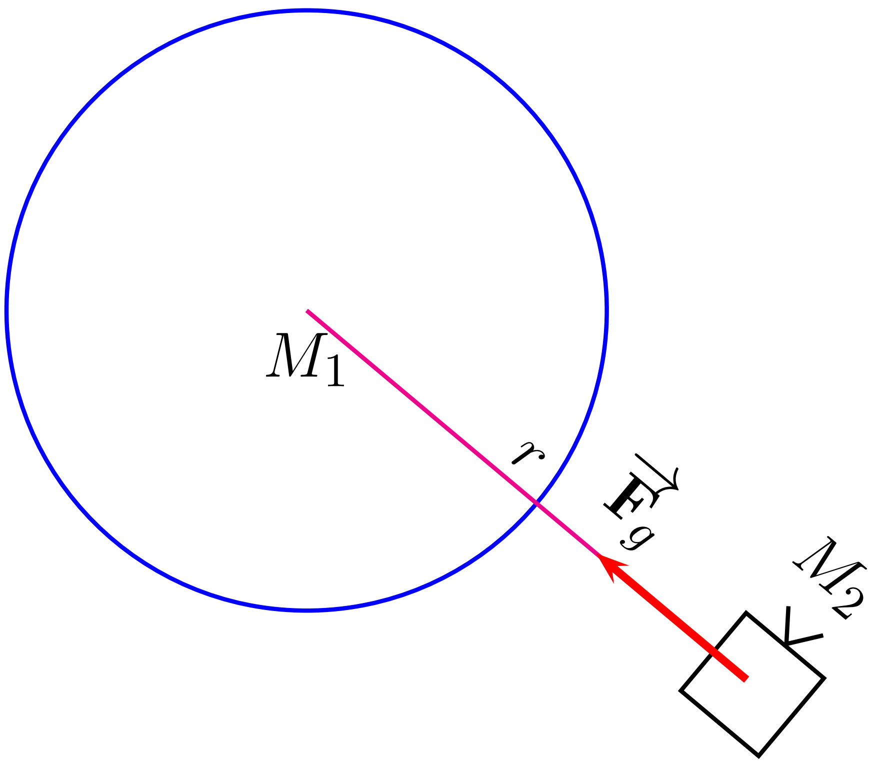

$$\sum \vec{\mathbf{F}}_2 = M_2 \vec{\mathbf{a}} \Rightarrow F_g = M_2 a_{rad}$$

$$\Rightarrow F_g = M_2 \frac{v^2}{r}$$

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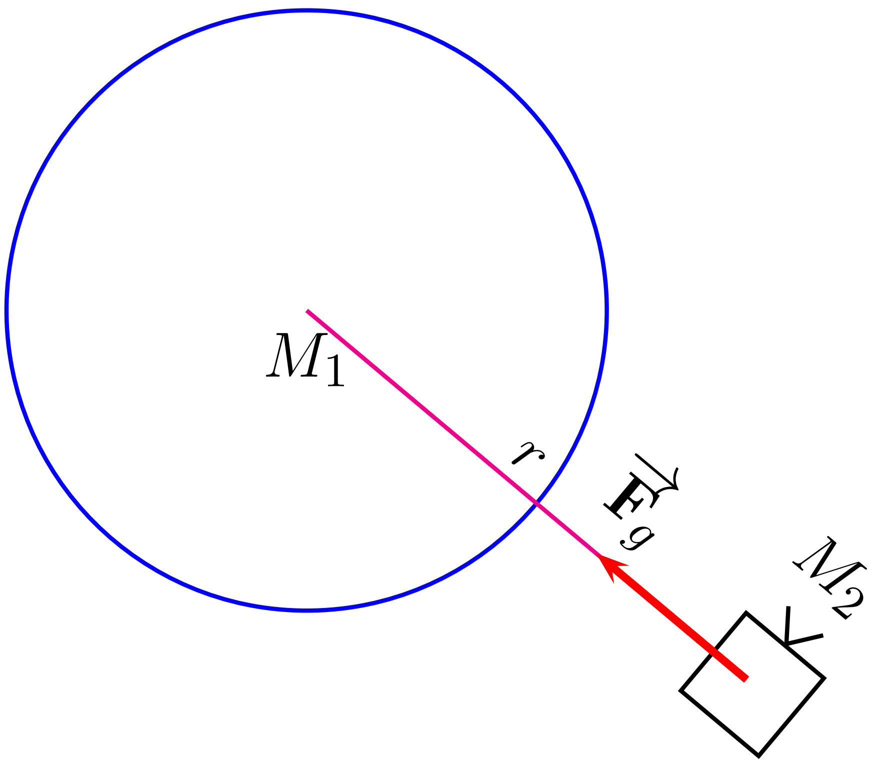

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$$\frac{GM_1 M_2}{r^2} = M_2 \frac{v^2}{r}$$

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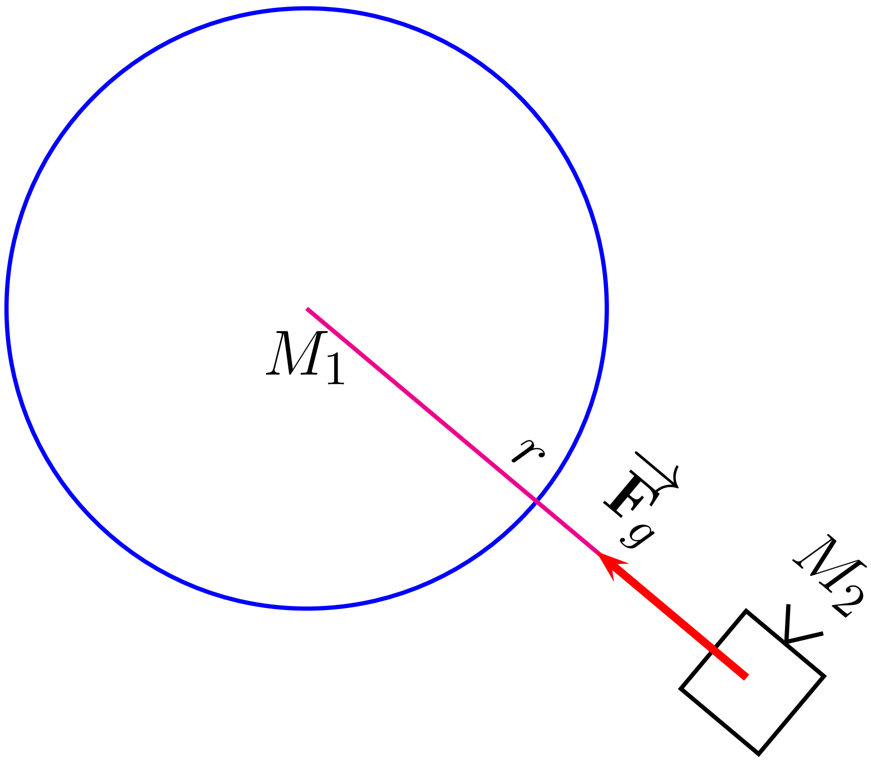

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$$\Rightarrow F_g = M_2 \frac{v^2}{r}$$

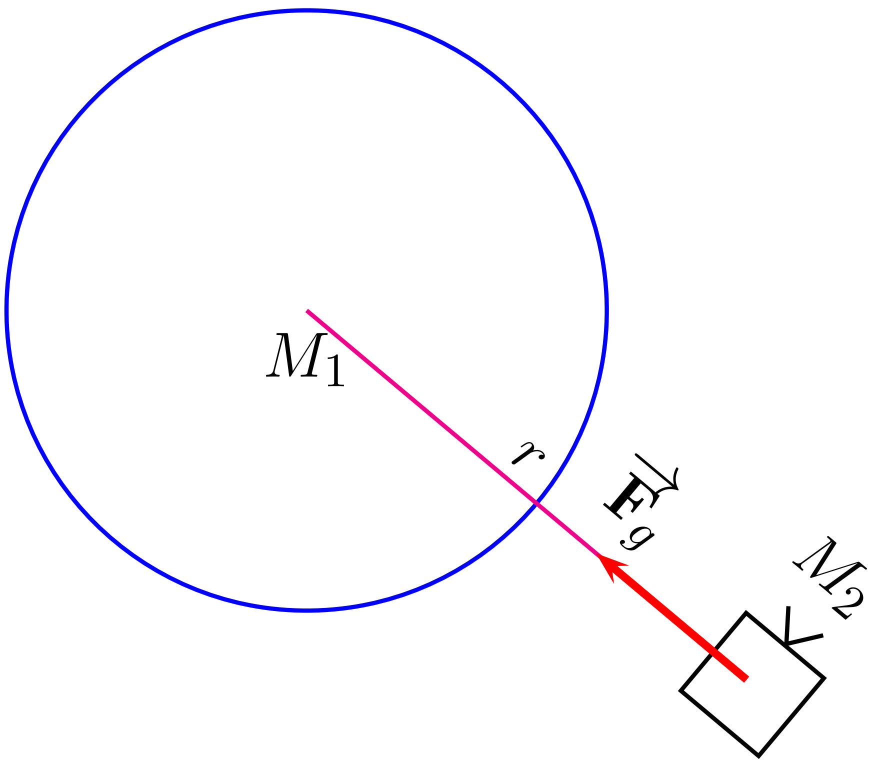
$$\frac{GM_1 M_2}{r^2} = M_2 \frac{v^2}{r}$$

$$v = \sqrt{\frac{GM_1}{r}}$$

Circular Orbits II

In circular orbit, gravity creates the centripetal acceleration.

Speed: $$v = \sqrt{\frac{GM_1}{r}}$$



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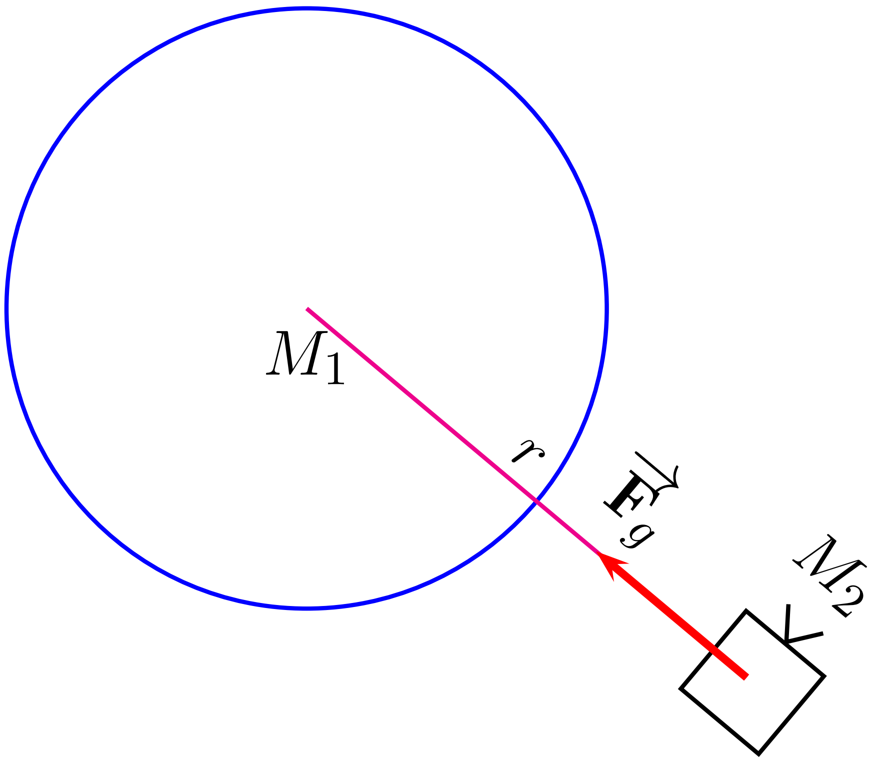

Speed: $\boxed{v = \sqrt{\frac{GM_1}{r}}}$

Constant Speed $\Rightarrow v = \frac{2\pi r}{T}$

Circular Orbits II

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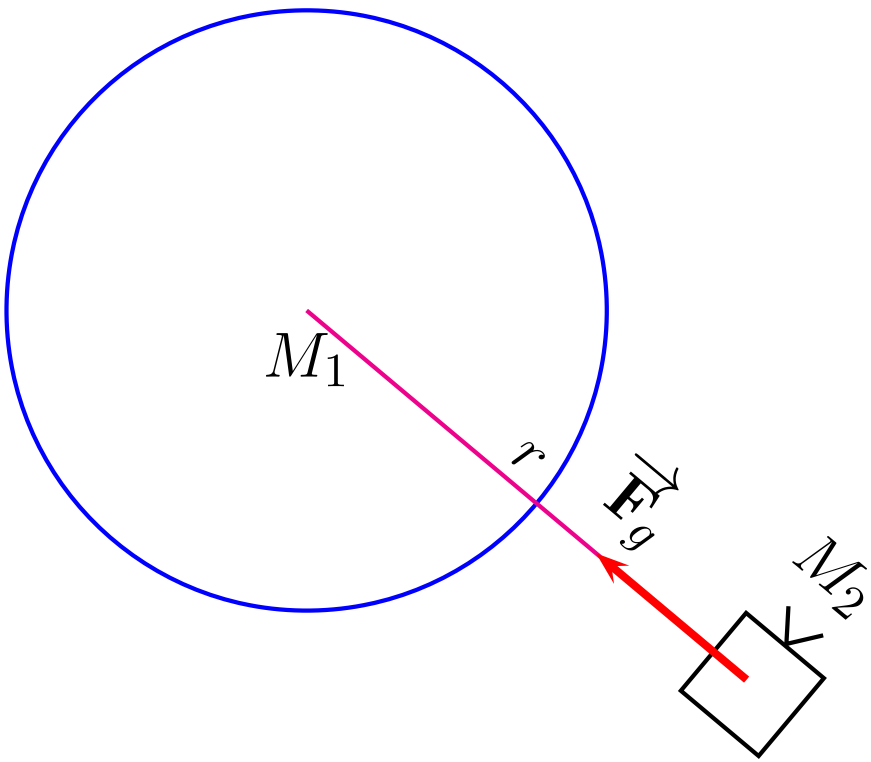

Speed: $\boxed{v = \sqrt{\frac{GM_1}{r}}}$

Constant Speed $\Rightarrow v = \frac{2\pi r}{T}$

Period: $\boxed{T = \frac{2\pi r^{3/2}}{\sqrt{GM_1}}}$

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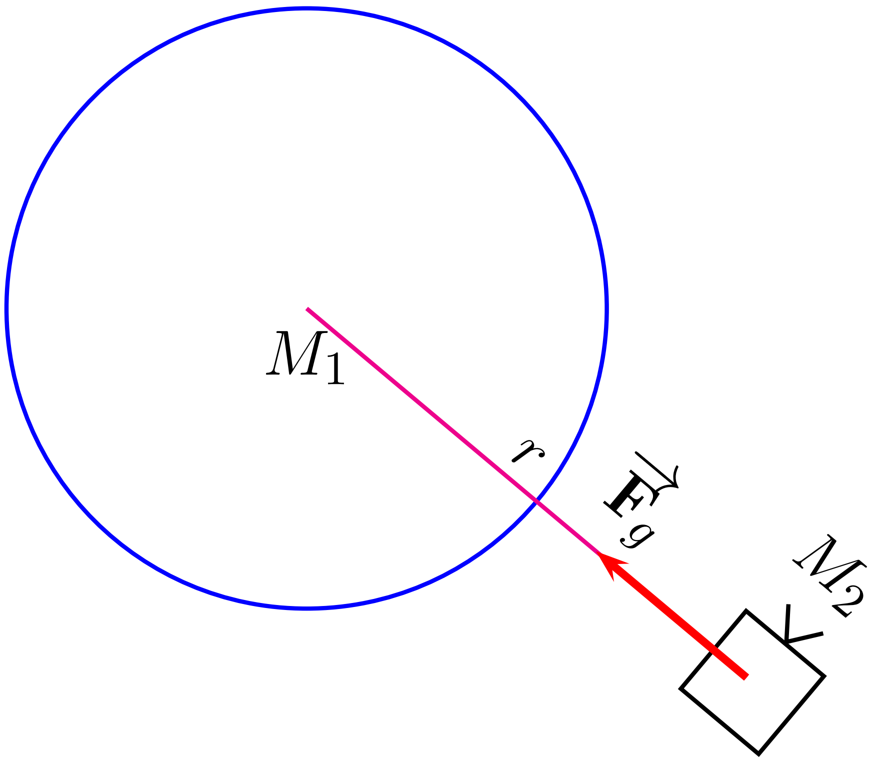

Speed: $$v = \sqrt{\frac{GM_1}{r}}$$

Constant Speed $\Rightarrow v = \dfrac{2\pi r}{T}$

Period: $$T = \frac{2\pi r^{3/2}}{\sqrt{GM_1}}$$

Energy: $$E = \frac{-GM_1 M_2}{2r}$$

Kepler's Laws

Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

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Before Newton, all astronomical work had been observational. Using the data of Danish astronomer Tycho Brahe (1546-1601), the German mathematician Johannes Kepler (1571-1630) was able to deduce (but not explain), three statements about planetary motion.

Kepler's Laws:

1: Each planet's orbit traces out the shape of an ellipse with the sun located at one focus.

2: The imaginary line from the sun to a planet sweeps out equal areas in equal times.

3: The period of the planet's motion is proportional to the orbit's semi-major axis to the $\frac{3}{2}$ power.

Ellipses - The Geometric Approach

Ellipse - ovals.

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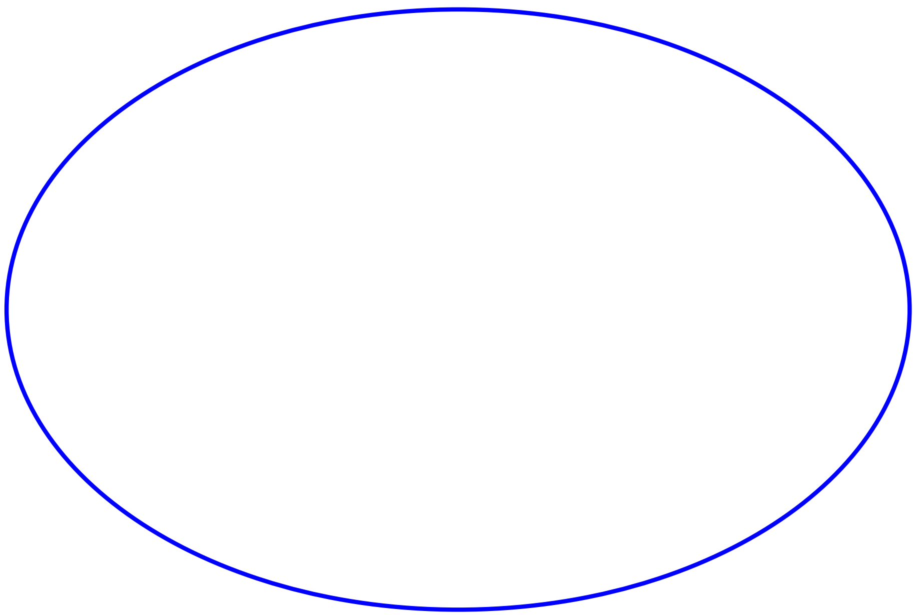

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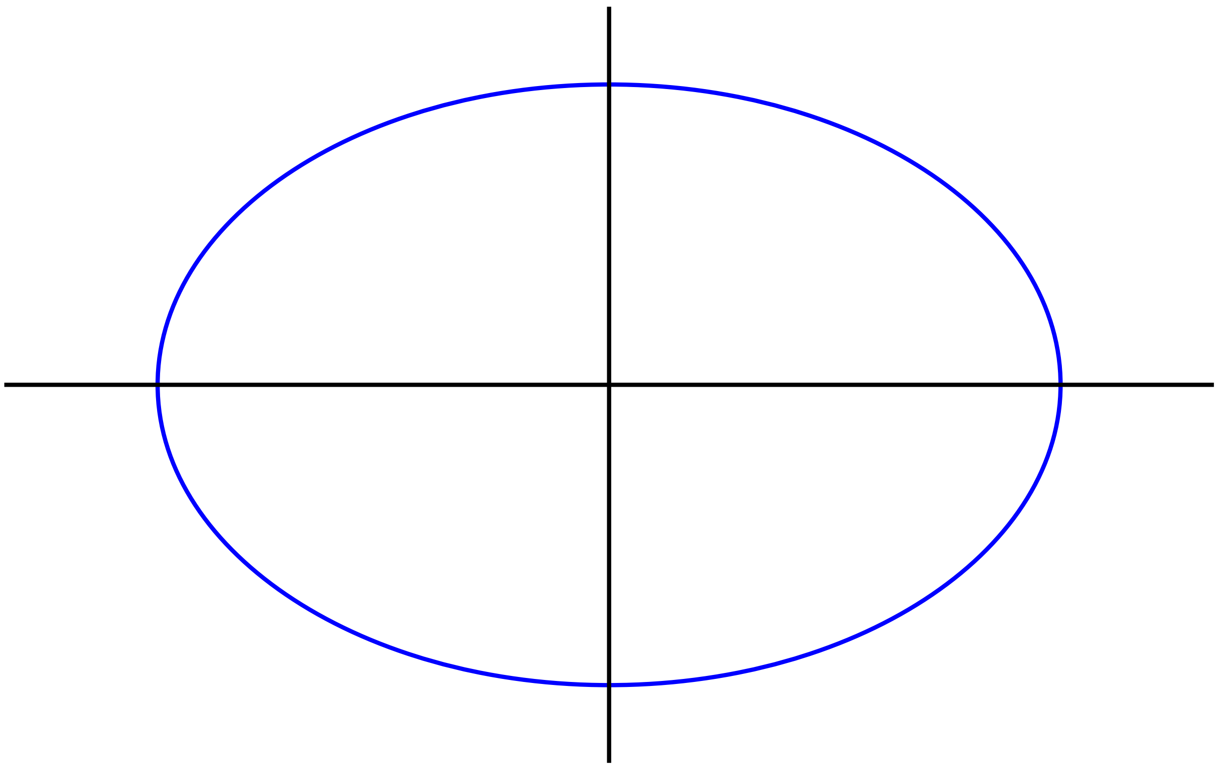

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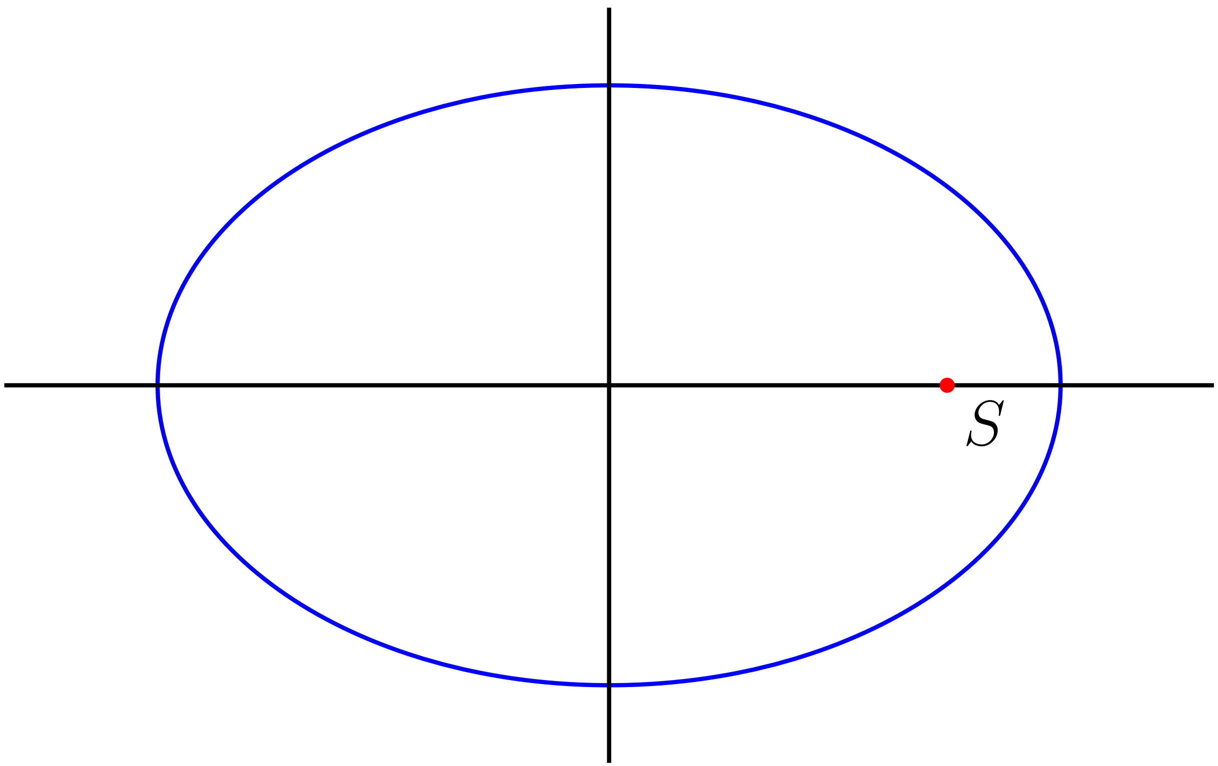

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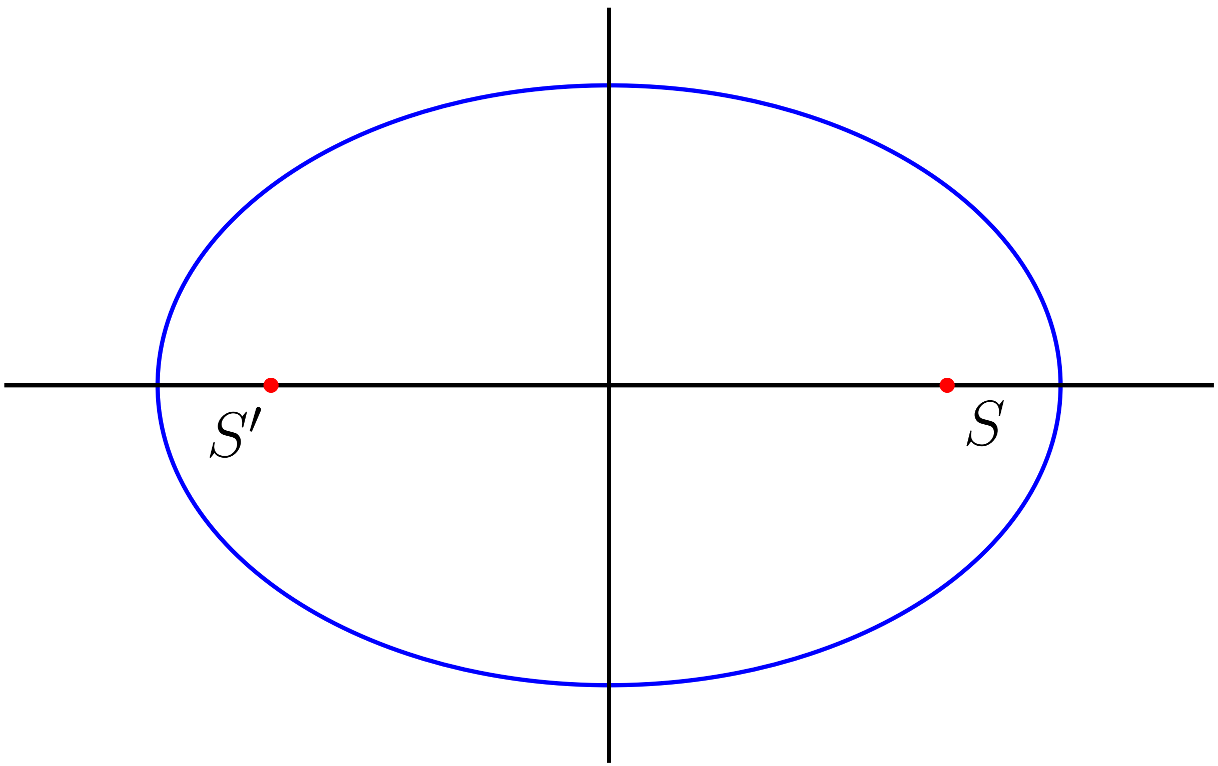

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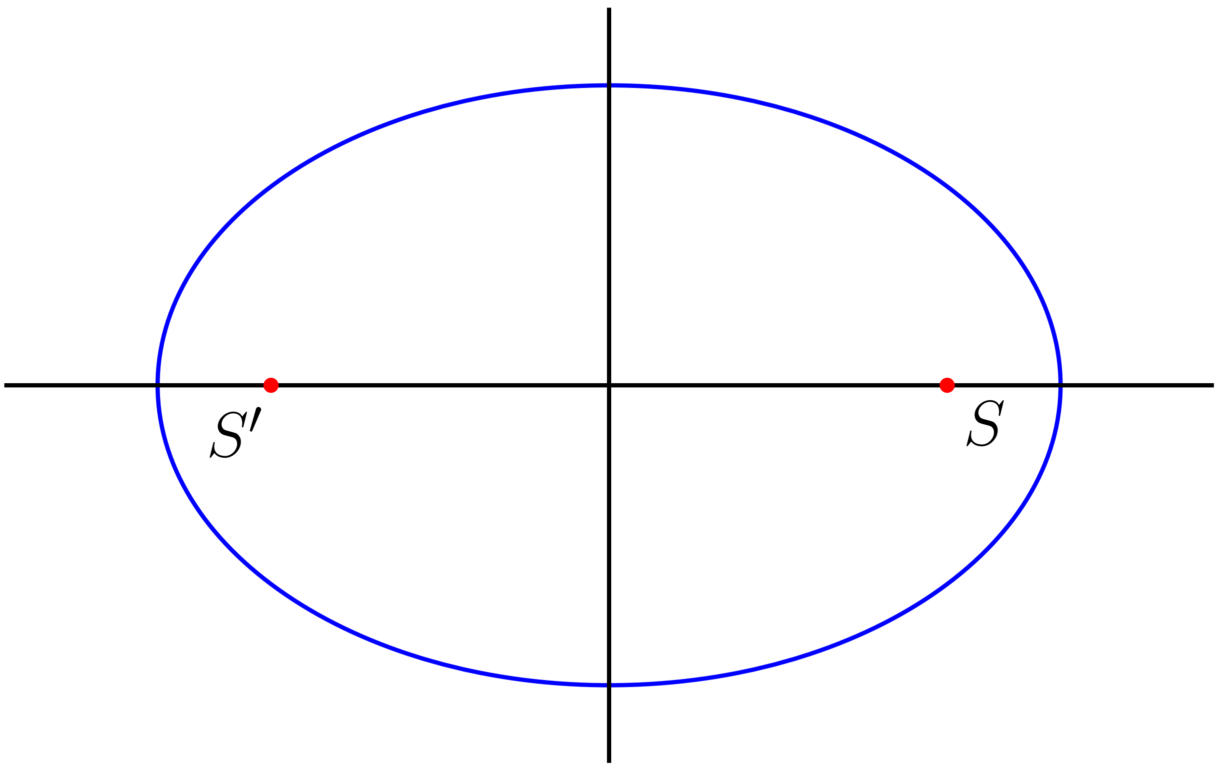

S, S' - foci

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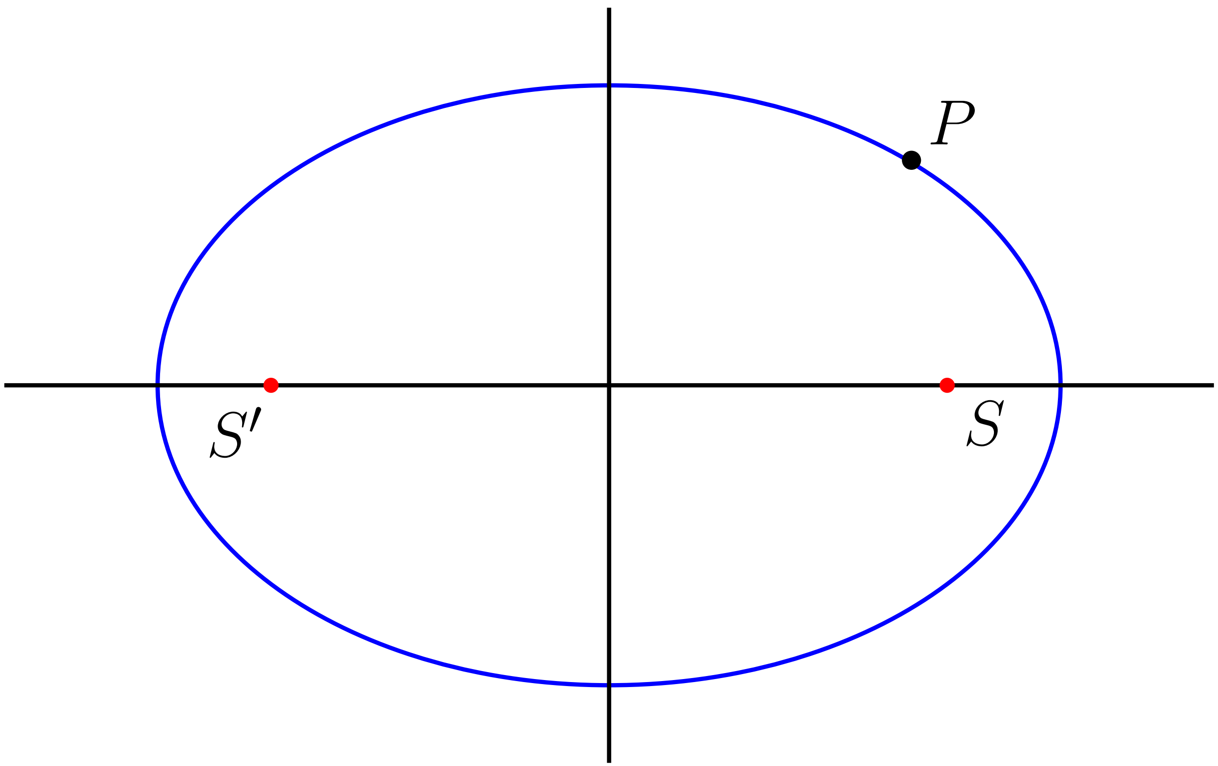

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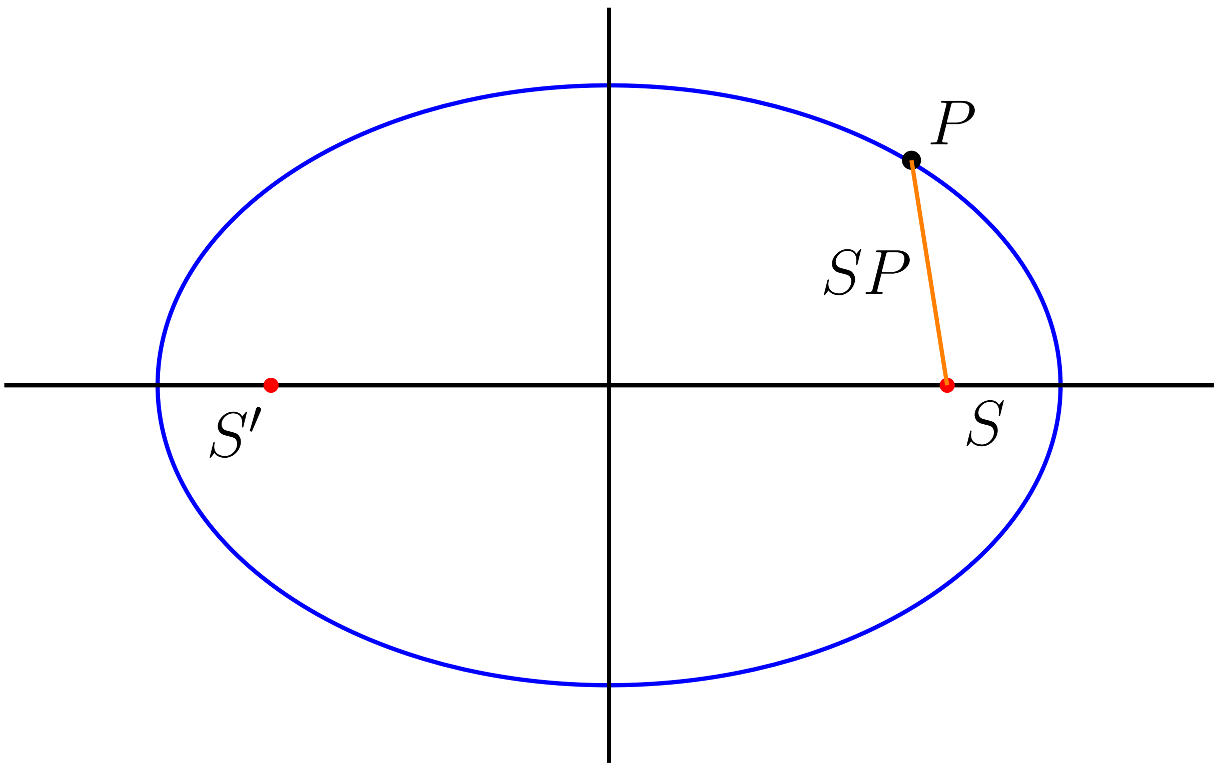

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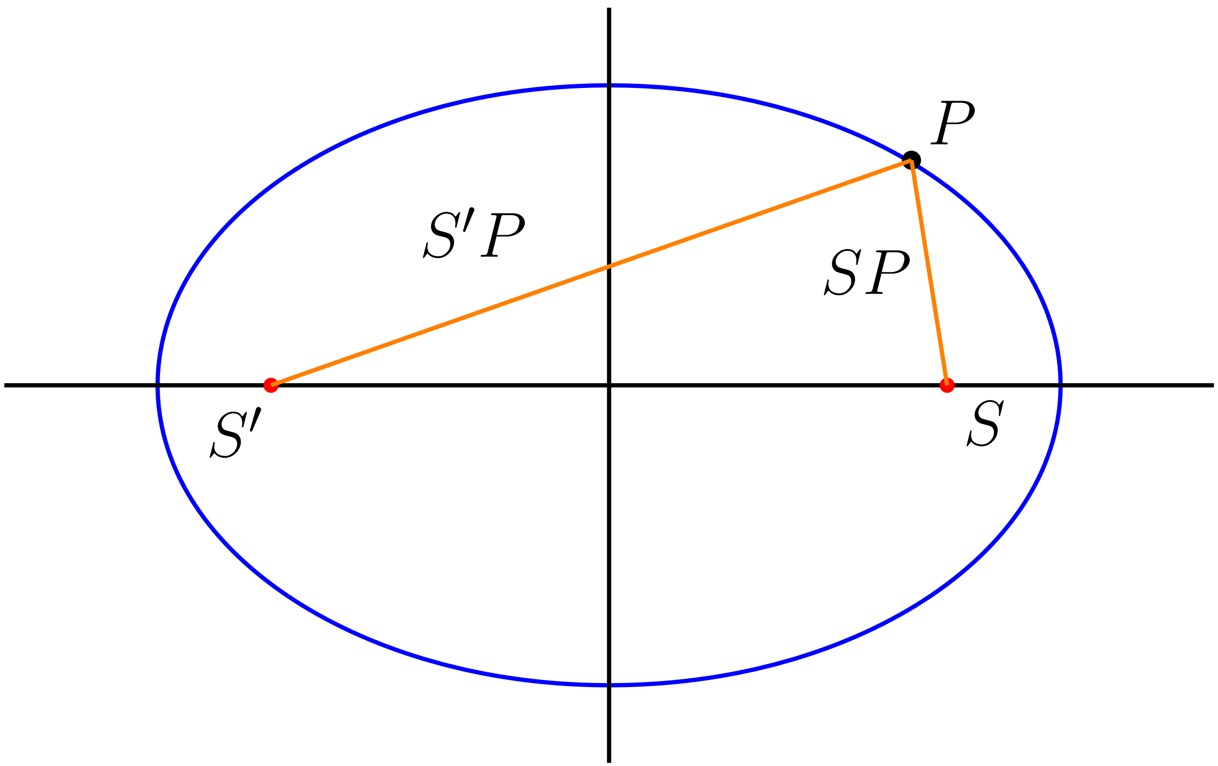

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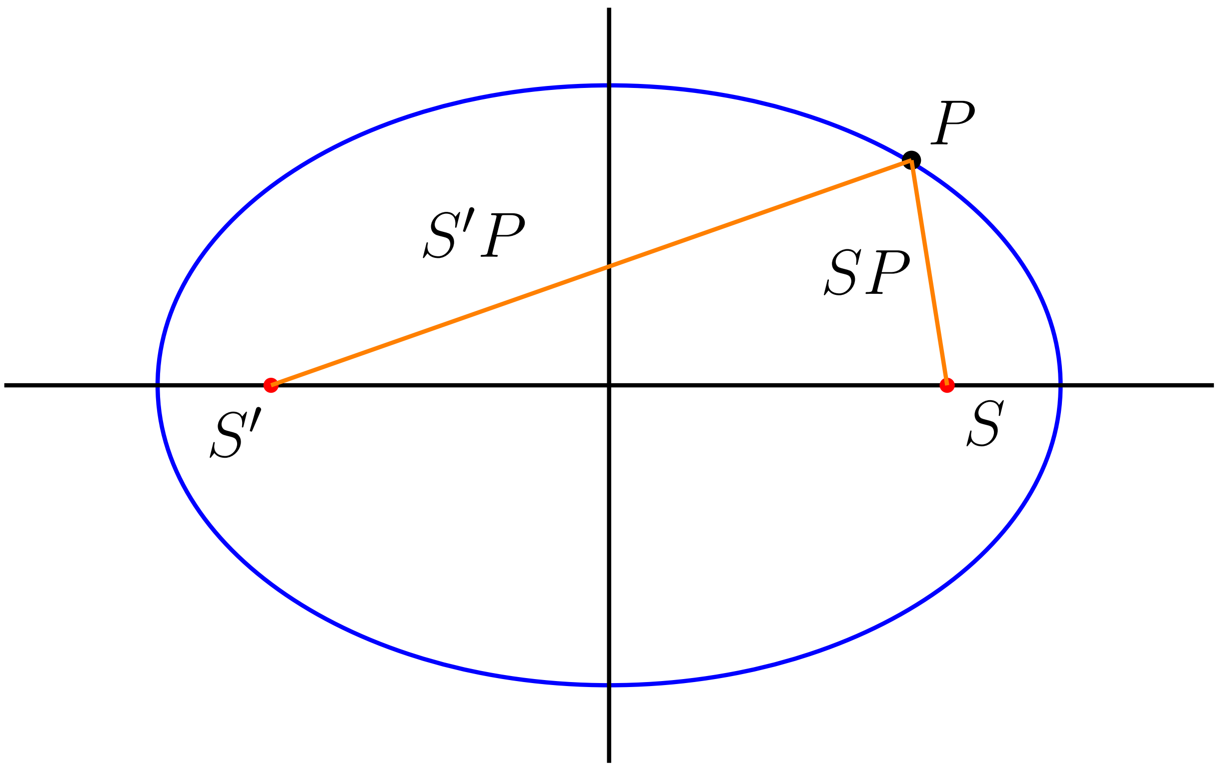


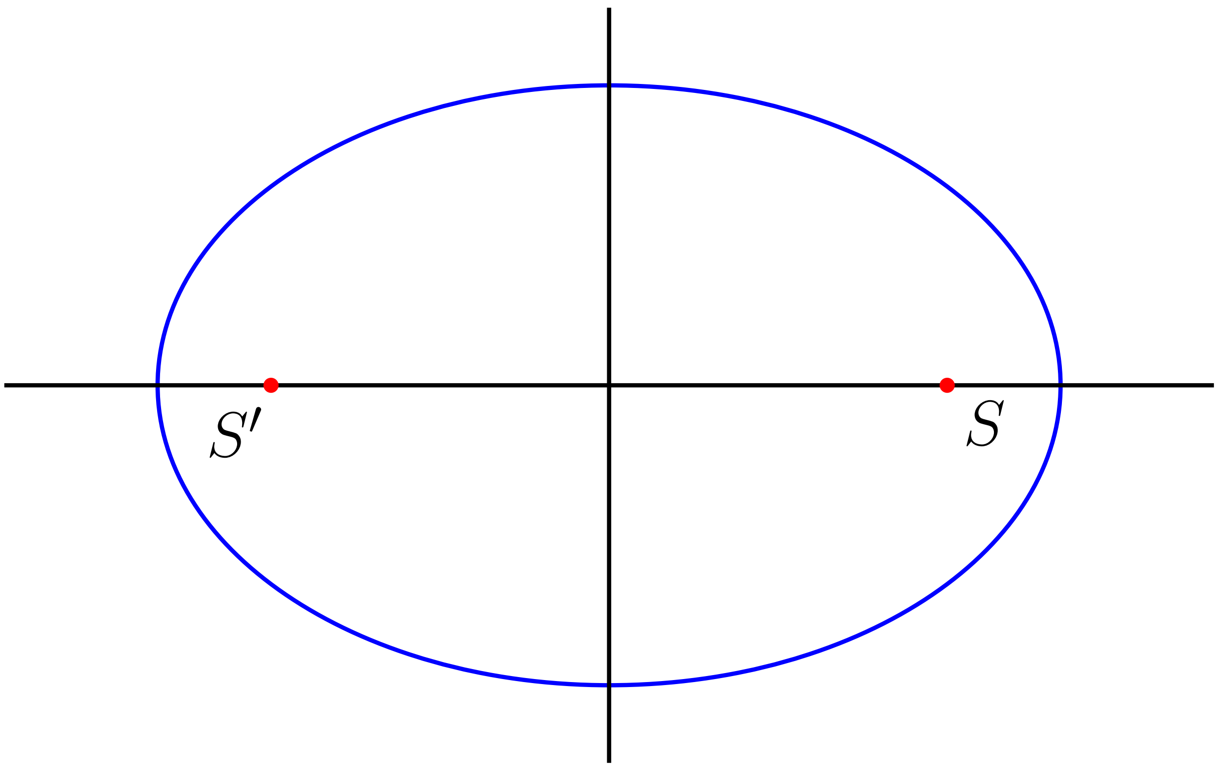
S, S' - foci

ellipse = all points P such that

$SP + S'P = constant$

Ellipses - The Algebraic Approach

Ellipse - ovals.



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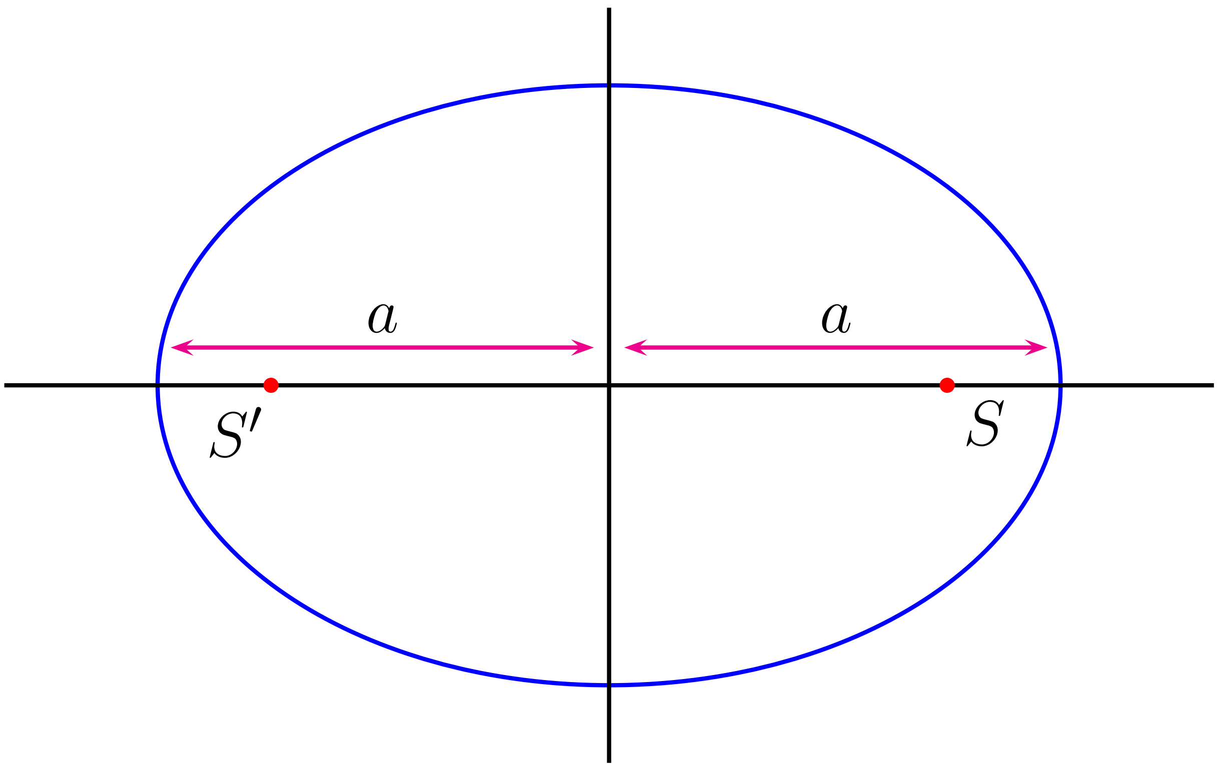

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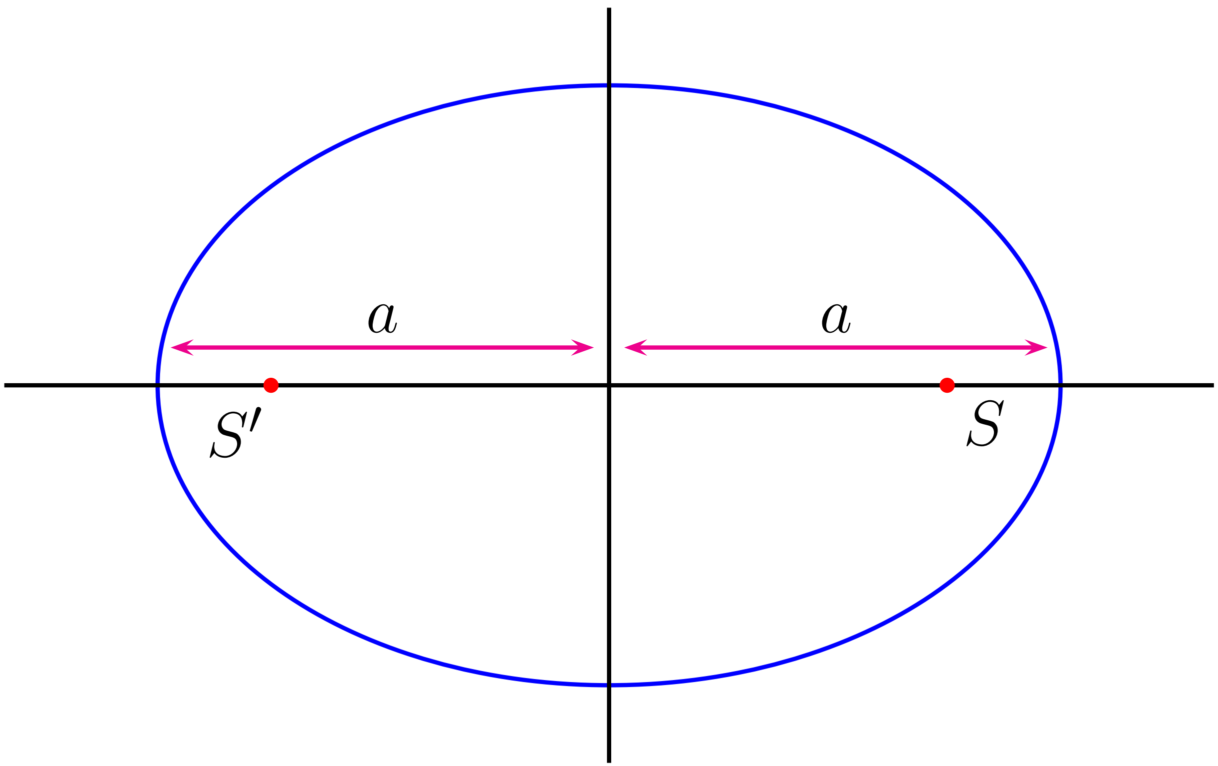

a: semi-major axis

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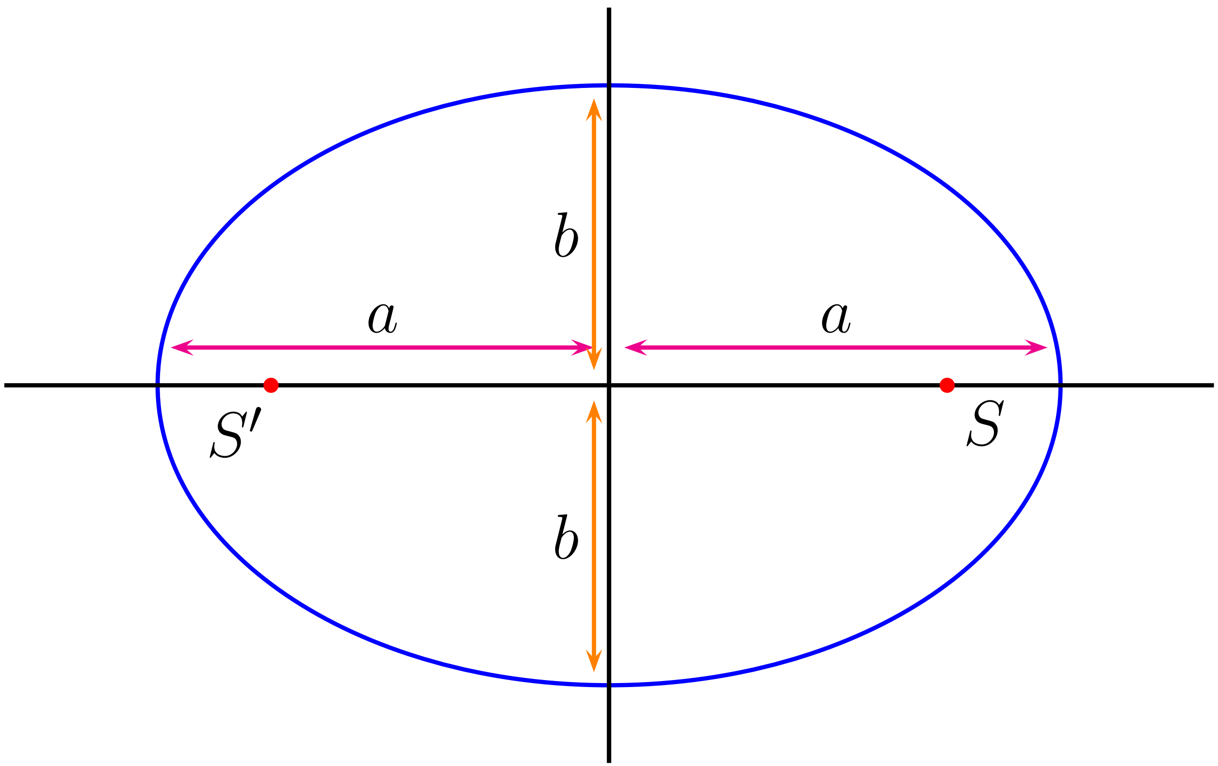

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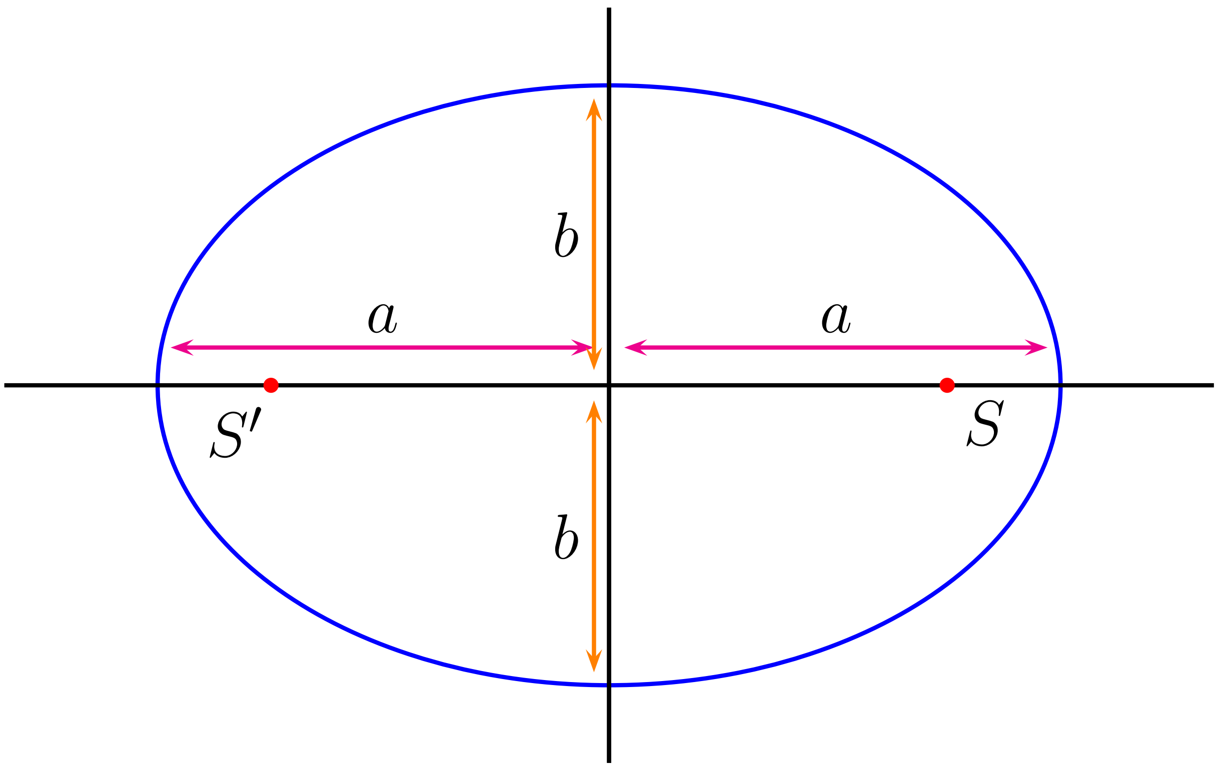

a: semi-major axis

b: semi-minor axis

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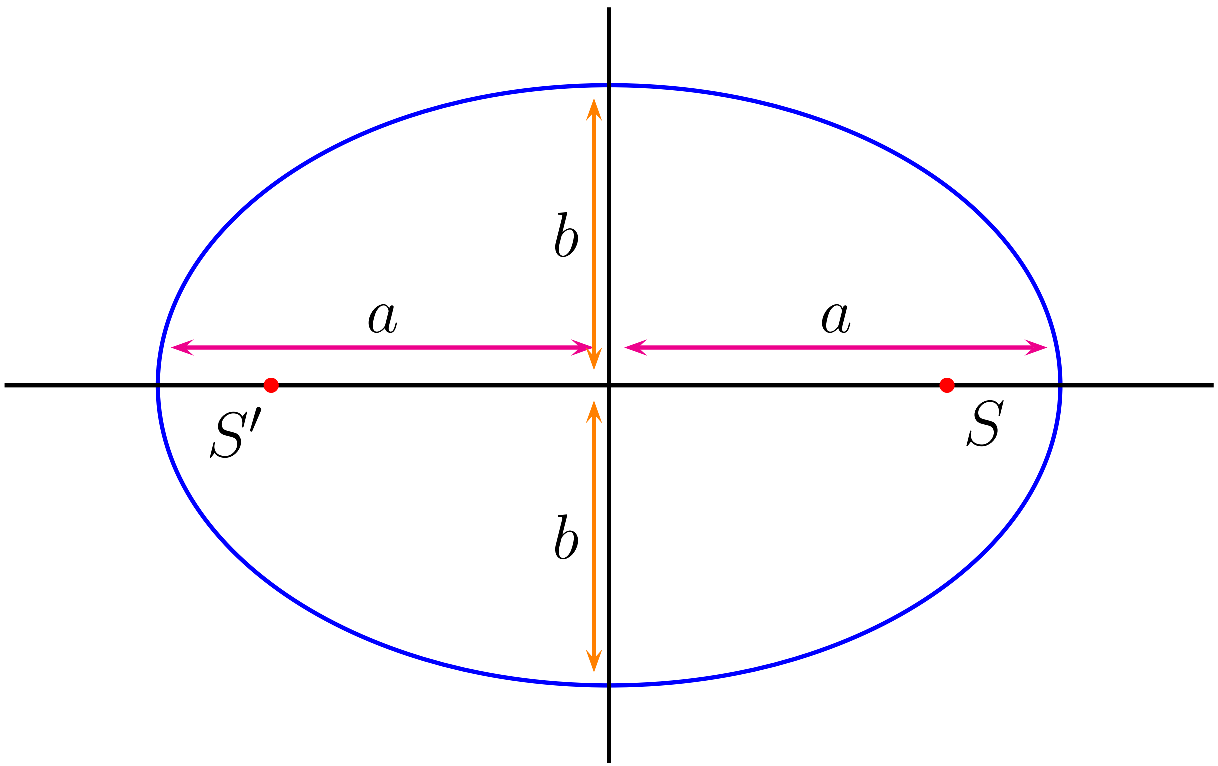

a: semi-major axis

b: semi-minor axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

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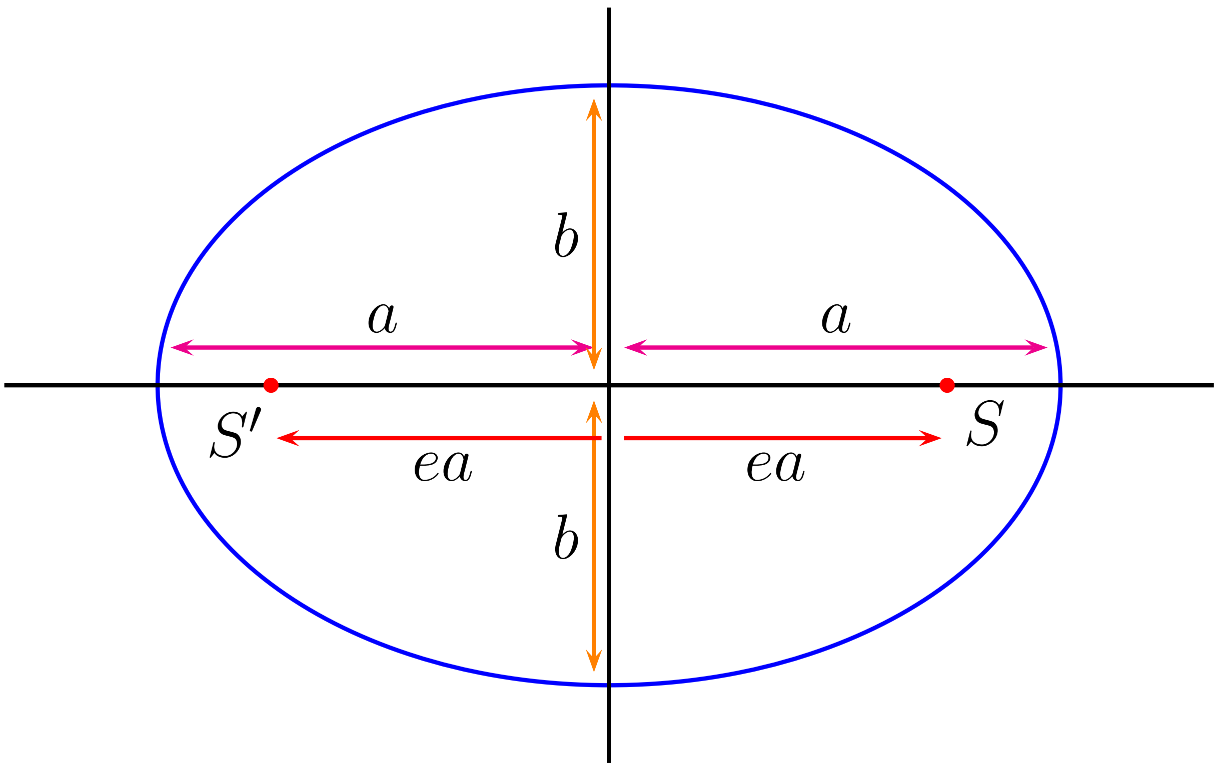

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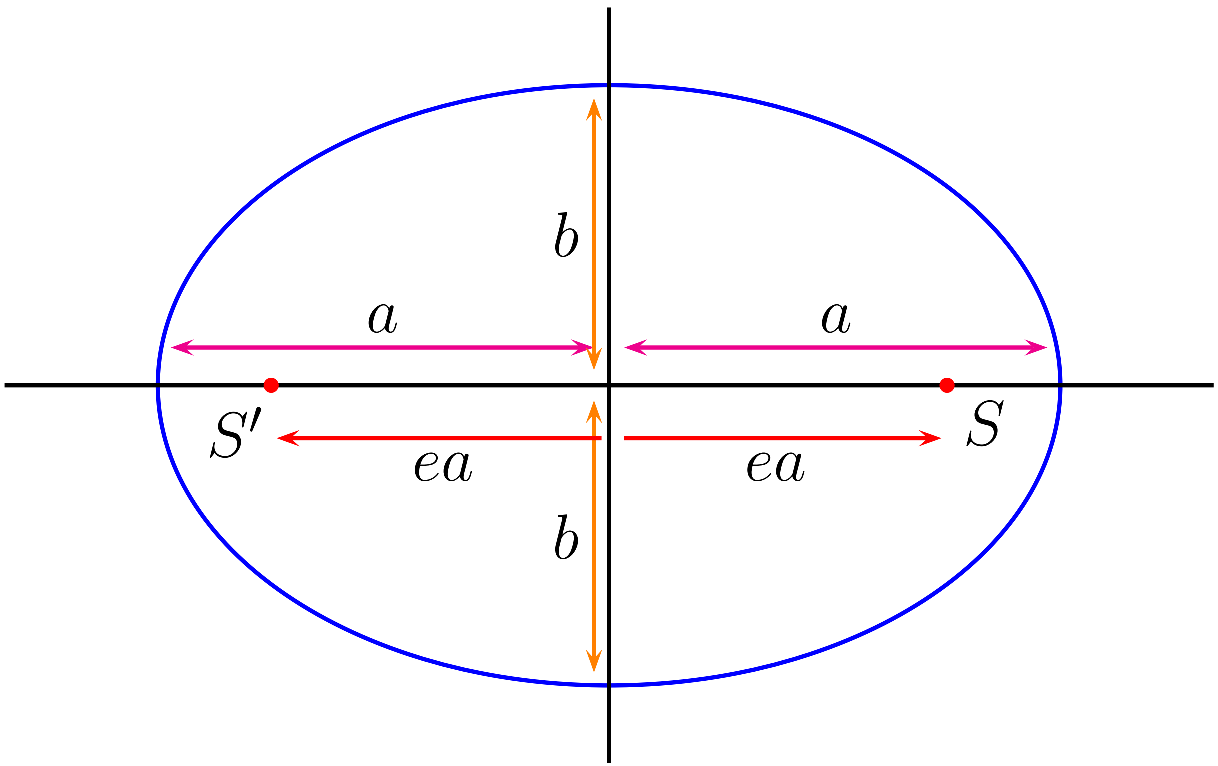


a: semi-major axis

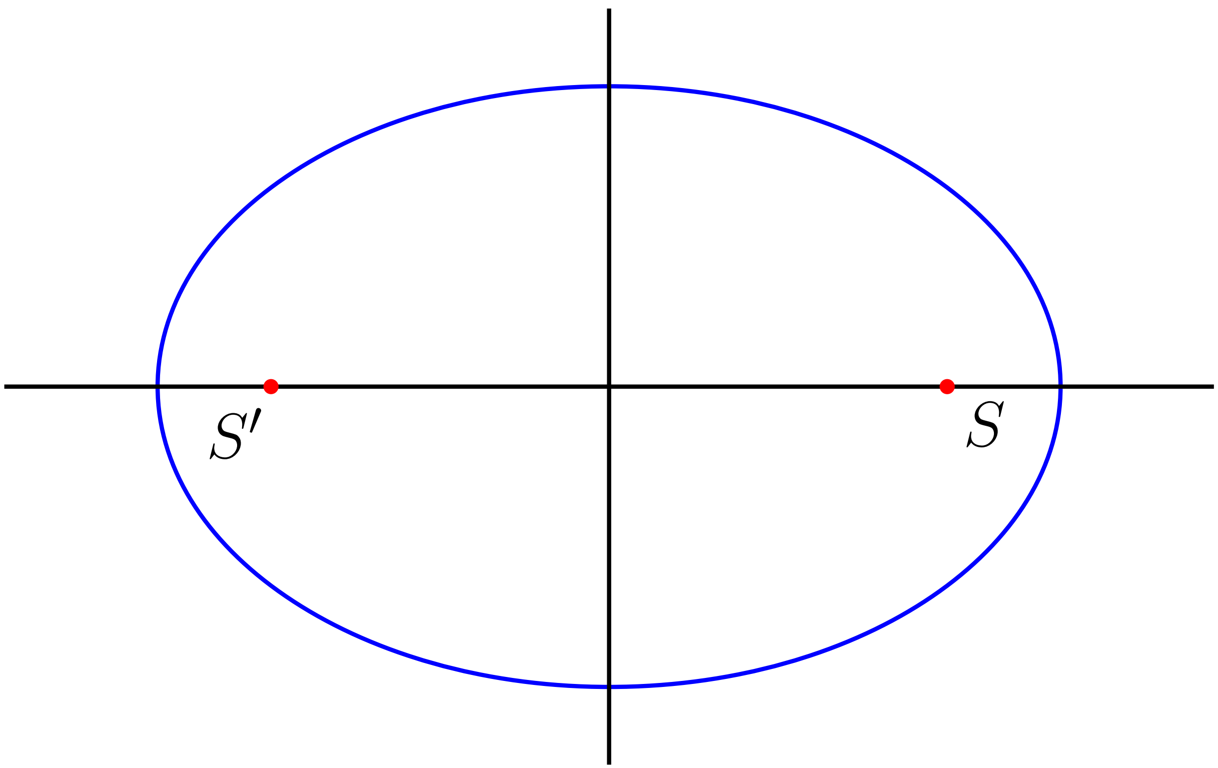
b: semi-minor axis

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

e: eccentricity

Eccentricity

The eccentricity gives the amount of “oval-ness" of the ellipse.



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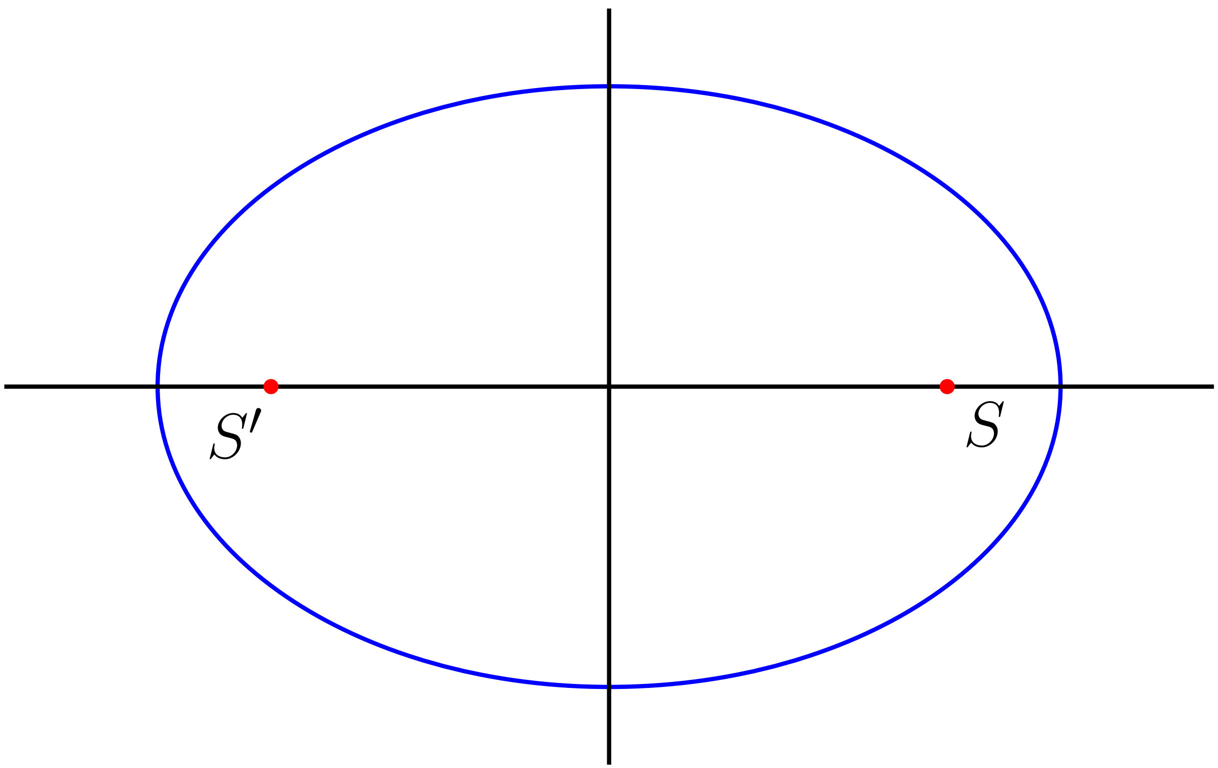

$a = 3$

$b = 2$

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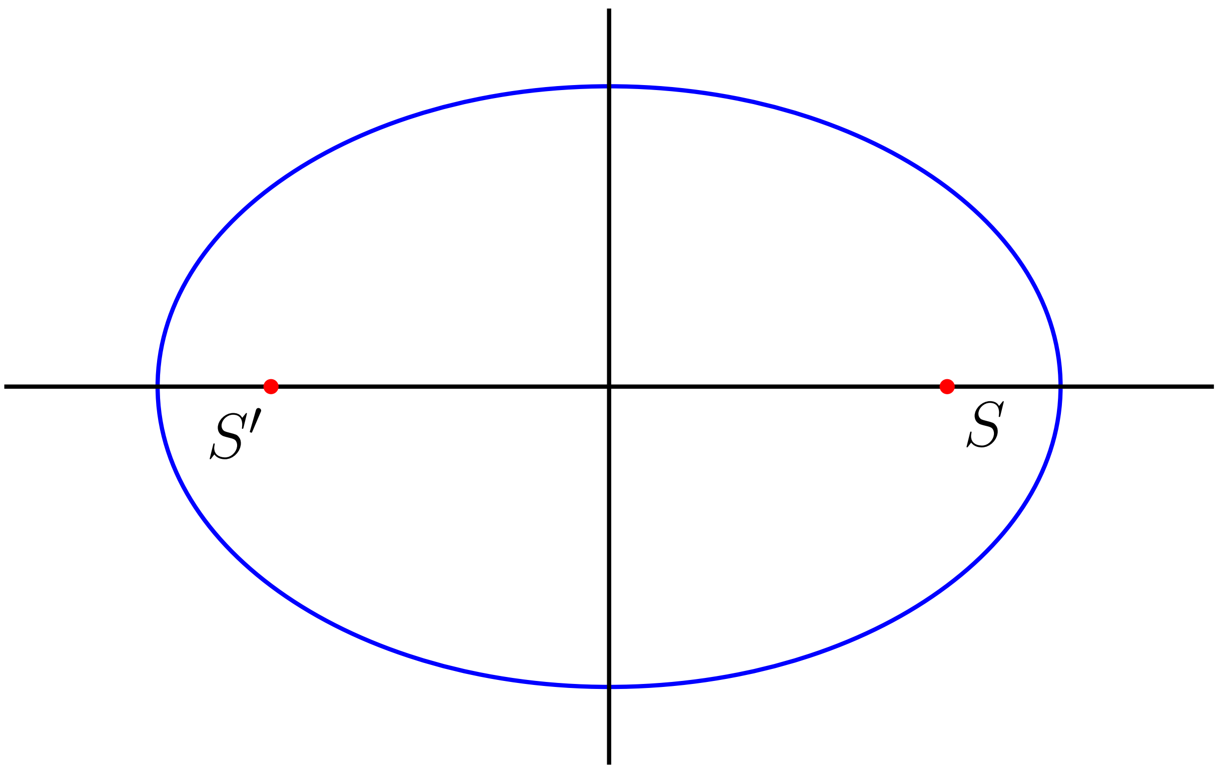

$$a = 3$$

$$b = 2$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

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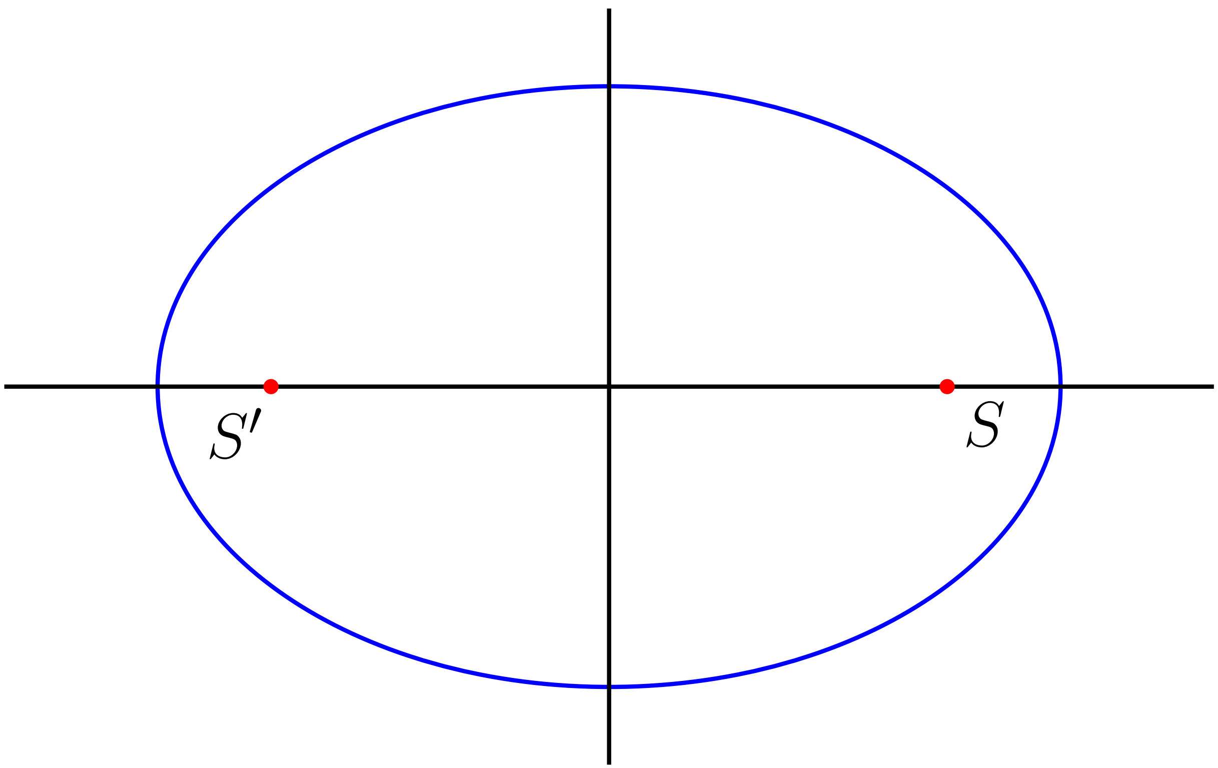

$$a = 3$$

$$b = 2$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.745$$

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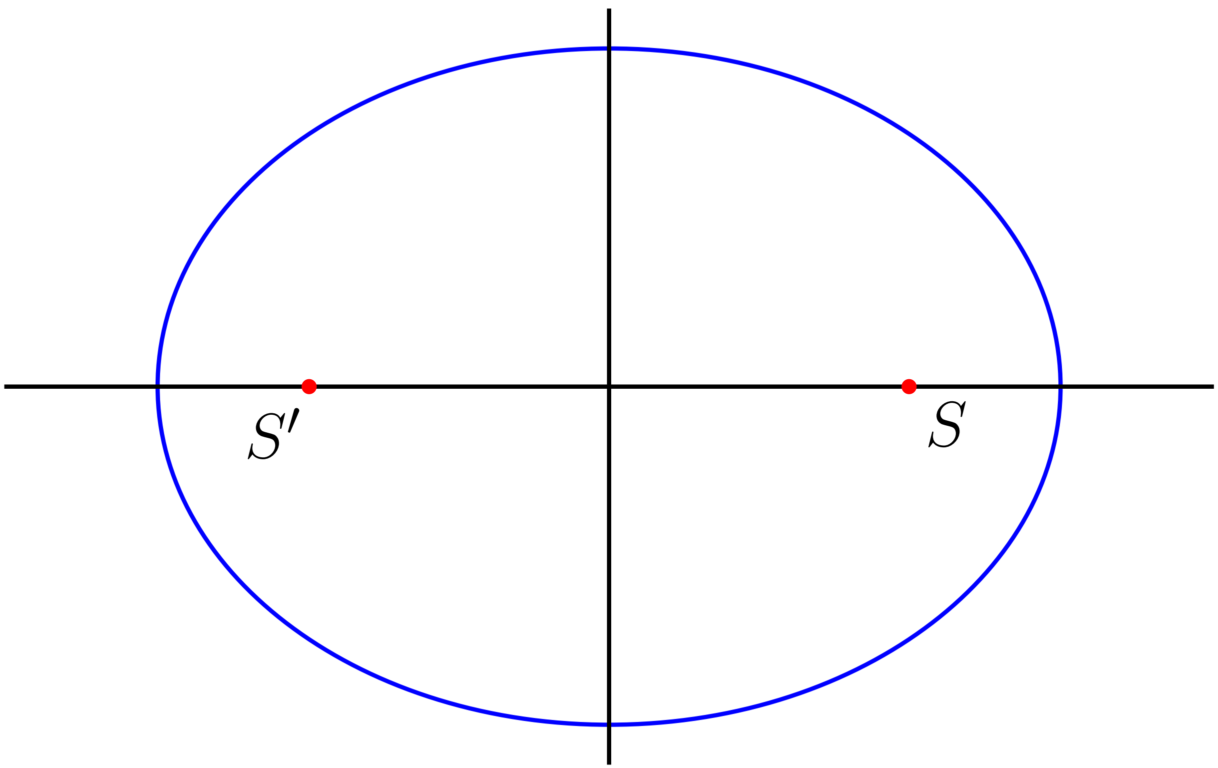

$$a = 3$$

$$b = 2.25$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.661$$

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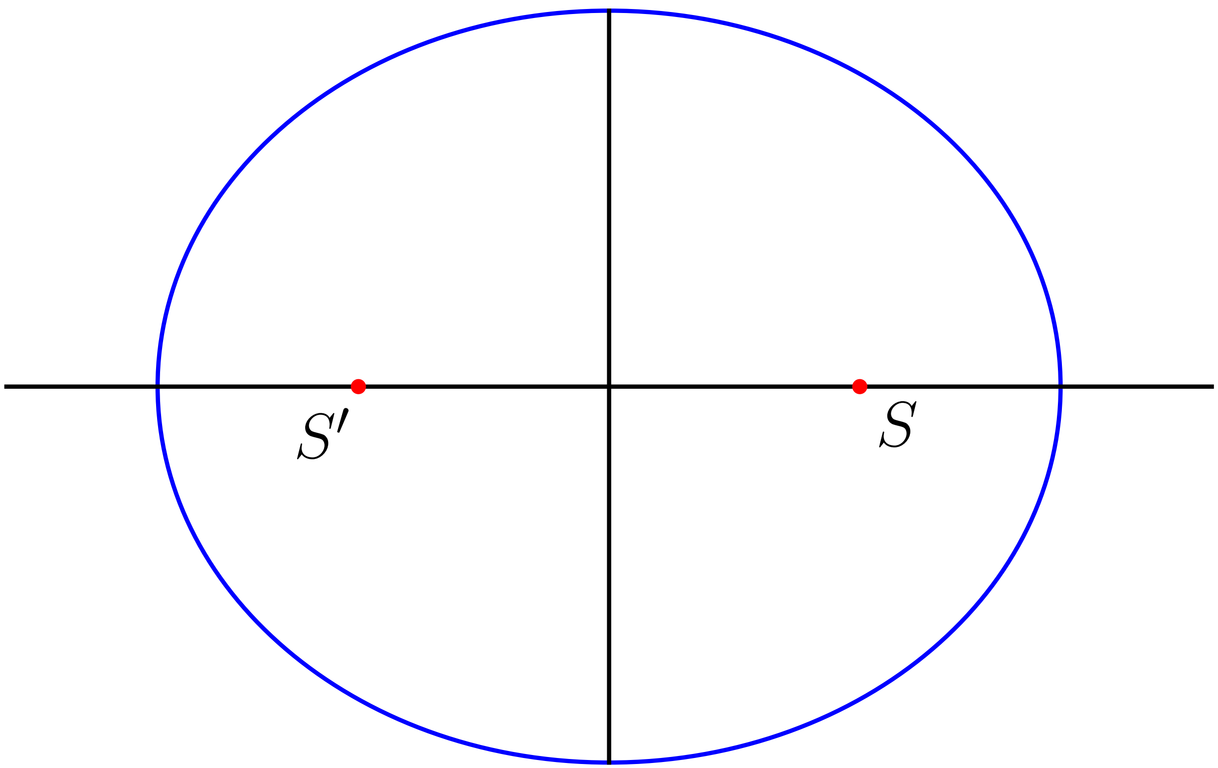

$$a = 3$$

$$b = 2.5$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.553$$

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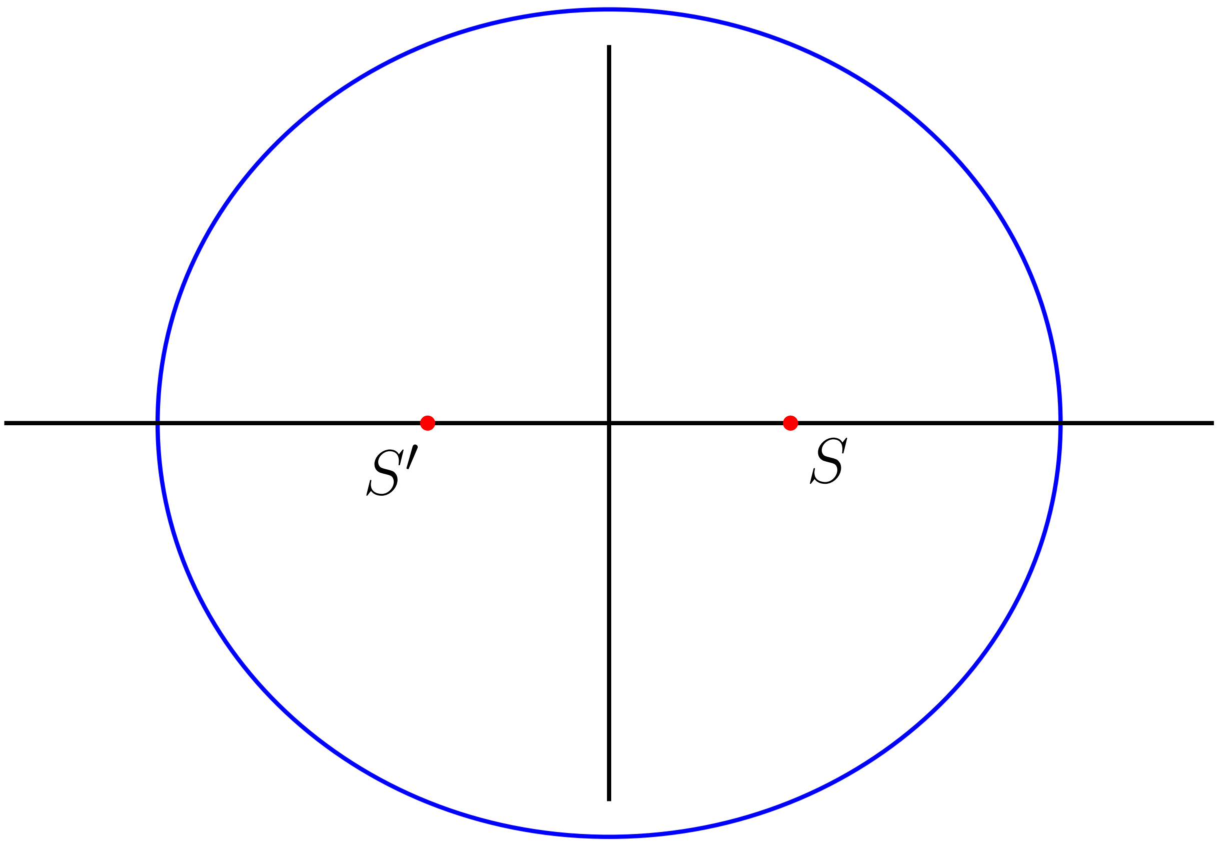

$$a = 3$$

$$b = 2.75$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0.400$$

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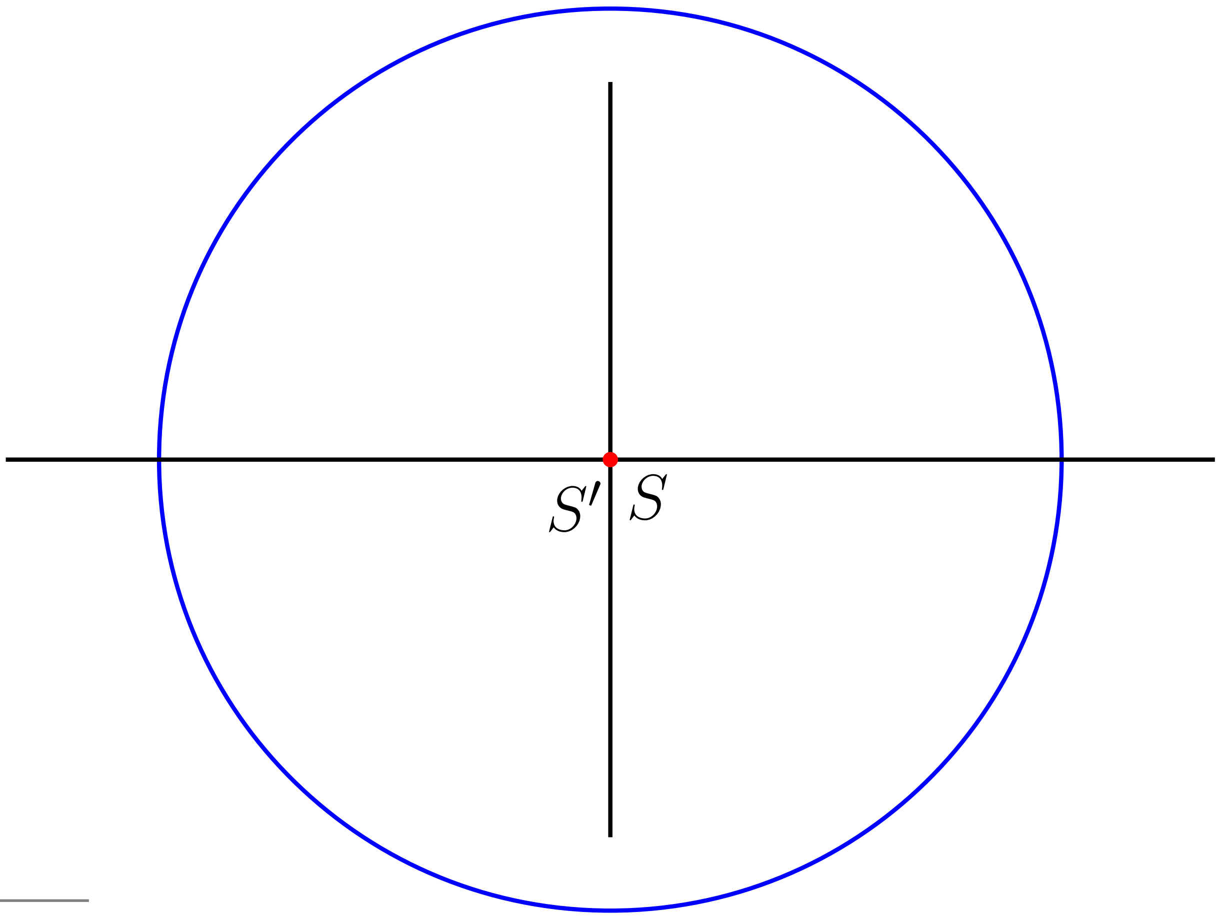

$$a = 3$$

$$b = 3$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0$$

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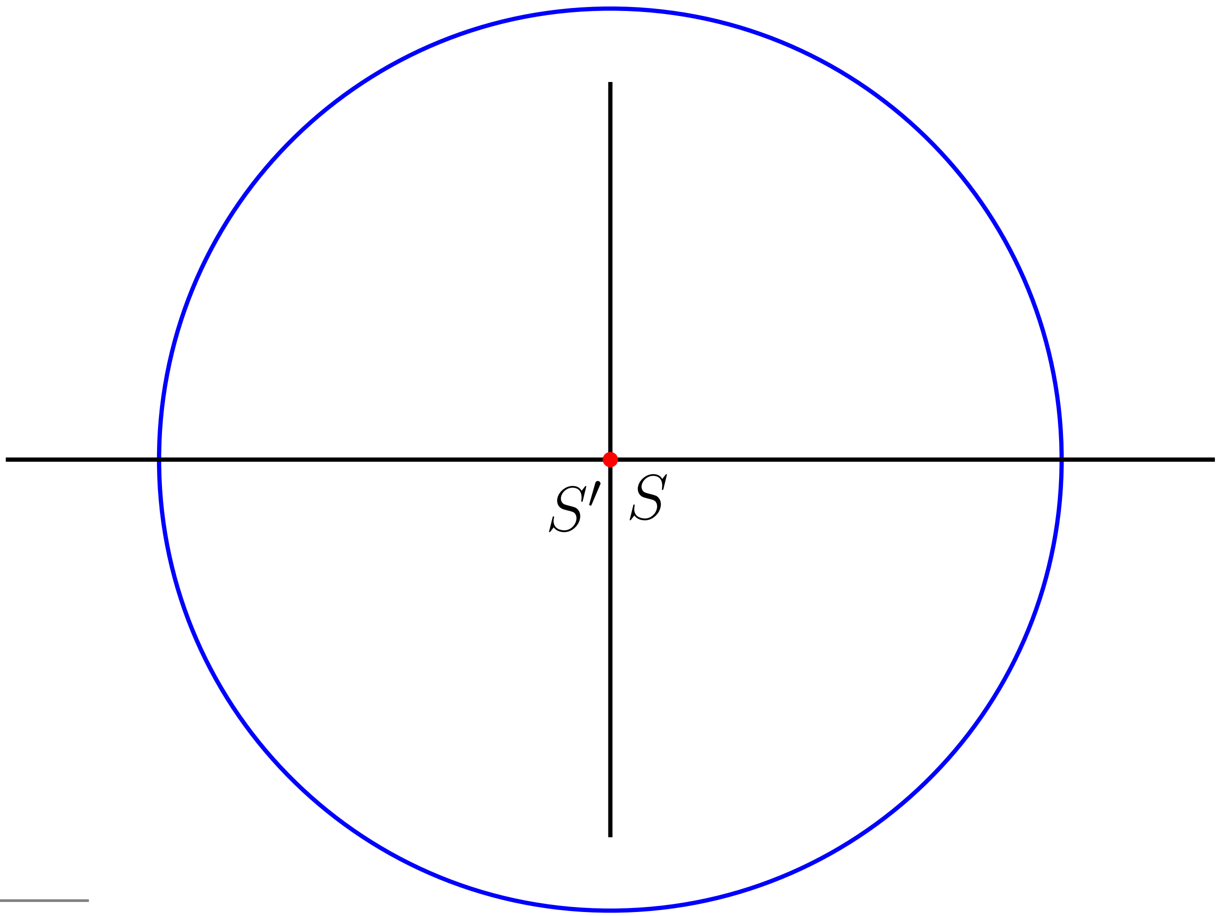

$$a = 3$$

$$b = 3$$

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$

$$e = 0$$

Circle

Kepler's First Law

Kepler's Laws:

1: Each planet's orbit traces out the shape of an ellipse with the sun located at one focus.

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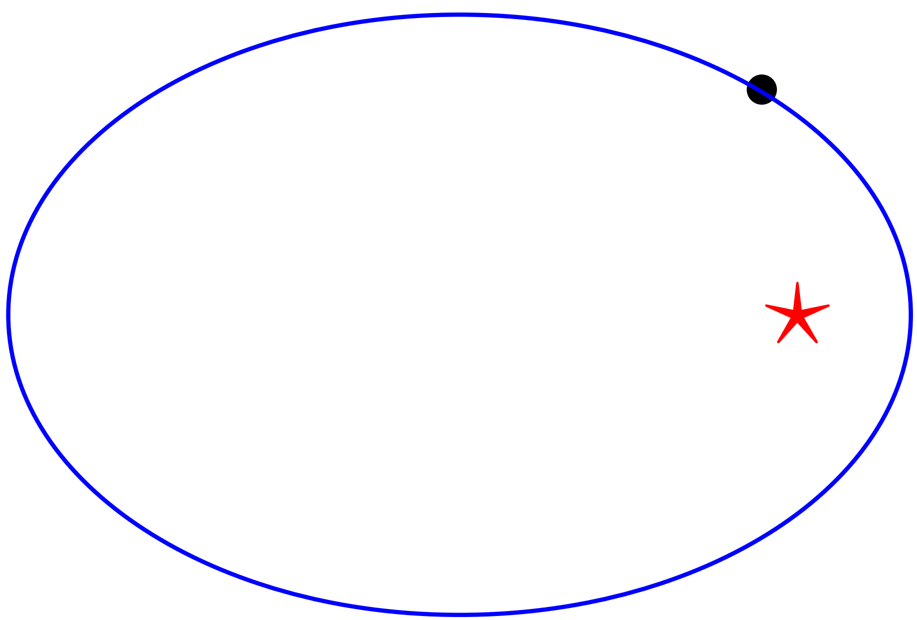

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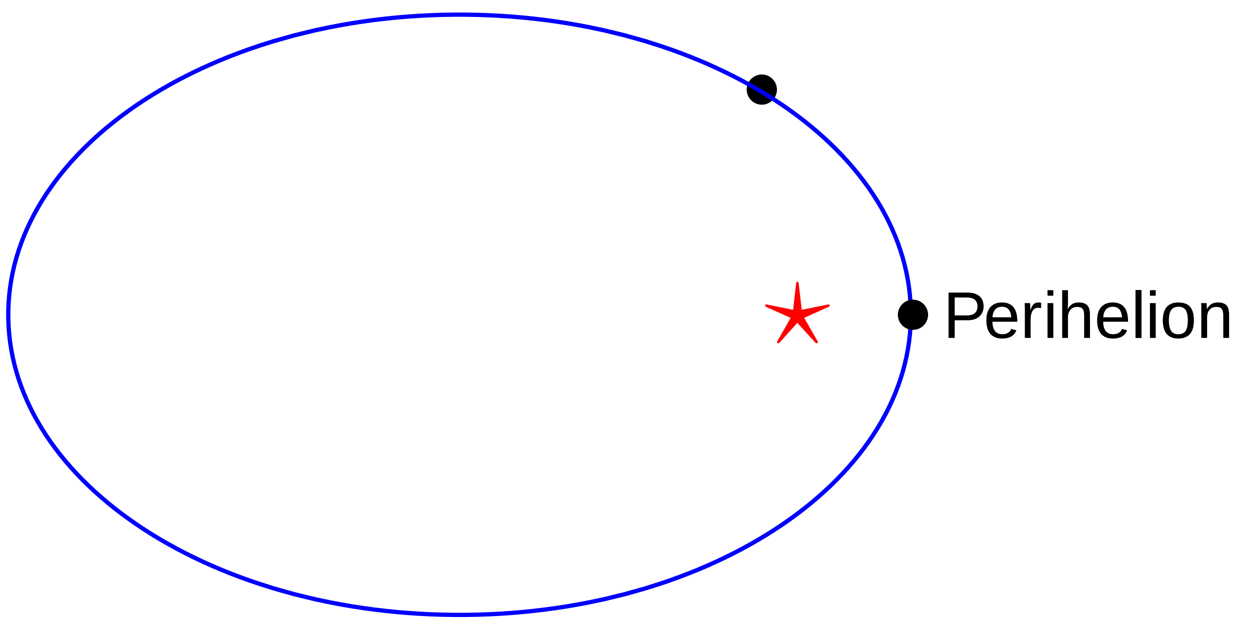

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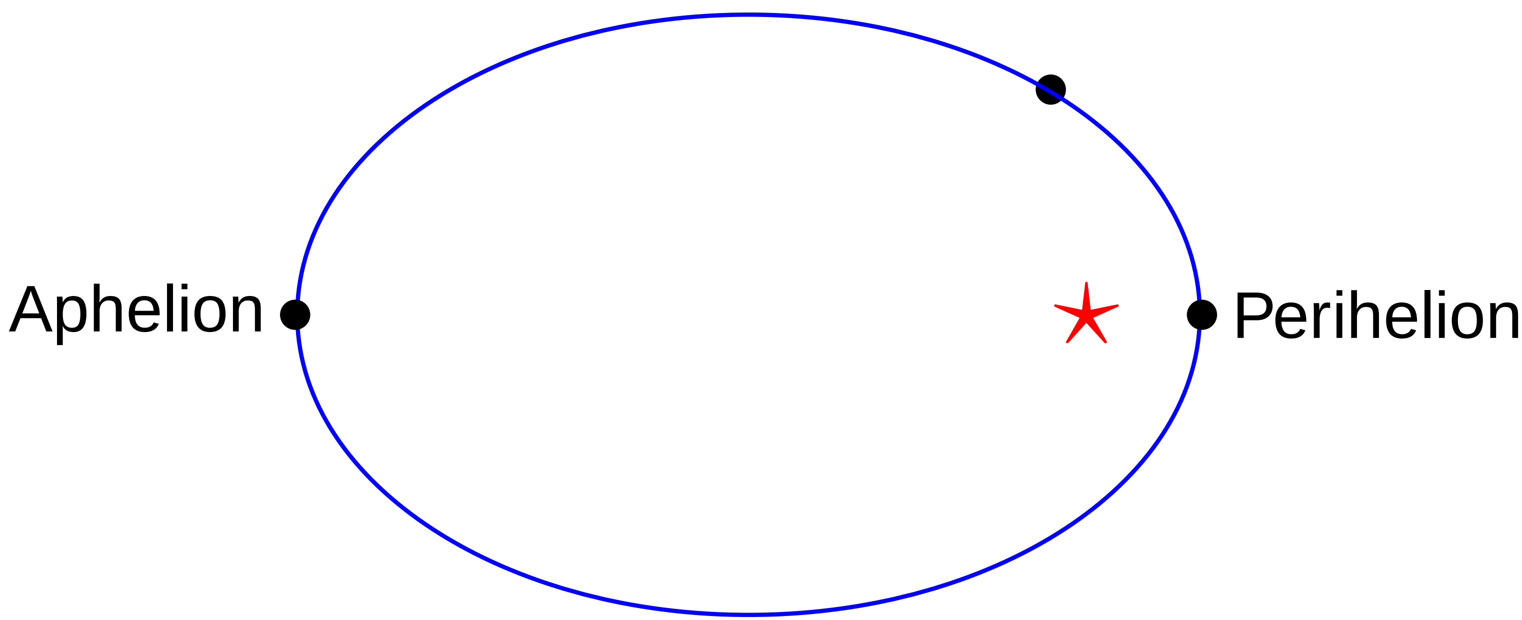

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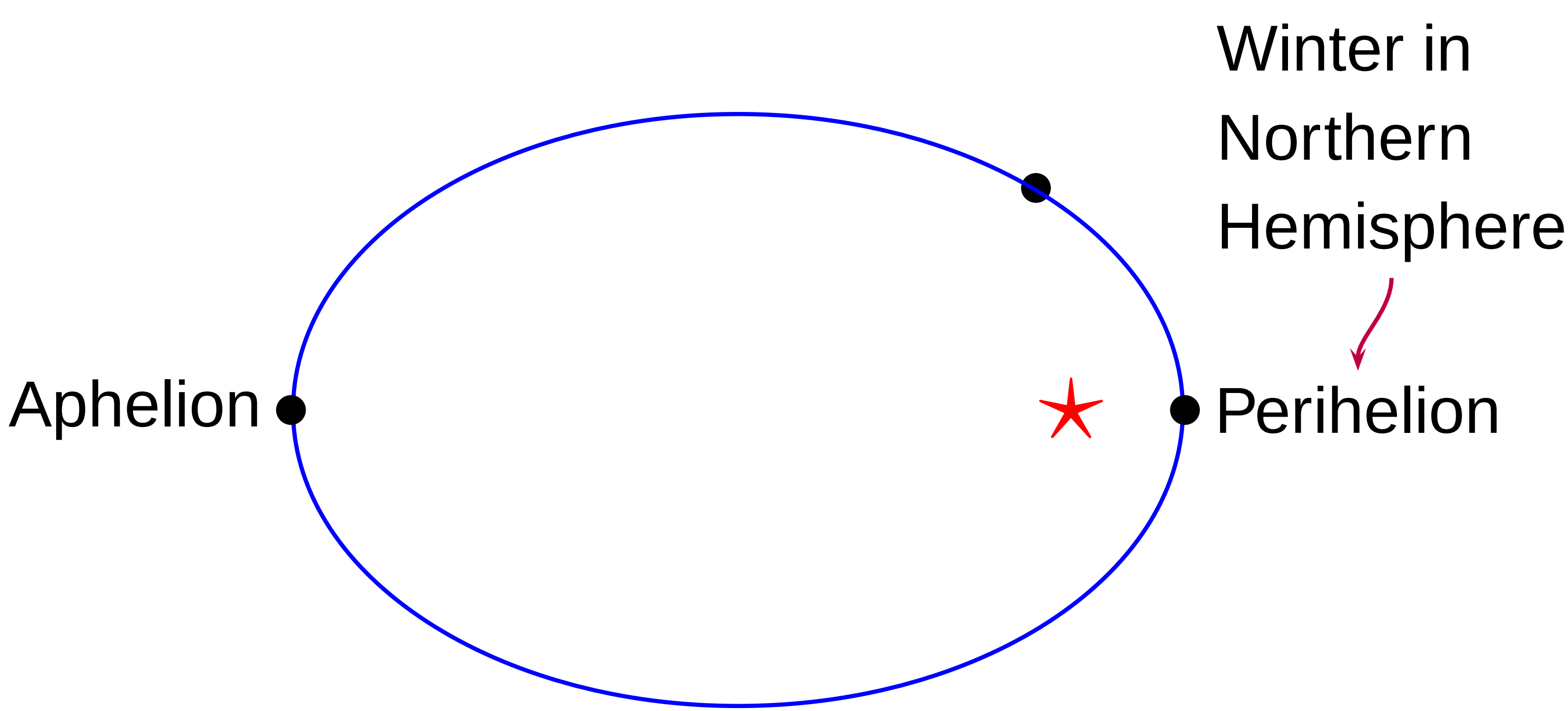

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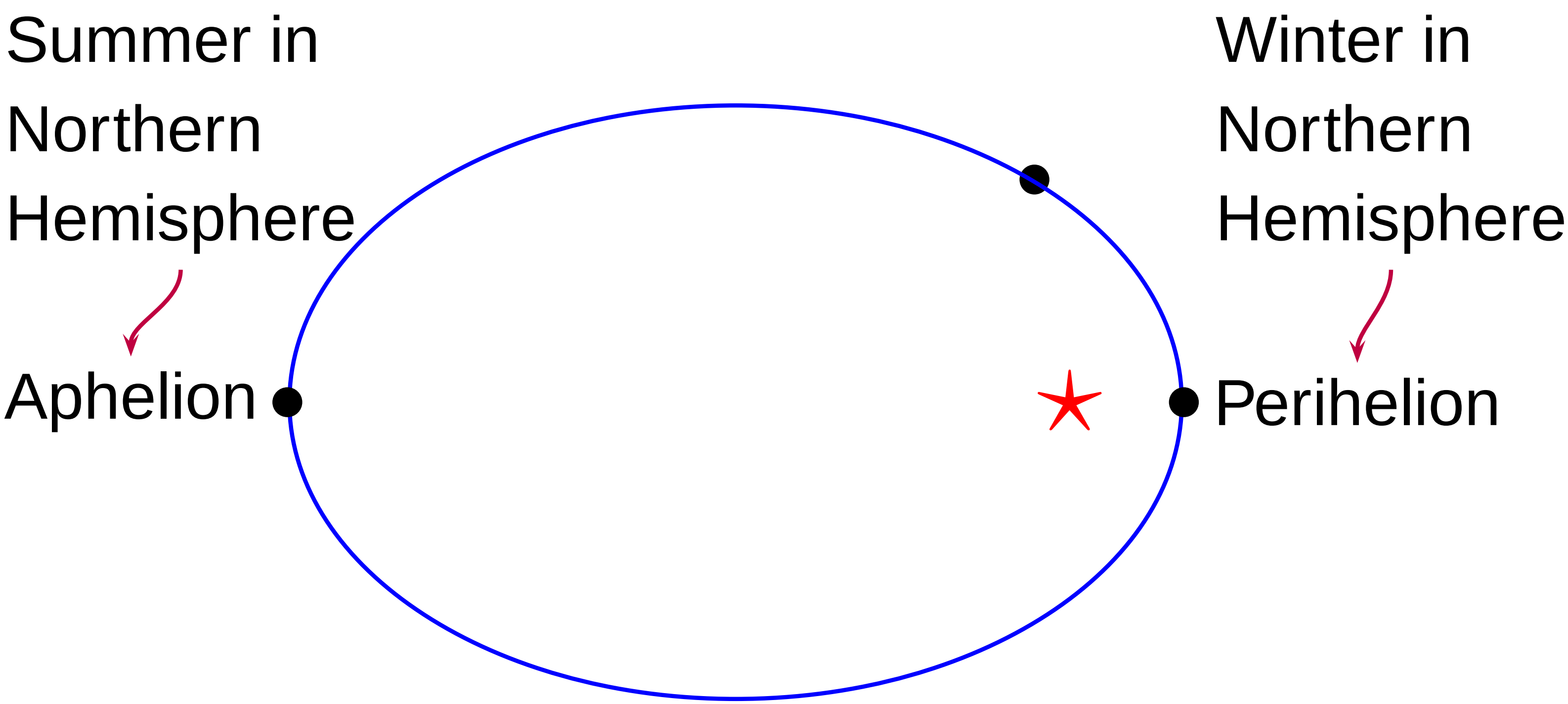
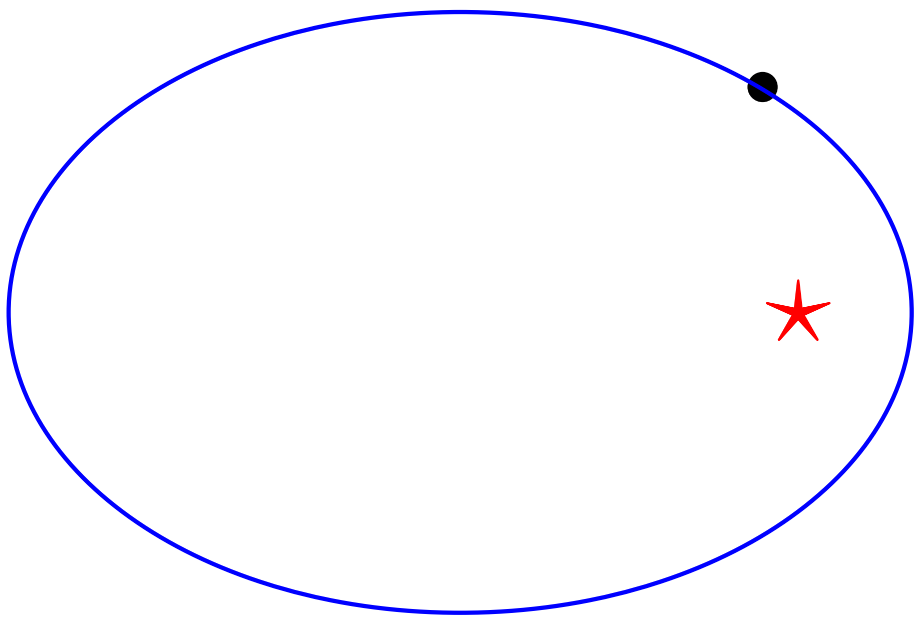


Kepler's First Law II



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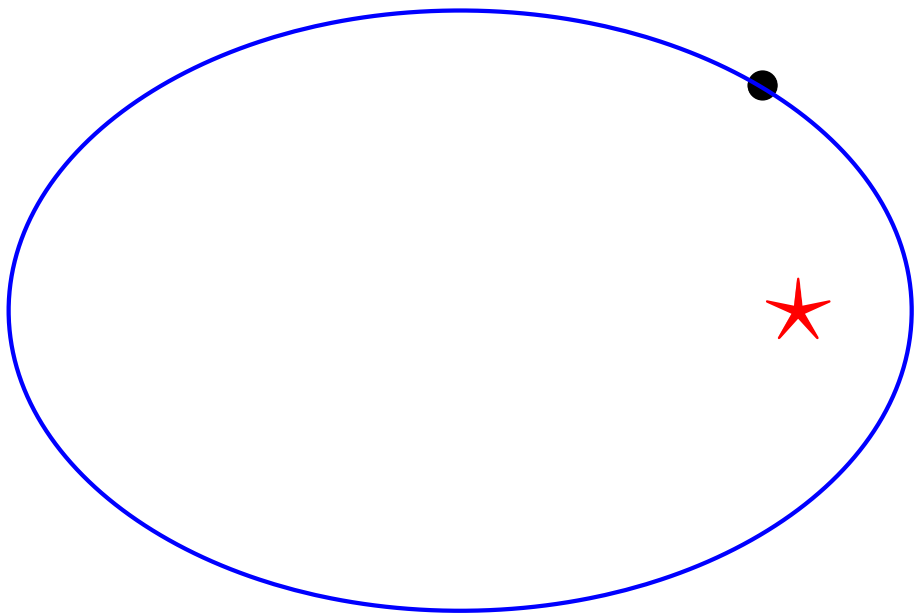
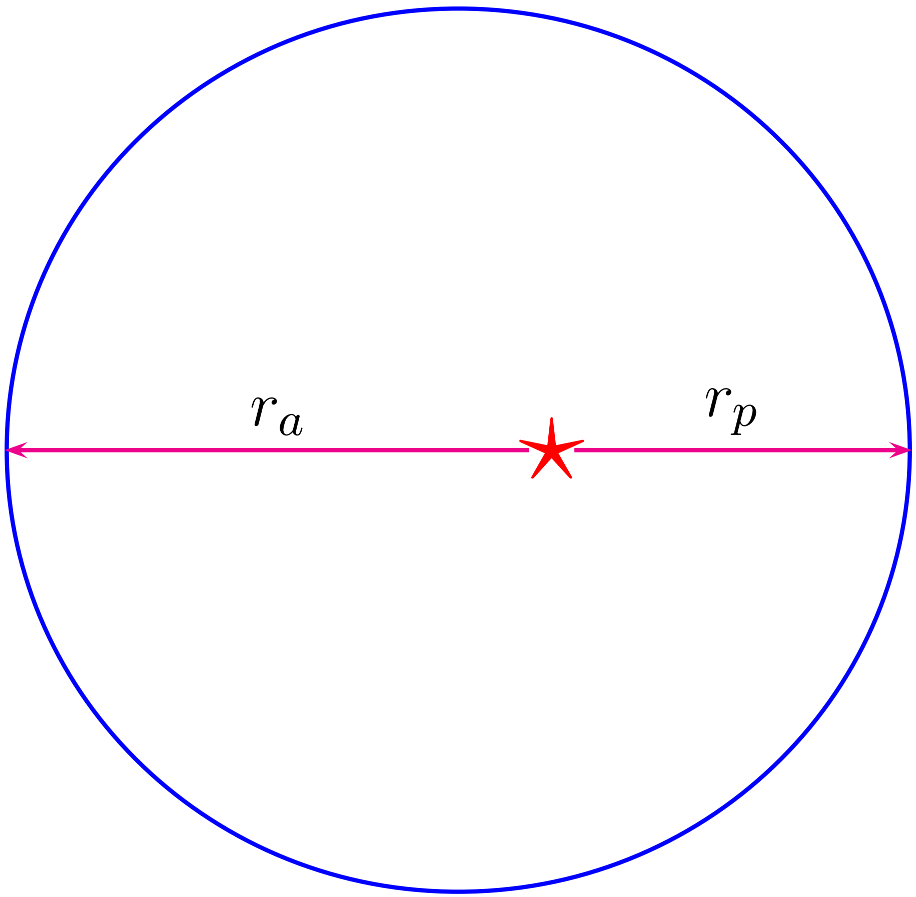


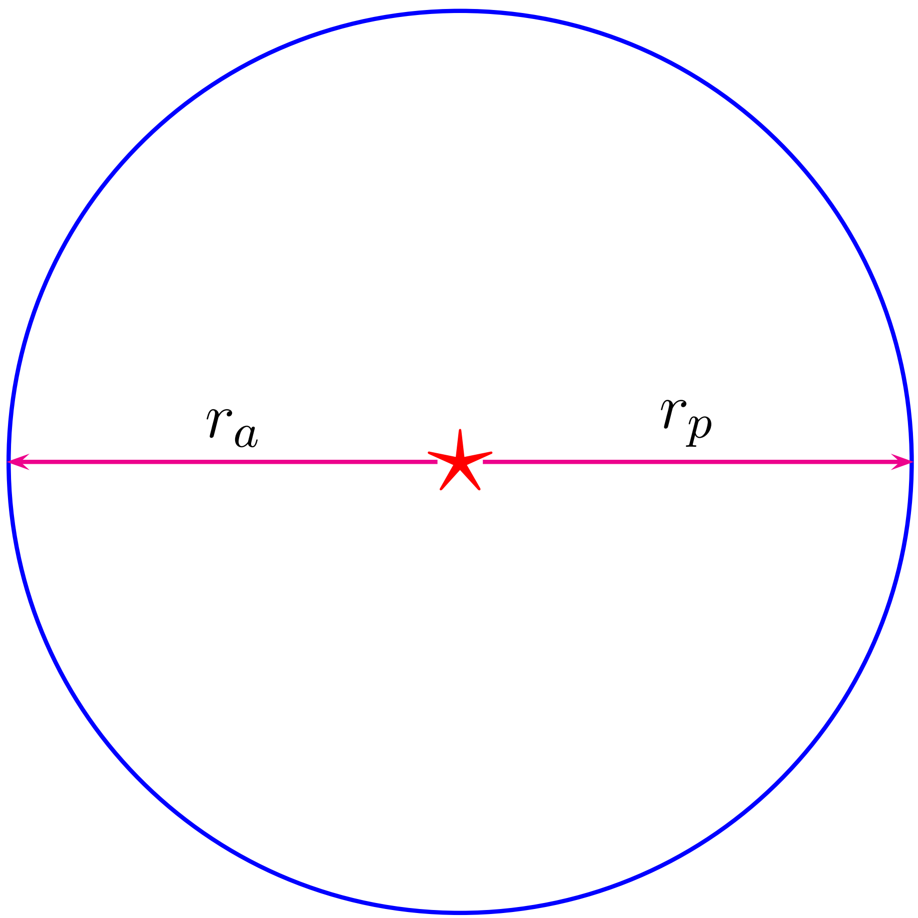
Planet	e

Kepler's First Law II



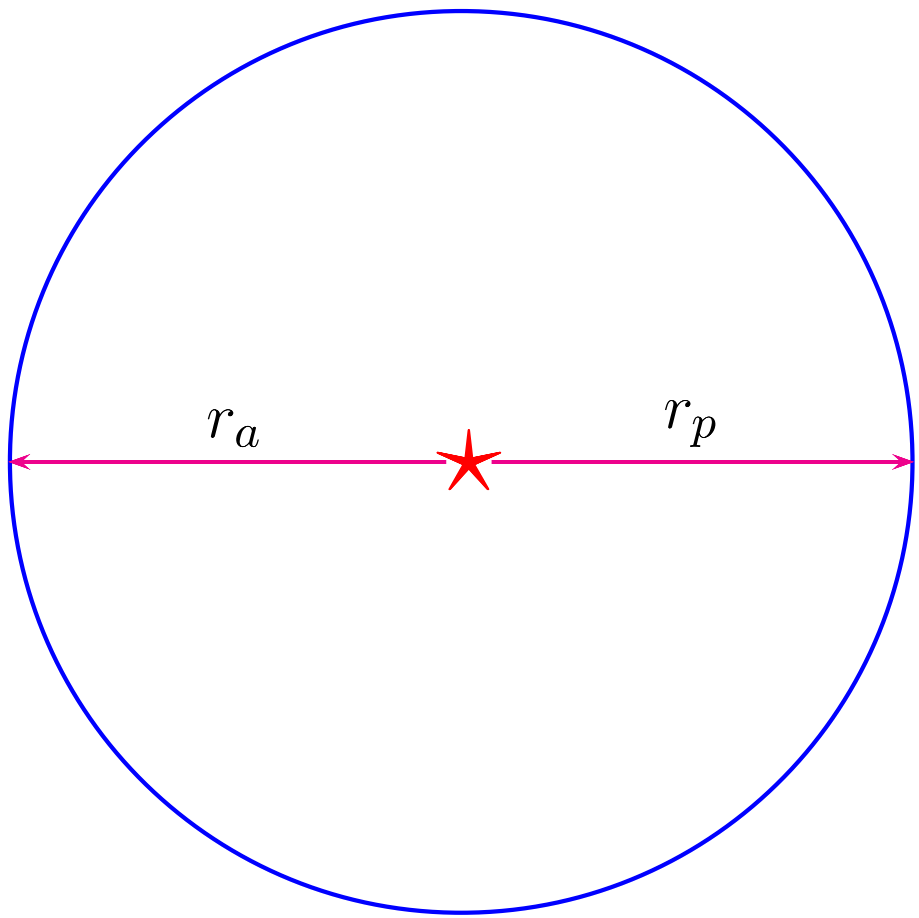
Planet	e
Mercury	0.206

Kepler's First Law II



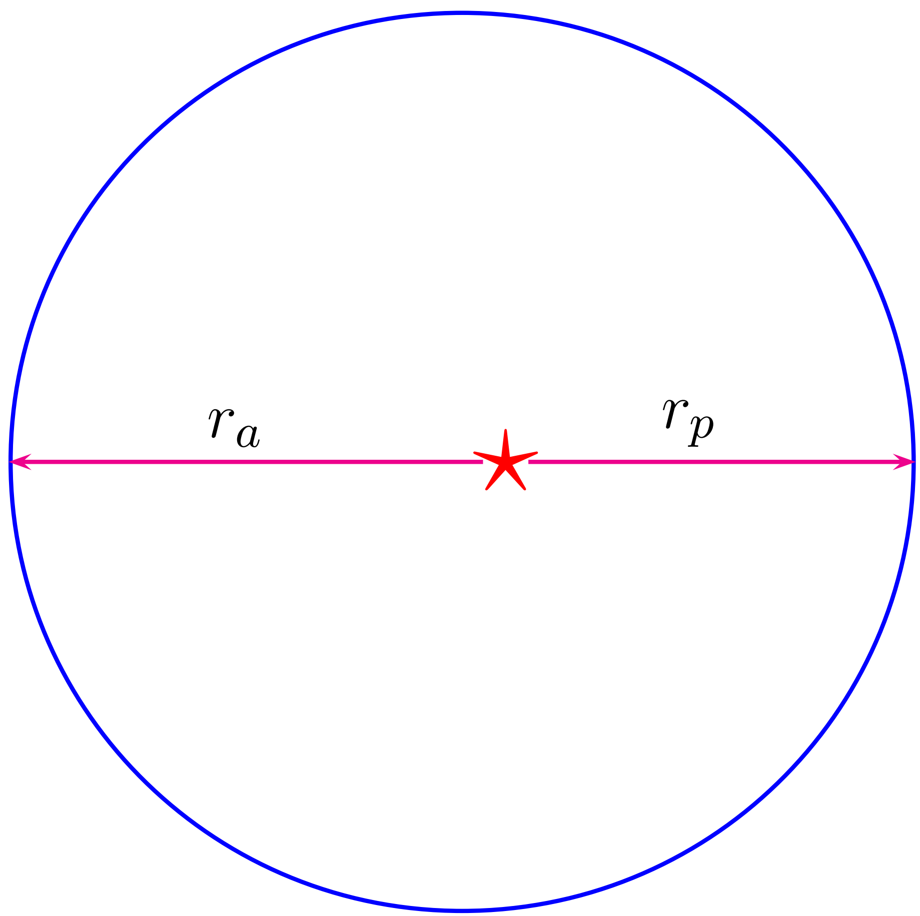
Planet	e
Mercury	0.206
Venus	0.007

Kepler's First Law II



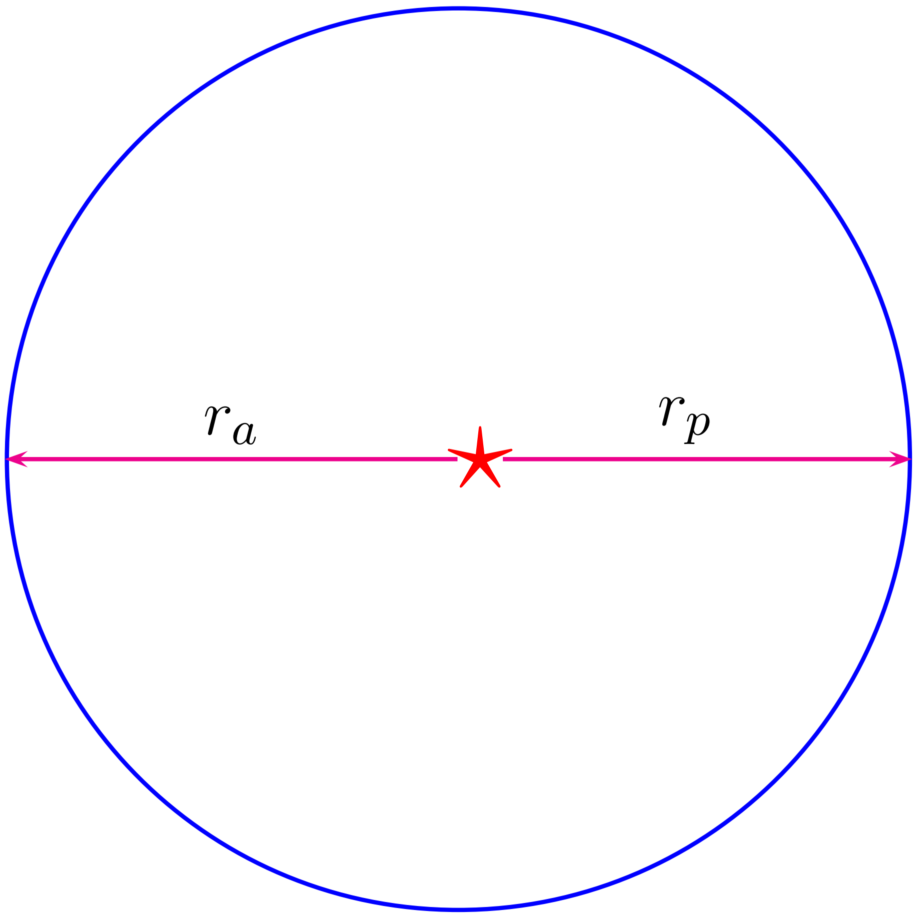
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017

Kepler’s First Law II



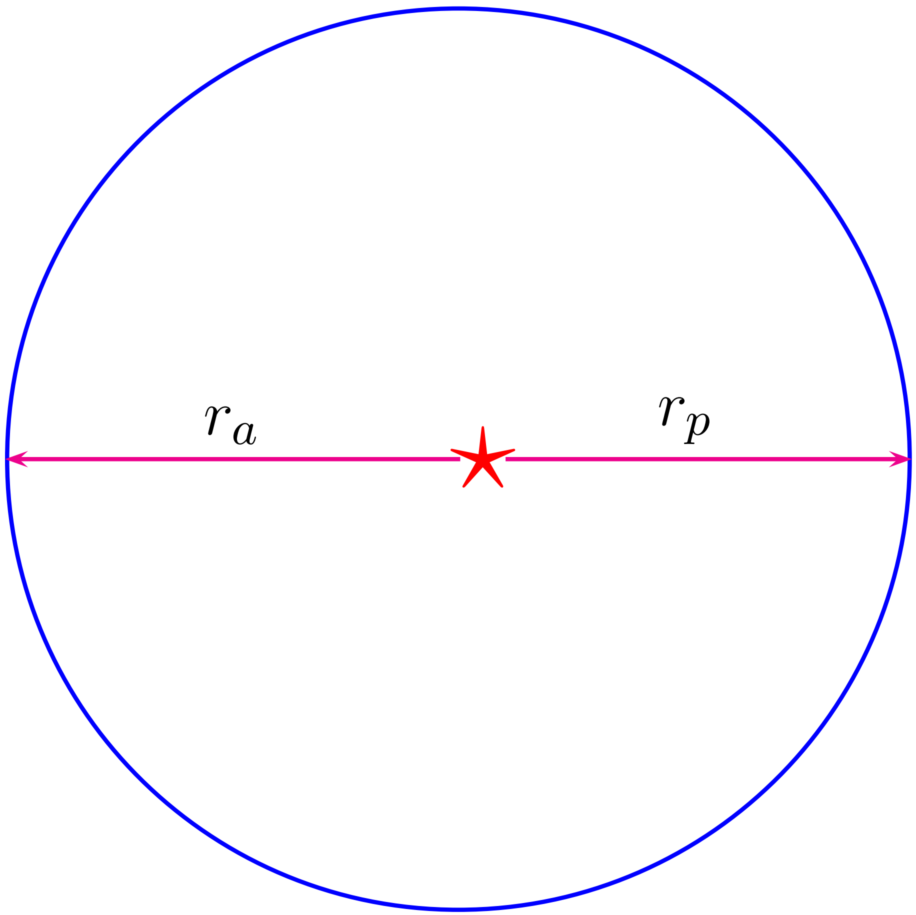
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093

Kepler's First Law II



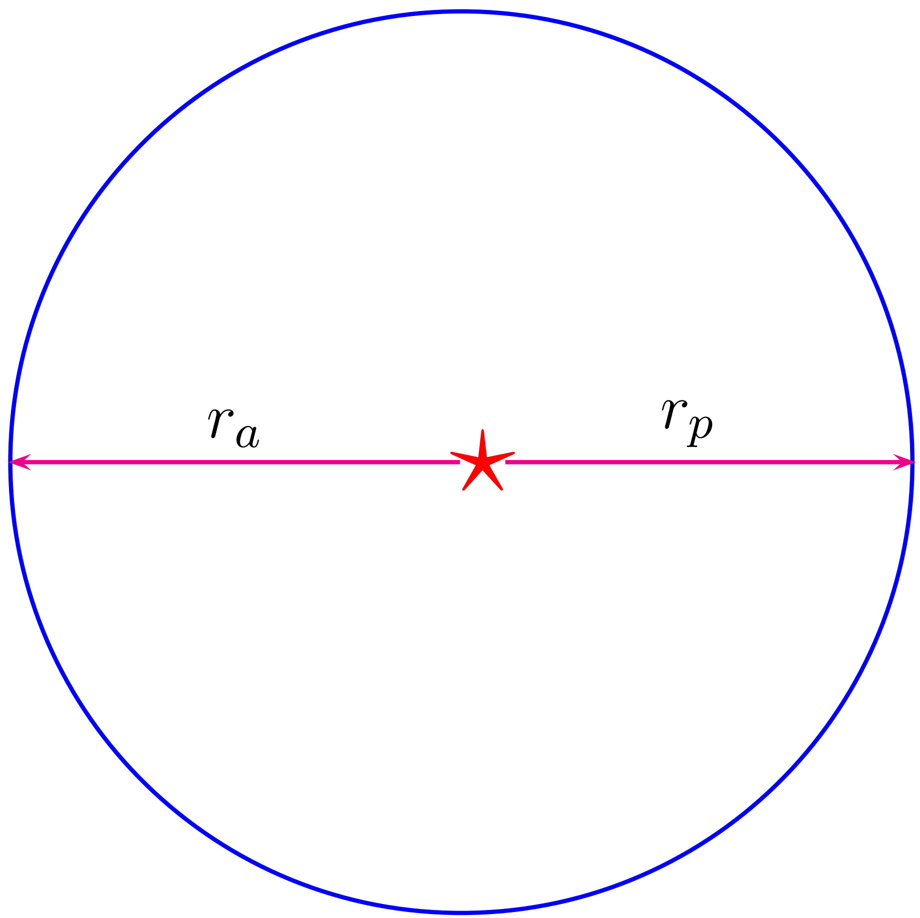
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048

Kepler's First Law II



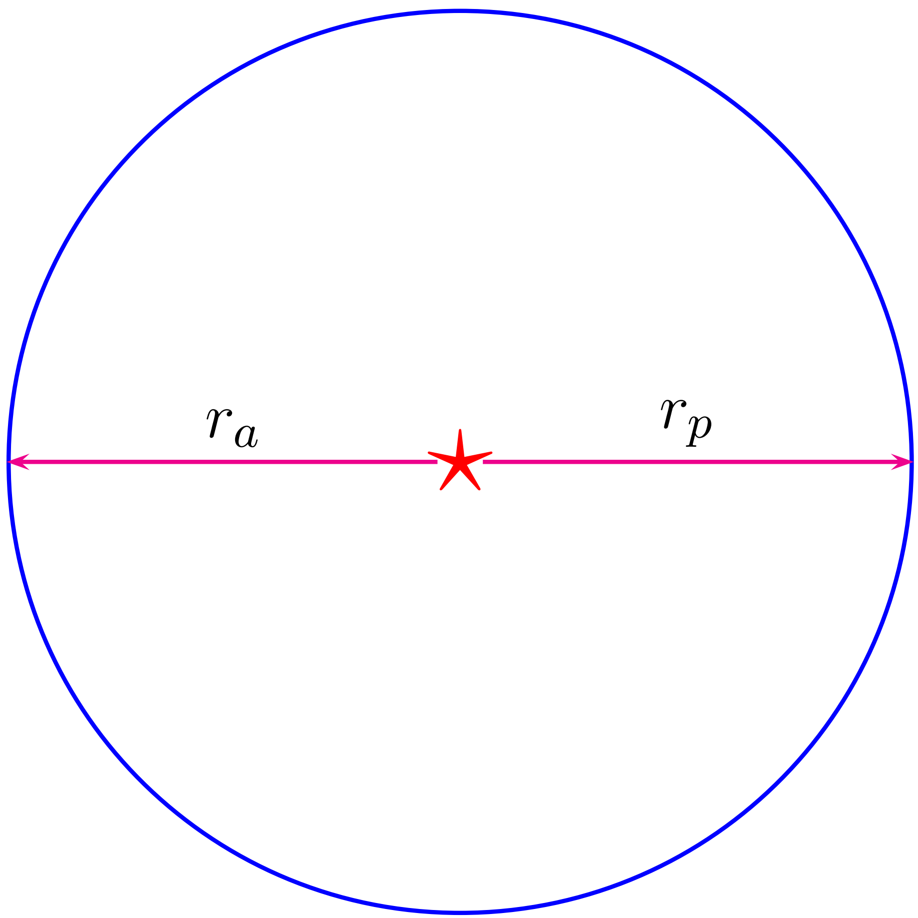
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054

Kepler's First Law II



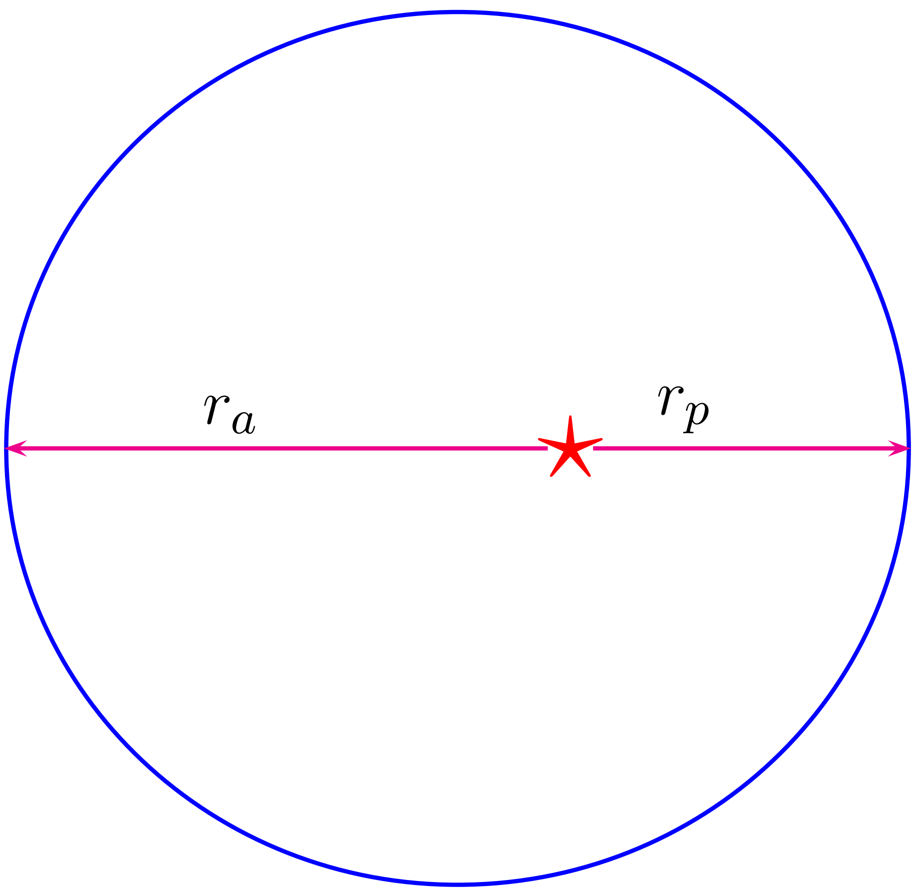
Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047

Kepler's First Law II



Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009

Kepler's First Law II



Planet	e
Mercury	0.206
Venus	0.007
Earth	0.017
Mars	0.093
Jupiter	0.048
Saturn	0.054
Uranus	0.047
Neptune	0.009
Pluto	0.249

Kepler's First Law III