April 18, Week 13

Today: Chapter 10, Angular Momentum

Homework #10 - Due April 26 at 11:59pm
Mastering Physics: 7 questions from chapter 10.
Written Question: 10.86

Exam #5, Friday, April 27
On Chapters 9 and 10

Angular Momentum

Angular Momentum, $\overrightarrow{\mathbf{L}}$ - the rotational counterpart to linear momentum, $\overrightarrow{\mathbf{p}}$.

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How much torque must be applied to cause a change in rotation.

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How much torque must be applied to cause a change in rotation.

For a point particle (an object with a single value of $\overrightarrow{\mathbf{v}}$), the angular momentum is:

$$\boxed{\overrightarrow{\mathbf{L}} = \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}}$$

Angular Momentum II

$\overrightarrow{\mathbf{L}} = \overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}$ can be shown by taking a derivative.

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$$\overrightarrow{\boldsymbol{\tau}} = \frac{d\overrightarrow{\mathbf{L}}}{dt} = \frac{d}{dt}\left(\overrightarrow{\mathbf{r}} \times \overrightarrow{\mathbf{p}}\right)$$

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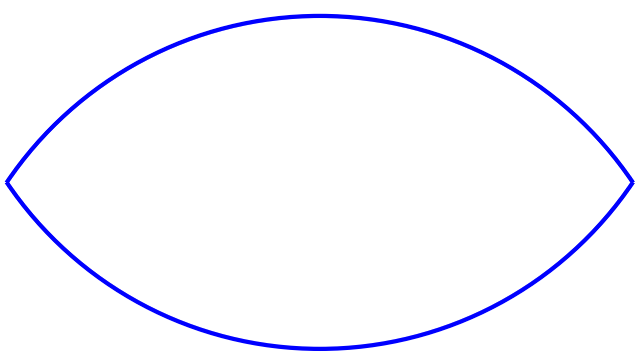

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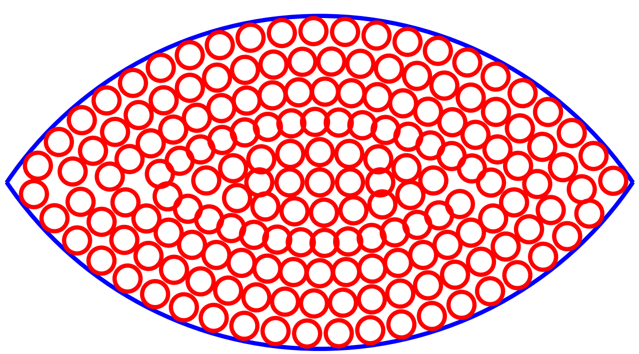

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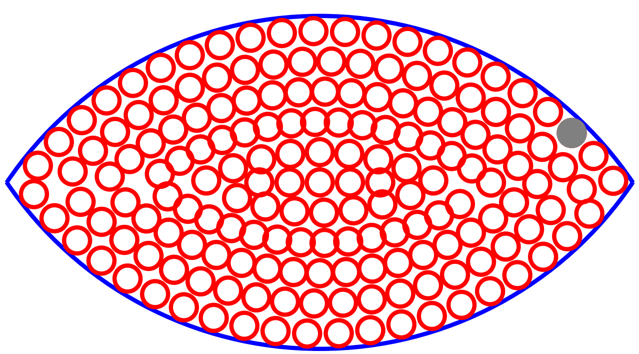

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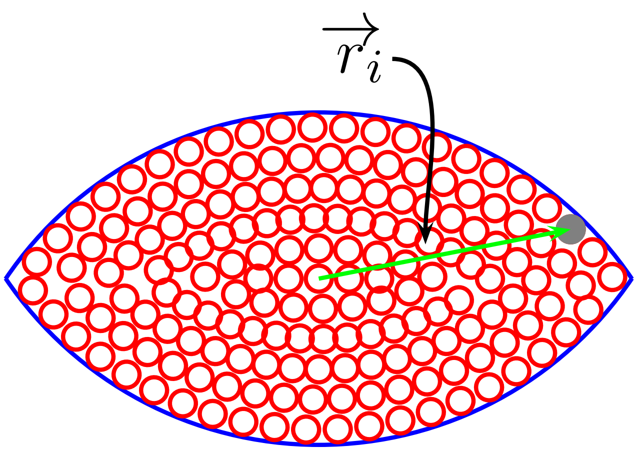

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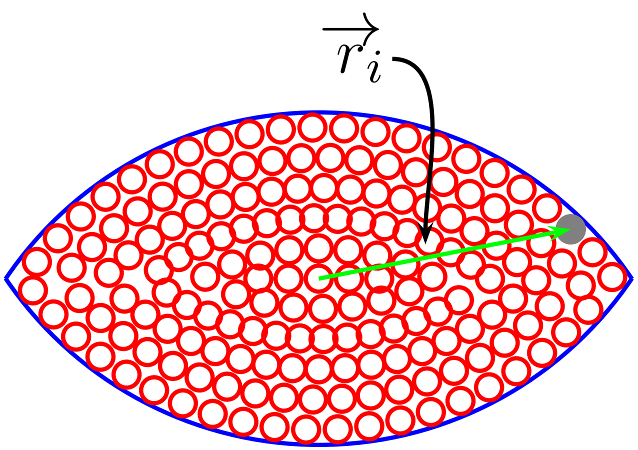

$$\overrightarrow{\mathbf{L}}_i = \overrightarrow{\mathbf{r}}_i \times \overrightarrow{\mathbf{p}}_i = \overrightarrow{\mathbf{r}}_i \times M_i \overrightarrow{\mathbf{v}}_i$$

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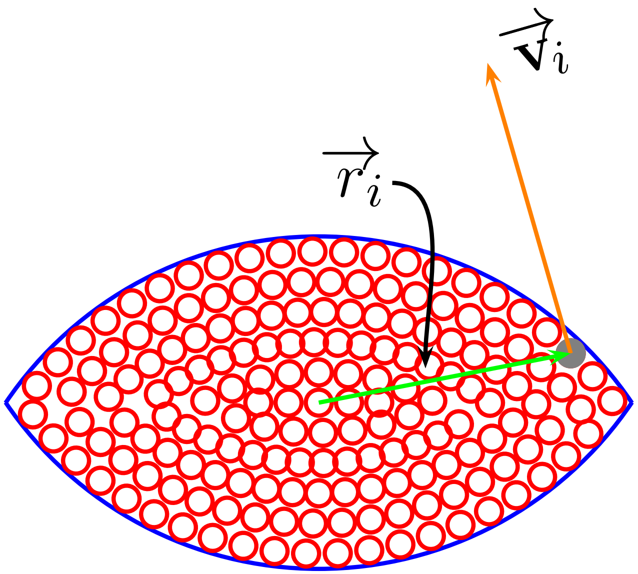

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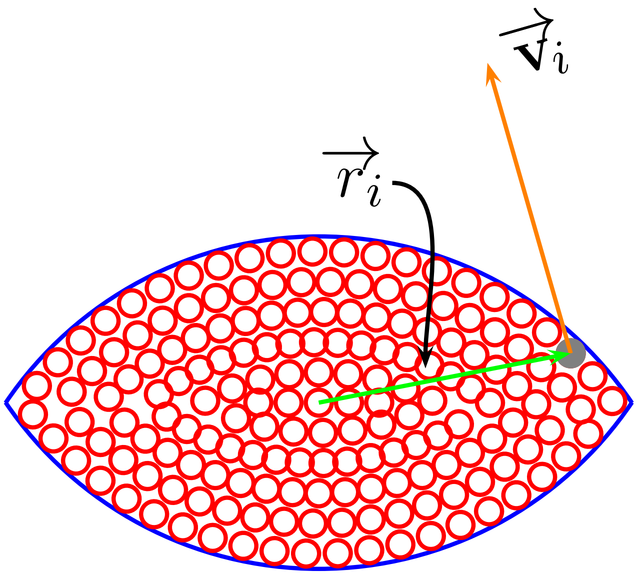

$\vec{\mathbf{L}}_i = \vec{\mathbf{r}}_i \times \vec{\mathbf{p}}_i = \vec{\mathbf{r}}_i \times M_i \vec{\mathbf{v}}_i$

Each piece going on circle
$\Rightarrow$ $\vec{\mathbf{r}}_i$ is 90° to $\vec{\mathbf{v}}_i$

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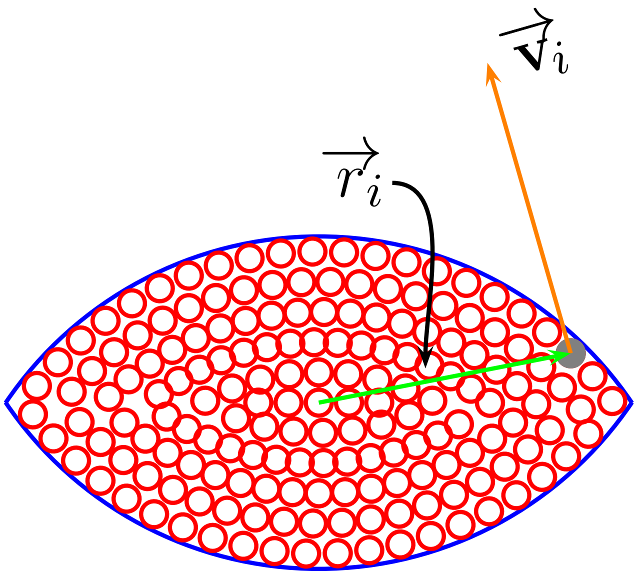

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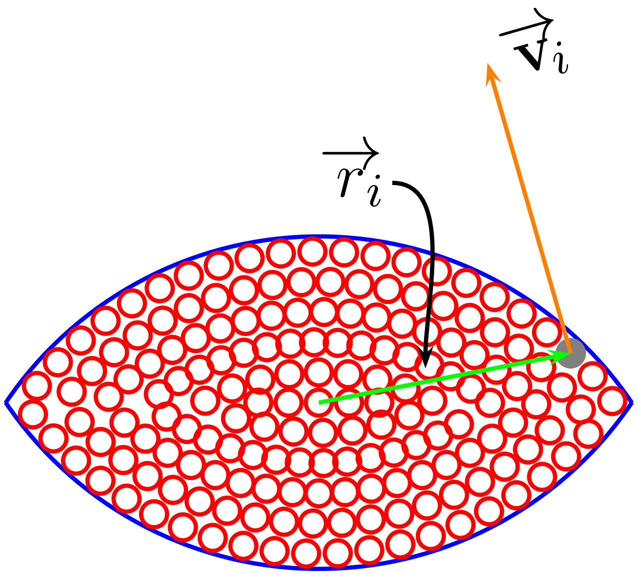

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$$L_i = r_i M_i v_i$$

$$v_i = r_i \omega$$

Angular Momentum IV

Units: $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} \Rightarrow m \cdot kg \cdot m/s = kg \cdot m^2/s$

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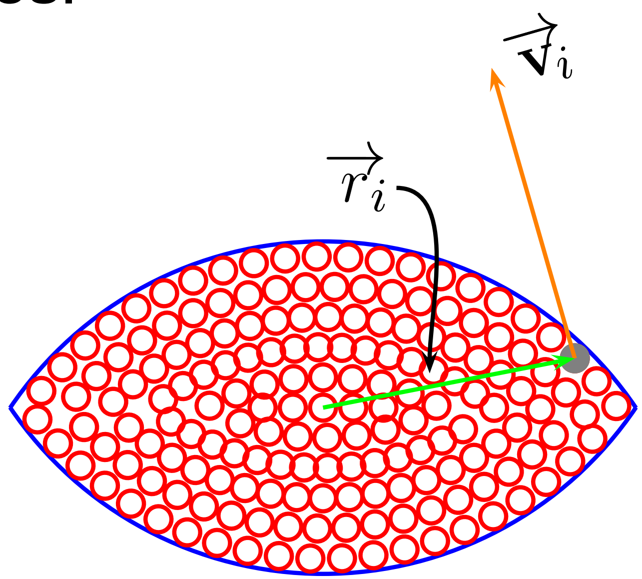

$$L_i = M_i r_i^2 \omega$$

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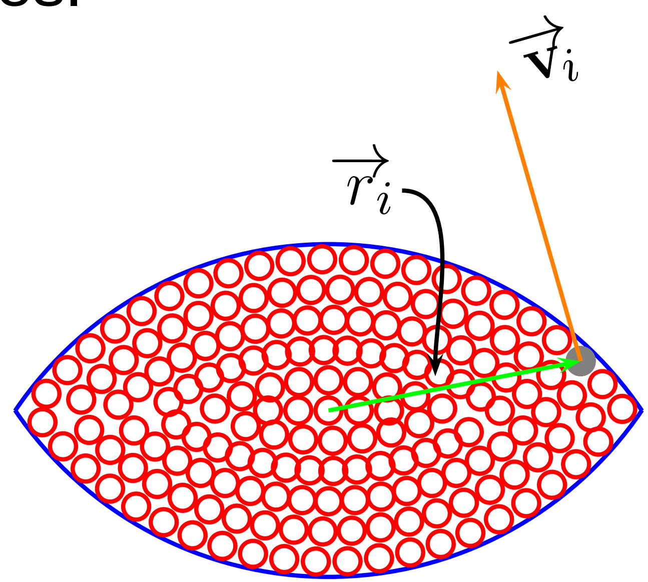

$$L_i = M_i r_i^2 \omega$$

$$L \approx \sum_i L_i = \left(\sum_i M_i r_i^2 \right) \omega$$

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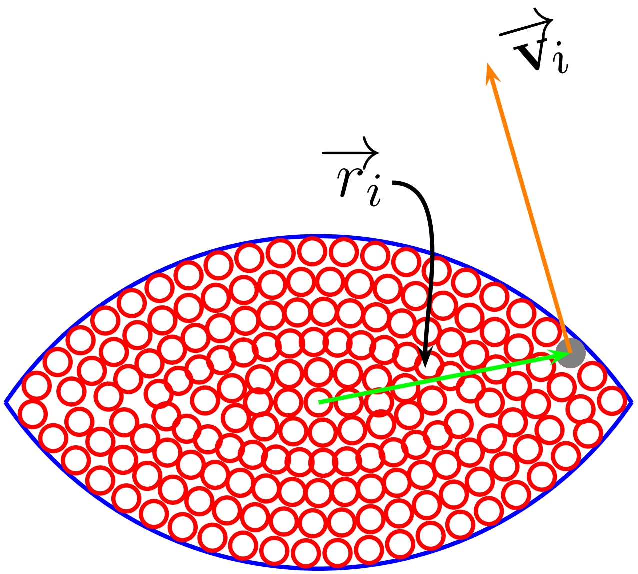

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As $i \to \infty, \quad \sum_i M_i r_i^2 \to I$

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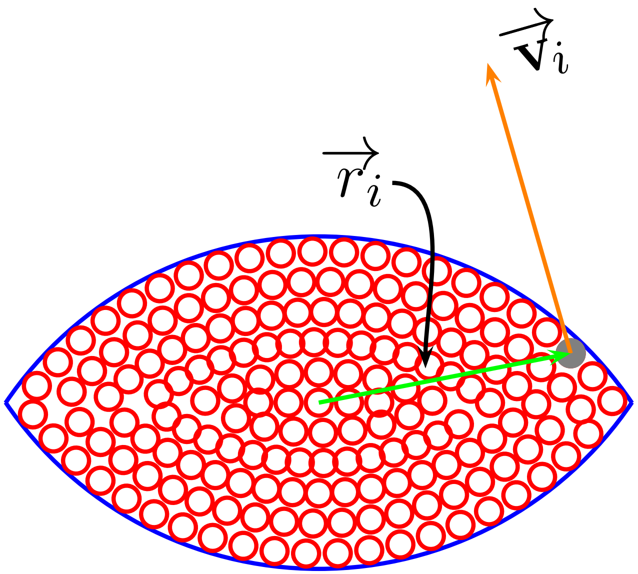

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$$\boxed{\vec{\mathbf{L}} = I \vec{\boldsymbol{\omega}}}$$

Clicker Quiz

A solid disk with moment of Inertia $I = 2\,kg \cdot m^2$ is rotating clockwise with angular speed $3\,rad/s$. What average torque must be exerted over $2\,s$ in order to make the disk spin counter-clockwise with angular speed $5\,rad/s$?

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(a) $2\,N \cdot m$

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(b) $4\,N \cdot m$

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(d) $8\,N \cdot m$

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Example

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Example: Find the angular momentum of the earth for its orbital motion around the sun. Assume the 5.97×10^{24}-kg earth is following a circular orbit of radius $1.5 \times 10^{11}\, m$.

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Example: Find the angular momentum of the earth for its orbital motion around the sun. Assume the 5.97×10^{24}-kg earth is following a circular orbit of radius $1.5 \times 10^{11}\,m$.

Example: Find the angular momentum of the earth for its 24-$hour$ daily motion. Treat the earth as a solid sphere of radius of $6.38 \times 10^{6}\,m$.

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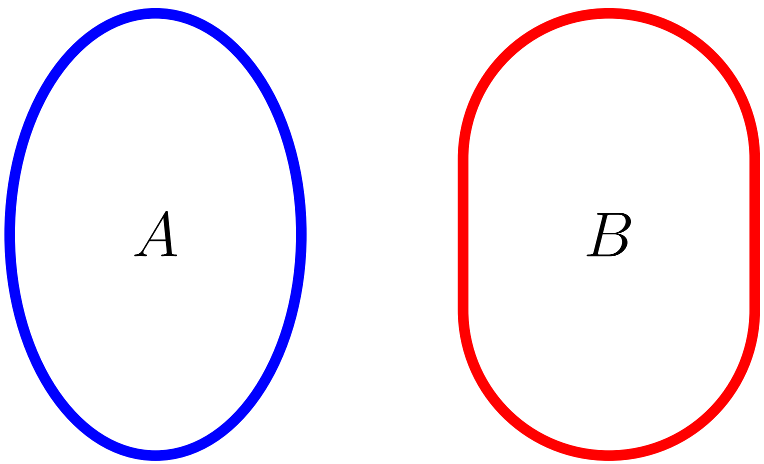

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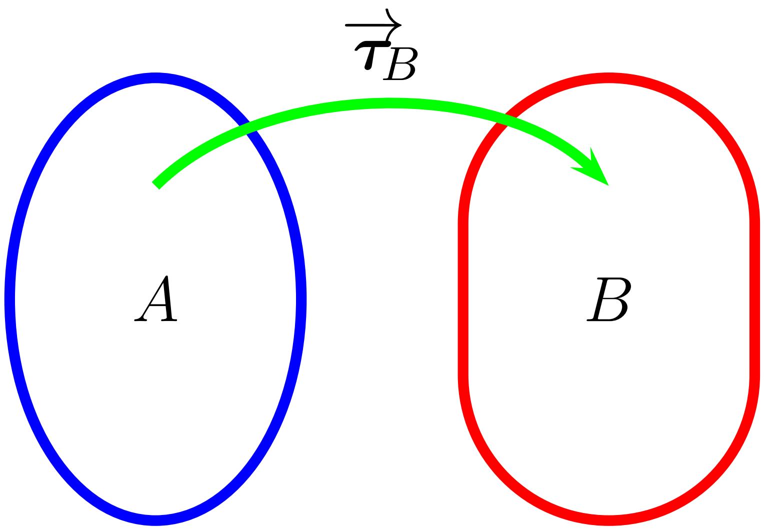

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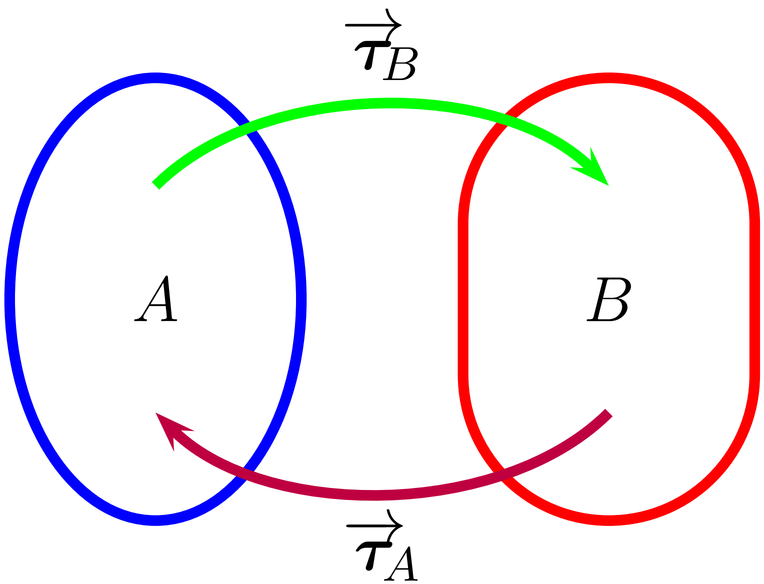

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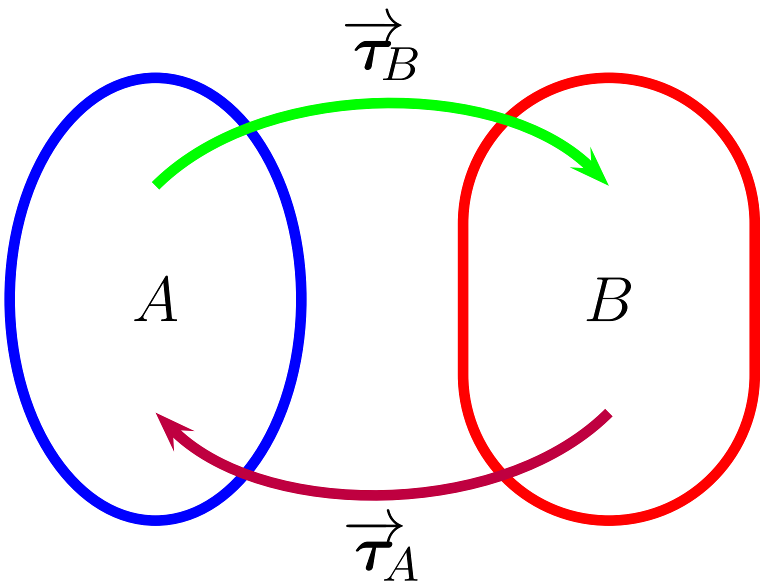

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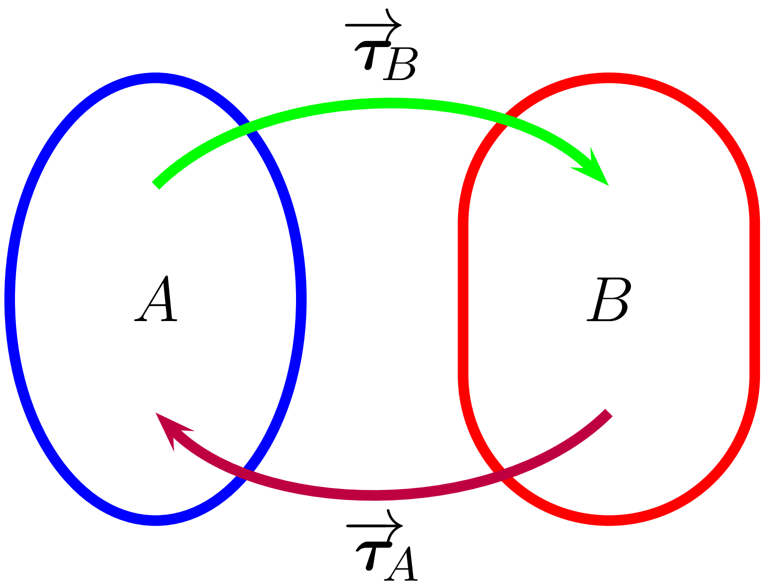

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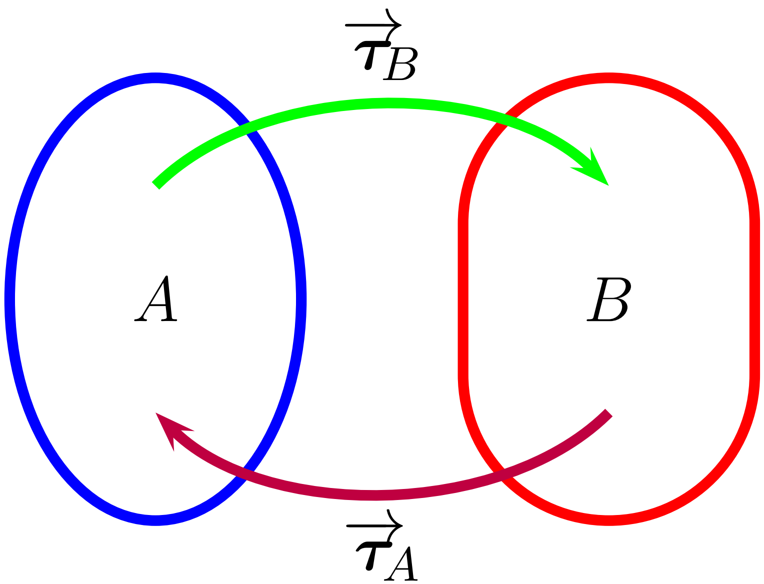

$$\overrightarrow{\boldsymbol{\tau}}_A = \frac{d\overrightarrow{\mathbf{L}_A}}{dt}$$

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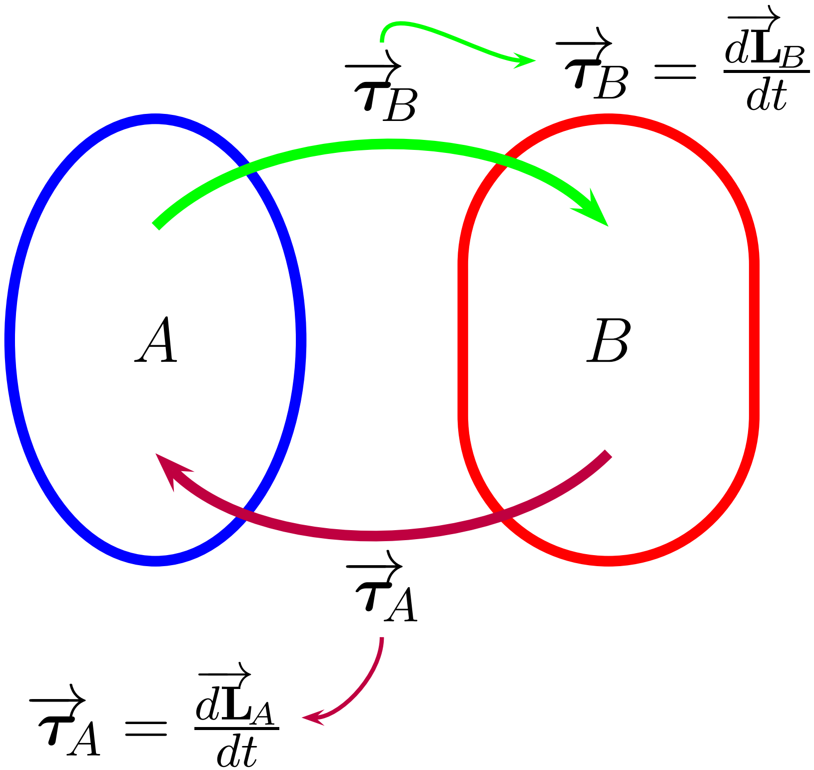


$\overrightarrow{\boldsymbol{\tau}}_B =$ Torque on B due to A

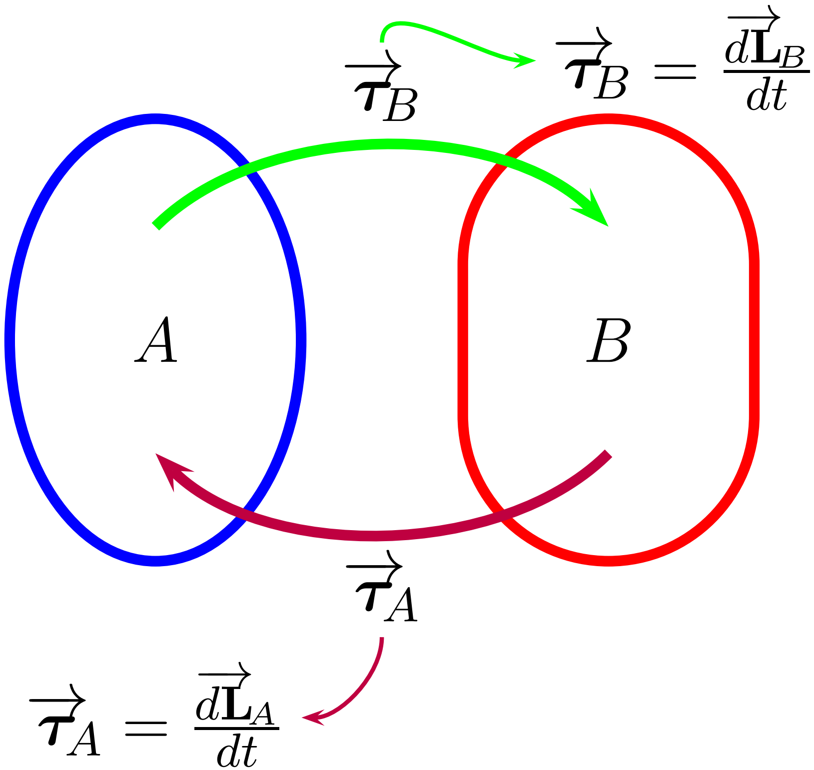
$\overrightarrow{\boldsymbol{\tau}}_A =$ Torque on A due to B

3rd Law for rotation: $\overrightarrow{\boldsymbol{\tau}}_A = -\overrightarrow{\boldsymbol{\tau}}_B$

$$\overrightarrow{\boldsymbol{\tau}}_A + \overrightarrow{\boldsymbol{\tau}}_B = 0$$

Conservation of Angular Momentum

In the absence of external torques, the total angular momentum of a system cannot change.



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$$\frac{d\vec{\mathbf{L}}_A}{dt} + \frac{d\vec{\mathbf{L}}_A}{dt} = 0$$

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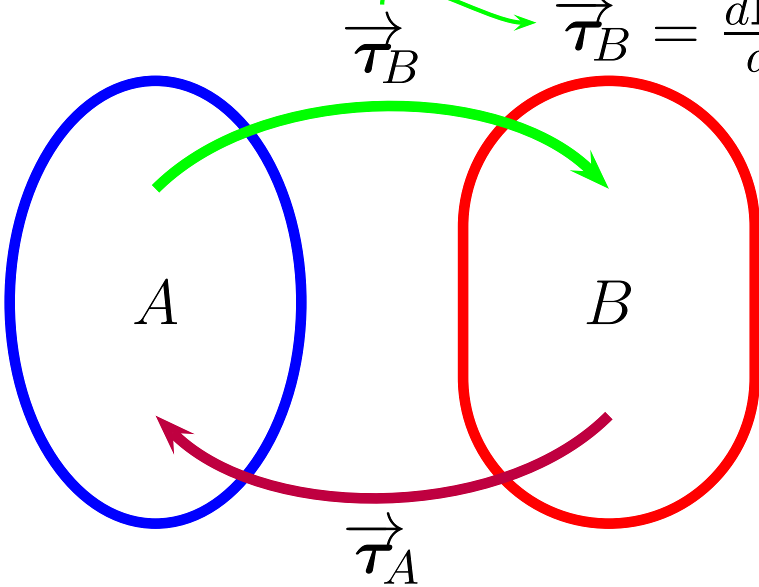
$$\vec{\boldsymbol{\tau}}_B = \frac{d\vec{\mathbf{L}}_B}{dt}$$



$$\vec{\boldsymbol{\tau}}_A = \frac{d\vec{\mathbf{L}}_A}{dt}$$

$\vec{\boldsymbol{\tau}}_B =$ Torque on B due to A

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$$\boxed{\vec{\mathbf{L}}_A + \vec{\mathbf{L}}_B = \text{constant}}$$