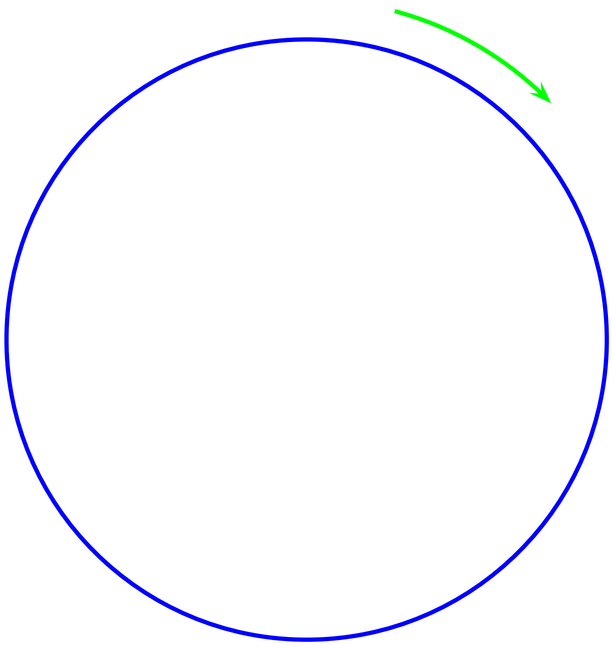
April 9, Week 12

Today: Chapter 9, Rotational Energy

Homework #9 - Due April 16 at 11:59pm
Mastering Physics: 7 questions from chapter 9. Written Question: 10.80

Review

The rate at which an objects spins is given by its angular velocity, $\vec{\omega}$, and angular acceleration $\vec{\alpha}$.



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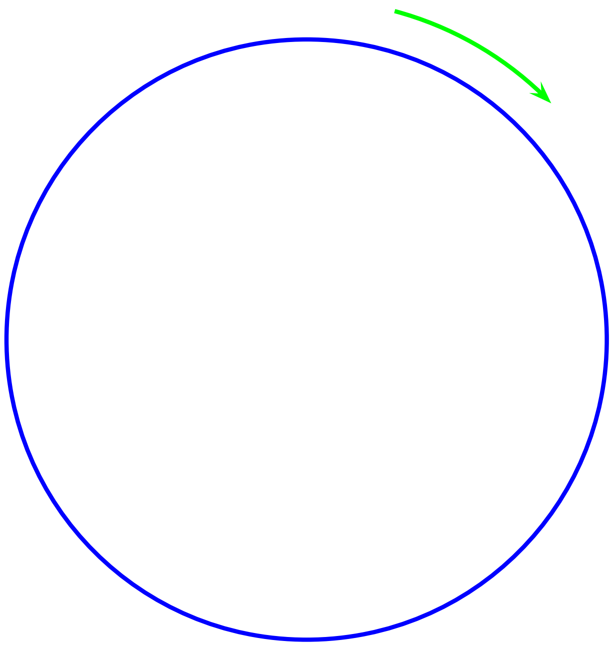

RHR I - Curl the fingers of your right hand in the “sense" of the rotation. Your extended thumb, points in direction of $\overrightarrow{\boldsymbol{\omega}}$.

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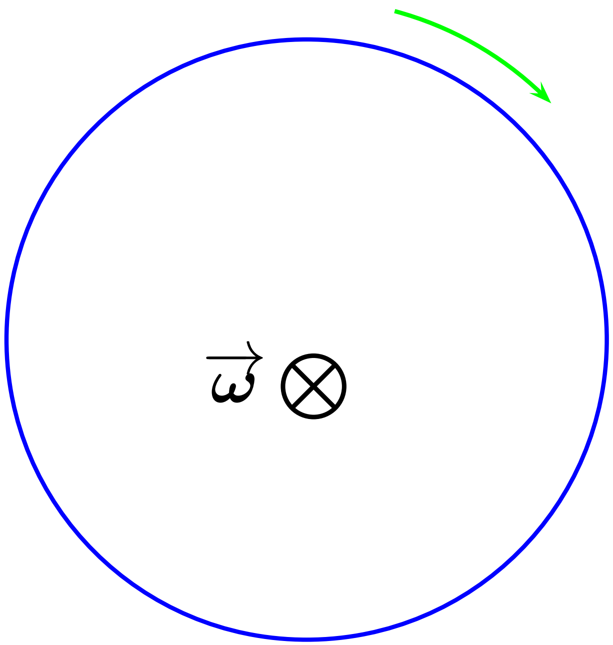

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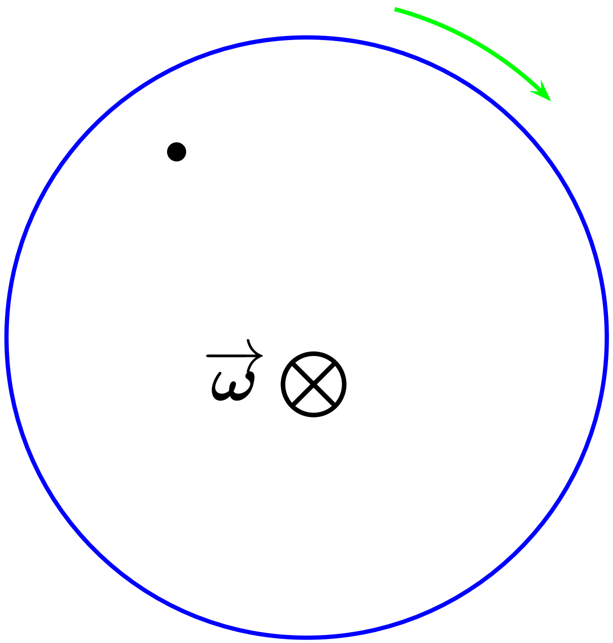

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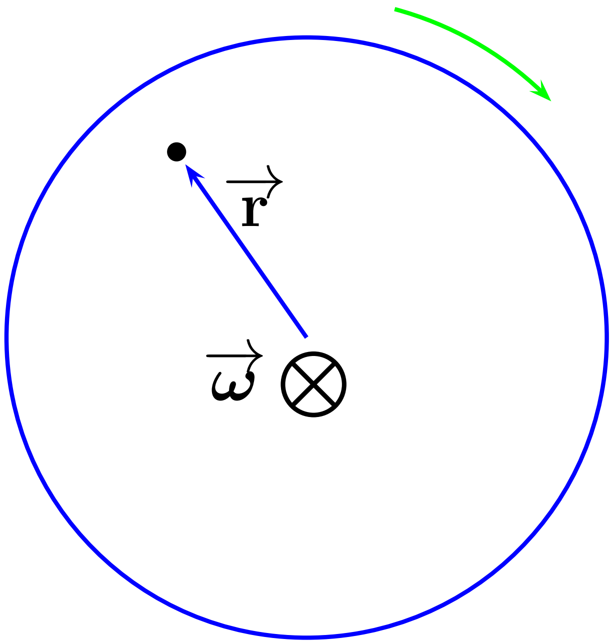

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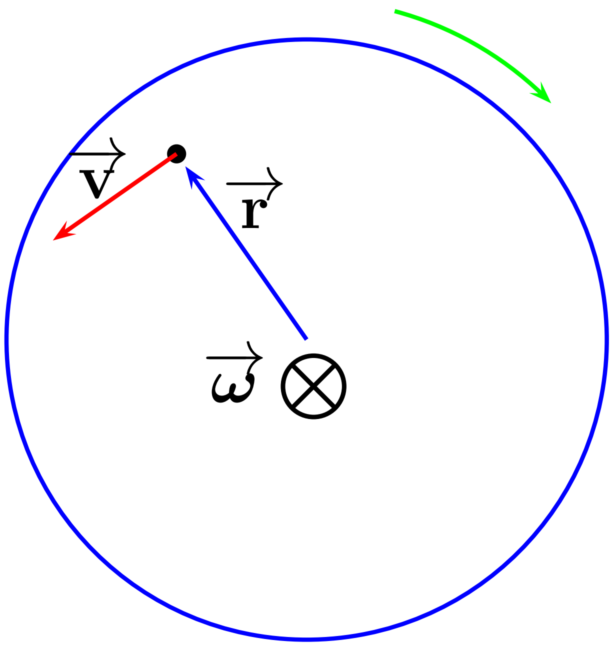

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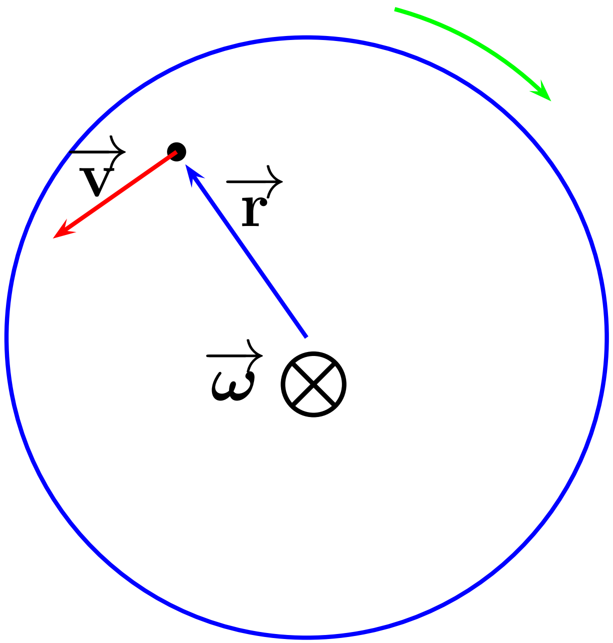

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$$\vec{\mathbf{v}} = \vec{\omega} \times \vec{\mathbf{r}}$$

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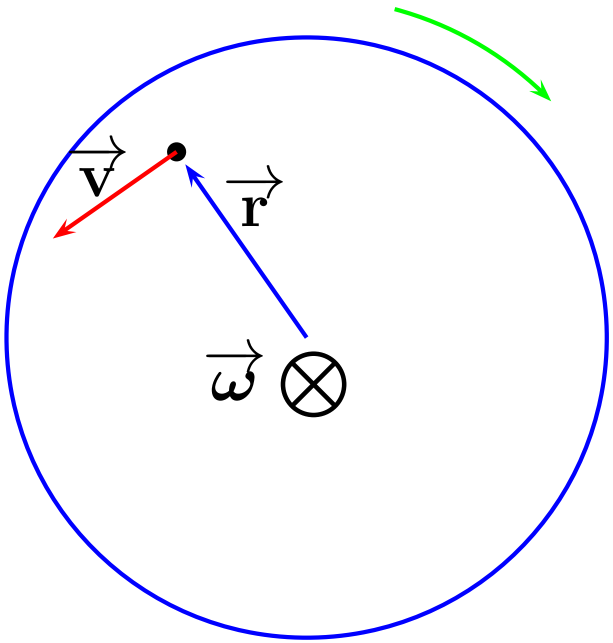

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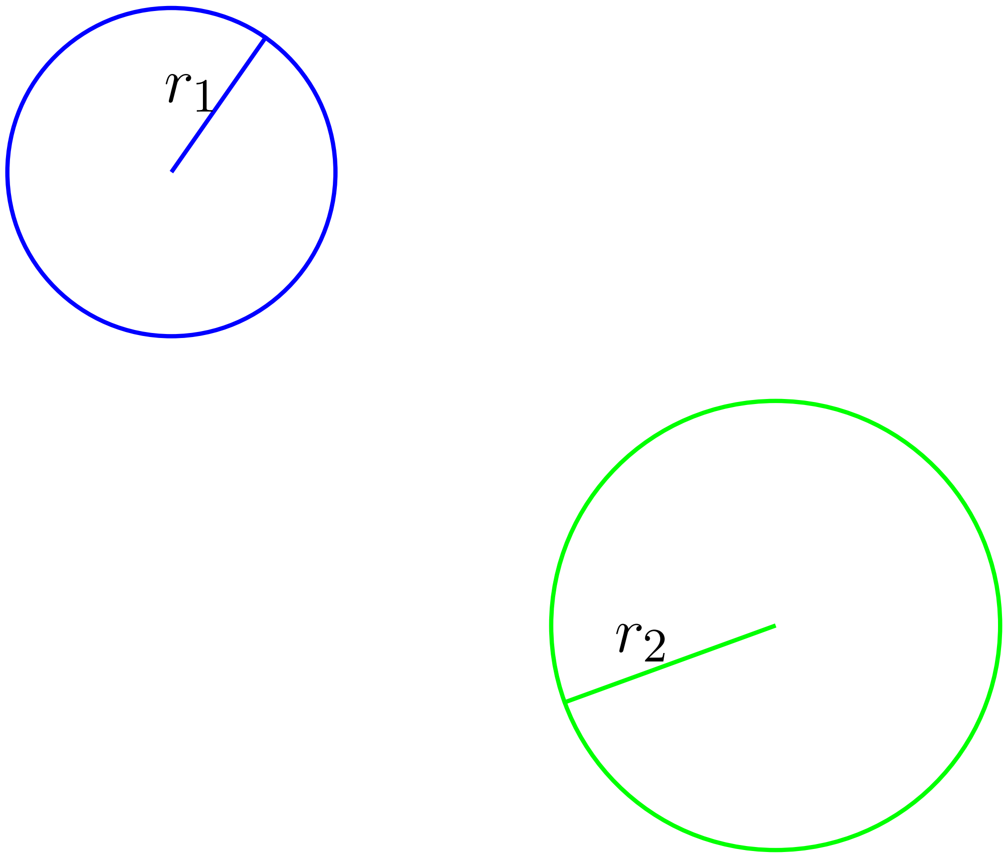
$$\overrightarrow{\mathbf{v}} = \overrightarrow{\omega} \times \overrightarrow{\mathbf{r}}$$

Take the fingers of the right hand and “sweep" $\overrightarrow{\mathrm{A}}$ into $\overrightarrow{\mathrm{B}}$, extended thumb points in direction of cross product

Connected Rotating Objects



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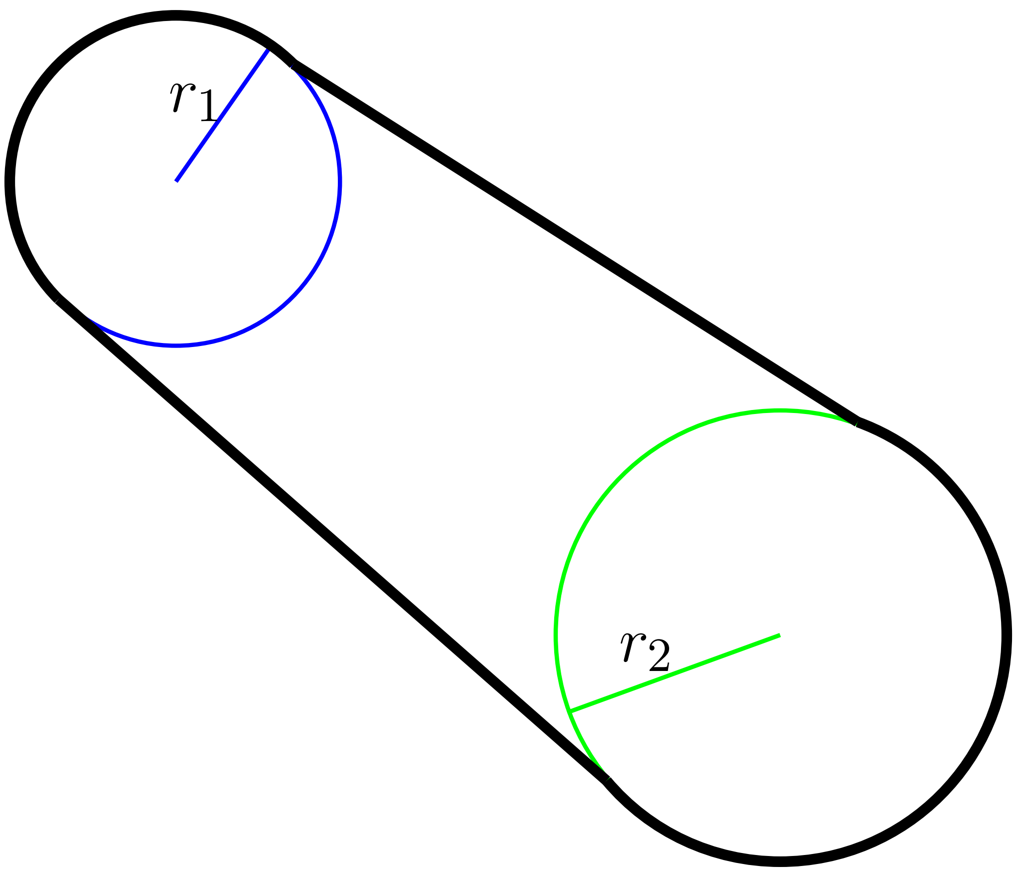

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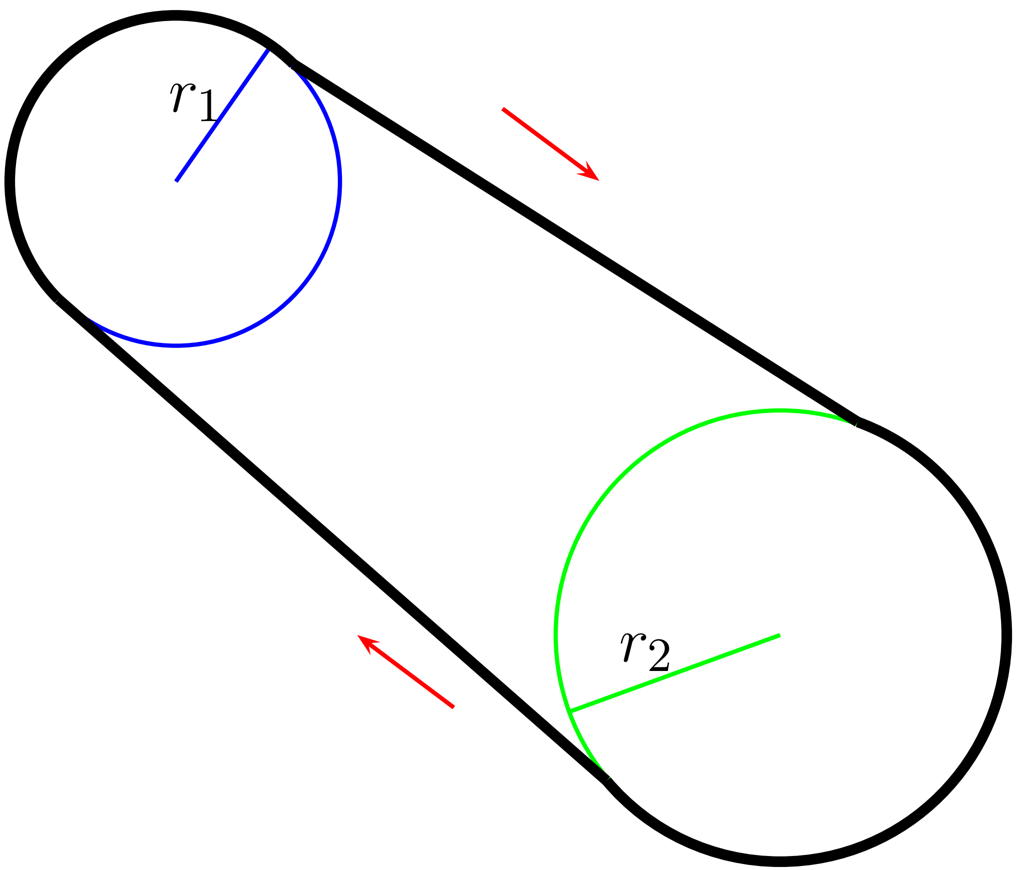

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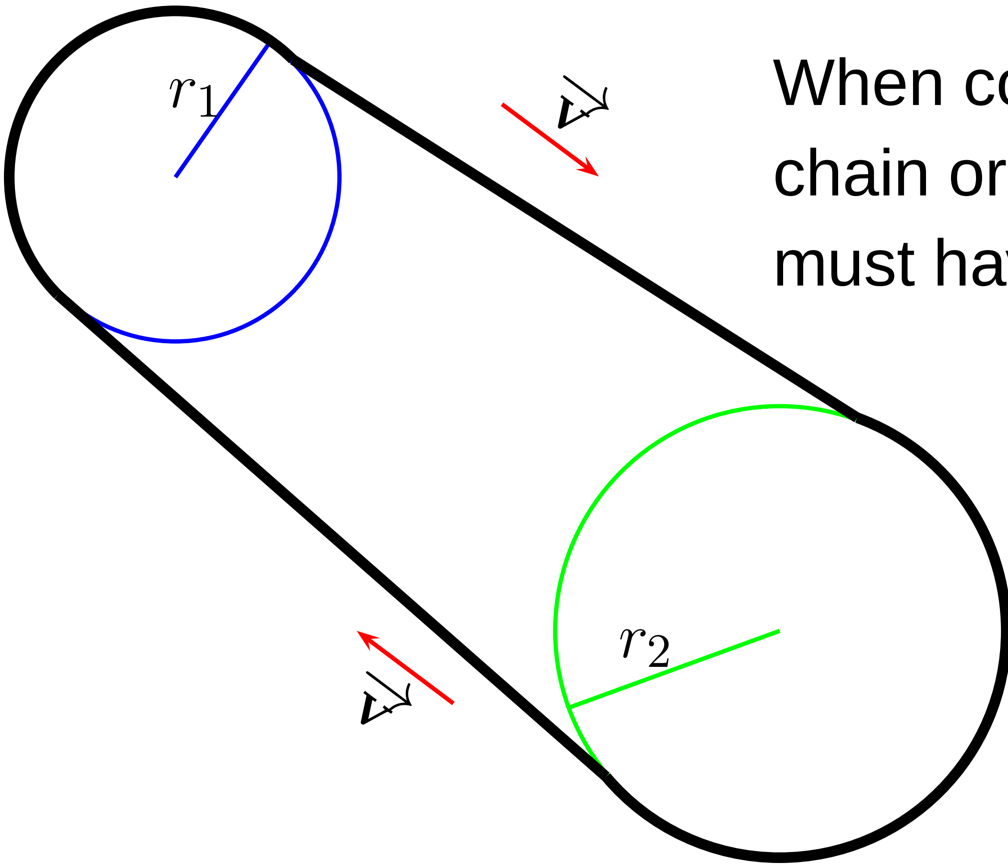
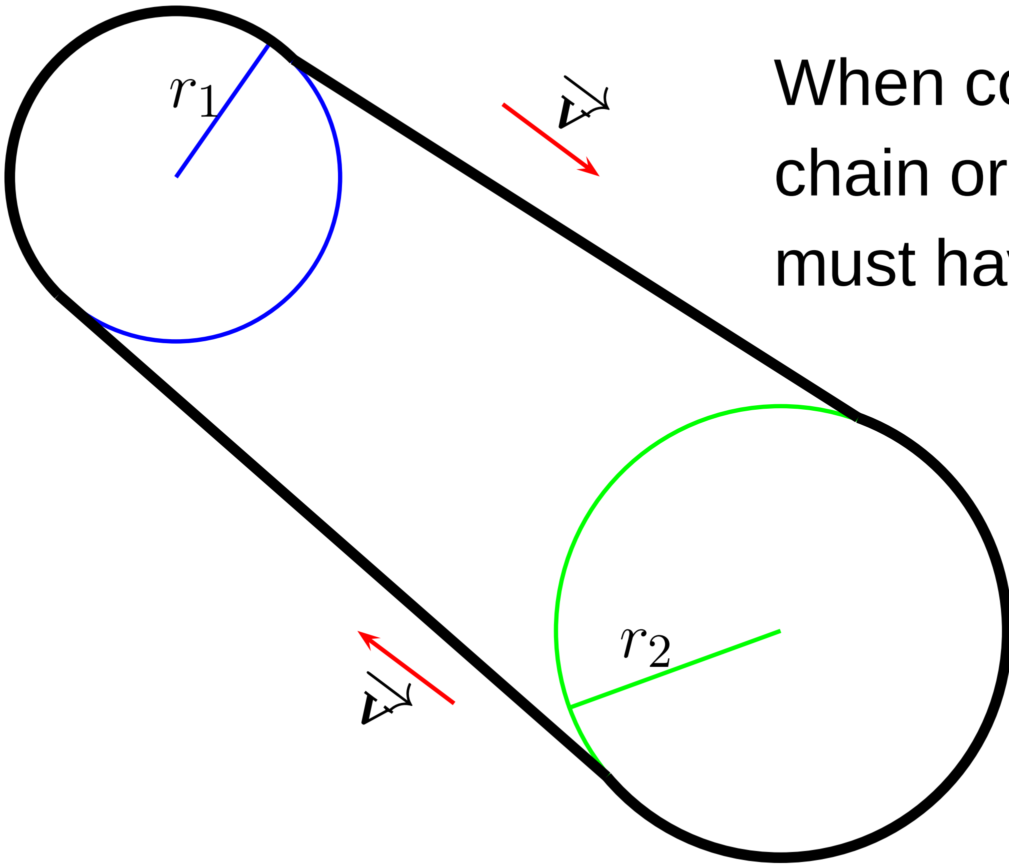


When connected by a non-slipping chain or belt, the two rotating objects must have the same linear velocity

Connected Rotating Objects



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$$v_1 = v_2$$

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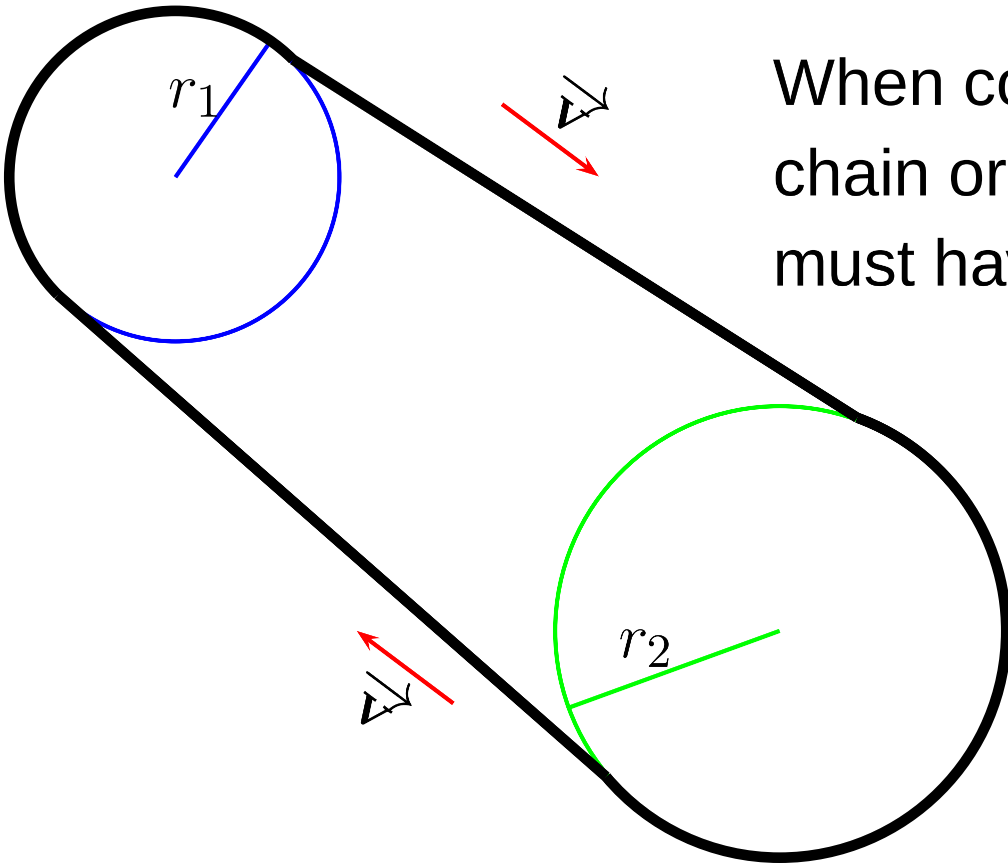

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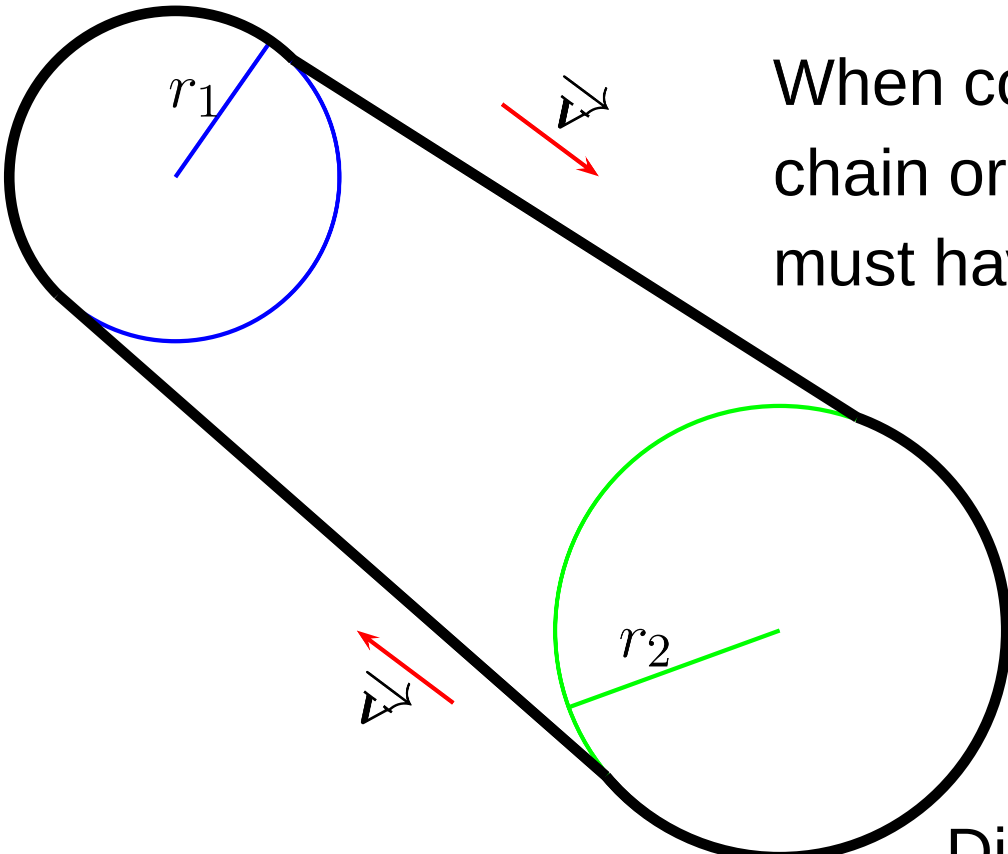

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Different Angular Velocities

Linear Accelerations

Every point on a rotating object has two acceleration components.

Linear Accelerations

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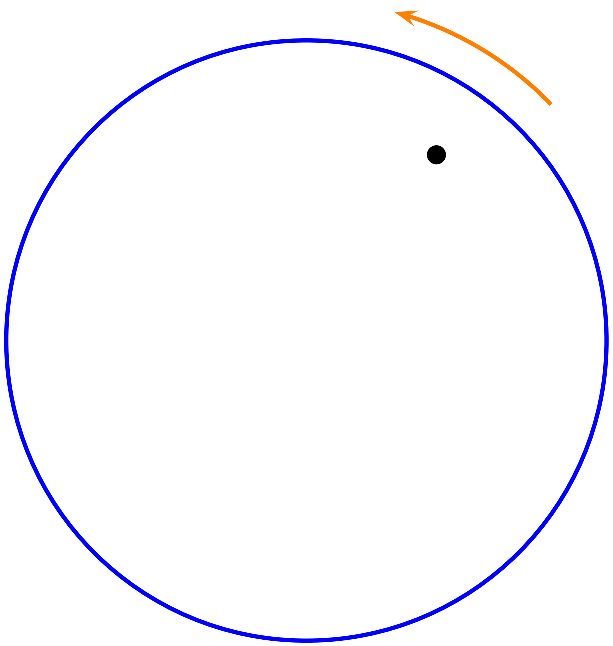

Linear Accelerations

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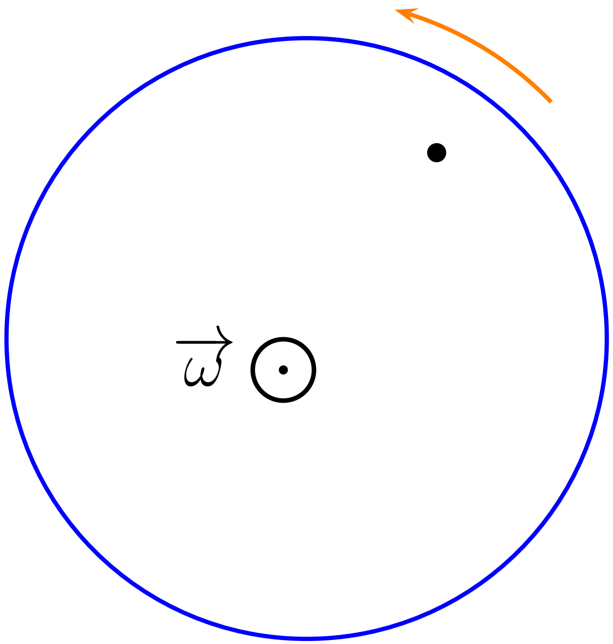

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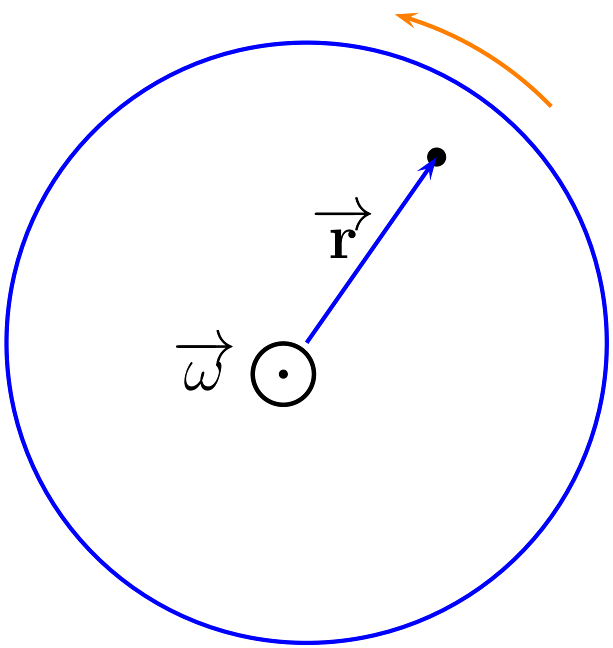

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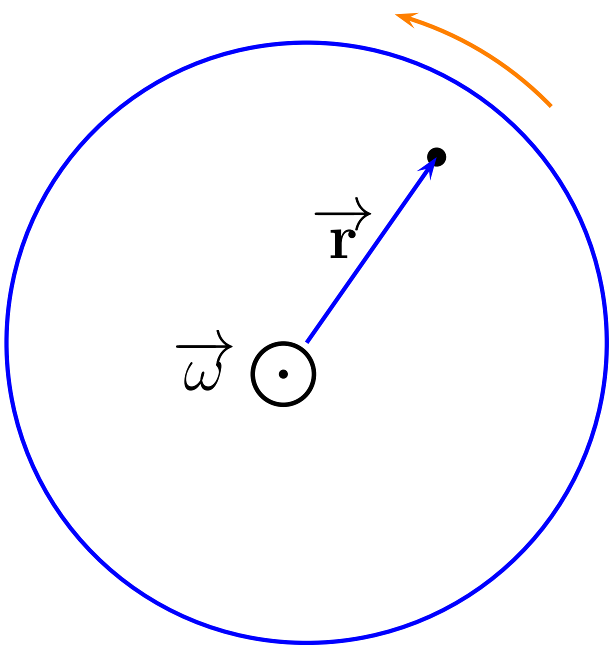

$\vec{\mathbf{a}}_{rad}$ - changes in direction

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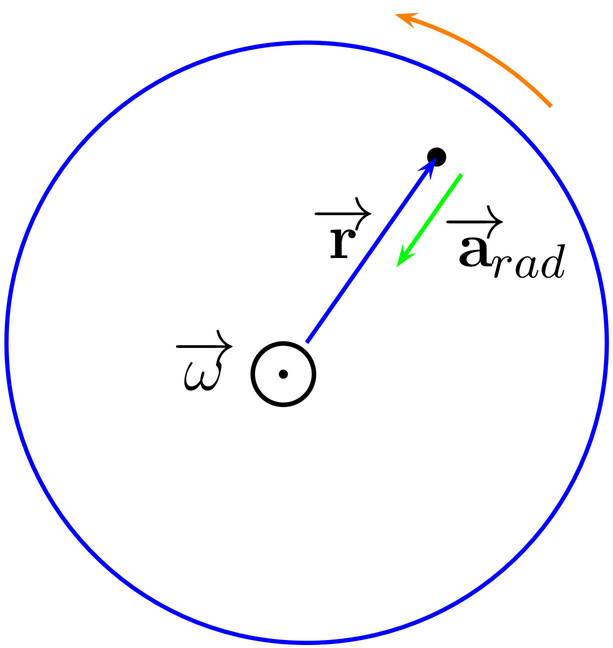

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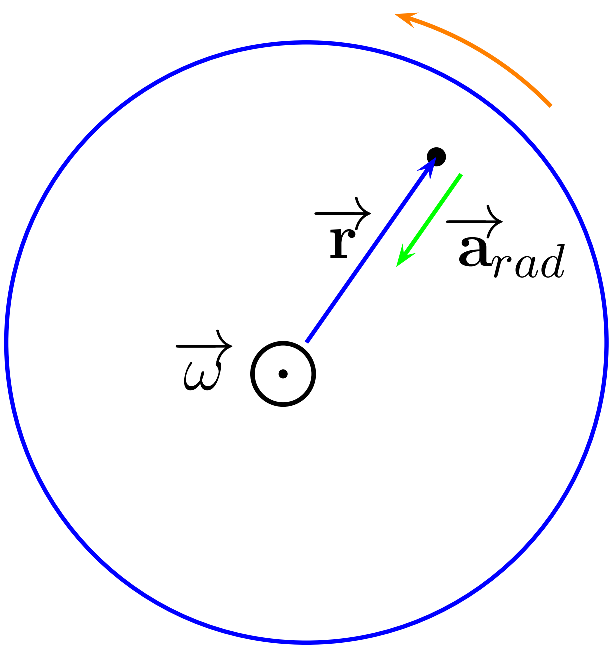

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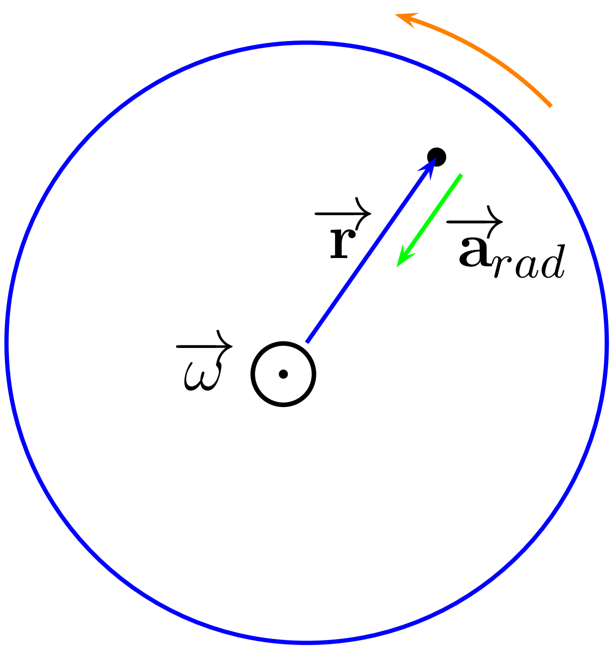

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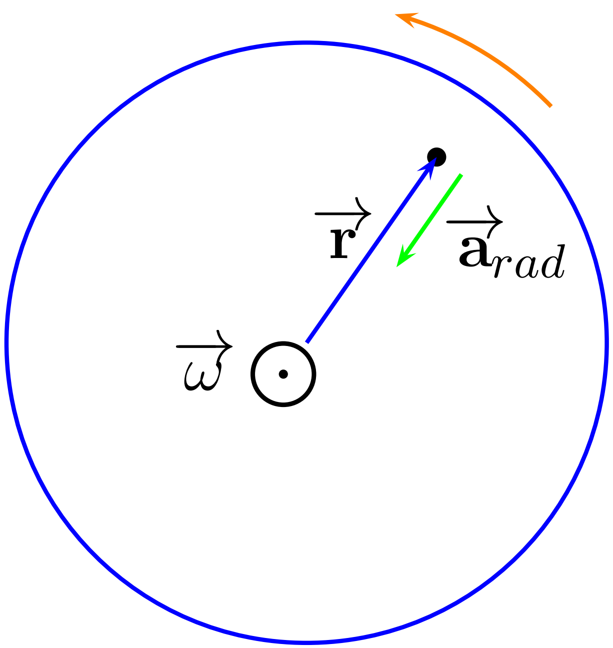

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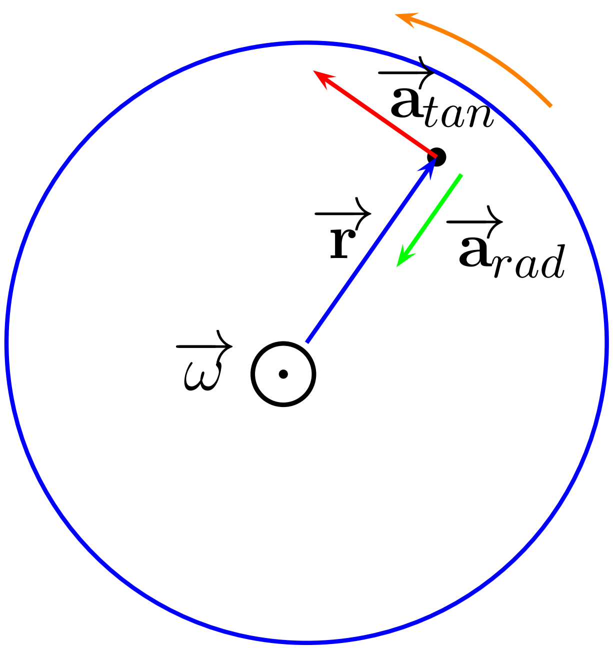

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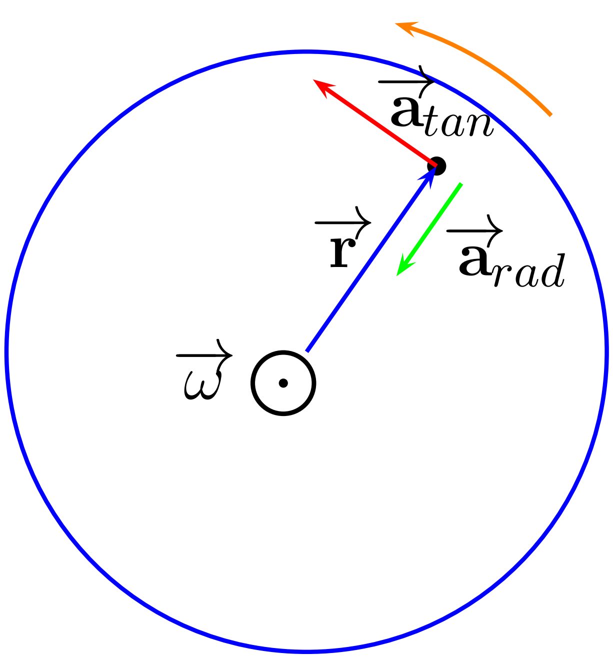

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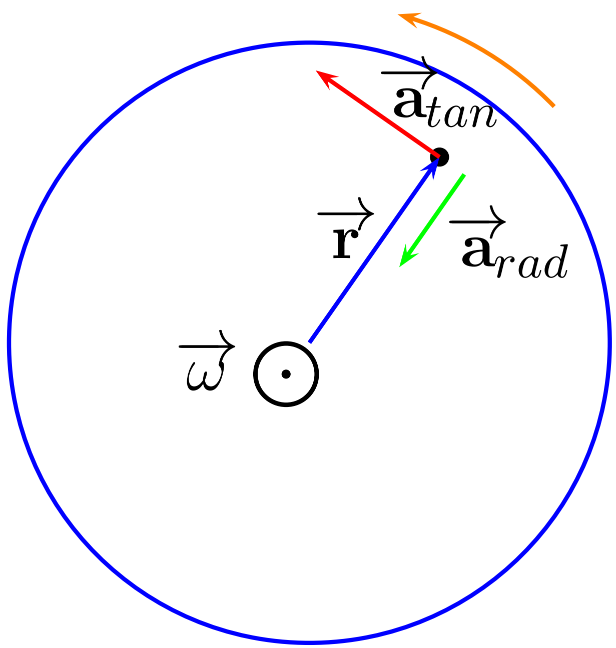

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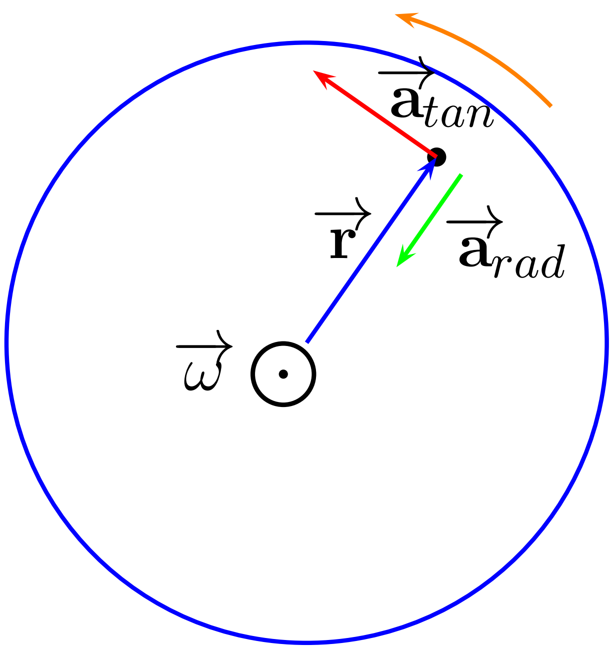

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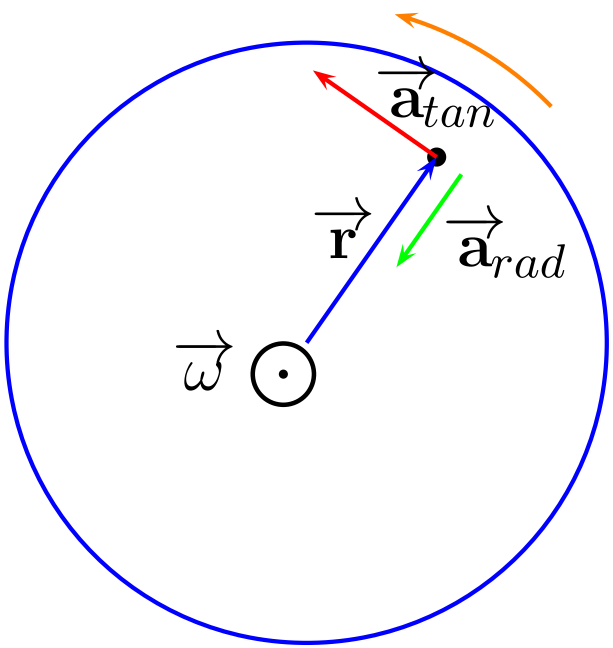

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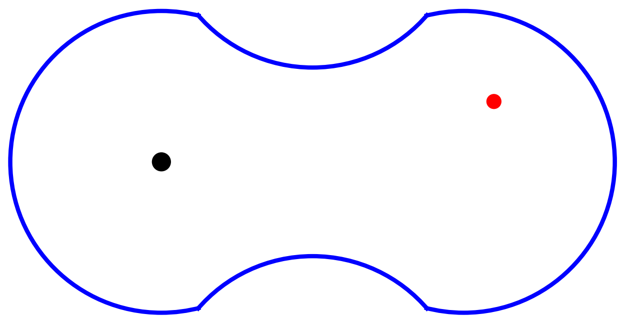
Non-Circular Objects

Putting the origin of the coordinate system at the axis of rotation allows us to use all of the equations for circular objects.



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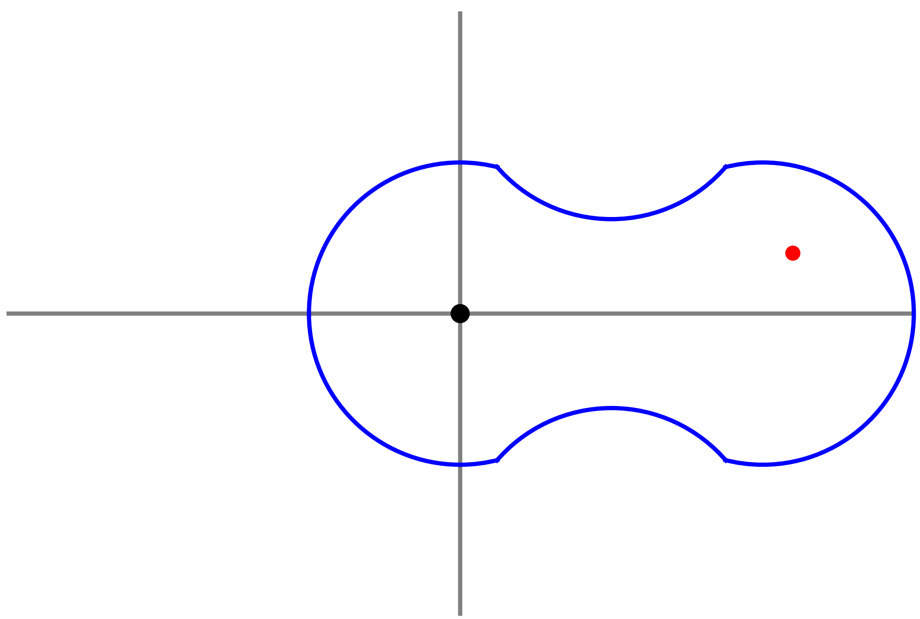

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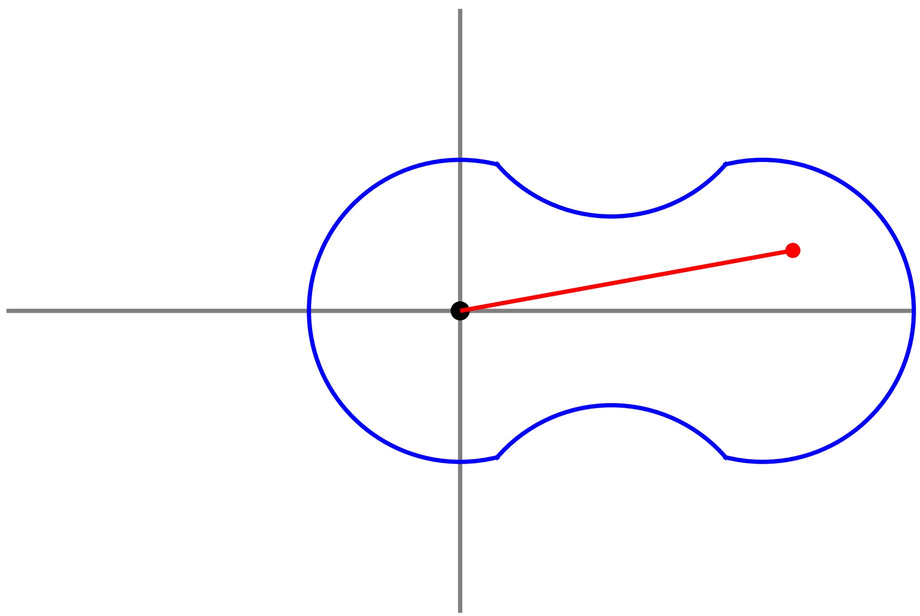

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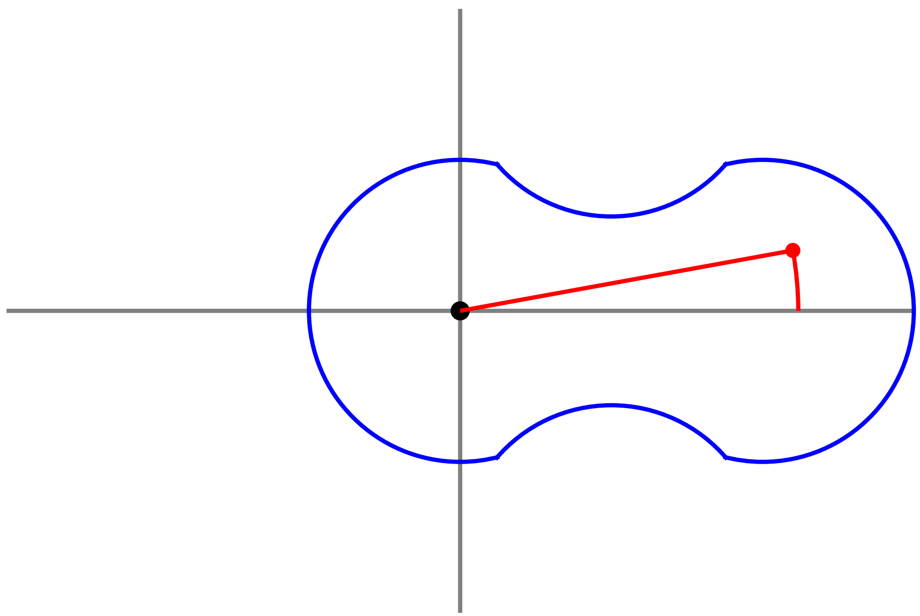

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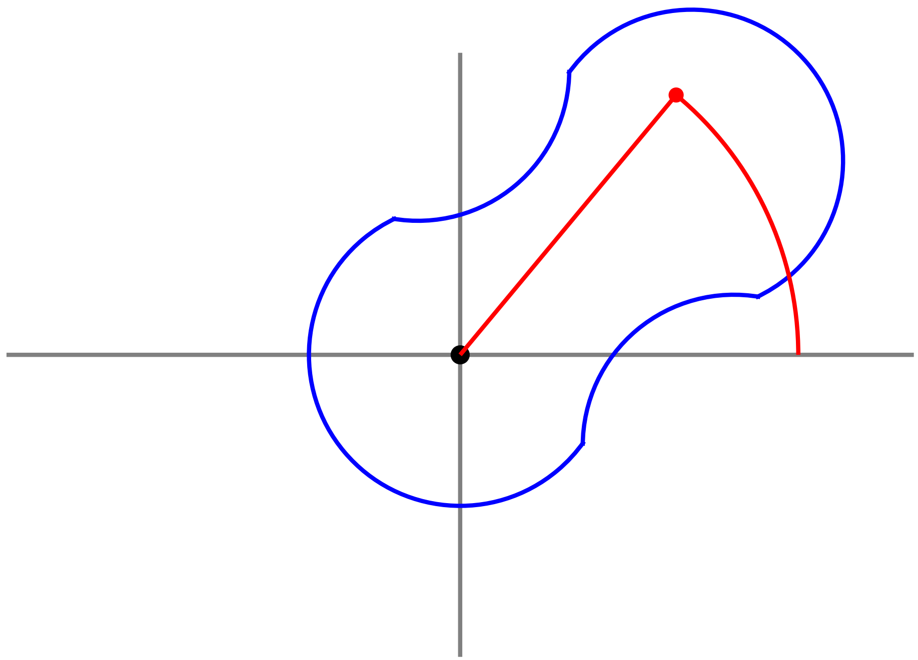

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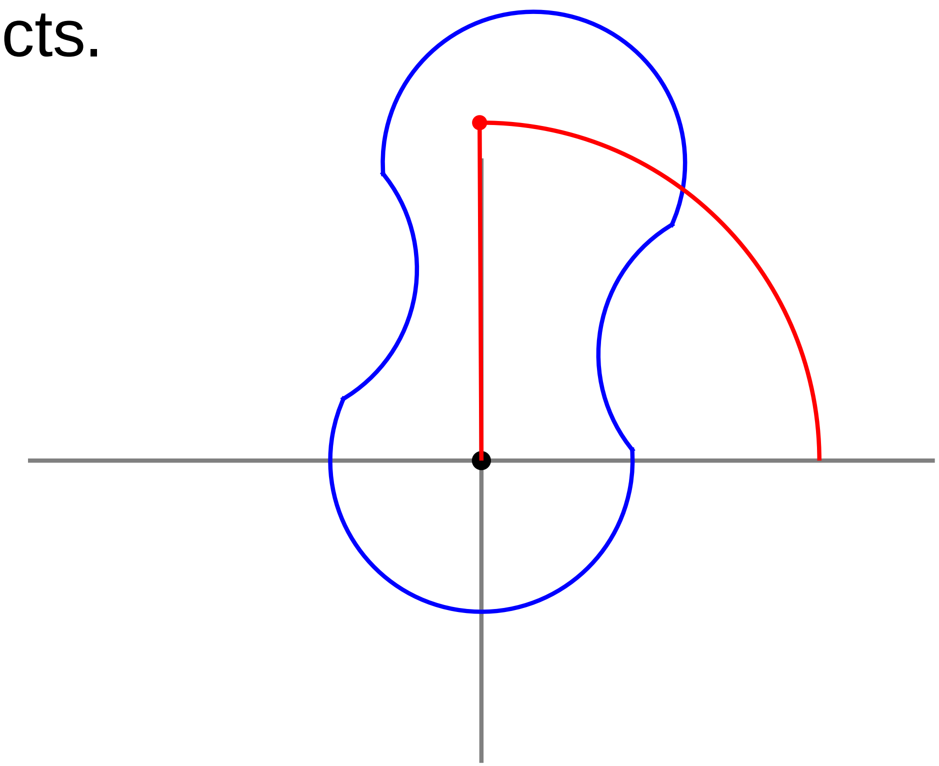

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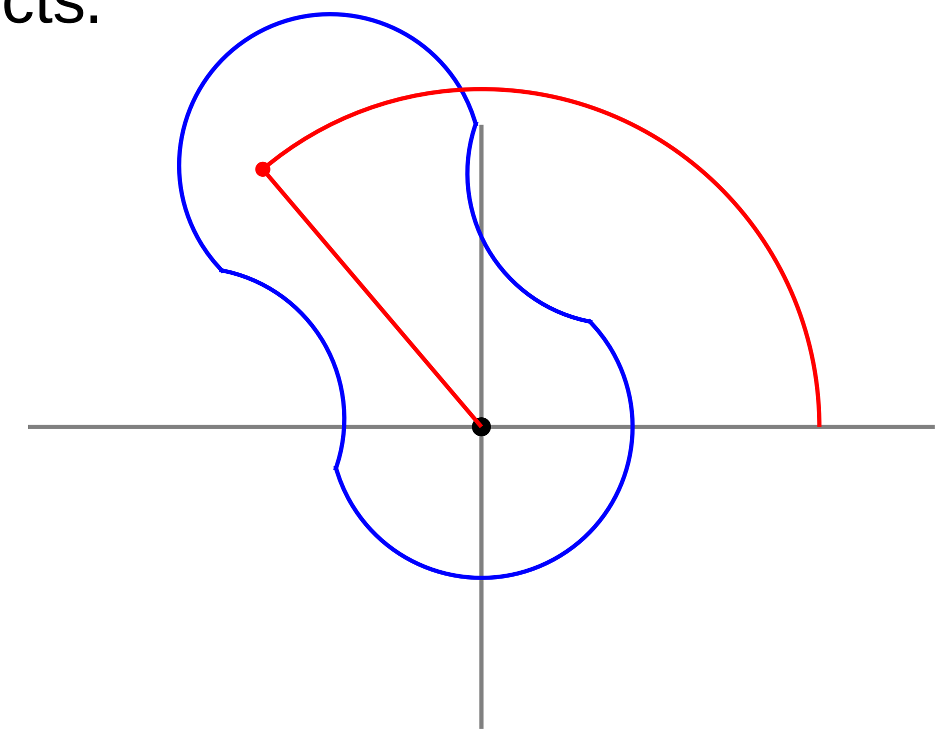

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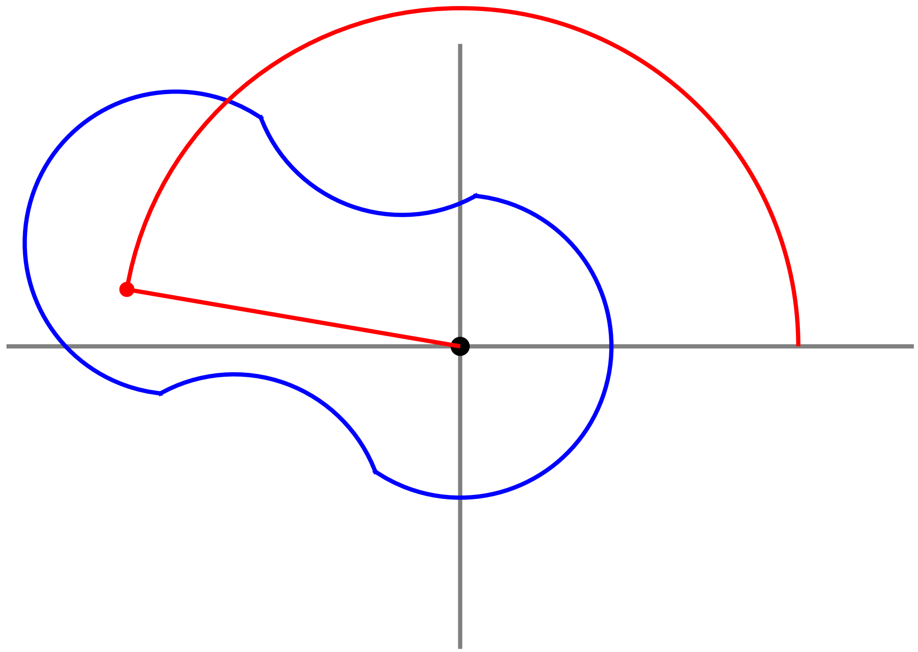

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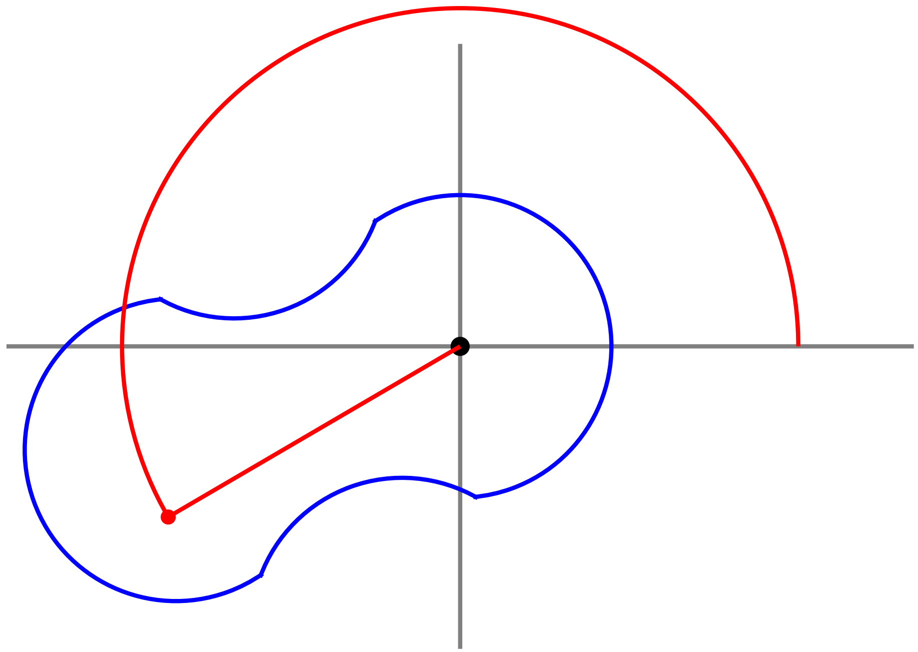

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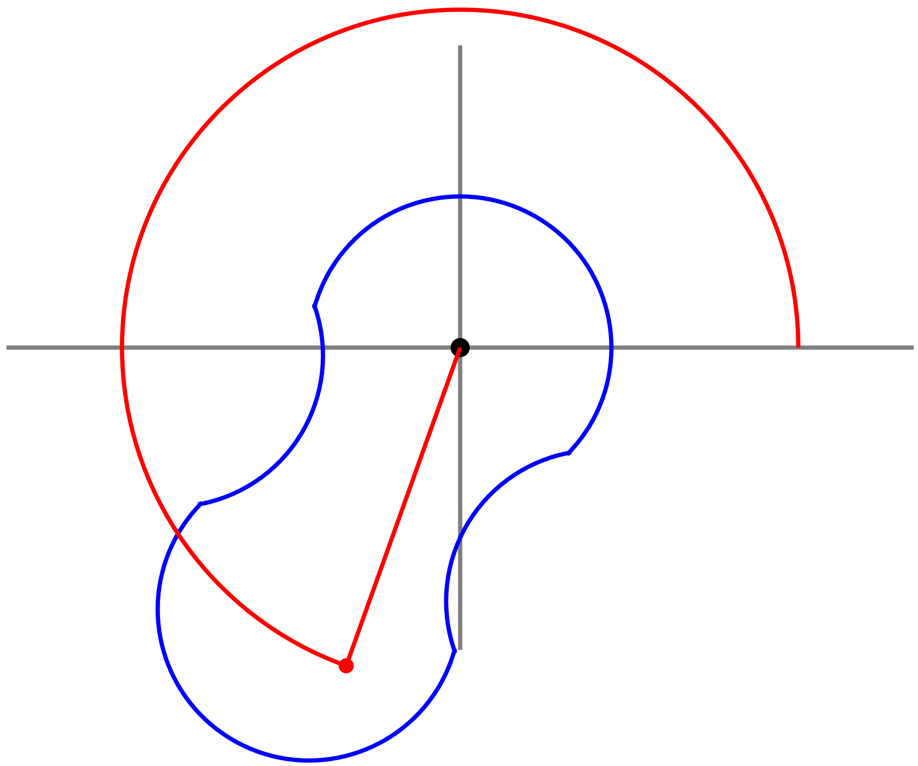

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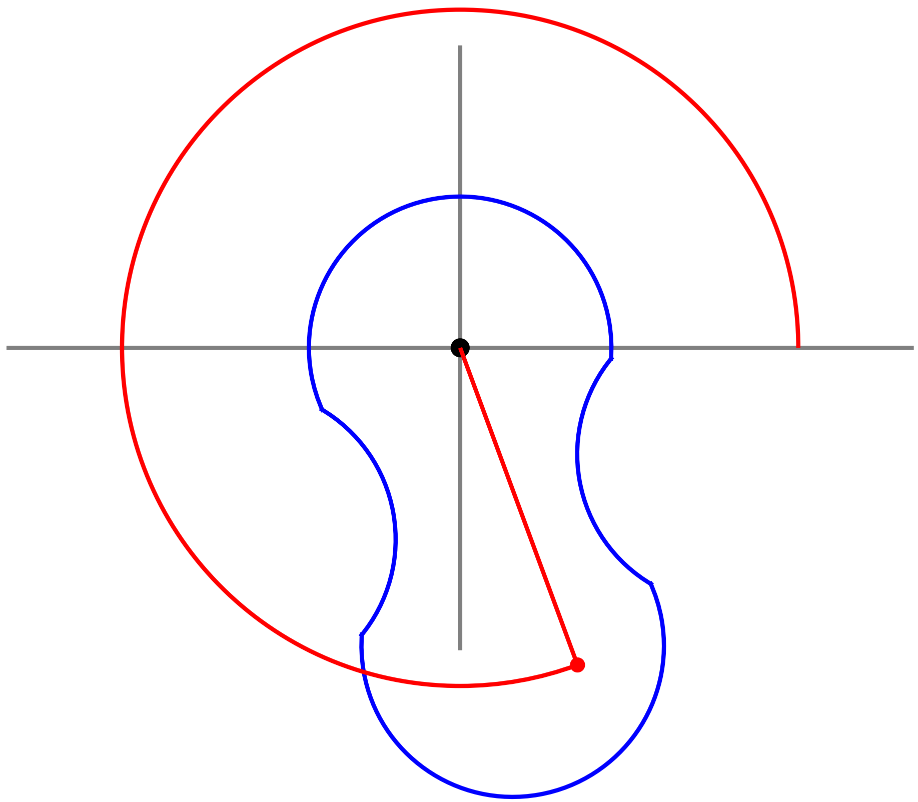

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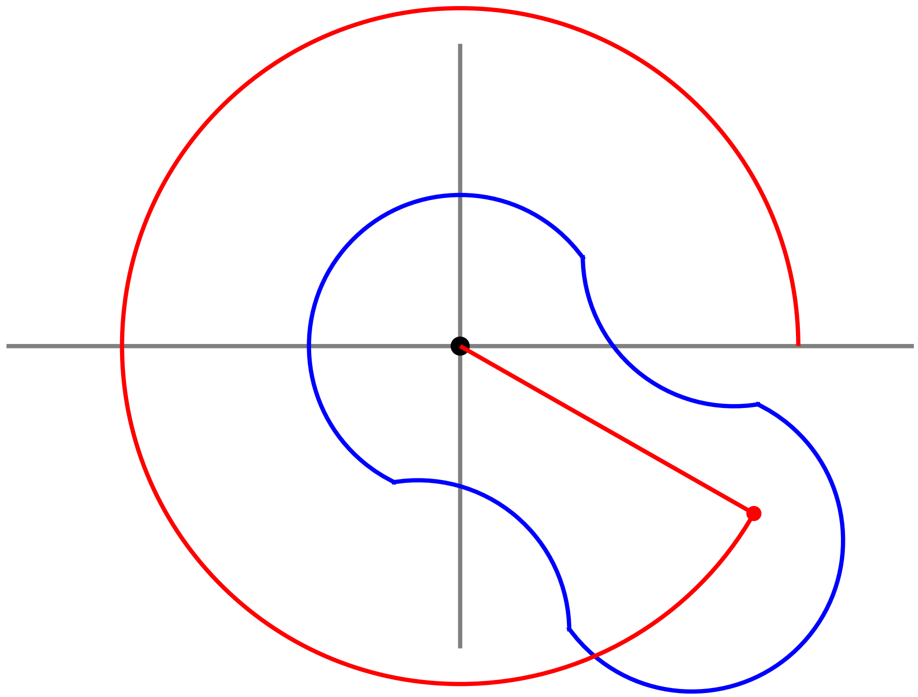

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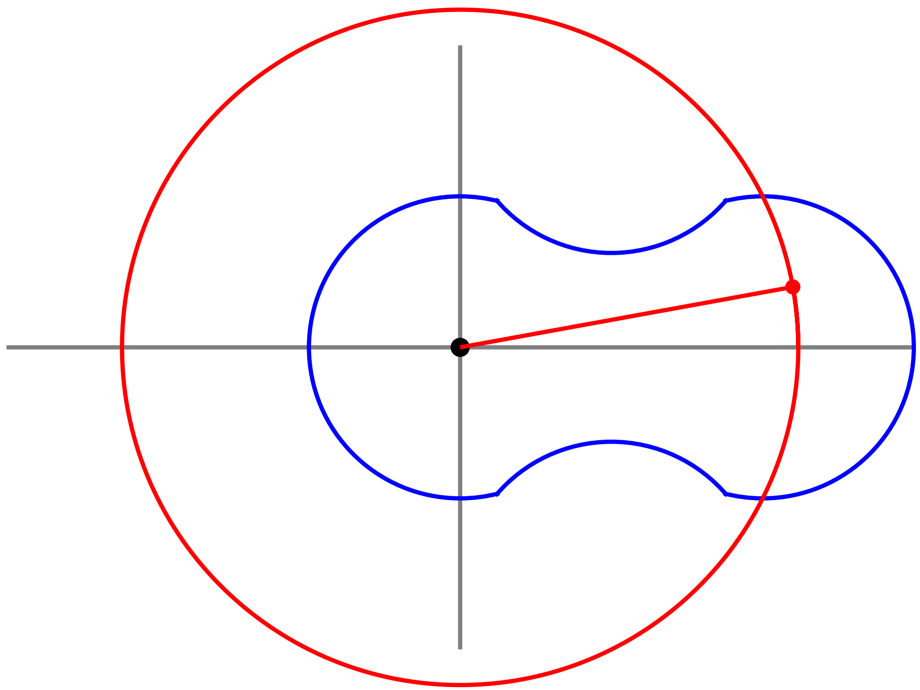

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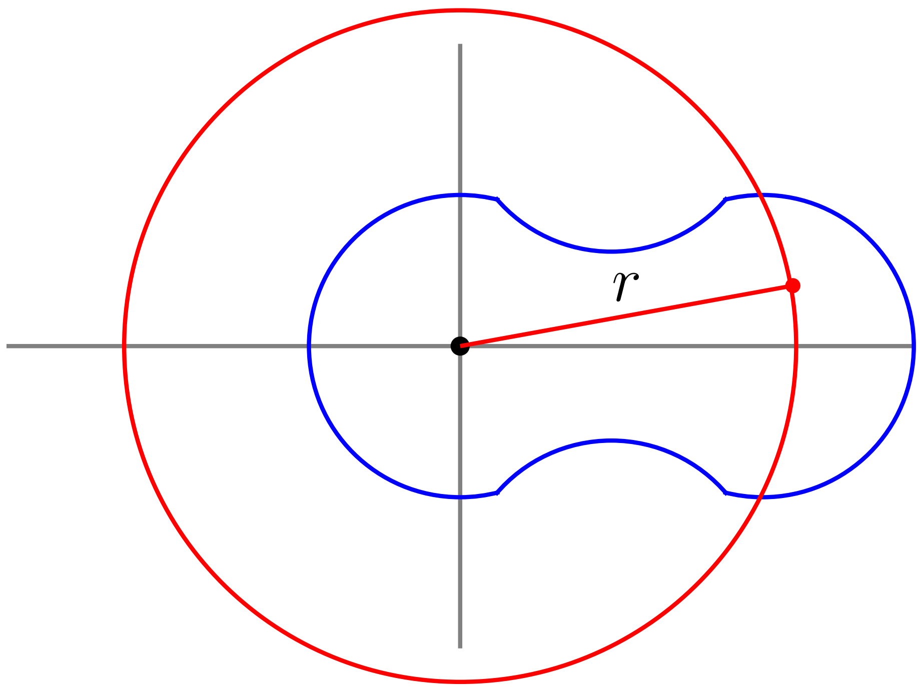

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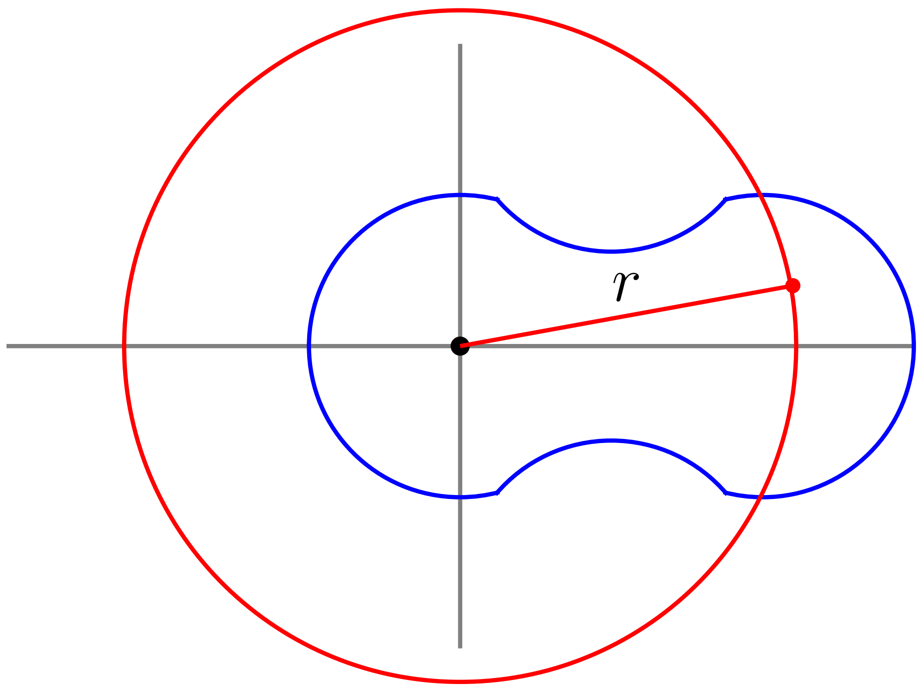
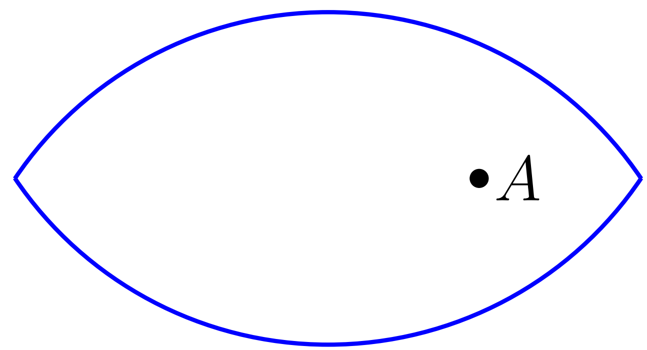


$r =$ distance from axis of rotation

Clicker Quiz

The following object is rotated about one end as shown. What is the linear speed of the point A?



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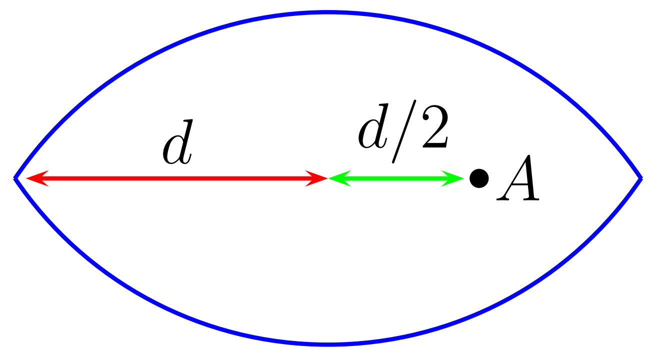

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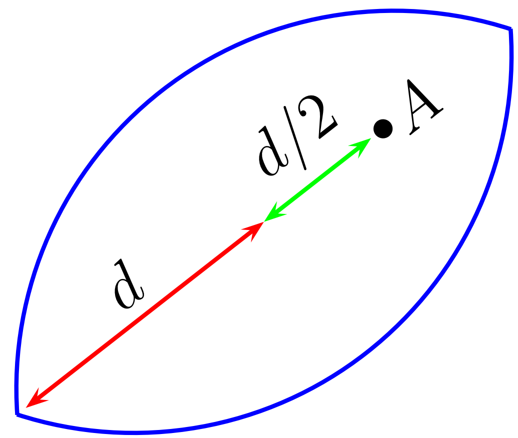

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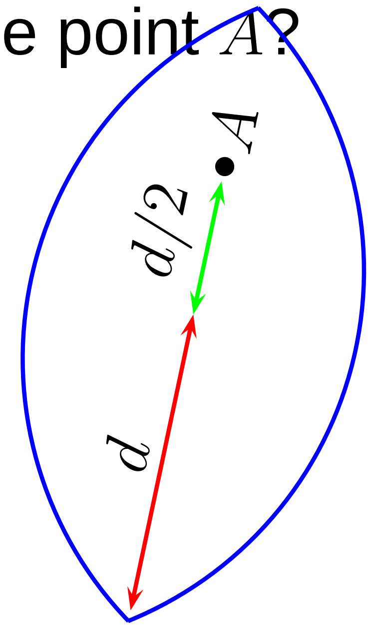

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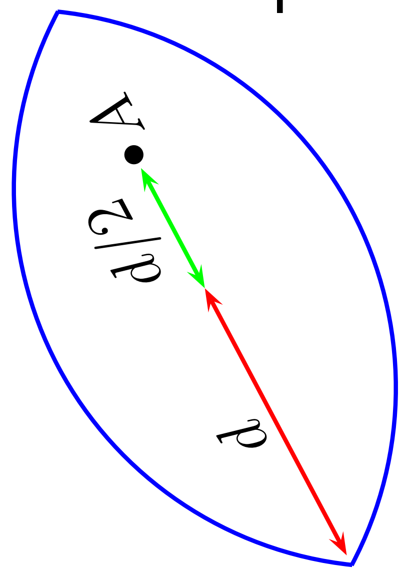

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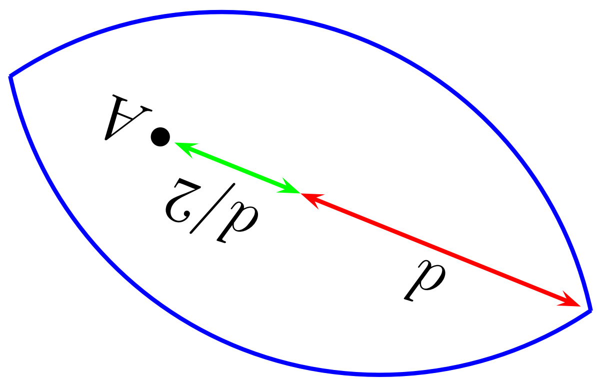

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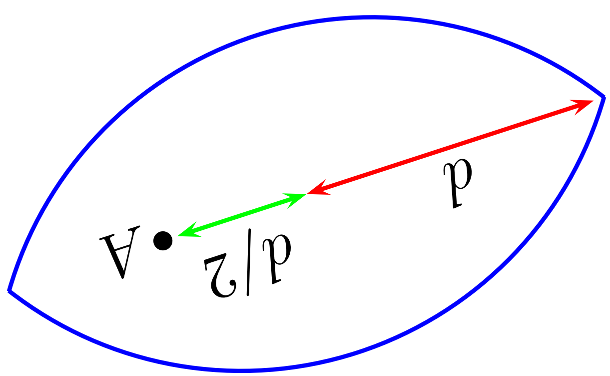

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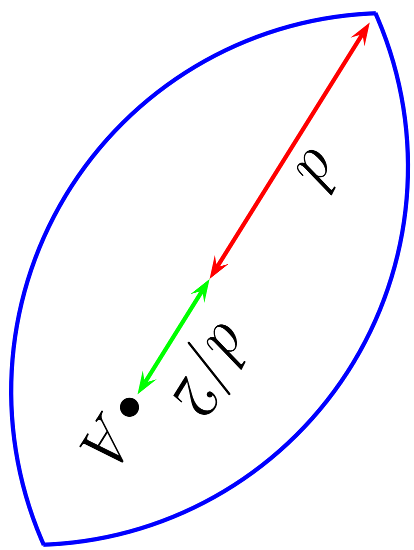

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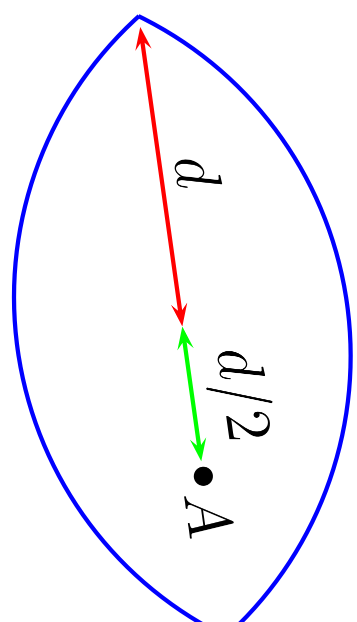

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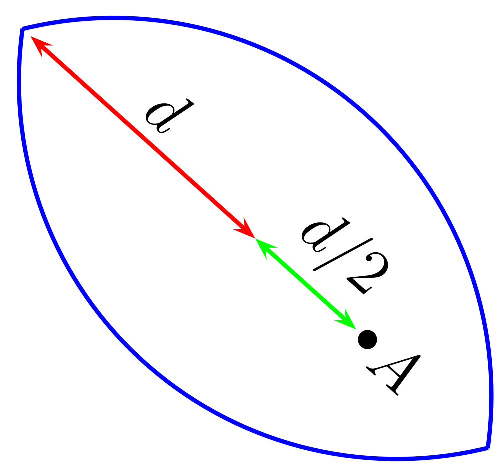

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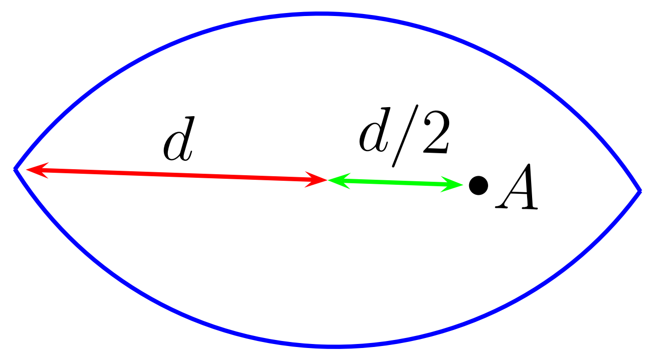

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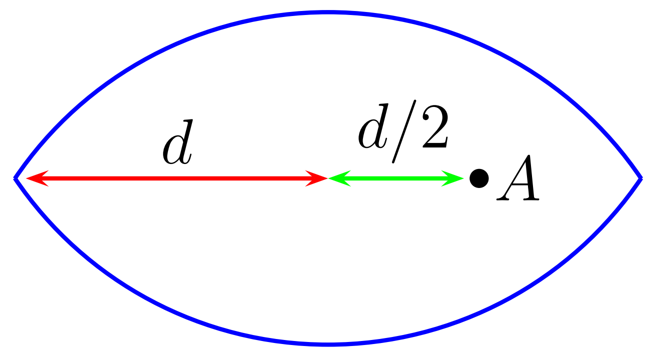
(a) $d\,\omega$



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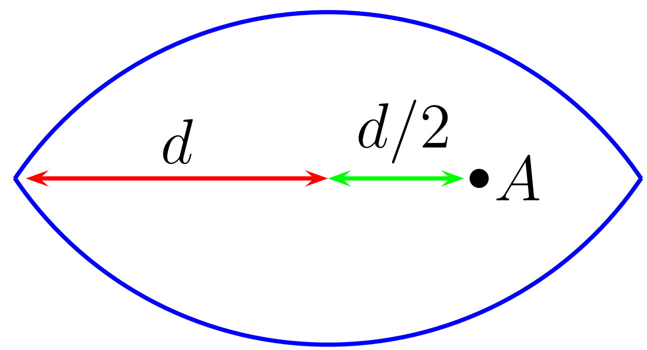

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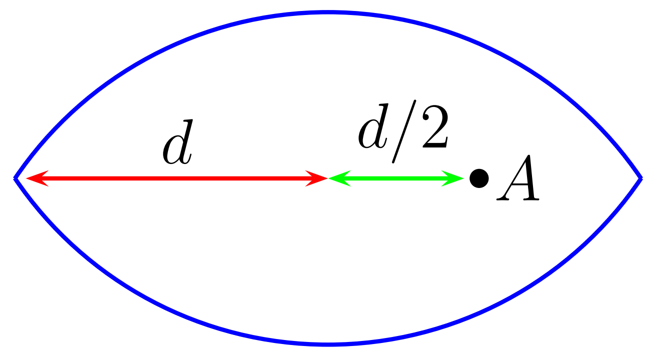

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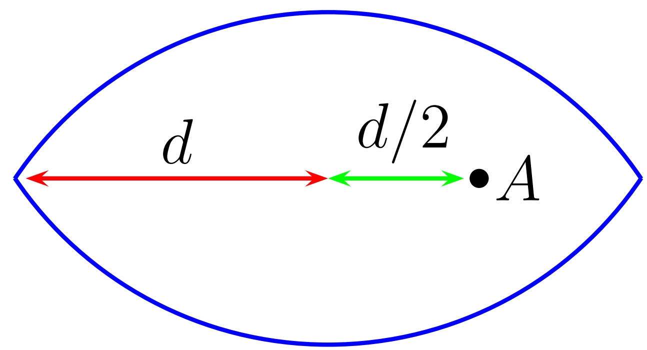

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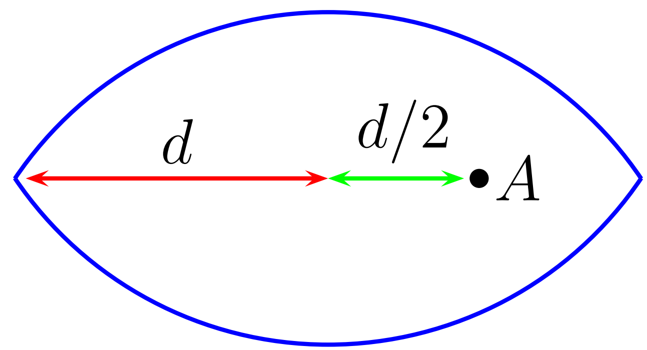

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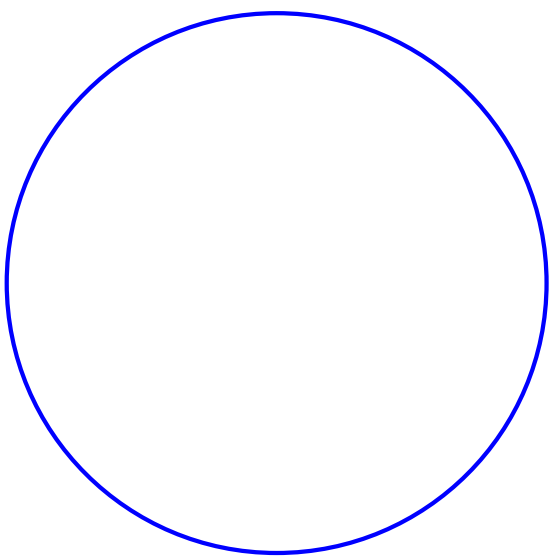

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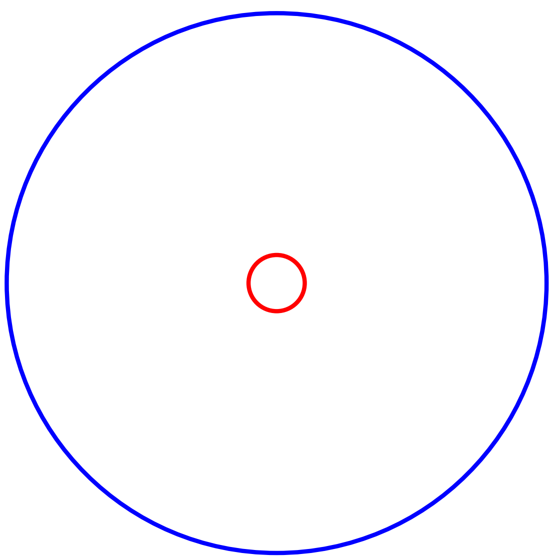

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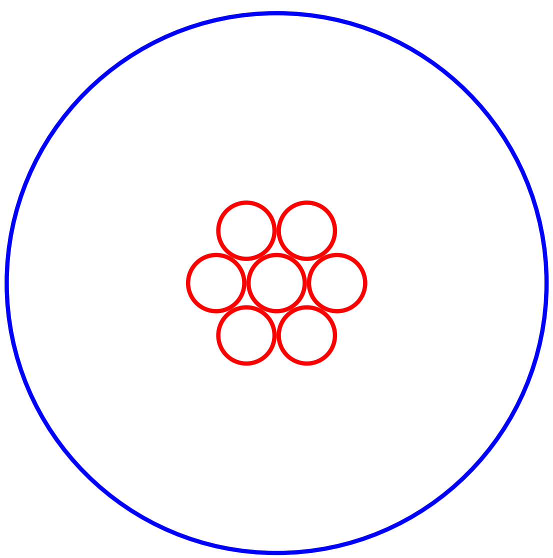

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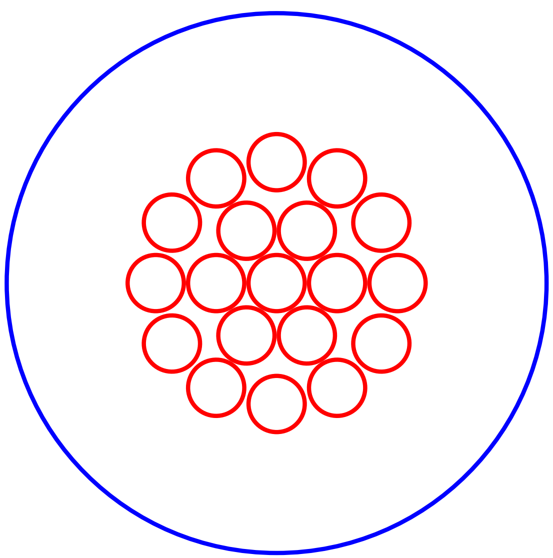

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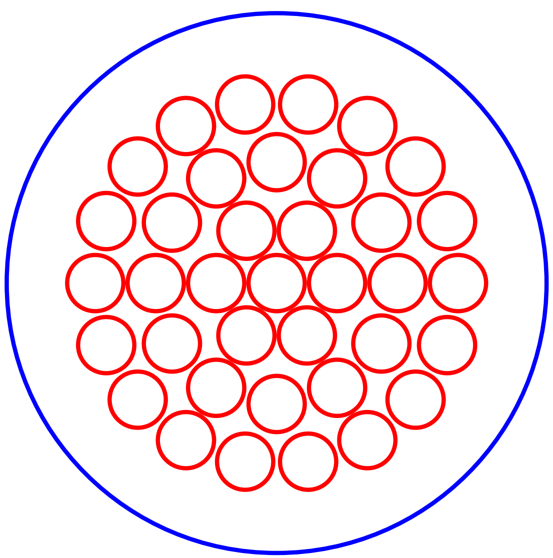

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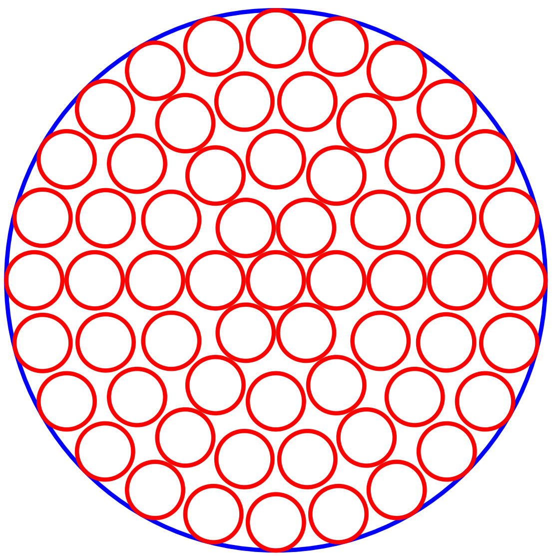

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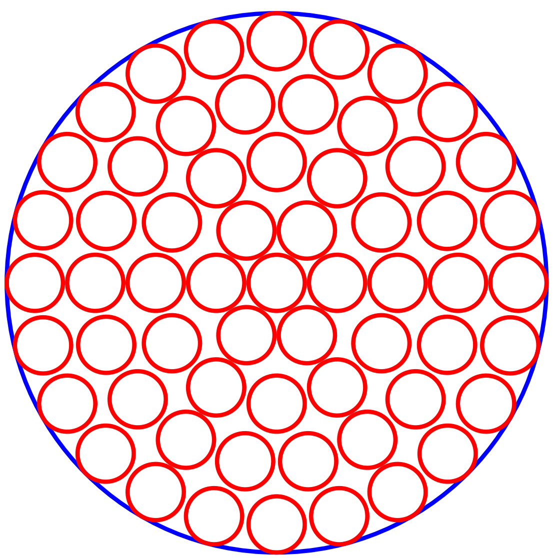

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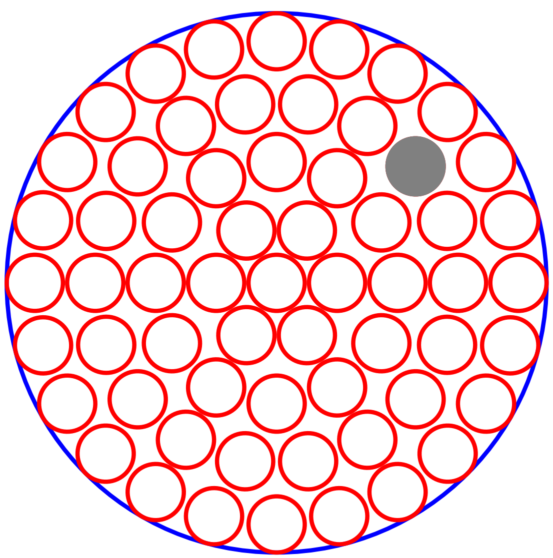

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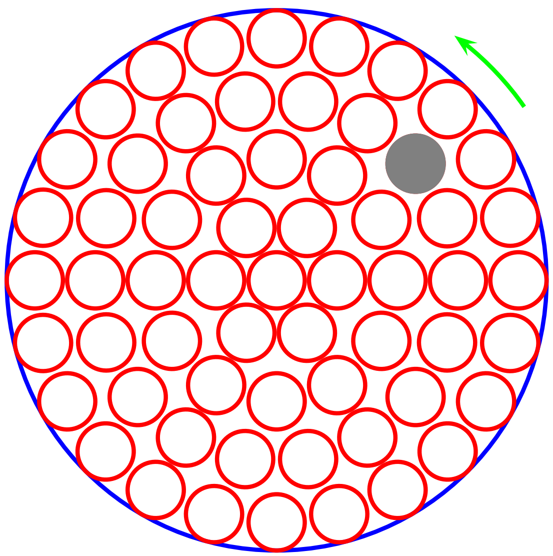

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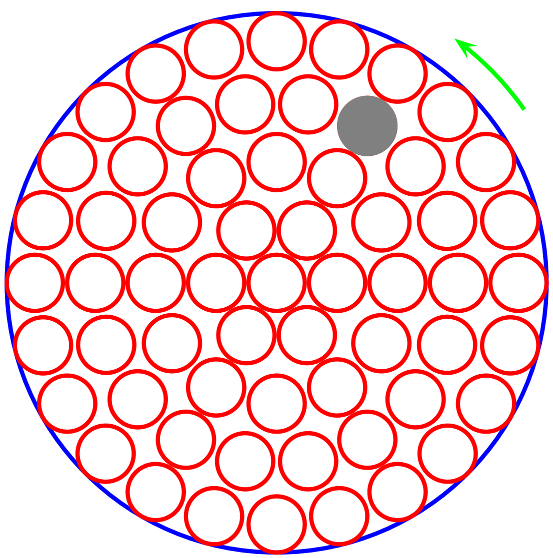

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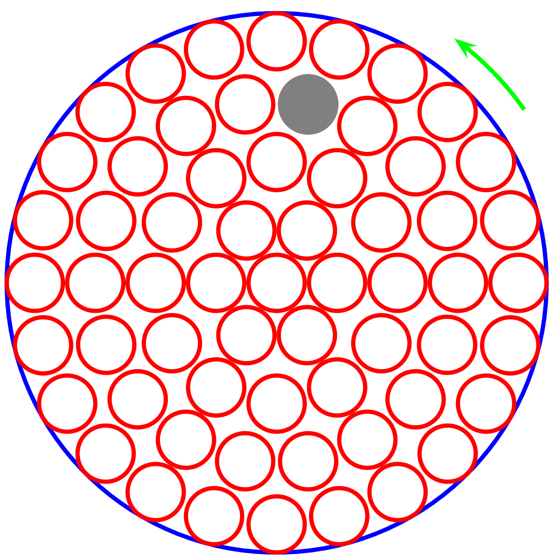

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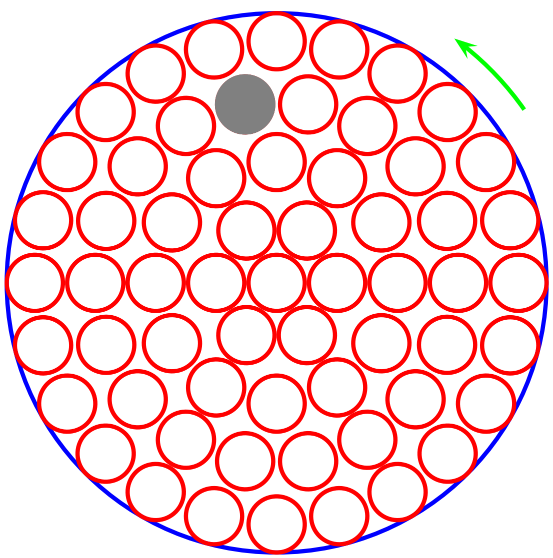

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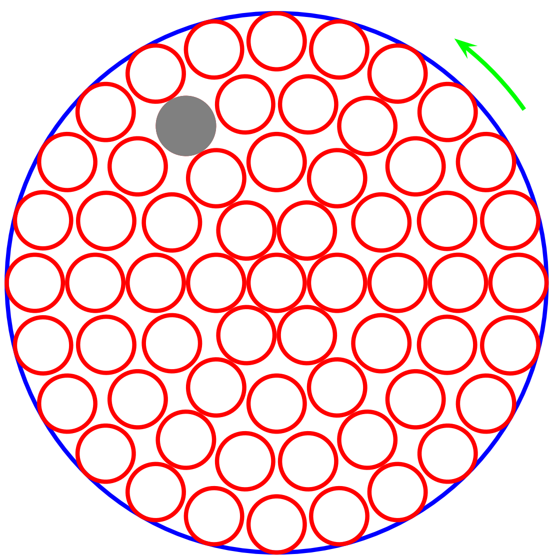

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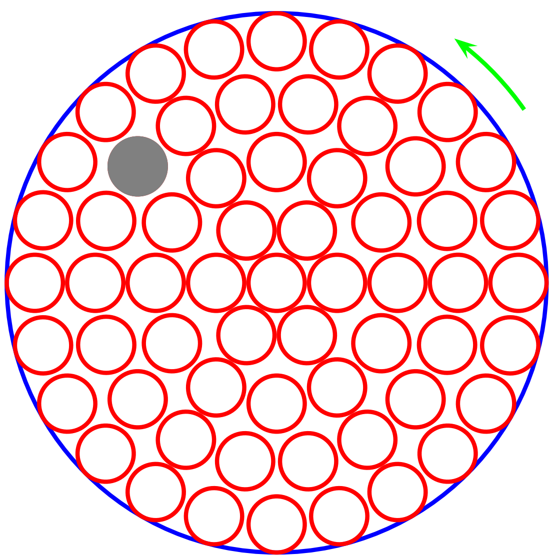

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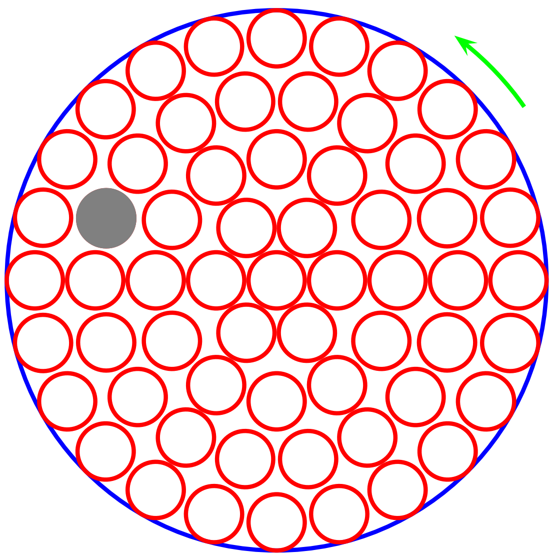

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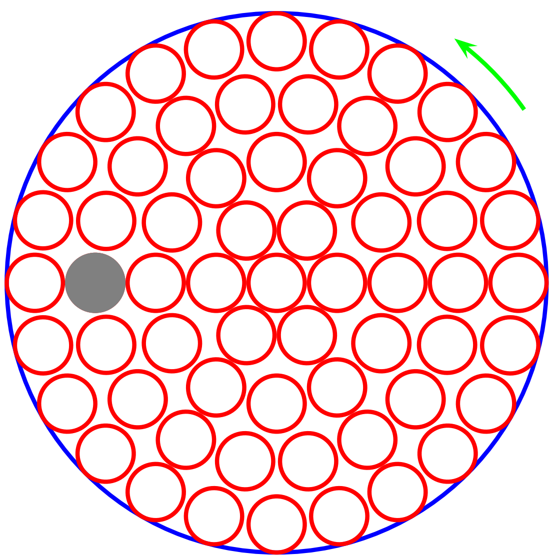

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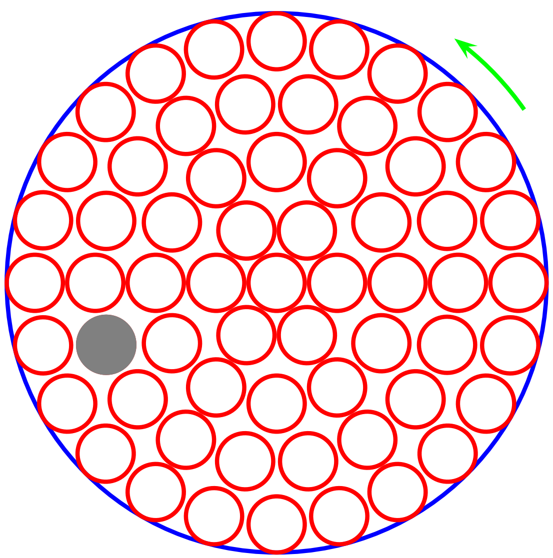

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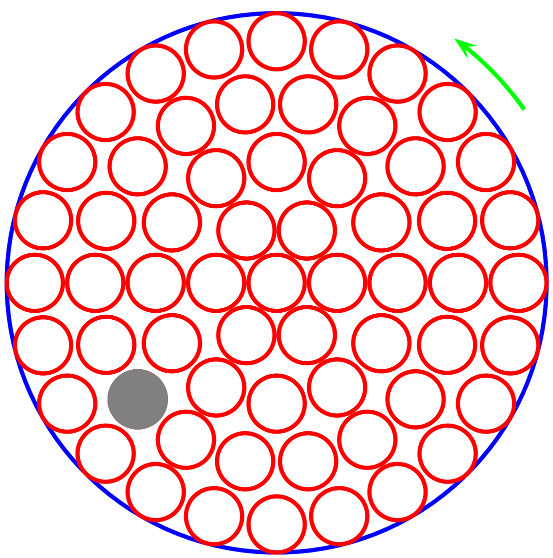

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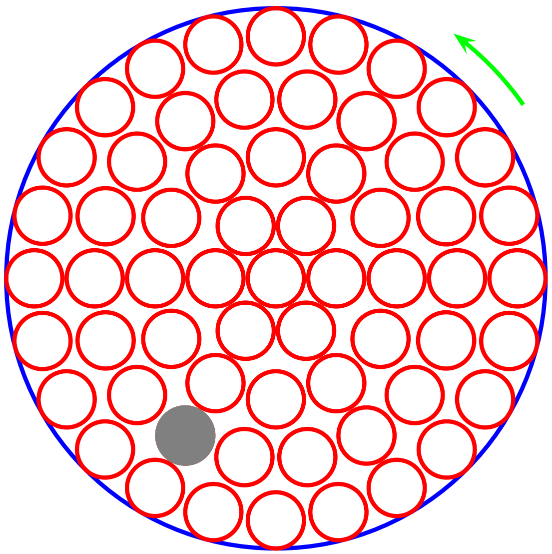

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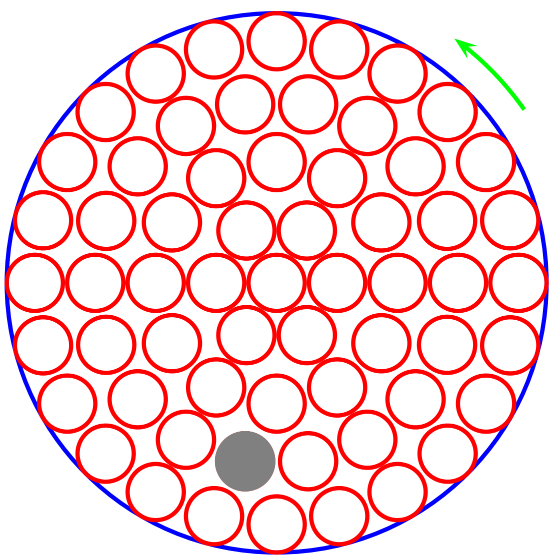

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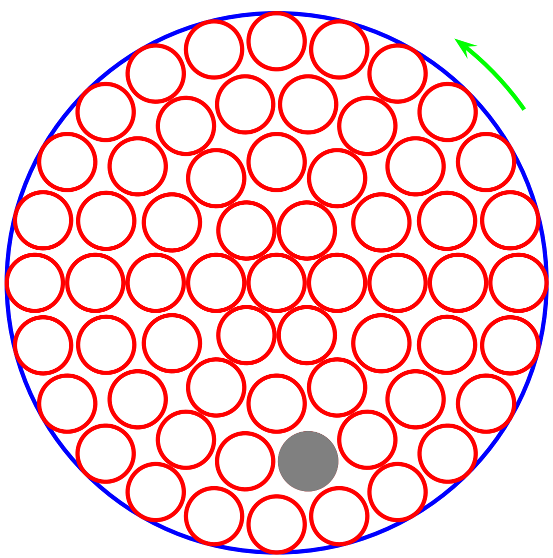

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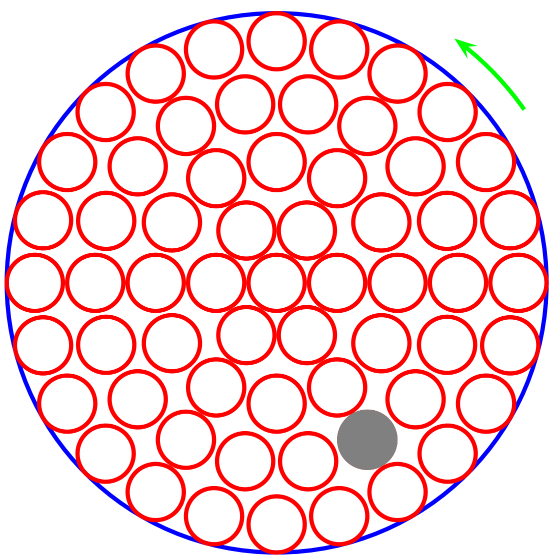

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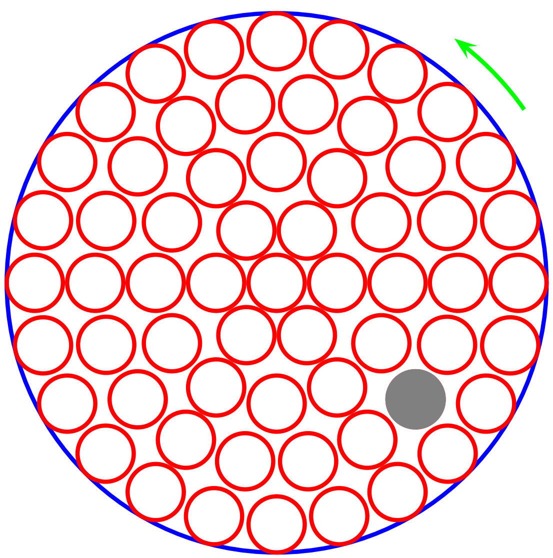

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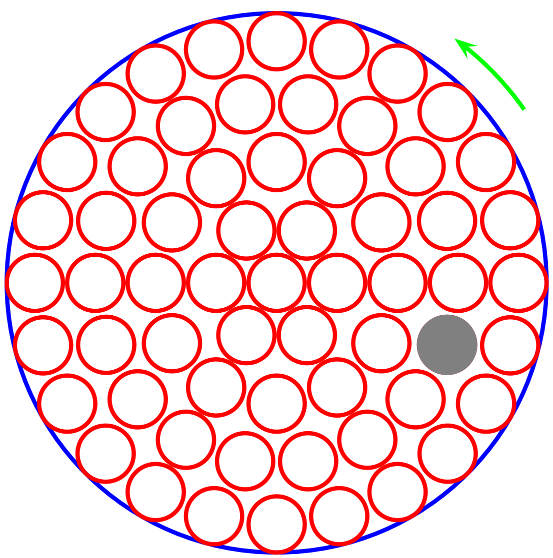

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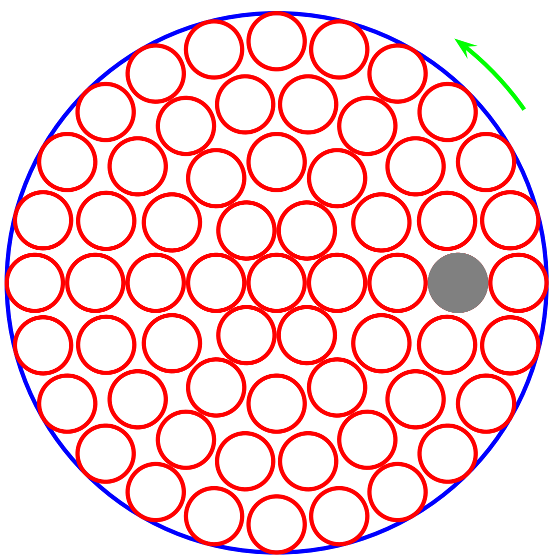

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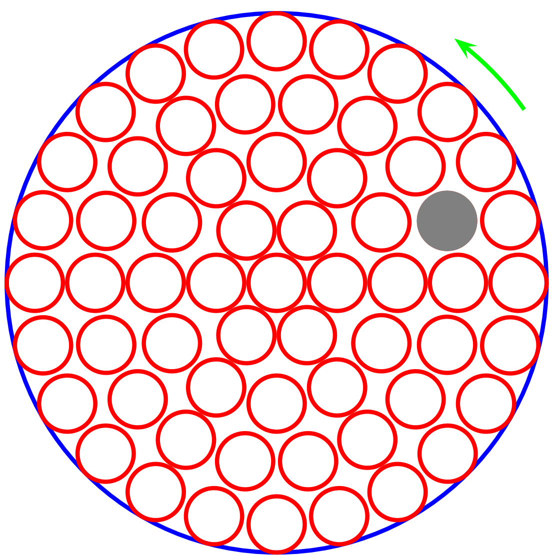

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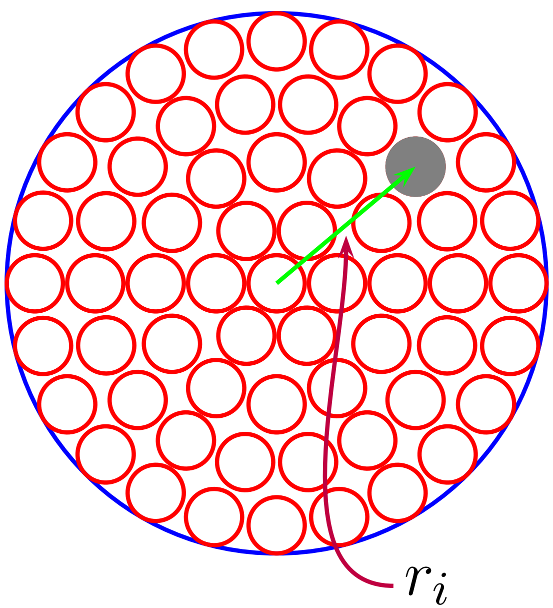

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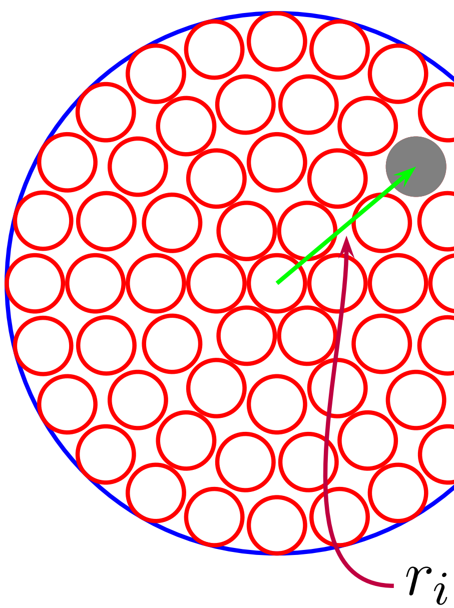

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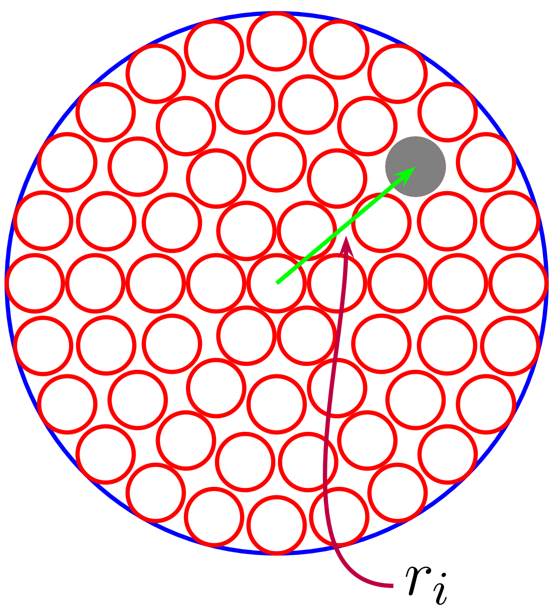

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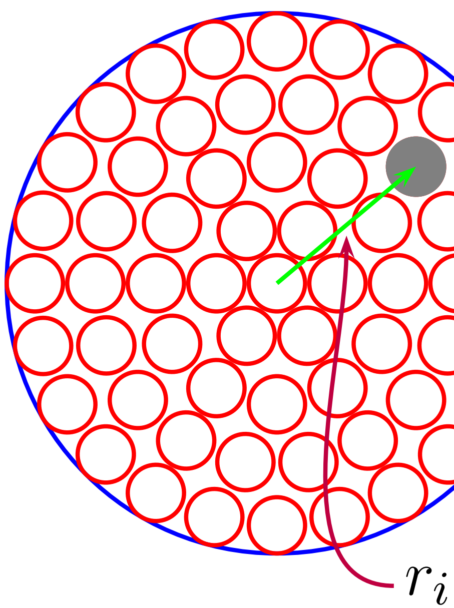

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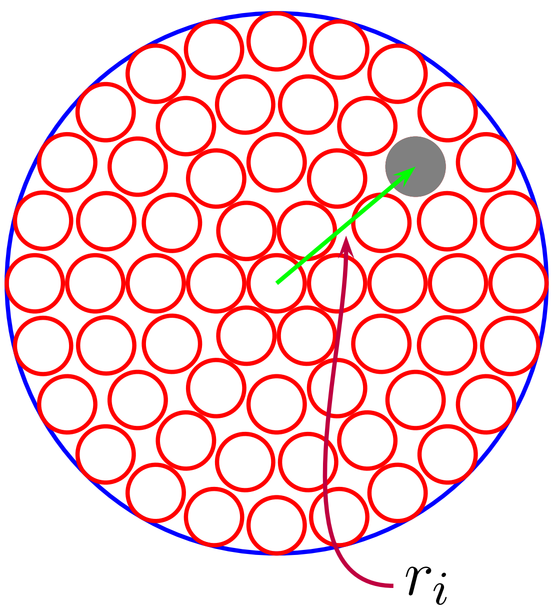

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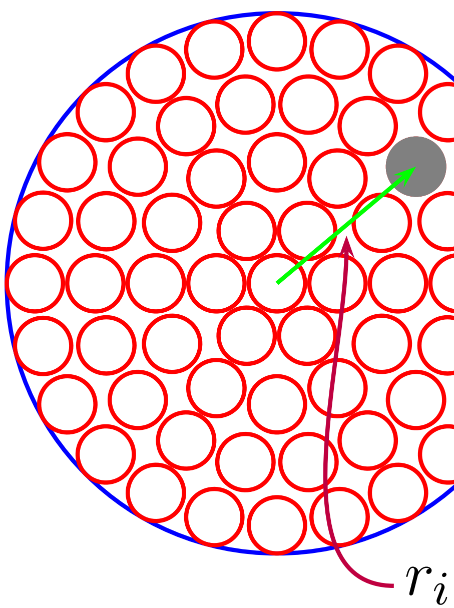

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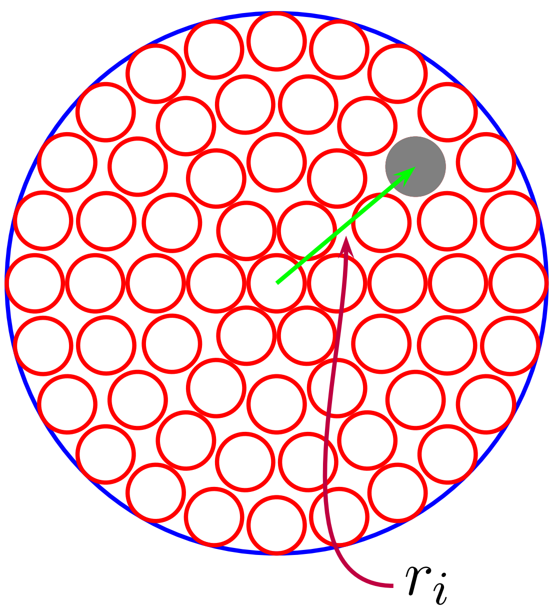

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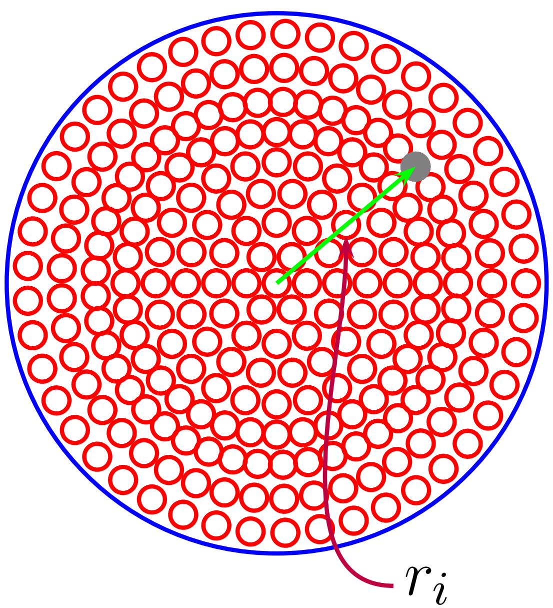

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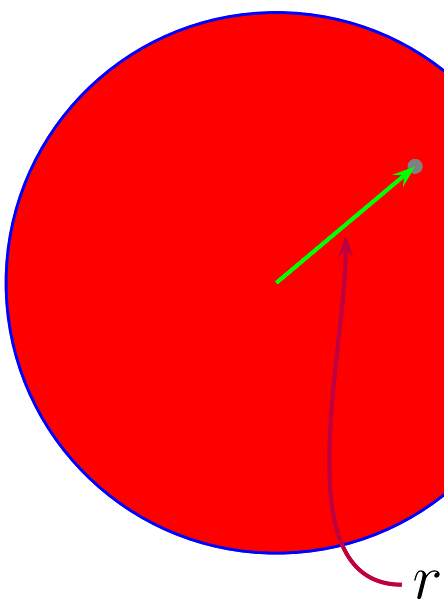

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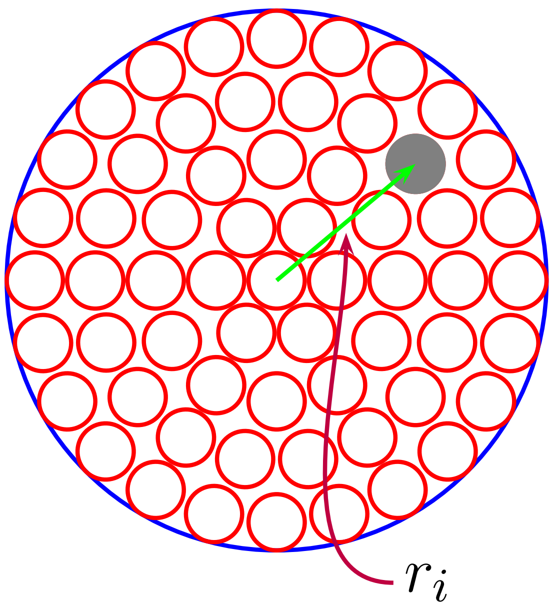

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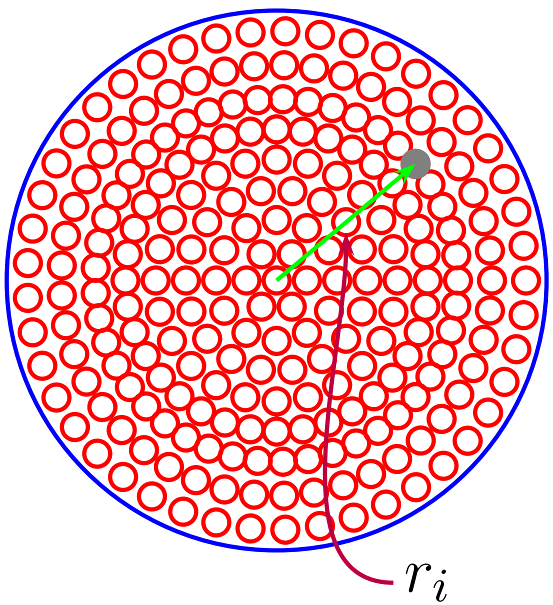

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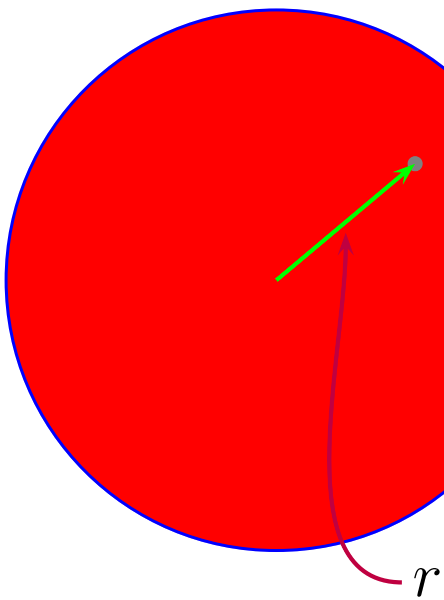

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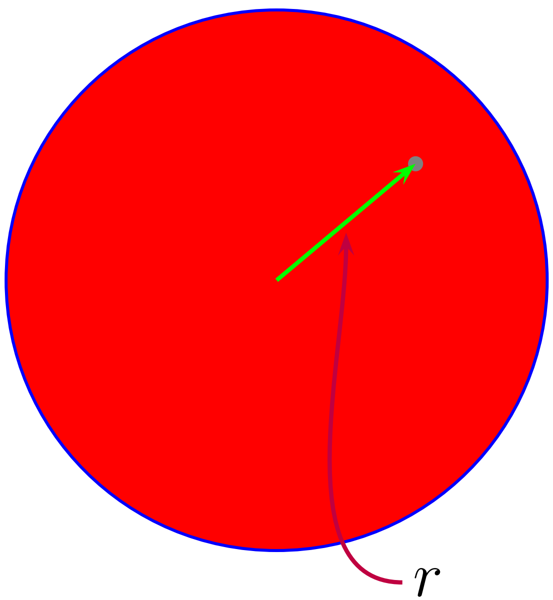
$$I = \lim_{i \to \infty} \sum_i M_i r_i^2$$

$$I = \int r^2 dM = \int r^2 \rho \, dV$$

($\rho =$ density)

Moment of Inertia

In the limit, the sum becomes the Moment of Inertia, I, for the rotating object.



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$$\boxed{K = \frac{1}{2} I \omega^2}$$

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(a) The object’s shape.
(b) The axis of rotation.
(c) The total mass of the object.