

April 4, Week 11

Today: Chapter 9, Rotation

Exam #4: Friday, April 6

Review Session: Thursday, April 5, 7:30PM in Regener 114

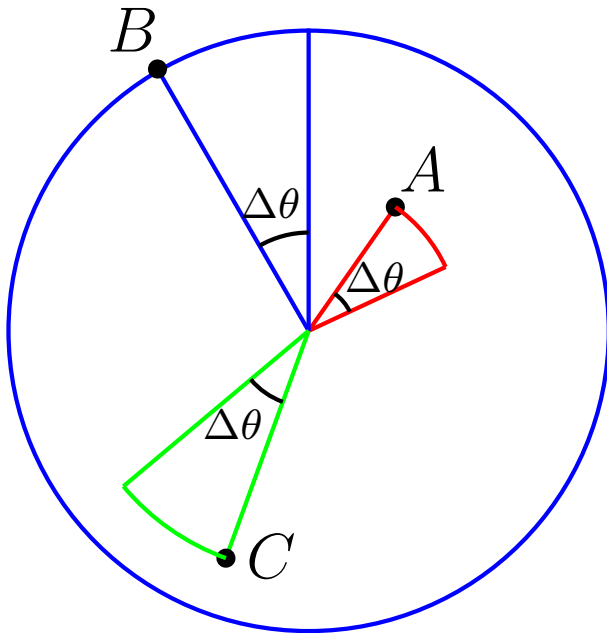
Practice Problems for chapters 5, 6, 7, and 8 available on Mastering Physics

Practice Exam on Website.

Review

All points rotate through the same angle

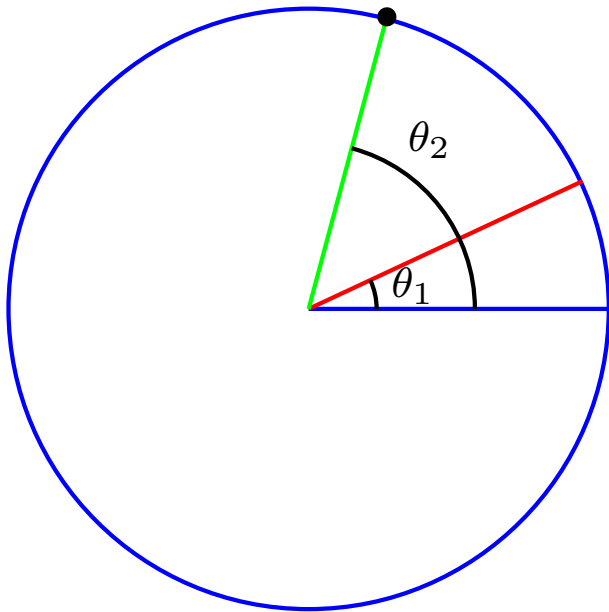
We must distinguish
linear motion = distance/time
from angular motion = angle/time



A rotating object has infinitely many linear speeds but only one angular speed

Angular Velocity

The rate at which an object spins is given by its angular velocity, $\vec{\omega}$.



$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Unit: *rad/s*

$$\omega = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

The Right-Hand-Rule

The angular velocity points along the axis of rotation. We use a right-hand-rule (RHR) to quickly determine which direction.

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RHR - Curl the fingers of your right hand in the “sense” of the rotation. Your extended thumb, points in direction of $\vec{\omega}$.

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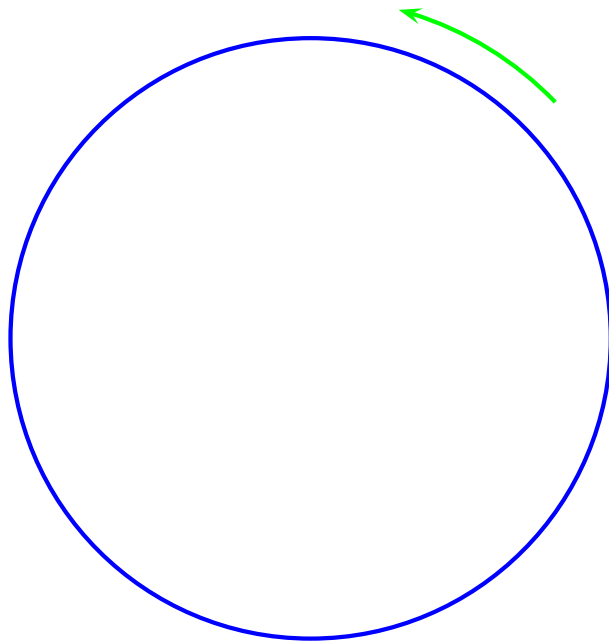
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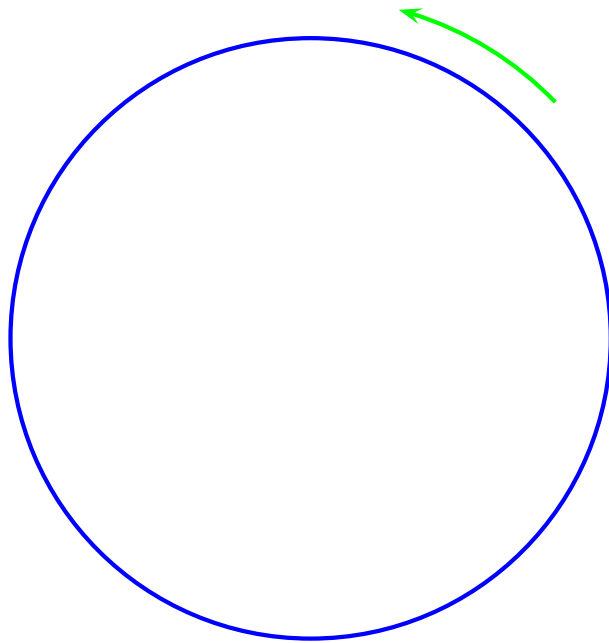
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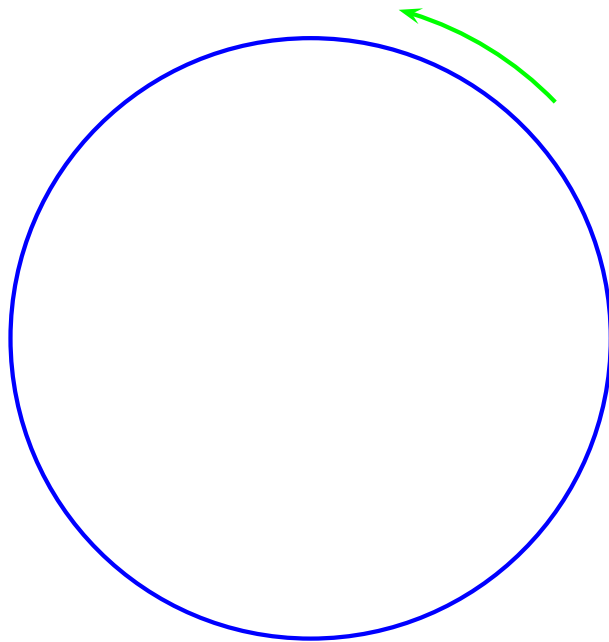
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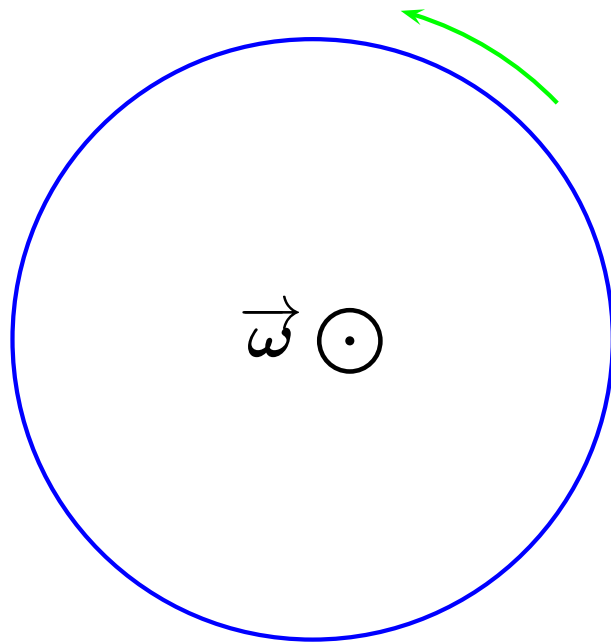
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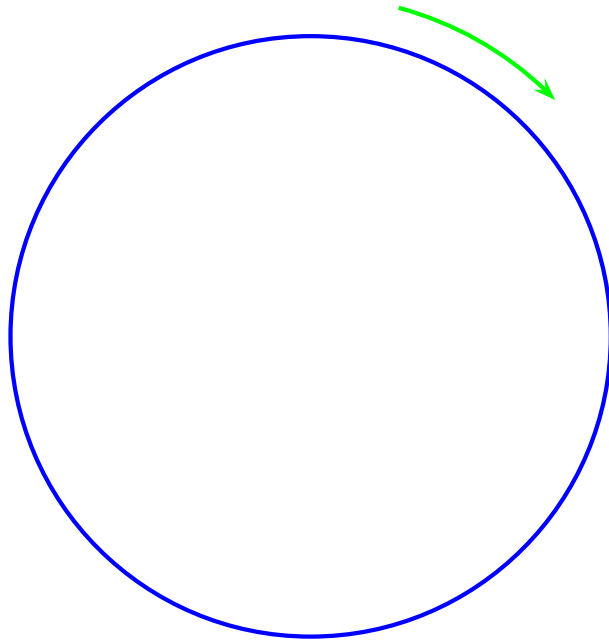
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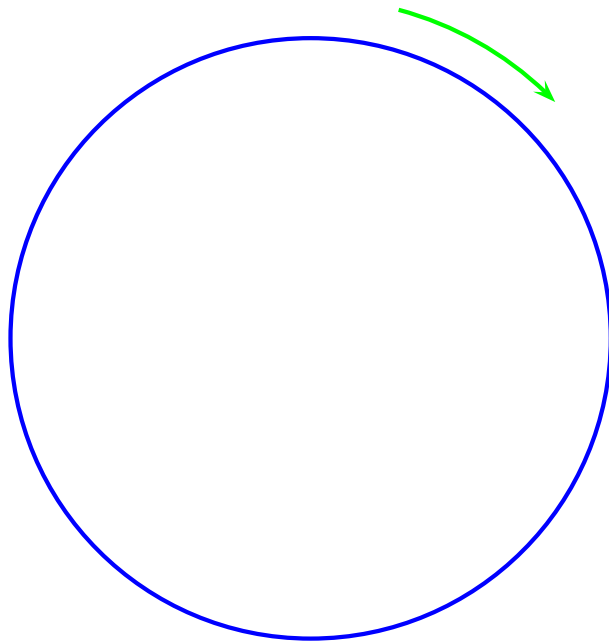
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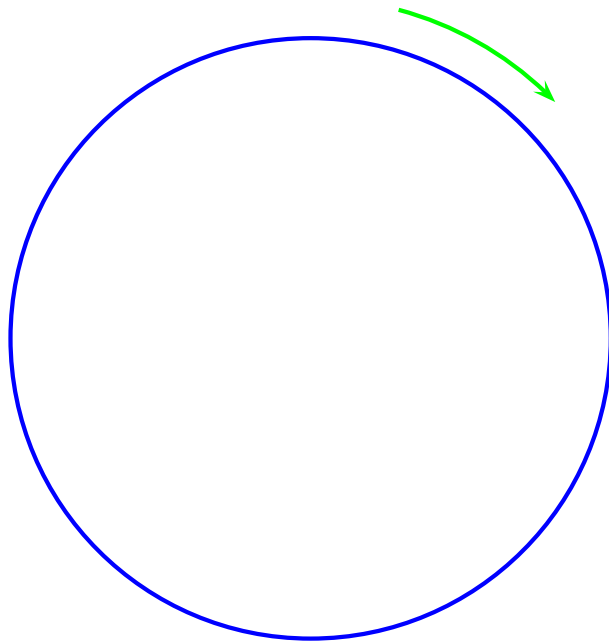
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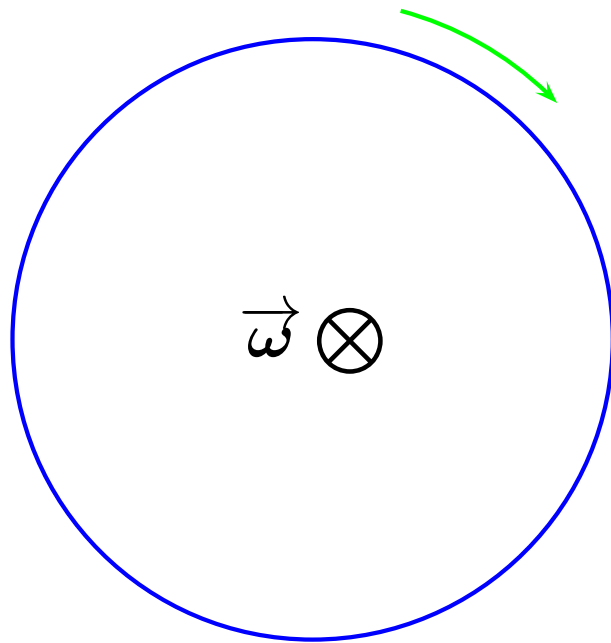
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Angular Acceleration

Any change in angular velocity must come from an angular acceleration, $\vec{\alpha}$.

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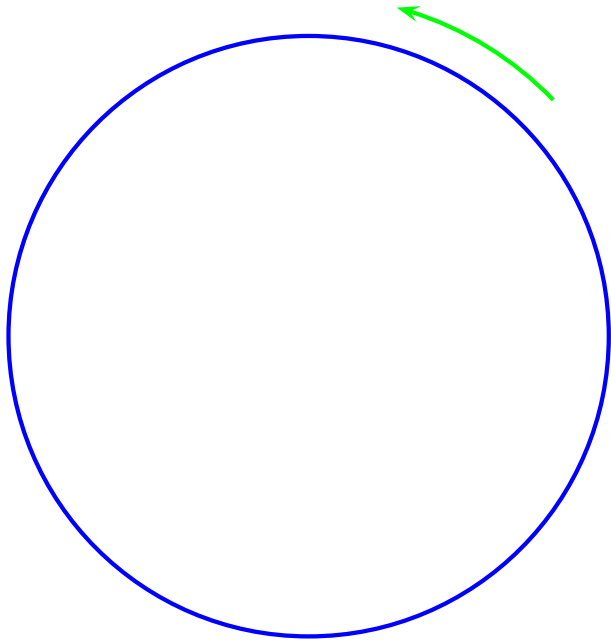
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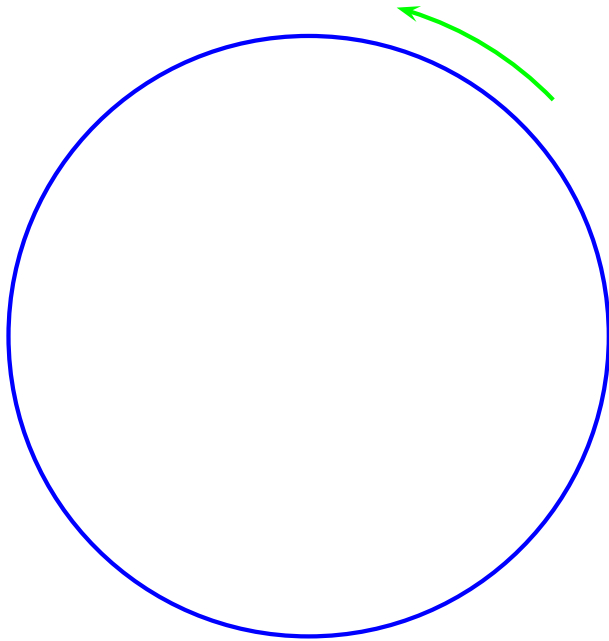
Clicker Quiz

A wheel, rotating counter-clockwise, has a decreasing angular speed. What direction is its angular acceleration vector, $\vec{\alpha}$?



Clicker Quiz

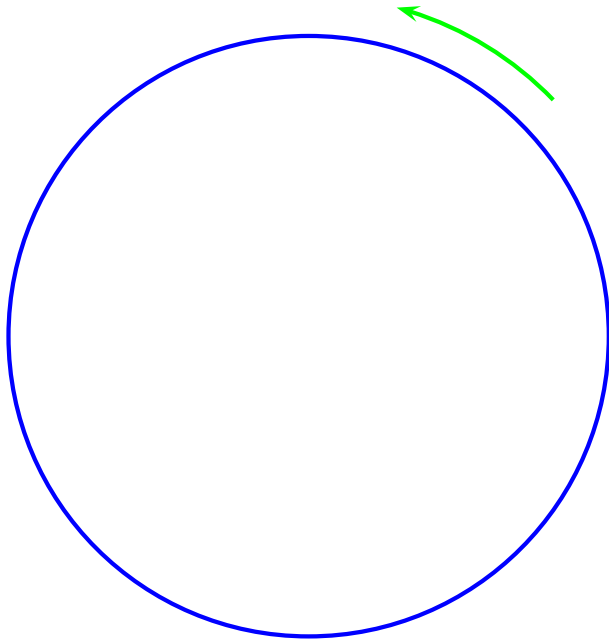
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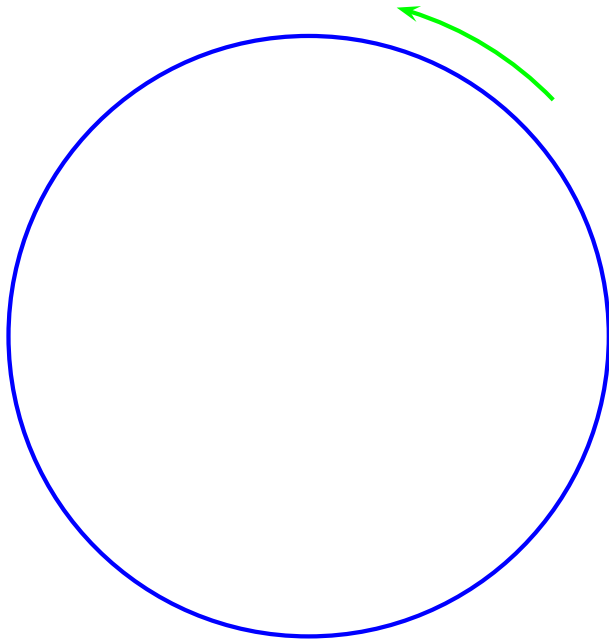


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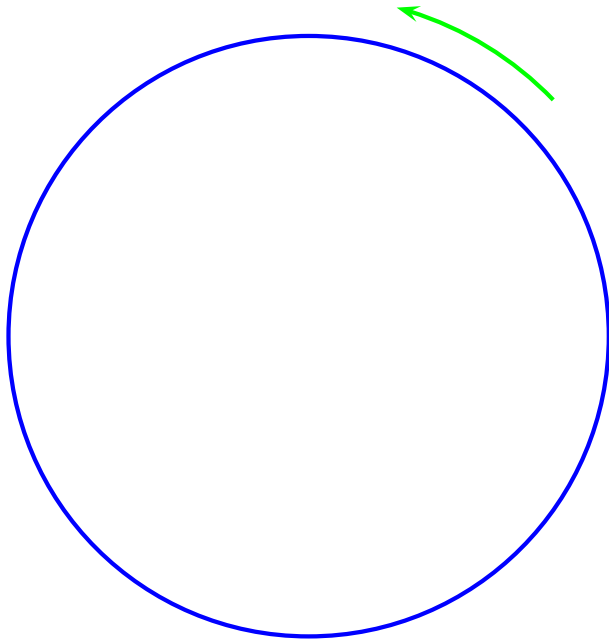
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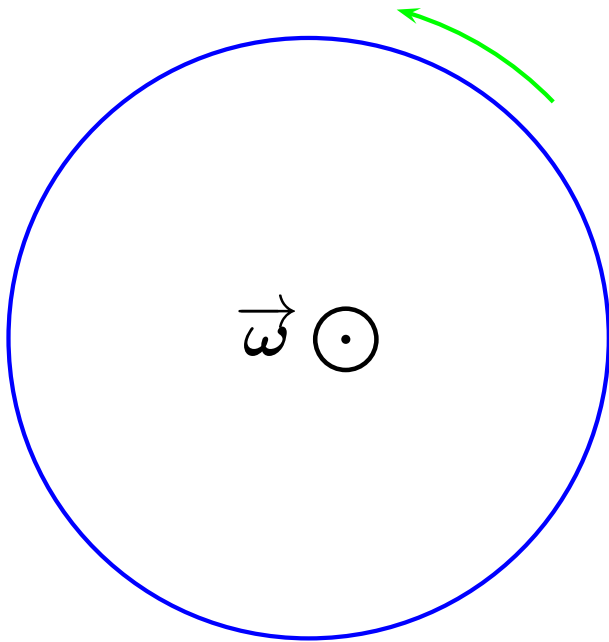
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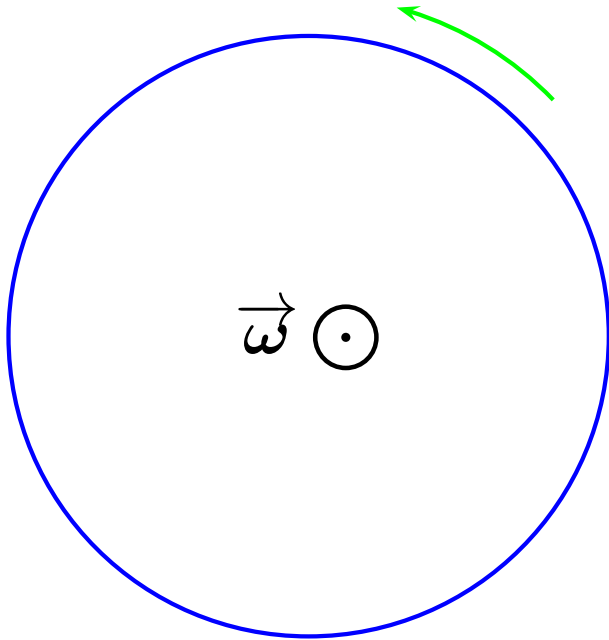
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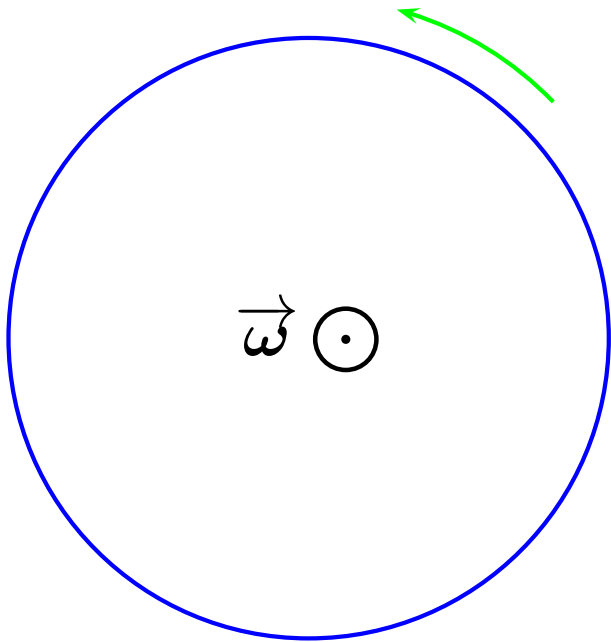
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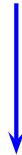
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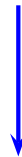
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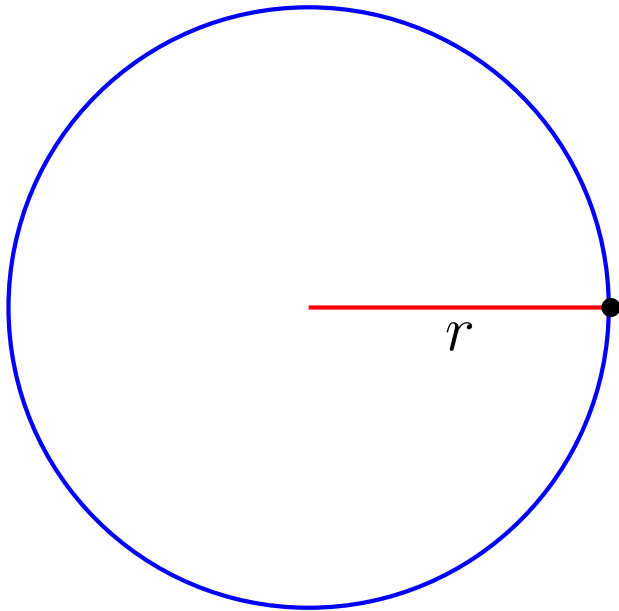
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Relating Linear and Angular Velocity

Use the relationship $s = r\theta$ to relate linear and angular speeds.

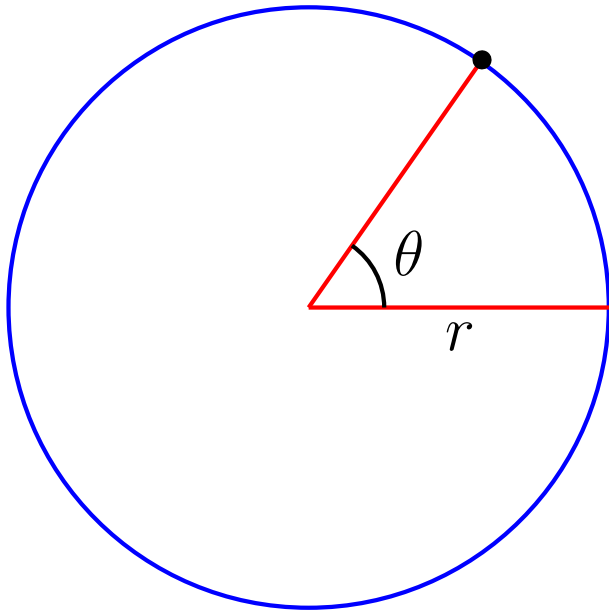
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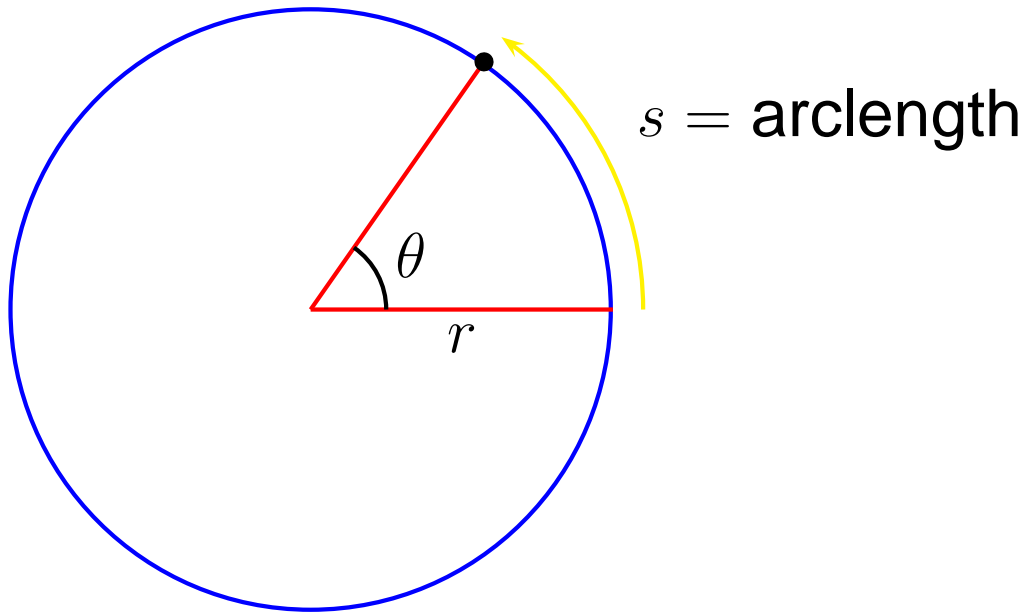
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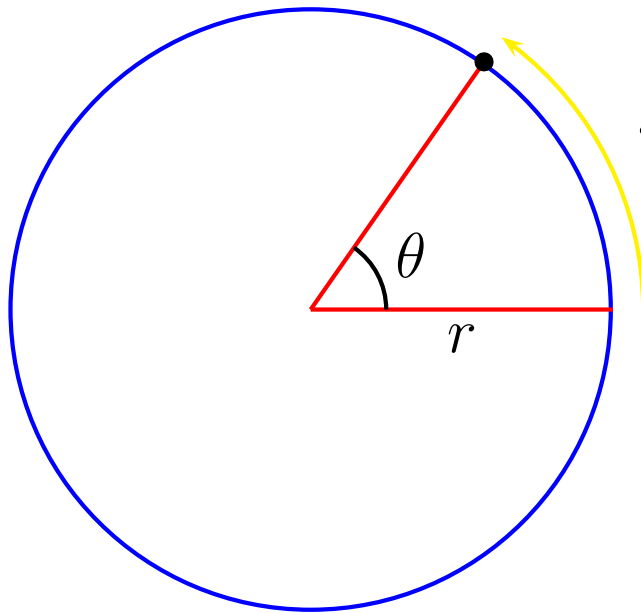
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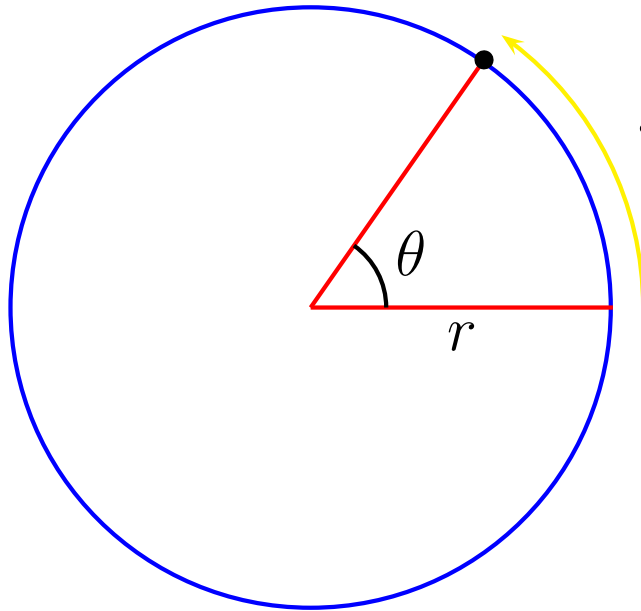


$s = \text{arclength}$

s is the linear distance traveled

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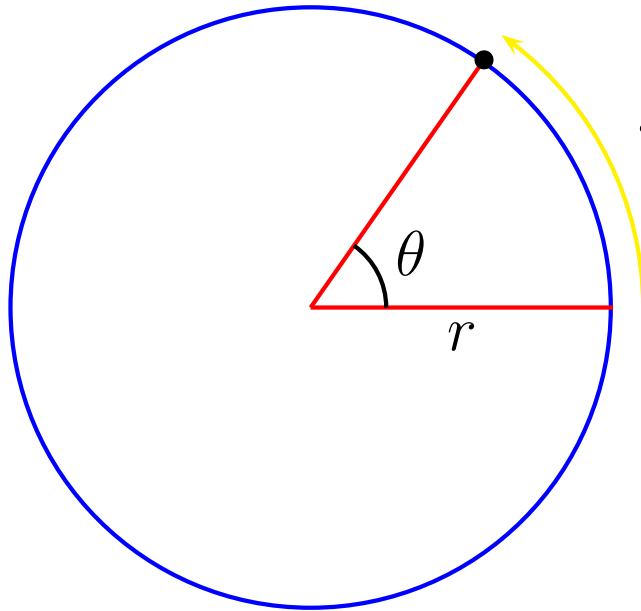
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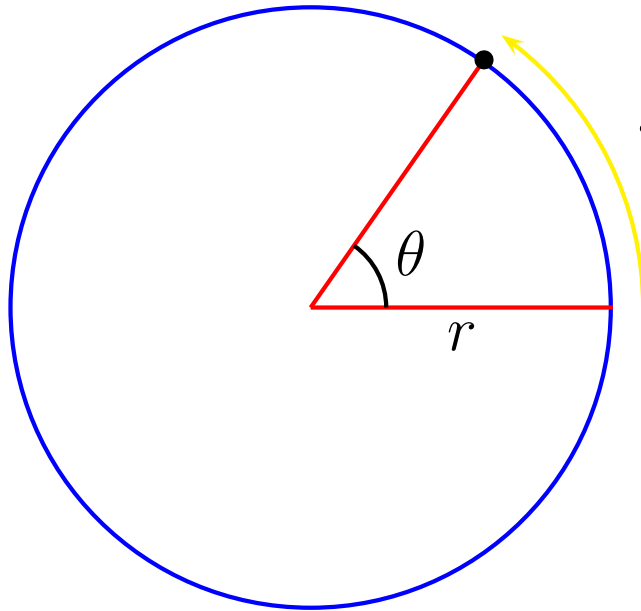
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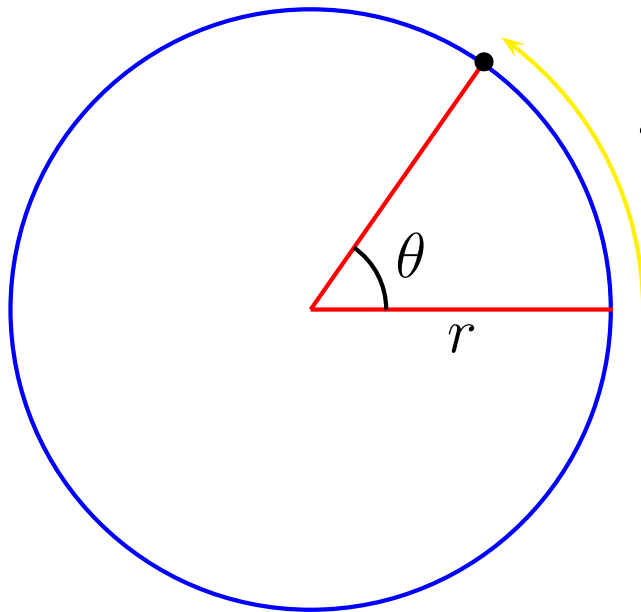
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$$\frac{d}{dt} (s = r\theta)$$

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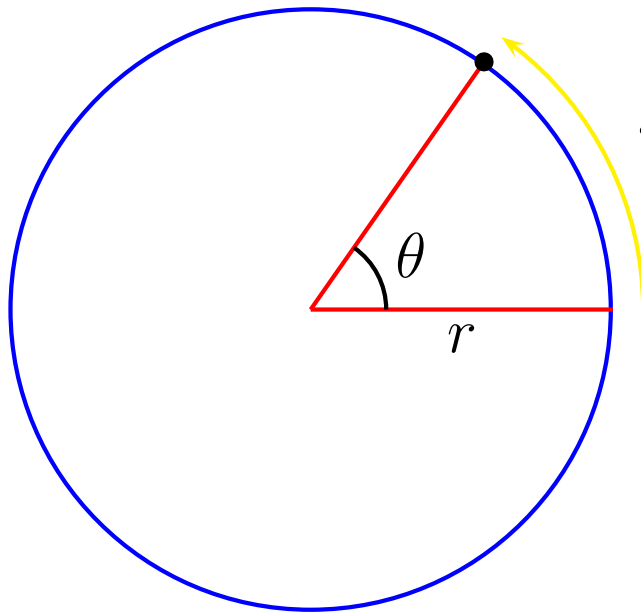
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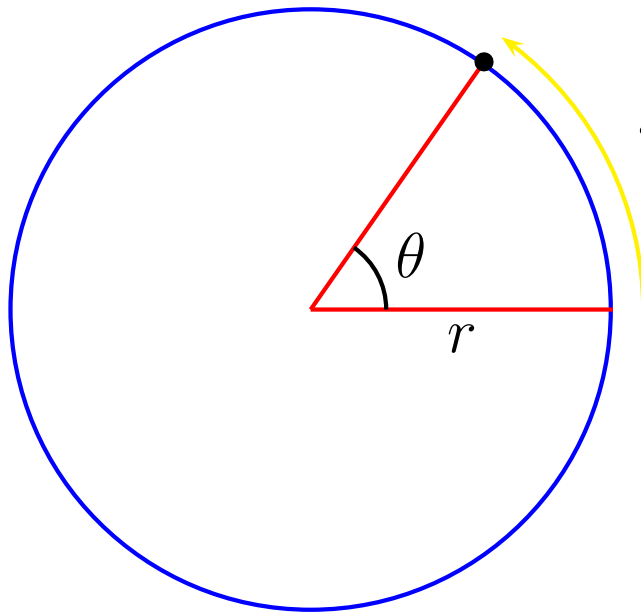
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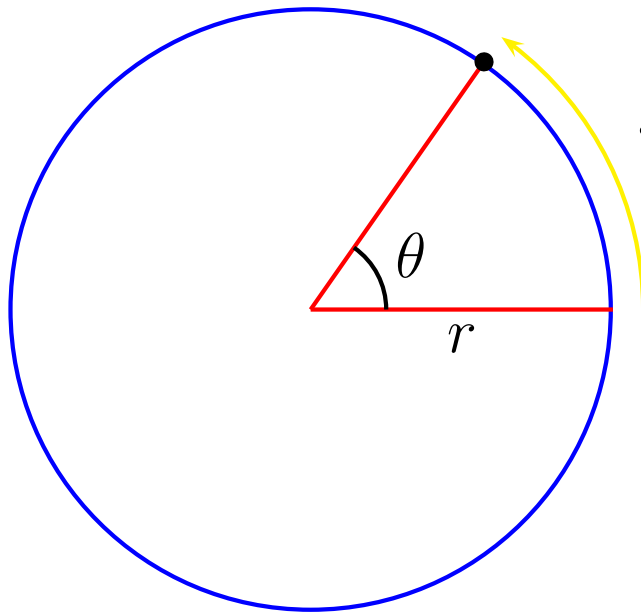
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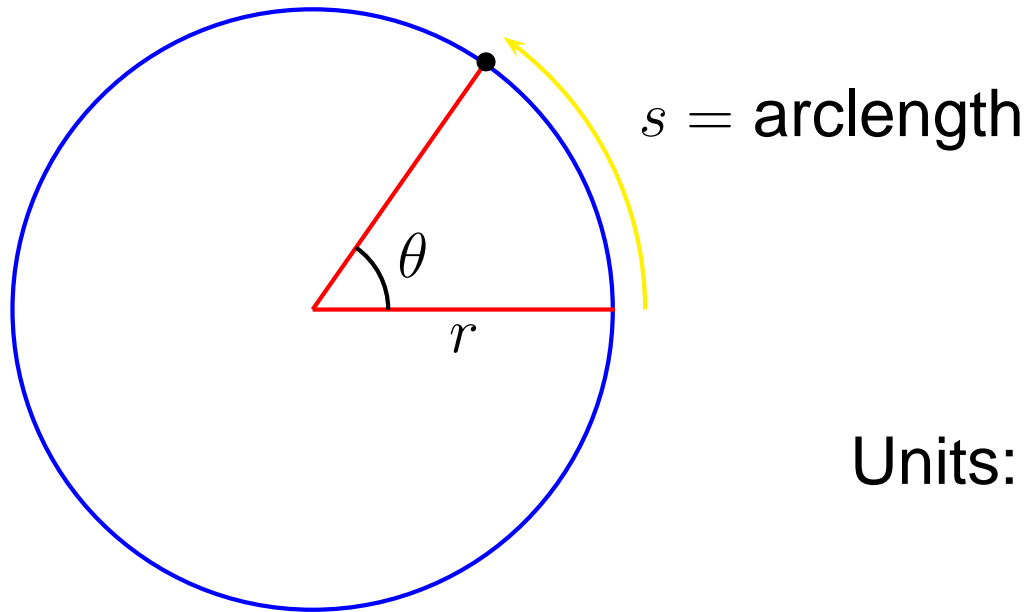
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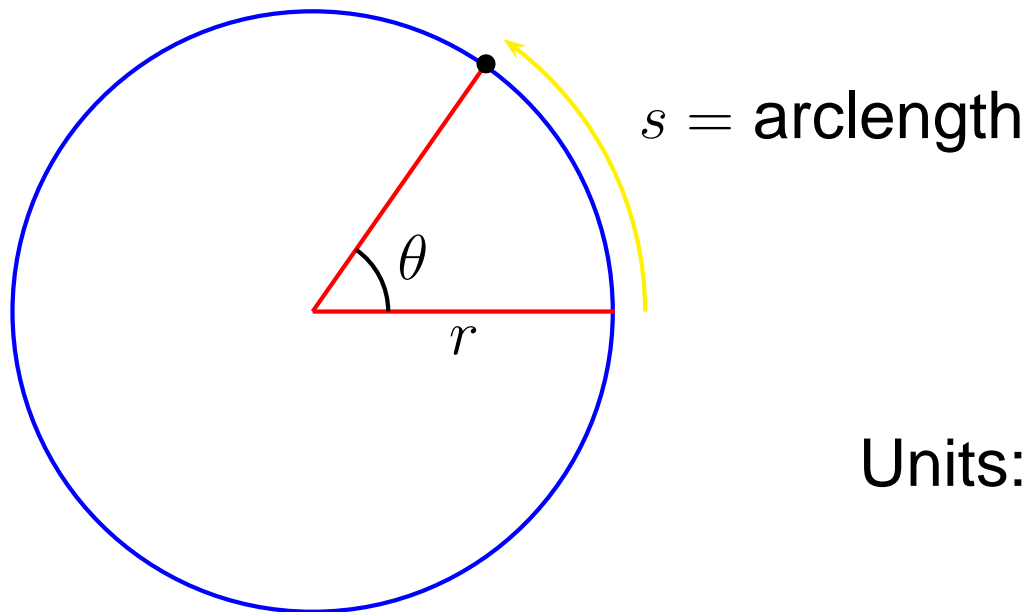


$$v = r\omega$$

Units:

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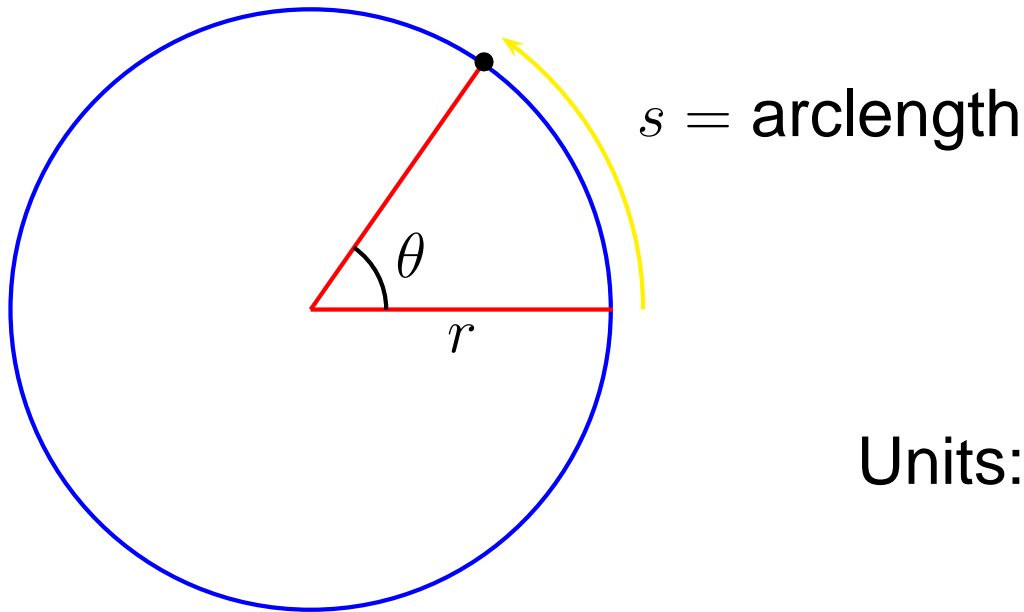


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Units: $\frac{m \cdot \text{rad}}{s}$

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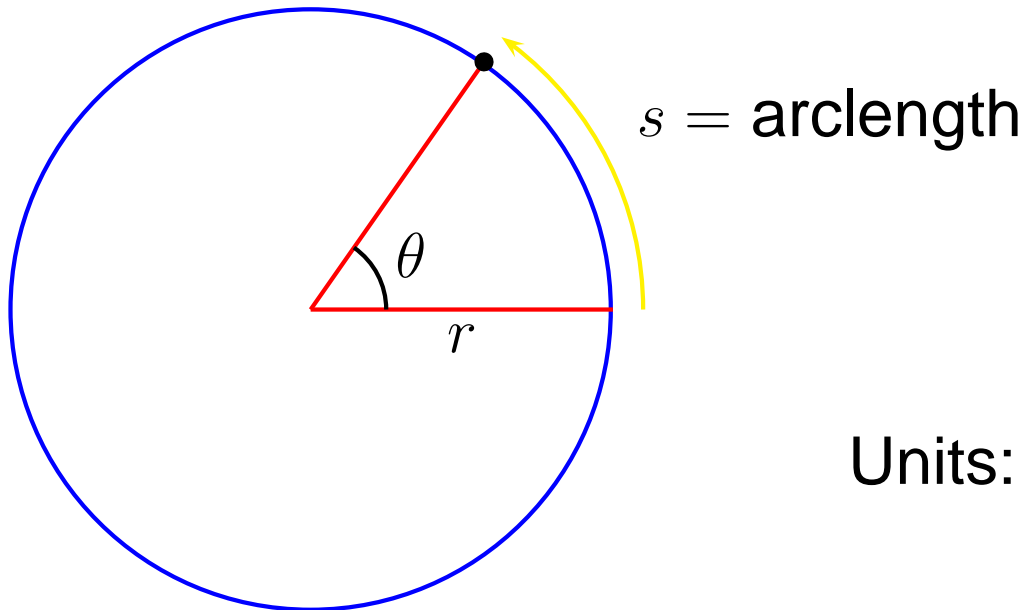


$$v = r\omega$$

Units: $\frac{m \cdot \text{rad}}{s} = \frac{m \cdot \cancel{\text{rad}}}{s}$

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$$v = r\omega$$

$$\text{Units: } \frac{m \cdot \text{rad}}{s} = \frac{m \cdot \cancel{\text{rad}}}{s} = m/s$$

The Cross Product

To get the correct direction, we use the cross product.

The Cross Product

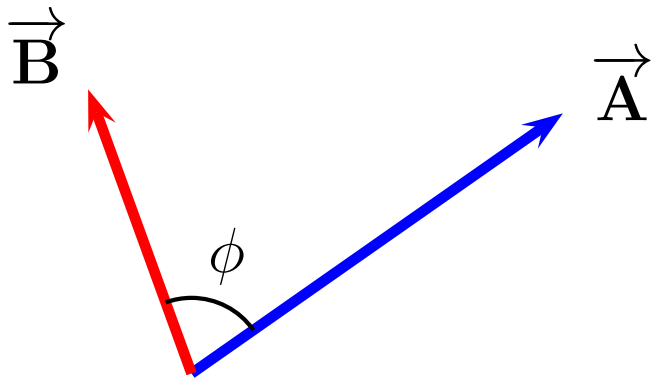
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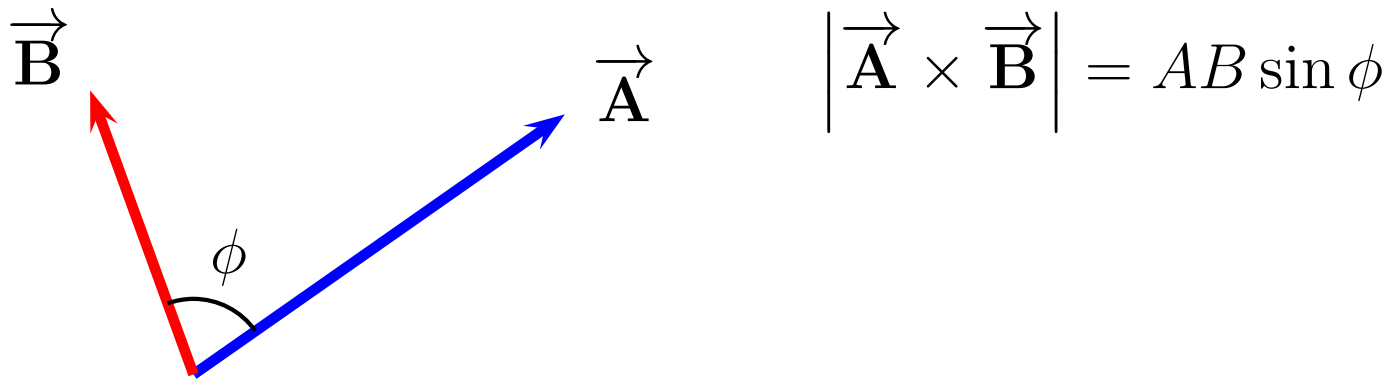
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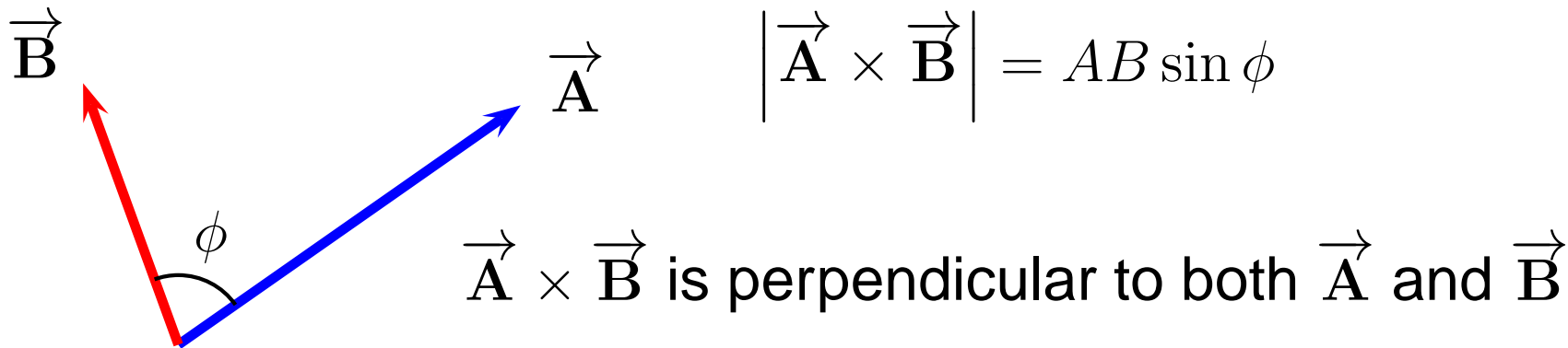
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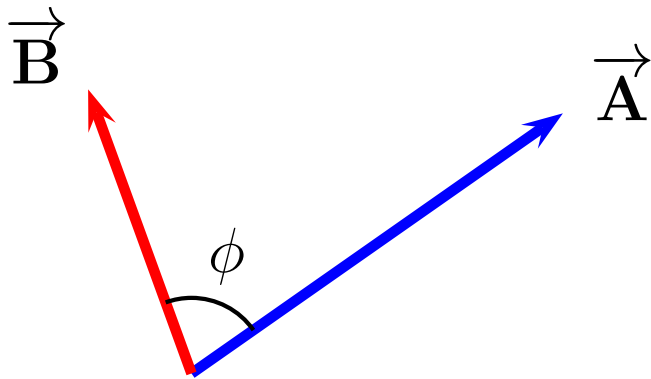


The Cross Product II

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Another Right-Hand-Rule (RHR) gives direction:

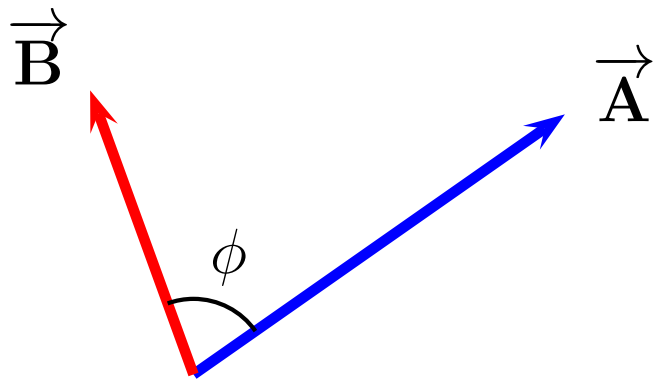


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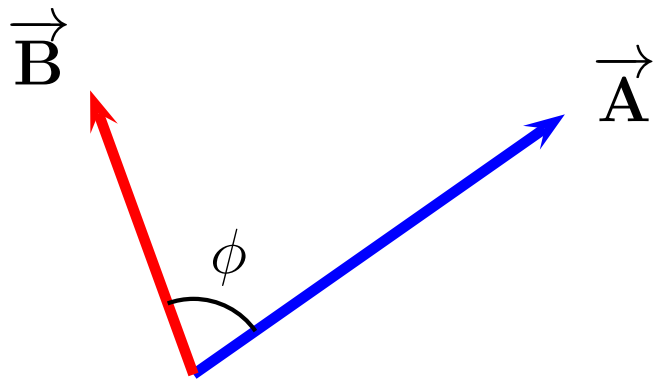
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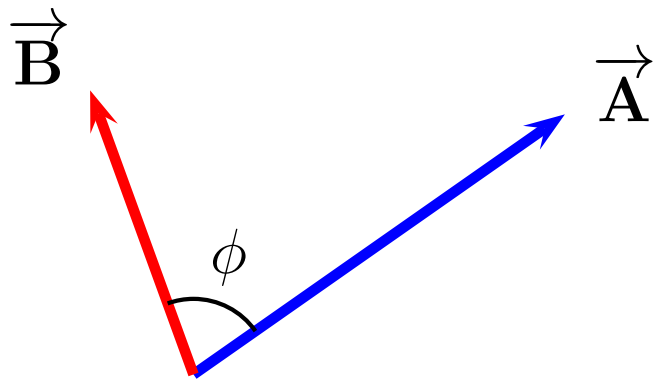
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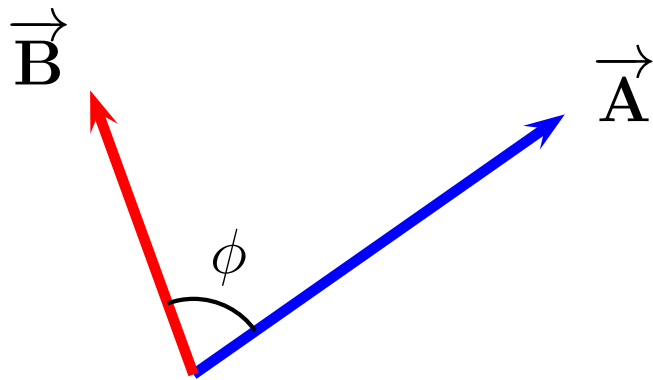
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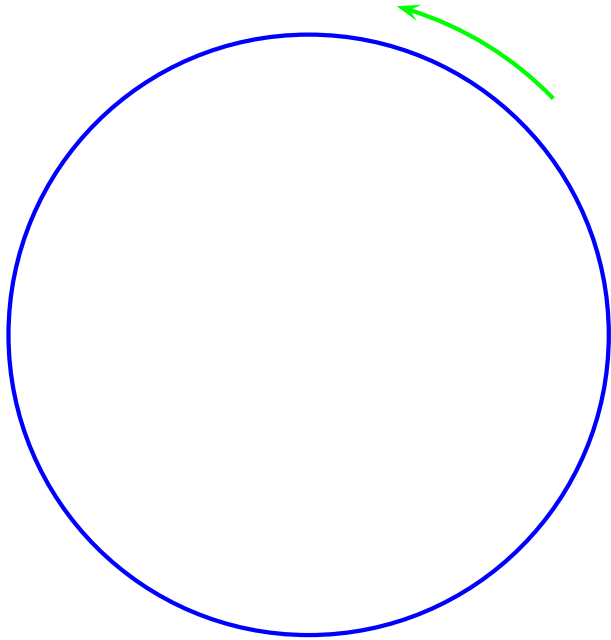
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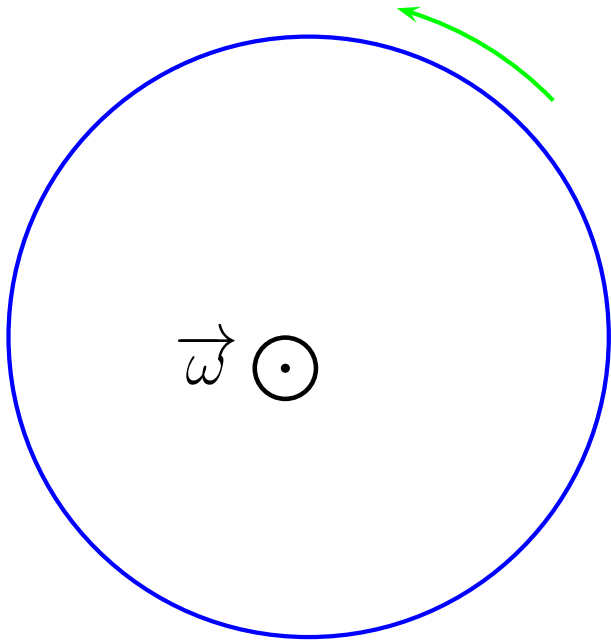
$$\vec{B} \times \vec{A} = \otimes$$

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

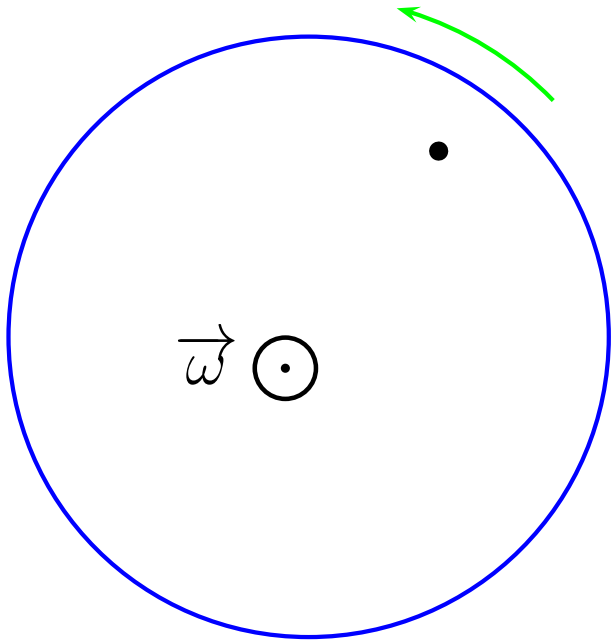
Linear and Angular Velocities



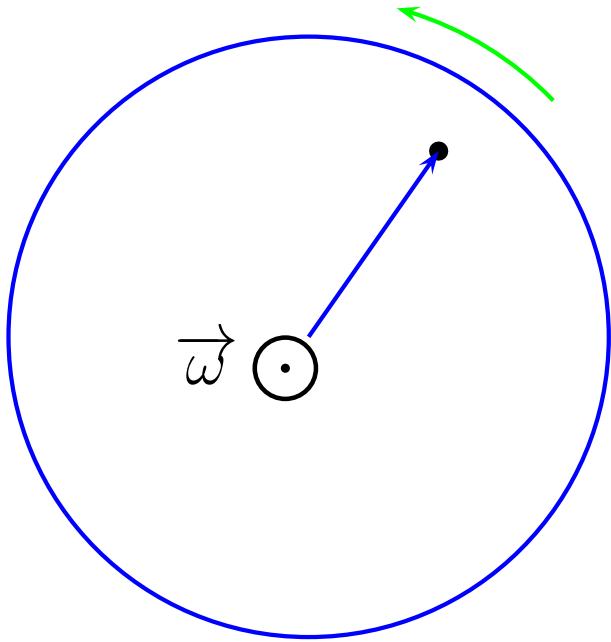
Linear and Angular Velocities



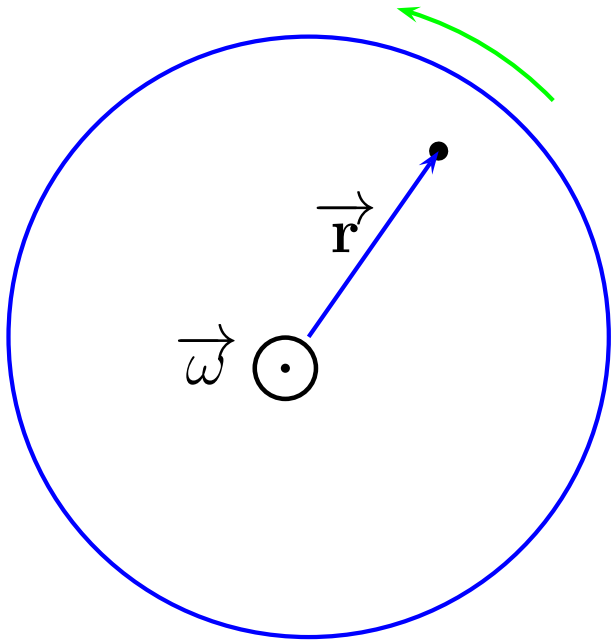
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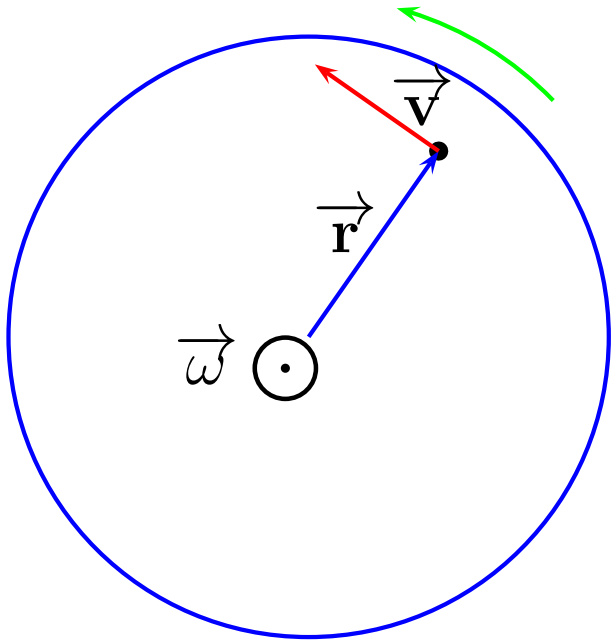
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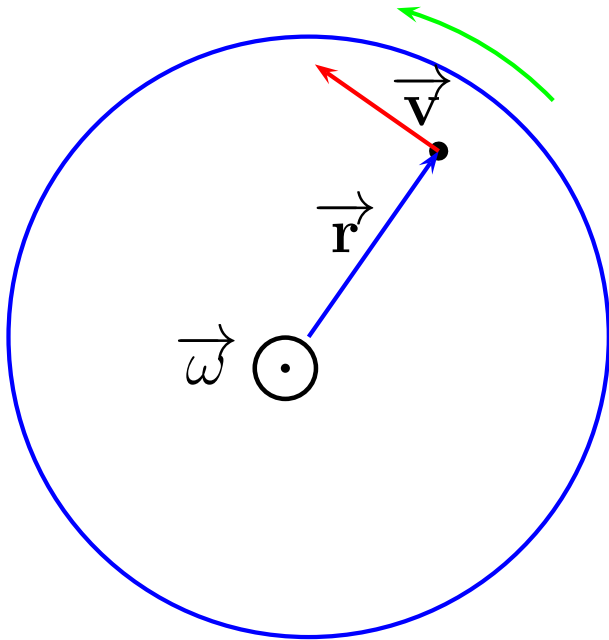
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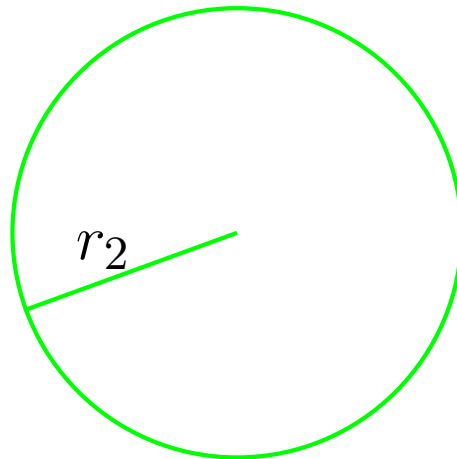
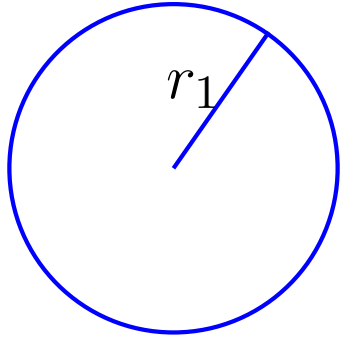


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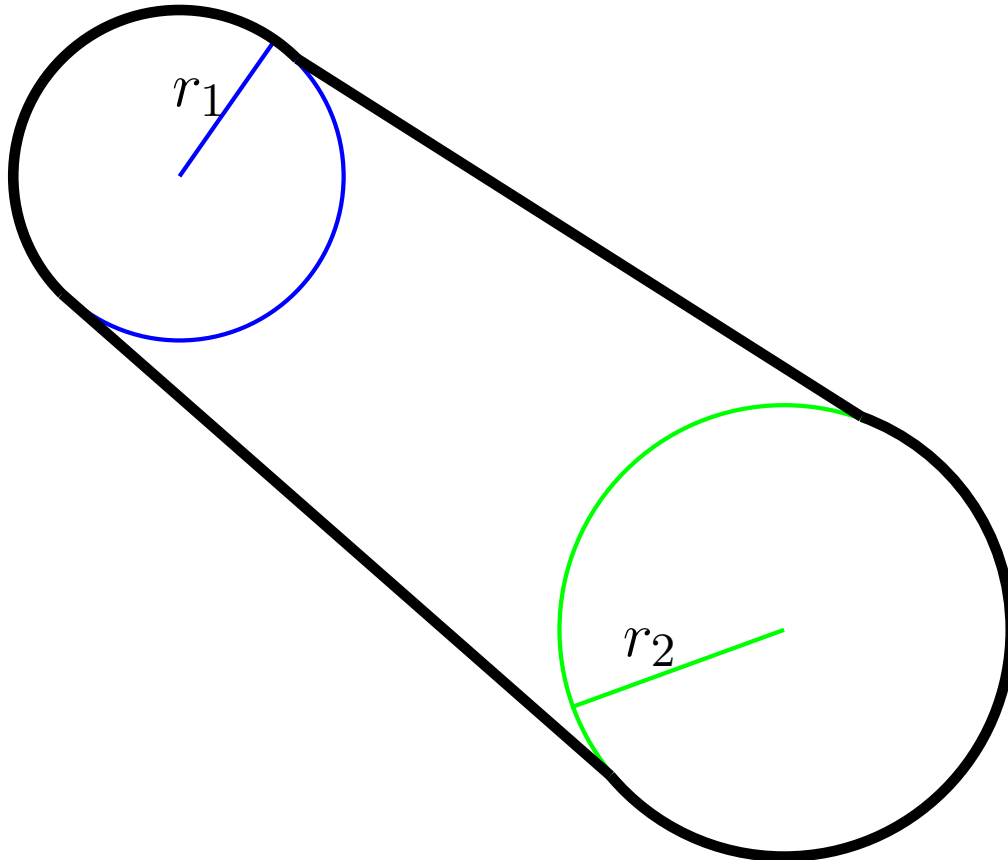


$$\vec{v} = \vec{\omega} \times \vec{r}$$

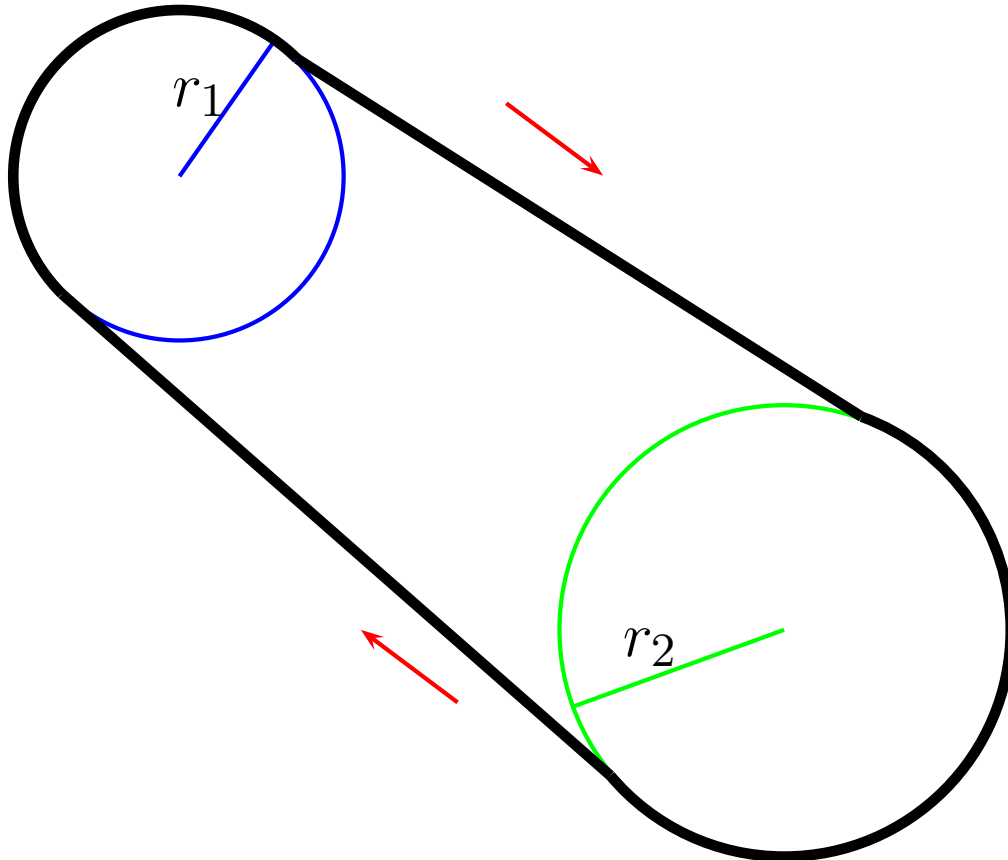
Connected Rotating Objects



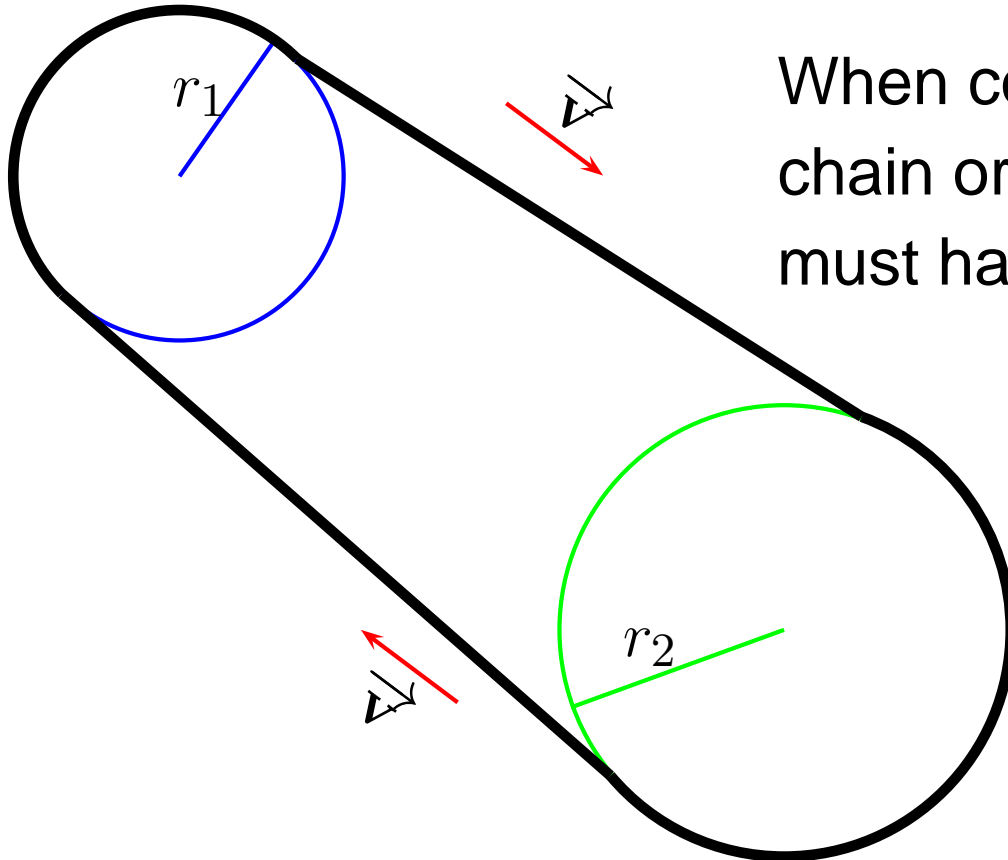
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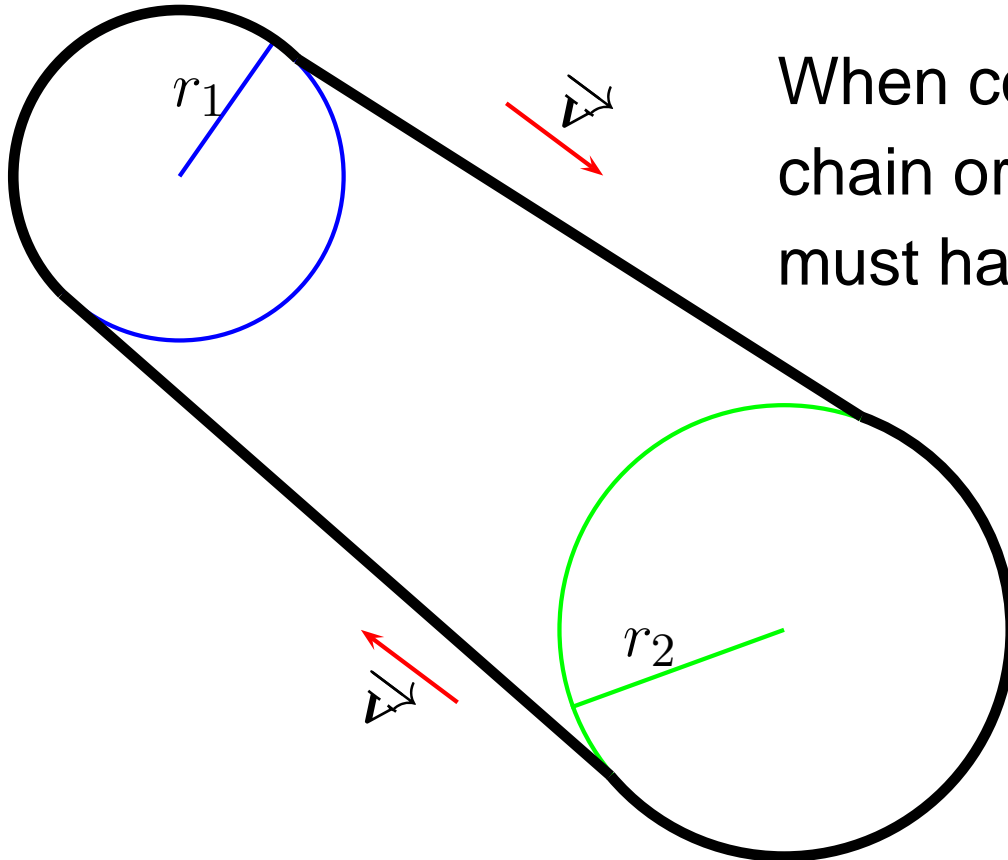


Connected Rotating Objects



When connected by a non-slipping chain or belt, the two rotating objects must have the same linear velocity

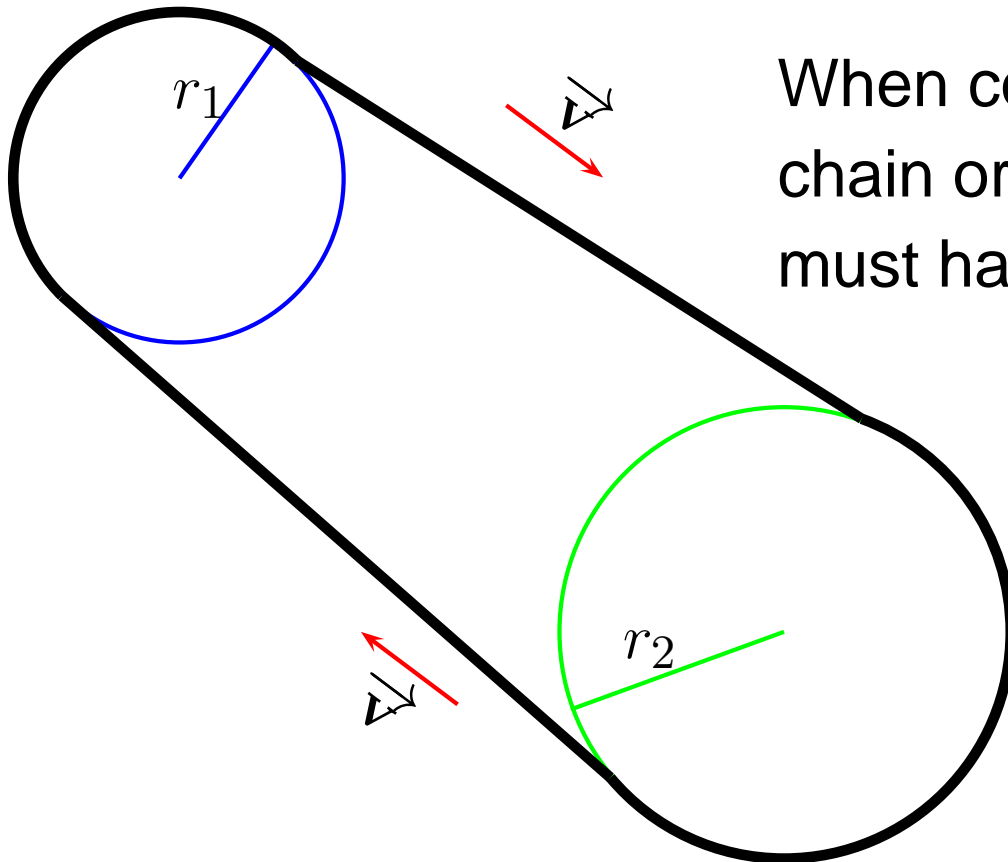
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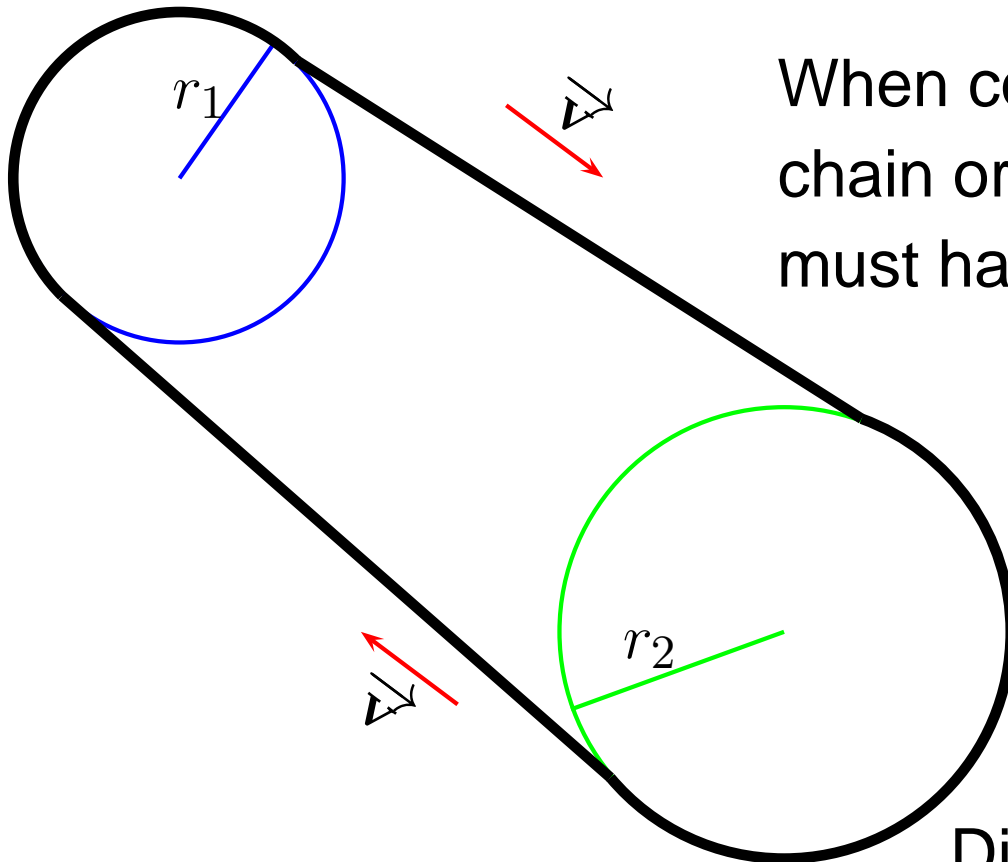


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Different Angular Velocities