March 30, Week 10

Today: Chapter 8, Collisions

Homework #8:
Mastering Physics: 8 problems from chapter 8
Written Question: 8.101
Due April 2 at 11:59pm

Exam #4: Friday, April 6

Review Session: Thursday, April 5, 7:30PM

Practice Problems for chapters 5, 6, and 7 available on Mastering Physics

Conservation of Momentum

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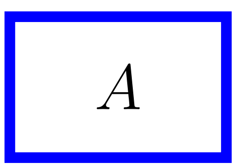

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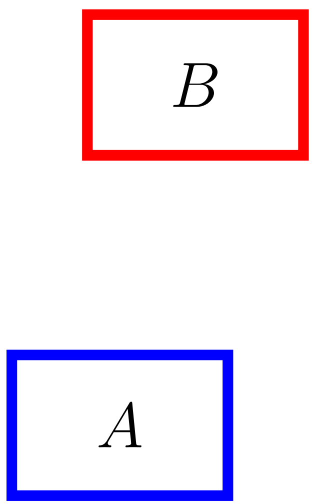

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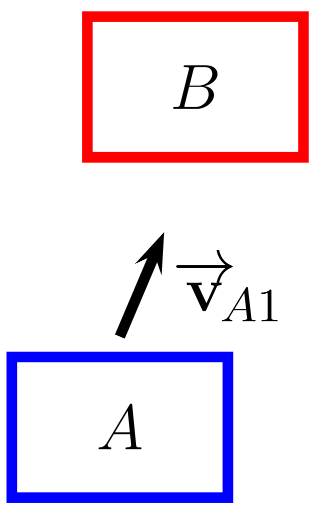

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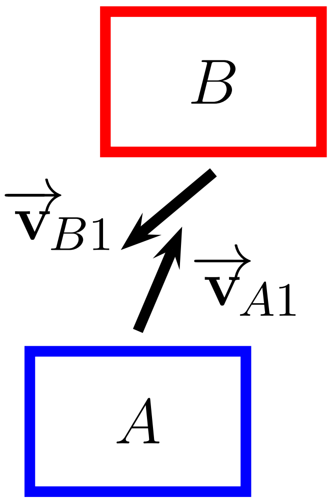

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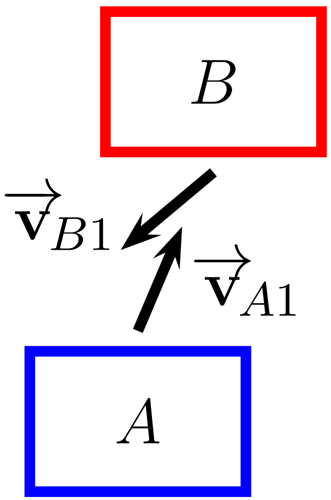

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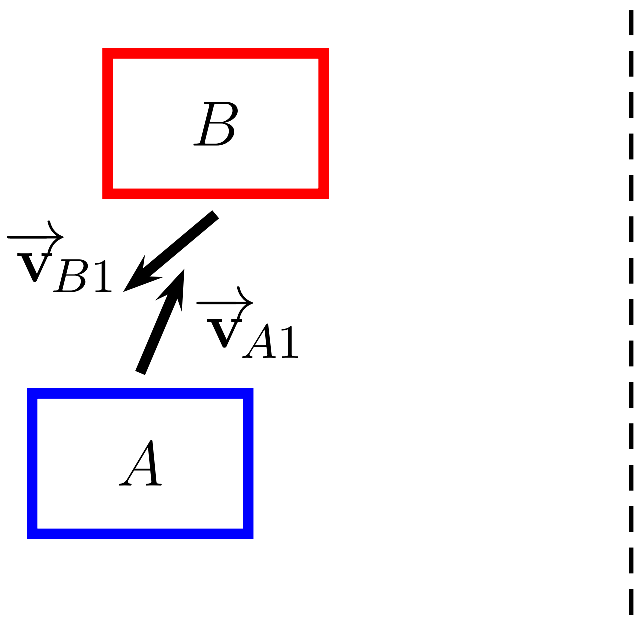

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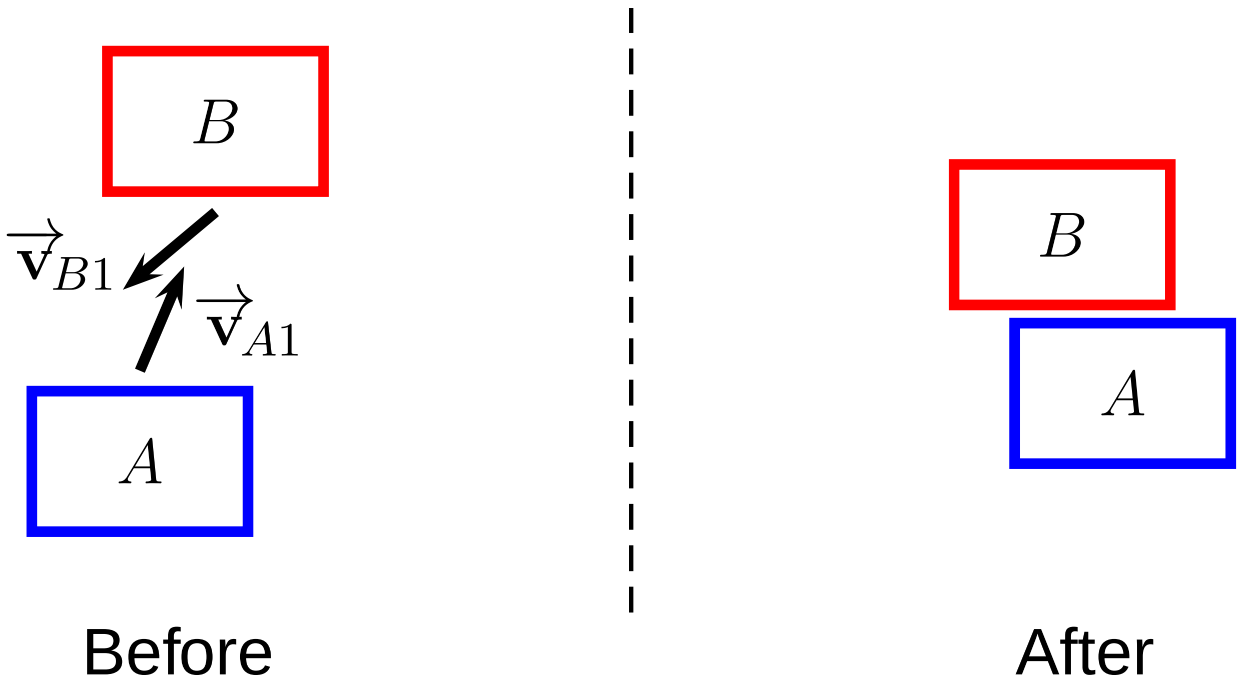

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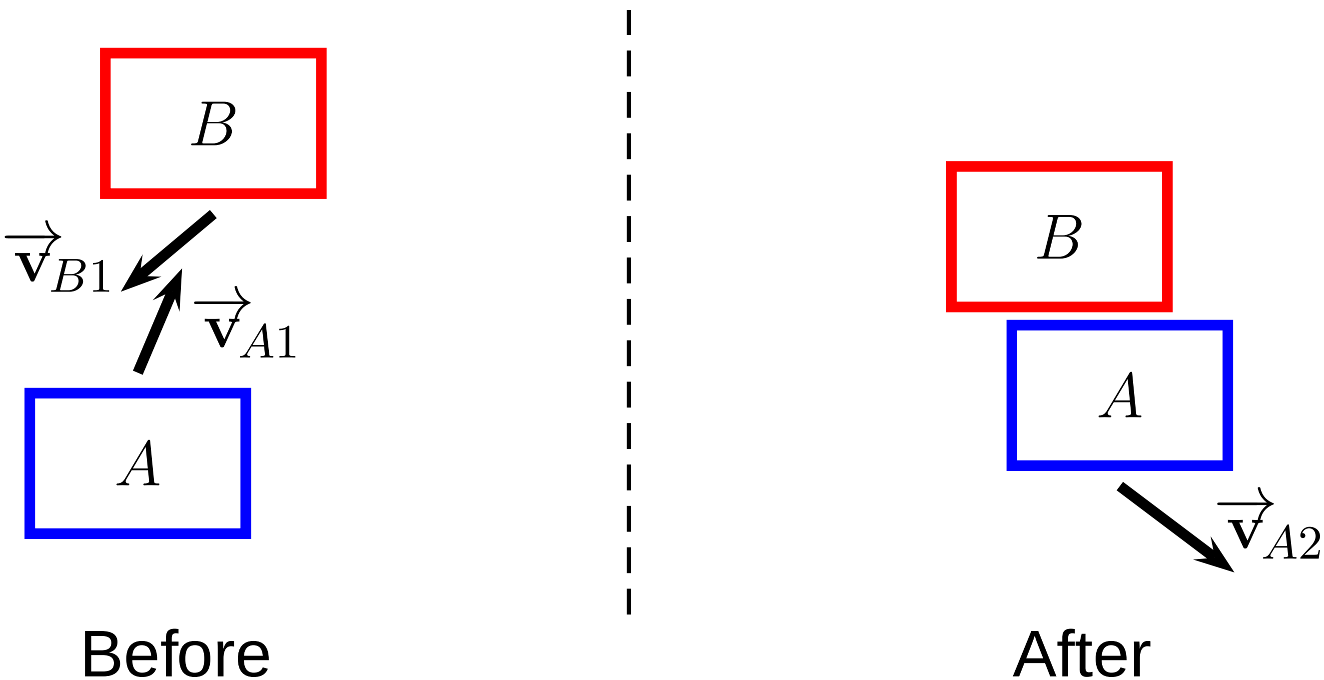

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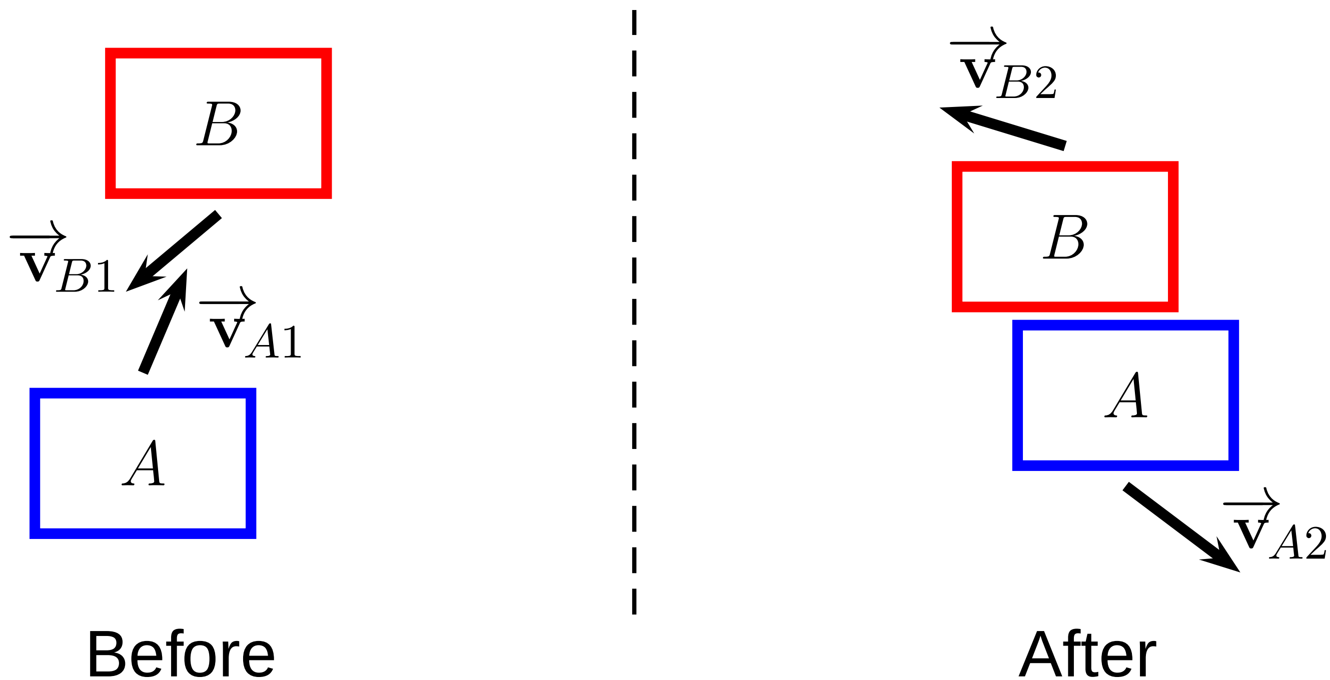

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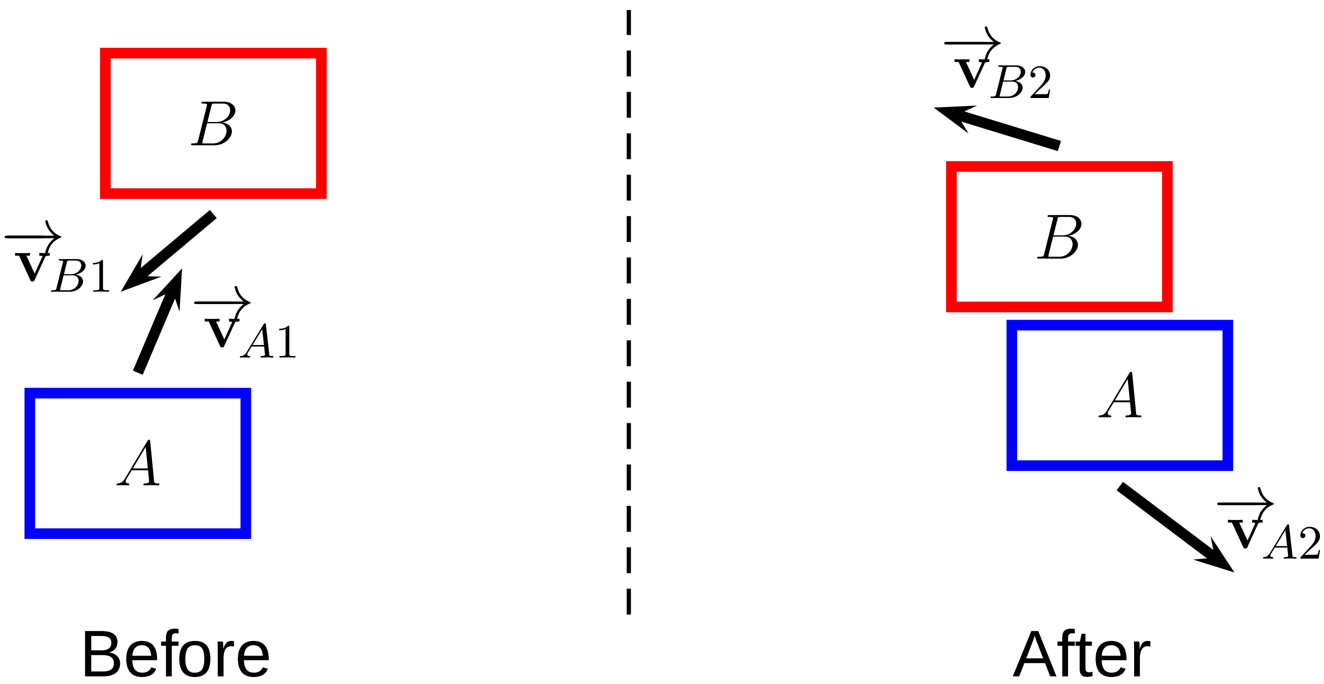

$$M_A \vec{\mathbf{v}}_{A1} + M_B \vec{\mathbf{v}}_{B1} \qquad M_A \vec{\mathbf{v}}_{A2} + M_B \vec{\mathbf{v}}_{B2}$$

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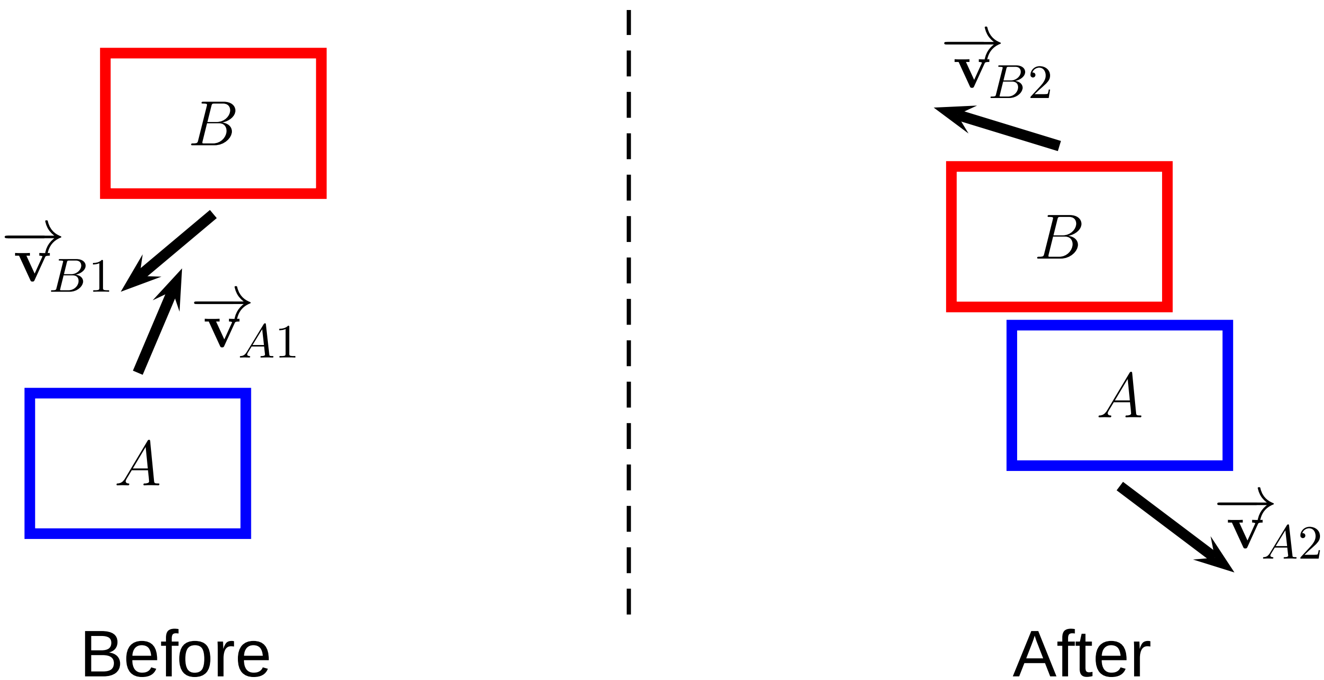

$$M_A \overrightarrow{\mathbf{v}}_{A1} + M_B \overrightarrow{\mathbf{v}}_{B1} = M_A \overrightarrow{\mathbf{v}}_{A2} + M_B \overrightarrow{\mathbf{v}}_{B2}$$

Non-Conservation of Momentum

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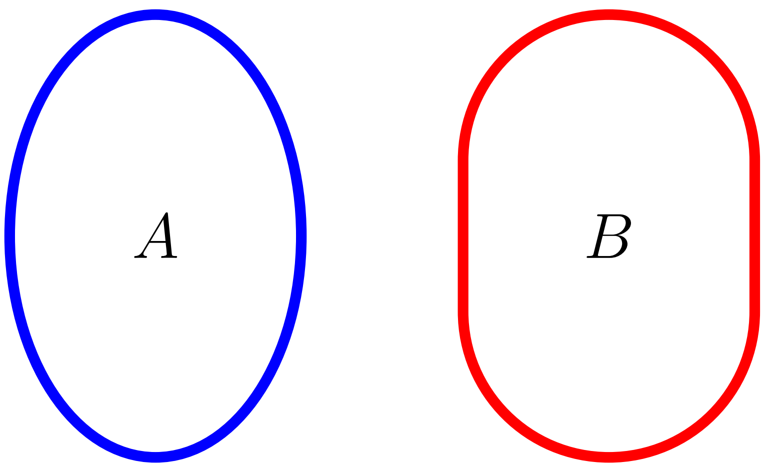

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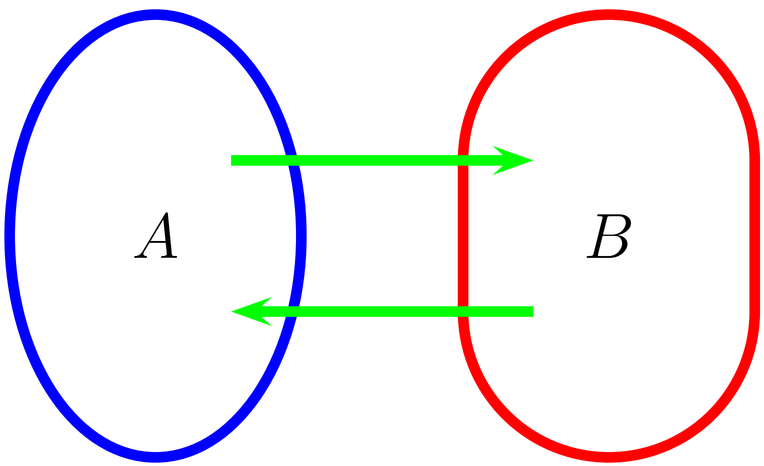

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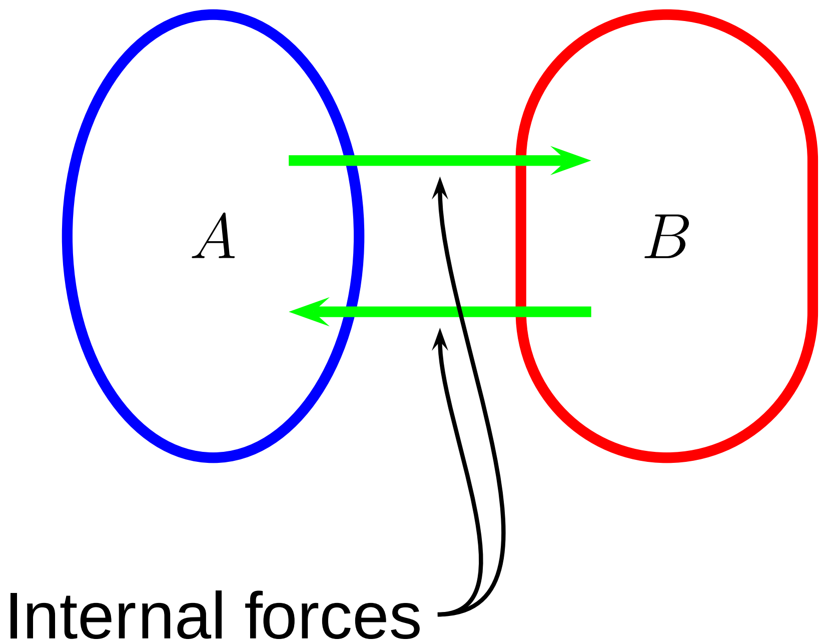

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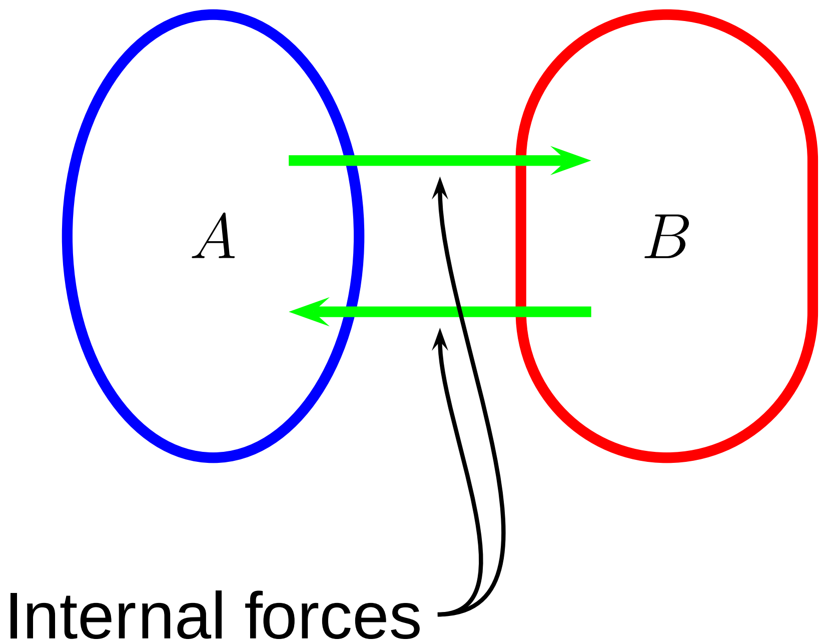

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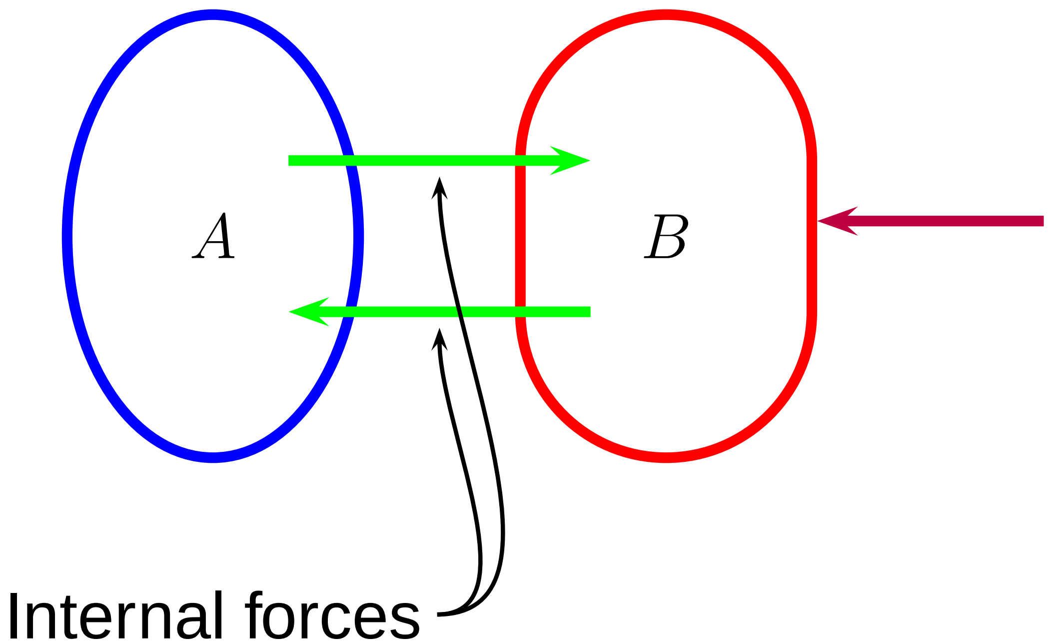

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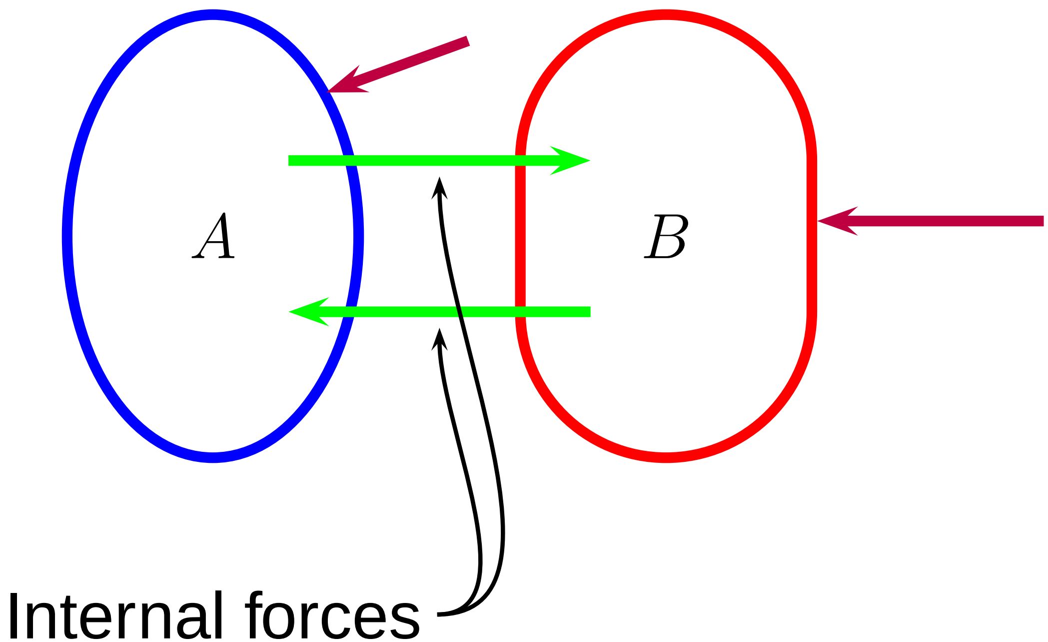

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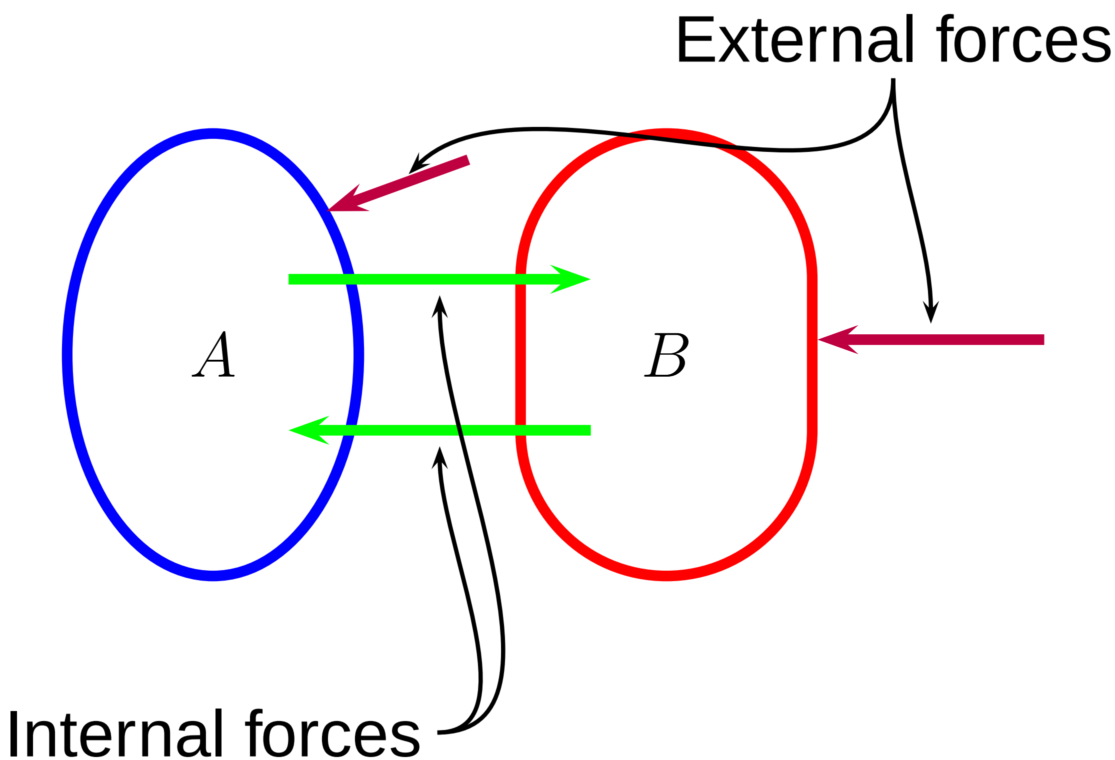

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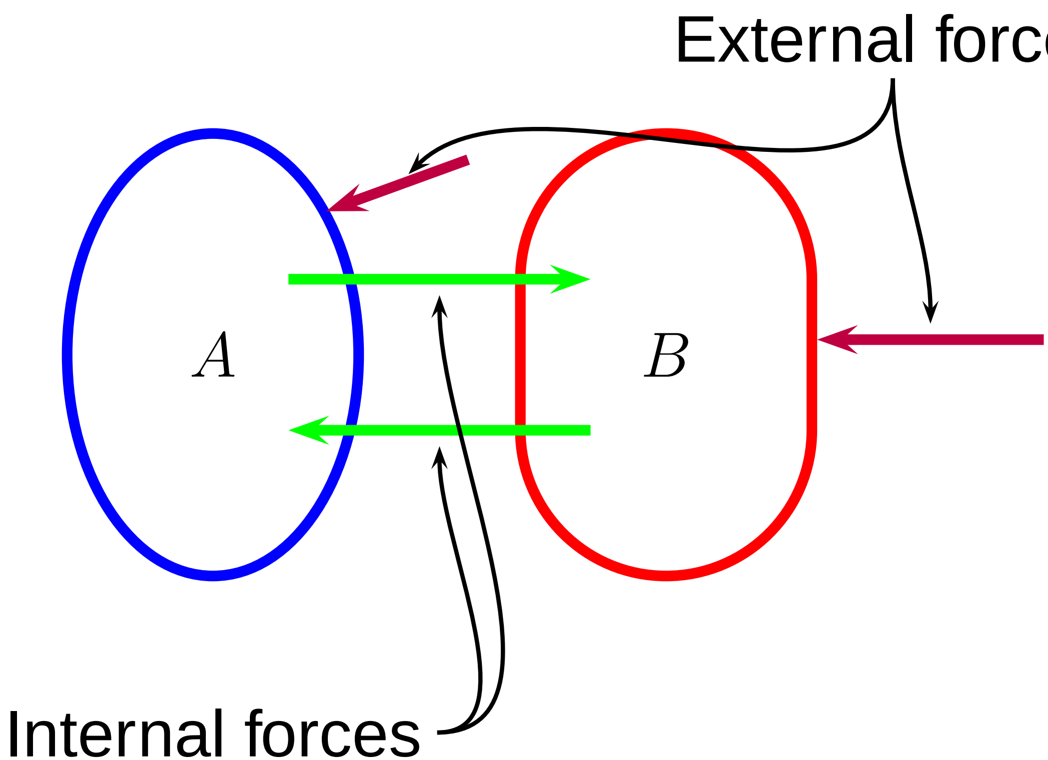

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External Forces - Forces from outside the system

Collisions

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Example: Are the collisions from the previous lecture elastic?

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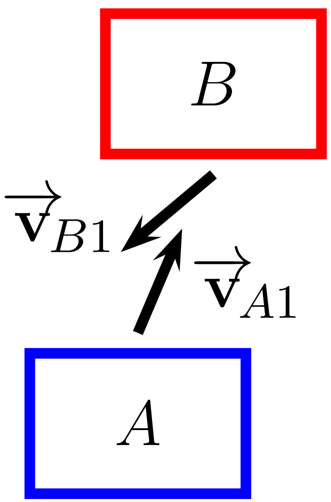

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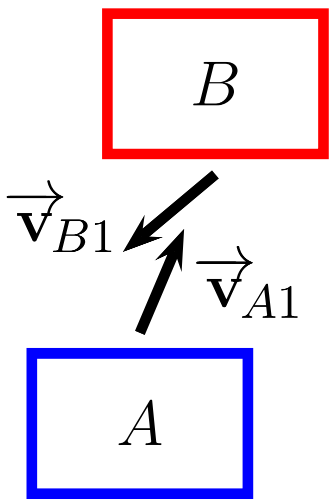

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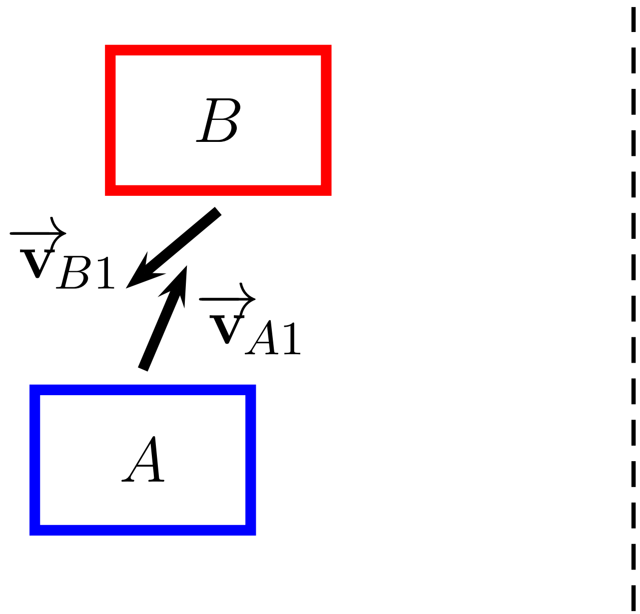

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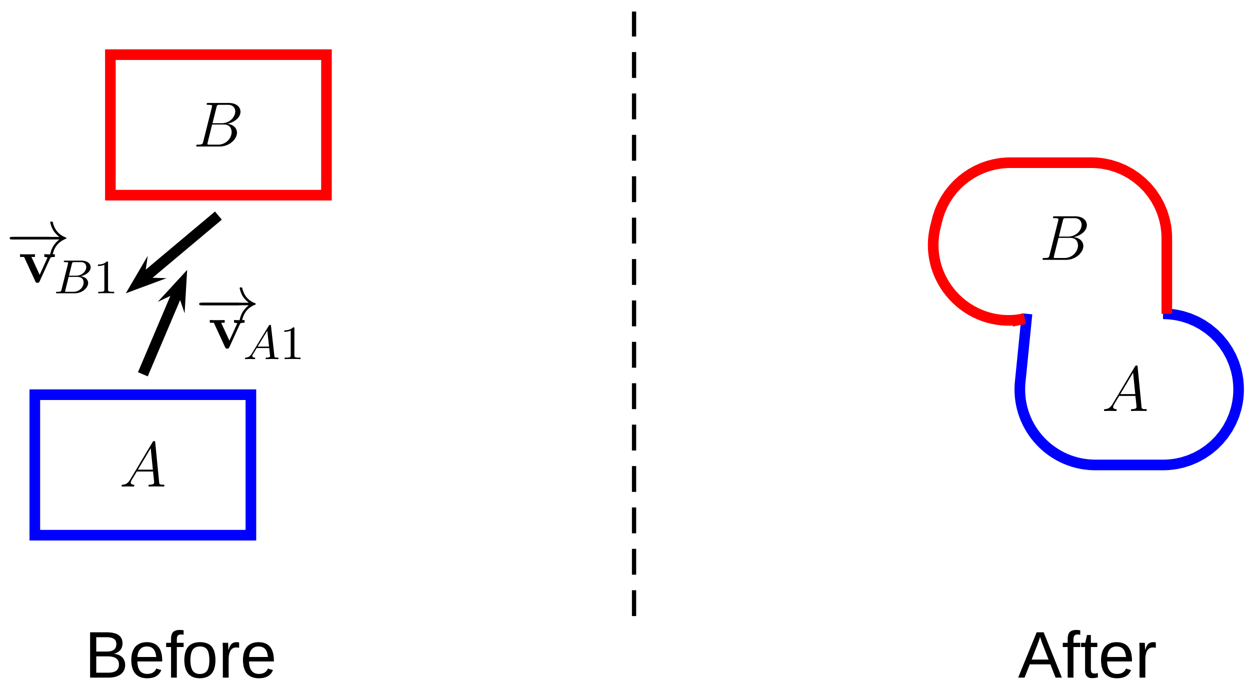

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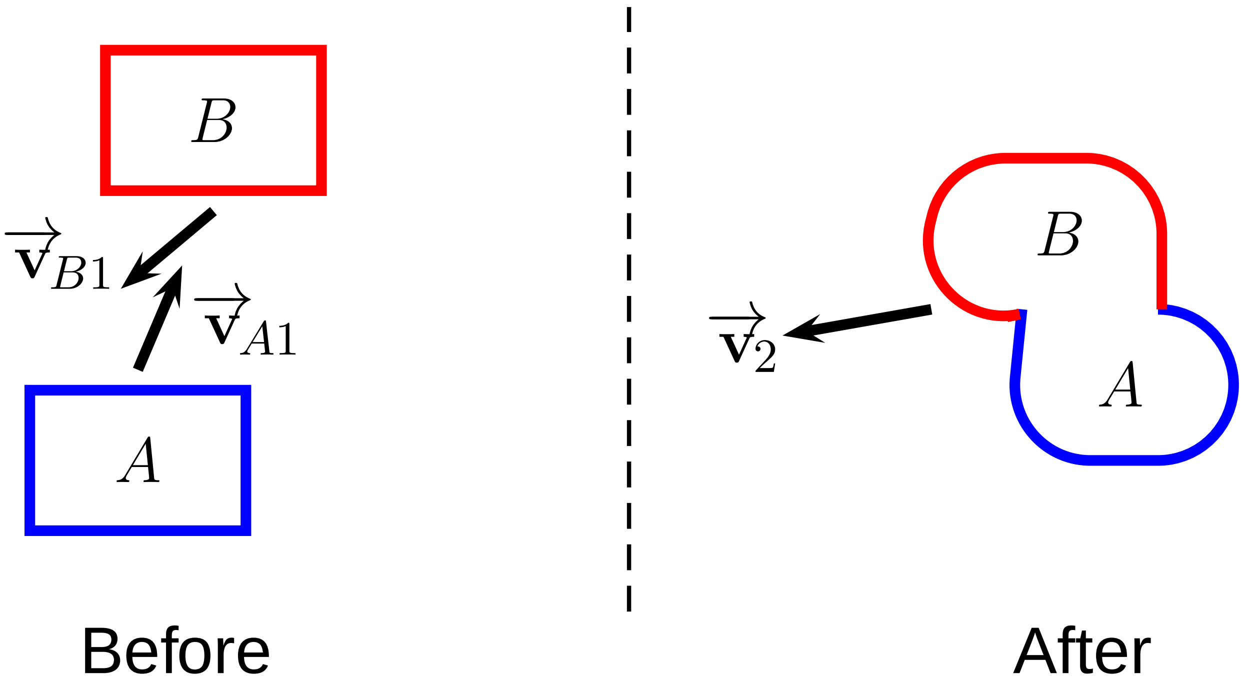

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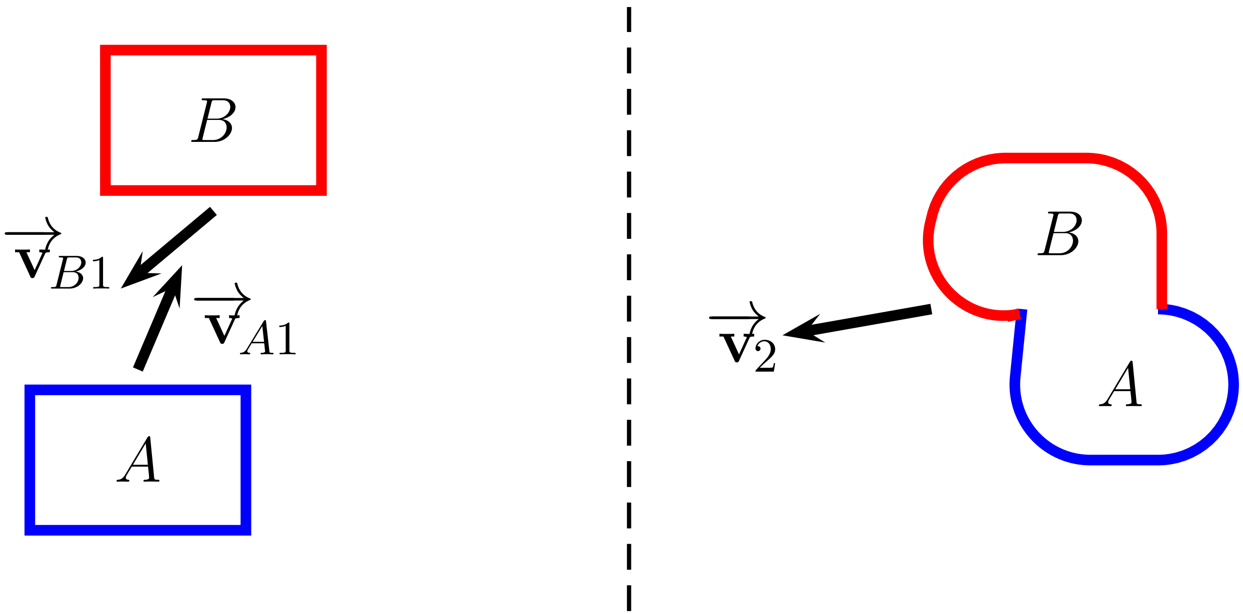

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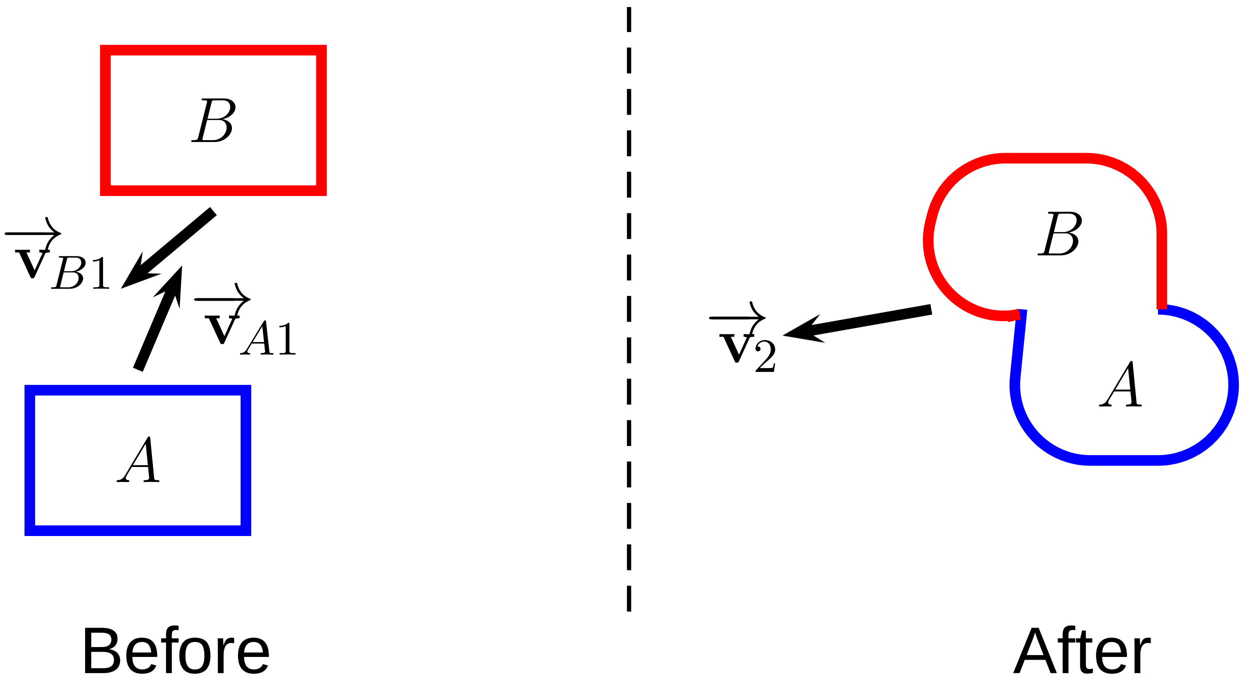

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Clicker Quiz

A block with $M_A = m$ and velocity $3\vec{\mathbf{v}}$ to the right has a completely inelastic collision with $M_B = 2m$ that is initially at rest. How fast must the masses be going the instant after their collision?

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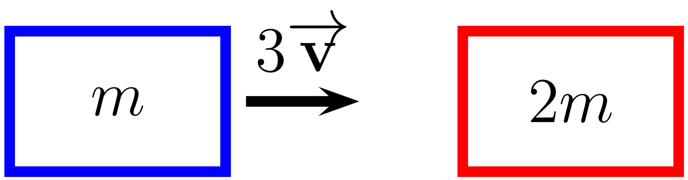

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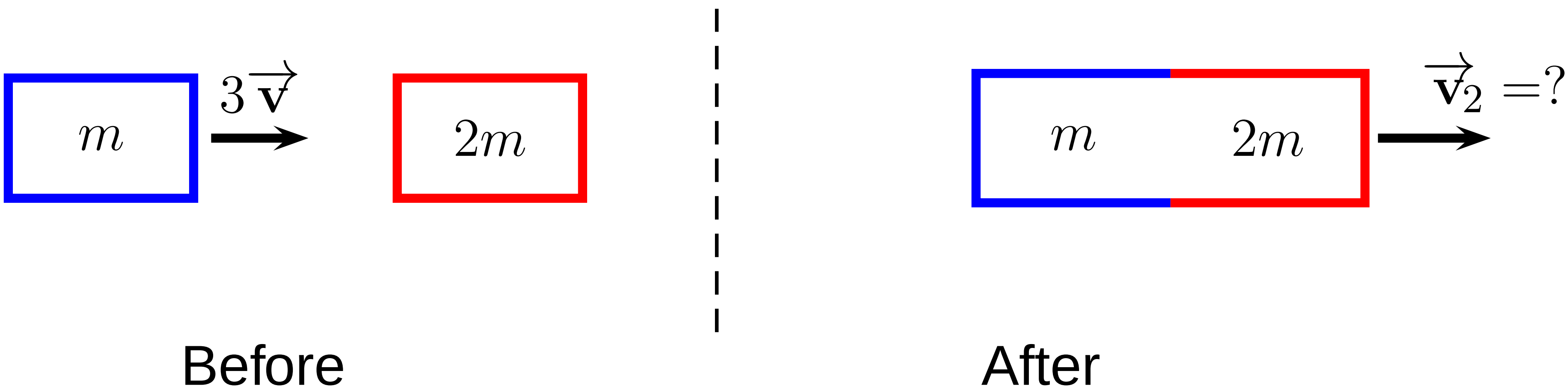

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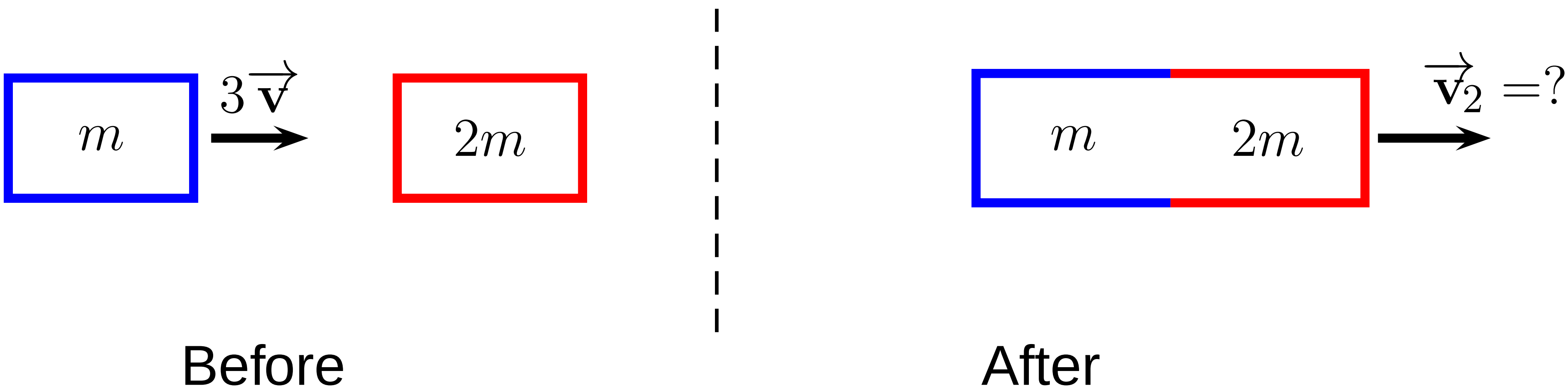

(a) $3v$

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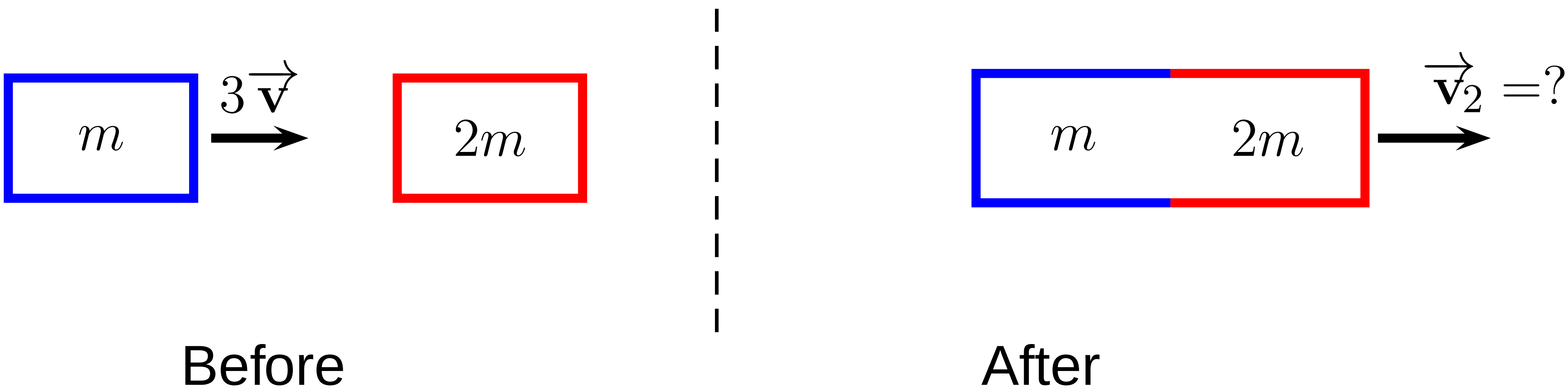

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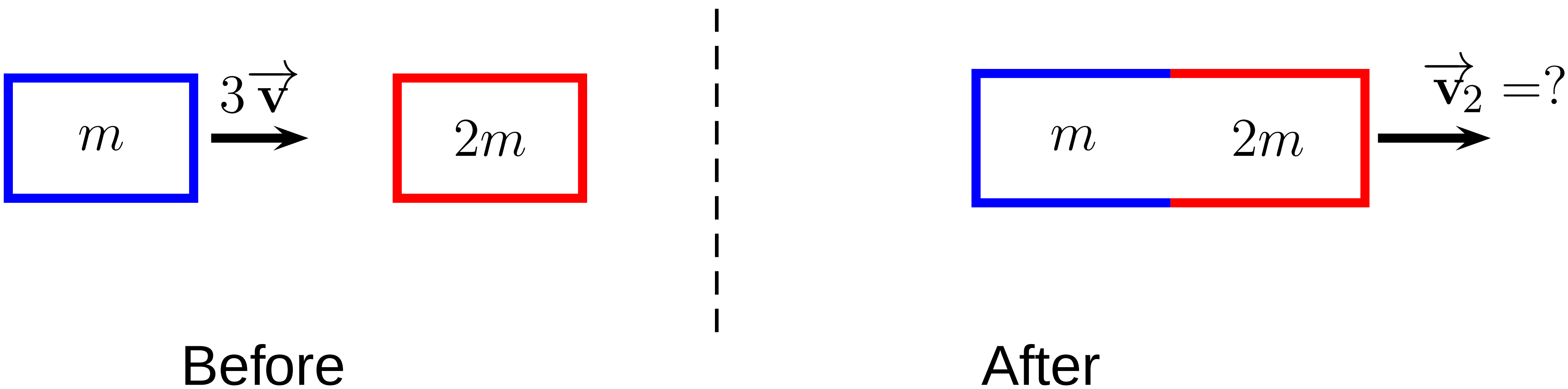

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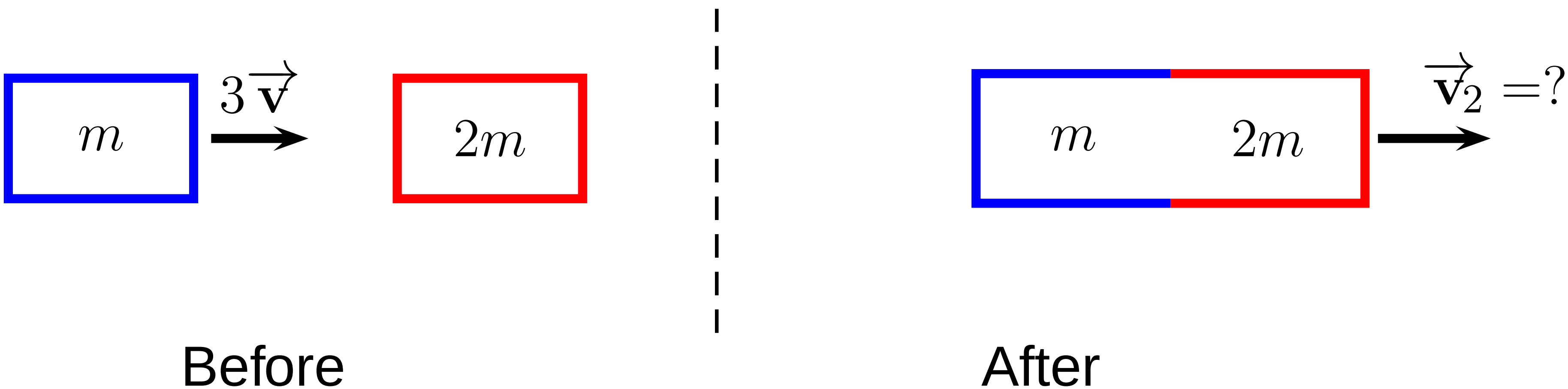

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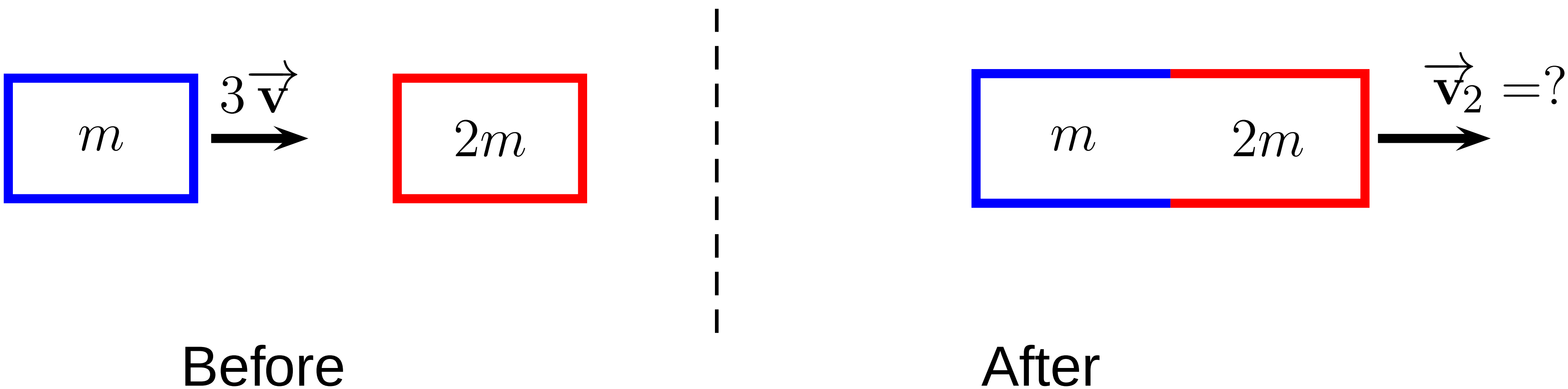

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Example

$$M_A \vec{\mathbf{v}}_{A1} + M_B \vec{\mathbf{v}}_{B1} = (M_A + M_B) \vec{\mathbf{v}}_2$$

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Example: A 2900-kg Hummer H_2 going East on Lomas Boulevard at $11\, m/s$ has a completely inelastic collision with a 730-kg smart-car going 25° East-of-North at $30\, m/s$ on Monte-Vista Boulevard. What is the speed and direction of the combination the instant after the collision?

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$$\frac{1}{2} M_A v_{A1}^2 + \frac{1}{2} M_B v_{B1}^2 = \frac{1}{2} M_A v_{A2}^2 + \frac{1}{2} M_B v_{B2}^2$$

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In 1D, the velocities have just one component $\Rightarrow \vec{\mathbf{v}}_{A1} = v_{A1}$, *etc.*

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Given M_A, M_B, v_{A1}, v_{B1} $\Rightarrow$ there are two unknowns (v_{A2} and v_{B2}) and two equations.

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Only one way to have an elastic collision in 1D!

1D Elastic Collisions Cont.

It can be shown (see textbook or lecture notes) that if a collision is elastic, the relative velocity, $v_A - v_B$, changes sign after the collision.

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1D Elastic Collisions Example

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Example: A 5-kg mass going $6\,m/s$ to the right has a one-dimensional elastic collision with a $30\,kg$ mass initially at rest. What are the speeds and direction of both afterwards?

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$$\frac{1}{2} M_A \left(v_{A1,x}^2 + v_{A1,y}^2\right) + \frac{1}{2} M_B \left(v_{B1,x}^2 + v_{B1,y}^2\right) = \frac{1}{2} M_A \left(v_{A2,x}^2 + v_{A2,y}^2\right) + \frac{1}{2} M_A \left(v_{B2,x}^2 + v_{B2,y}^2\right)$$

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Infinitely many ways to have a 2D elastic collision

2D Example

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Example: A 2900-kg Hummer H_2 going East on Lomas Boulevard at $11\,m/s$ has an elastic collision with a 730-kg smart-car going 25° East-of-North at $30\,m/s$ on Monte-Vista Boulevard. If the Hummer has a speed of $9\,m/s$ afterwards, what is the speed of the smart-car and direction for both, the instant after the collision?