March 7, Week 8

Today: Chapter 6, Work

If interested in Physics 110, please see me after lecture.

Exam 3: Friday, March 9

Review Session: Thursday, March 8, 7:30PM in Room 114 of Regener Hall

Solution for practice exam now available on website

Practice Problems on Mastering Physics

Review

For constant force and straight-line displacement:

$$\boxed{W = \overrightarrow{\mathbf{F}} \cdot \overrightarrow{\mathbf{s}}}$$

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Negative work causes an object to slow down

or, by Newton's Third Law:

$$\boxed{W_{\text{Done to object}} = -W_{\text{Done by object}}}$$

Component Dot Product

The dot product can also be written in terms of the components of the individual vectors.

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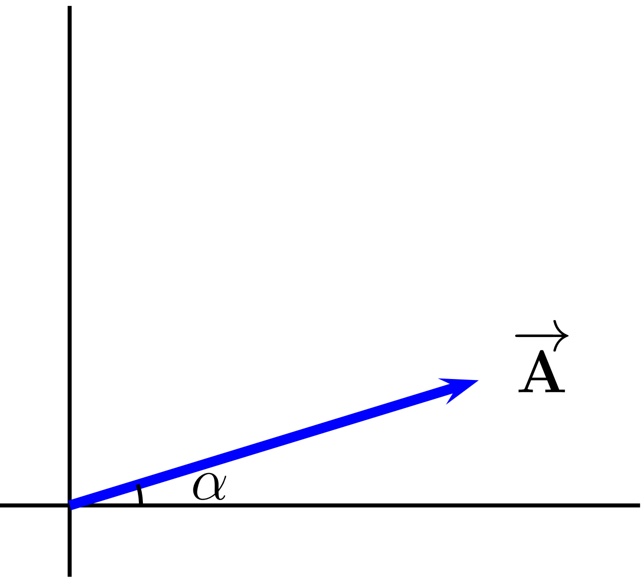

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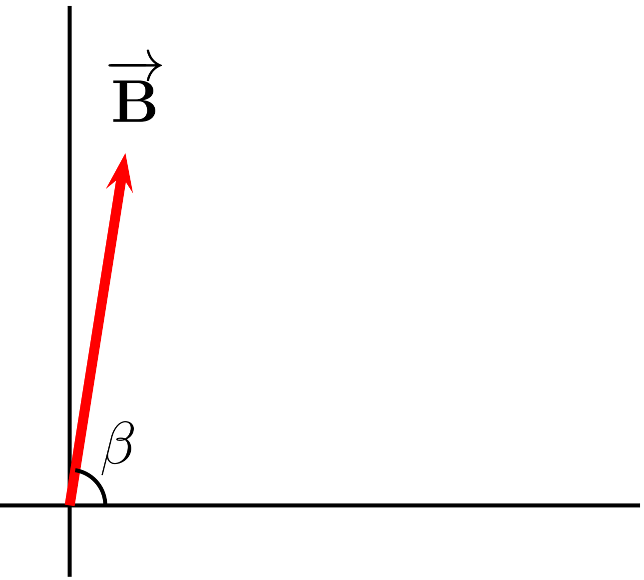

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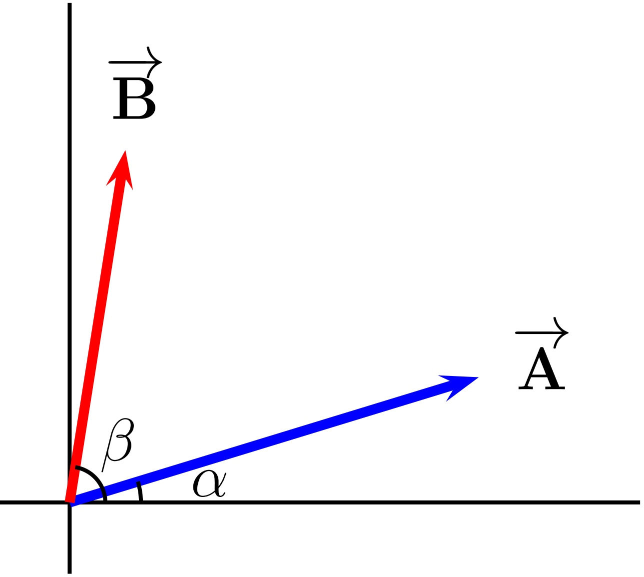

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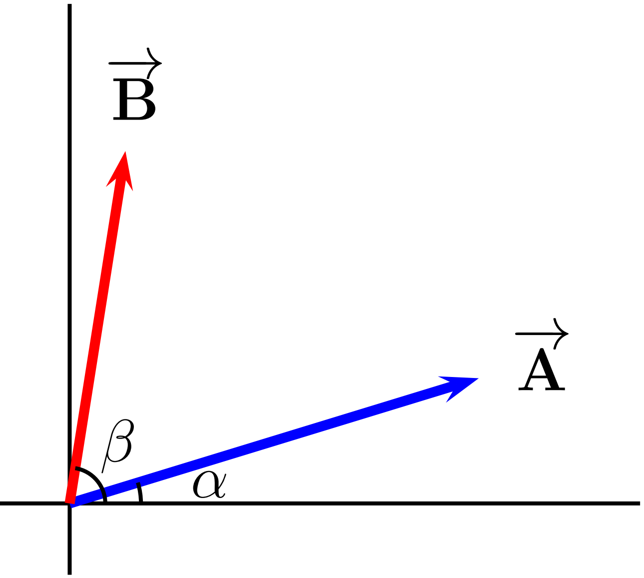

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi$$

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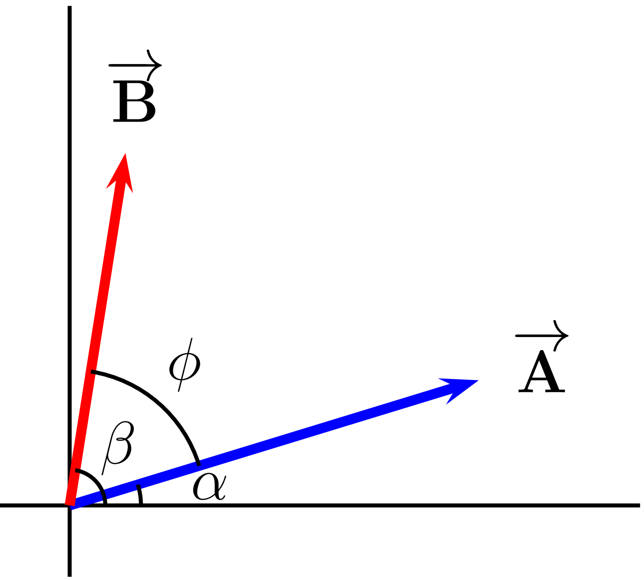

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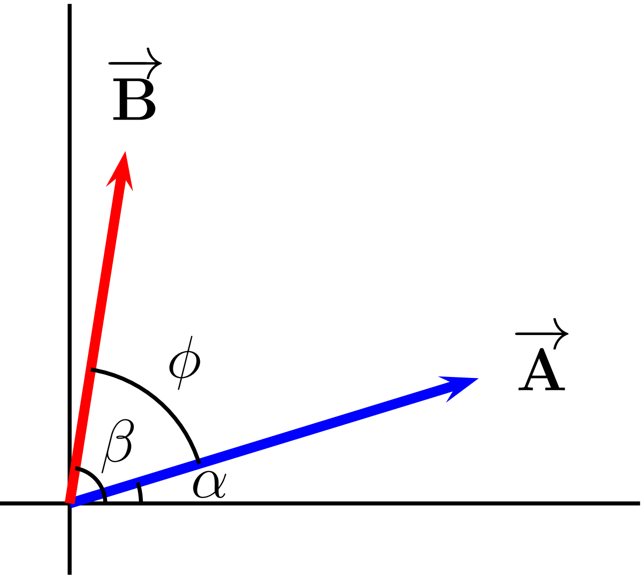

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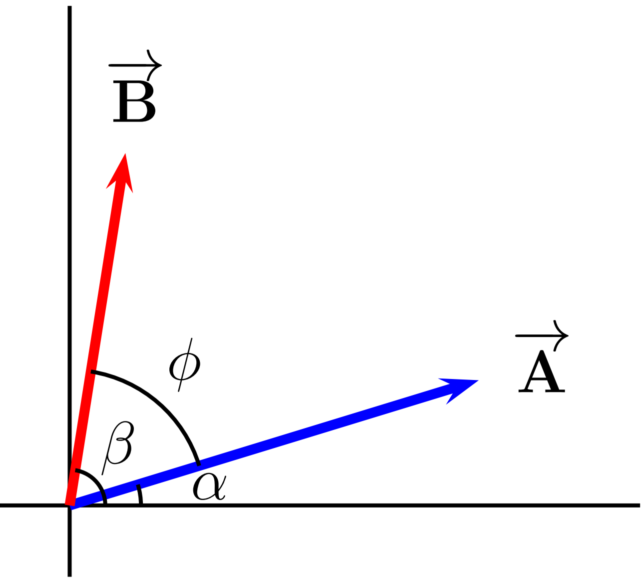

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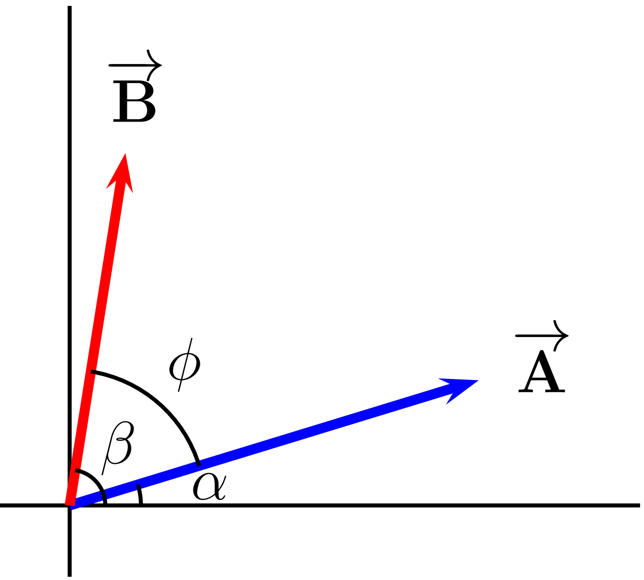

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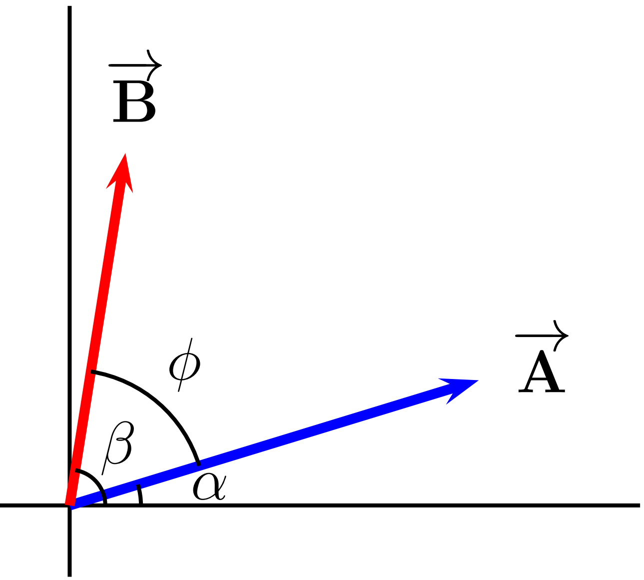

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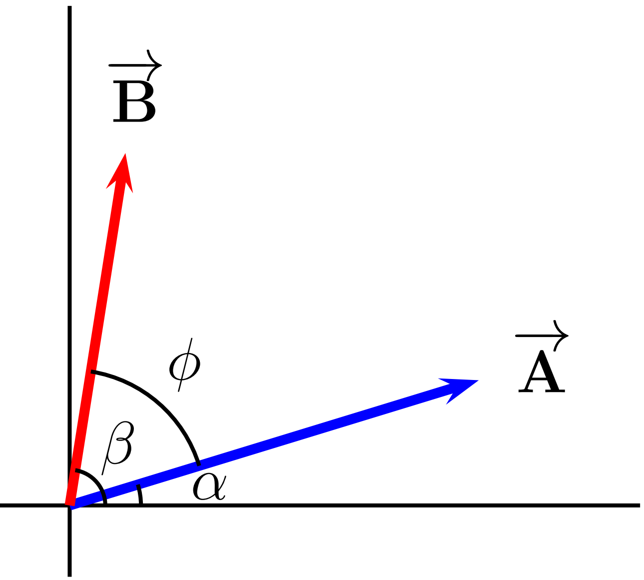

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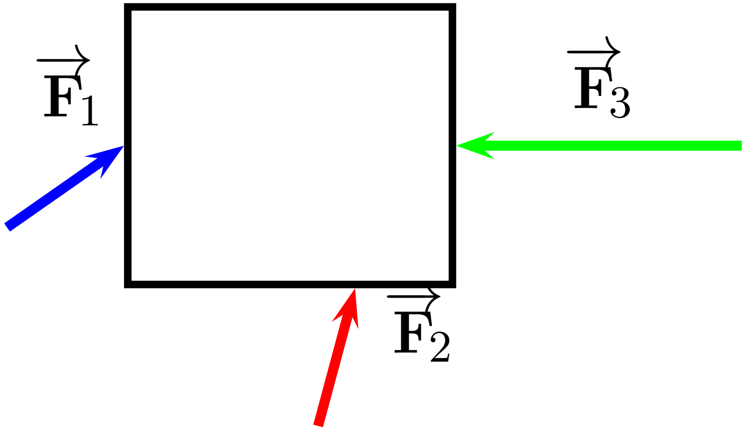
$$\boxed{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y}$$

Total Work

Since work is a scalar, the total work done by a collection of forces is given by the sum of the individual works.



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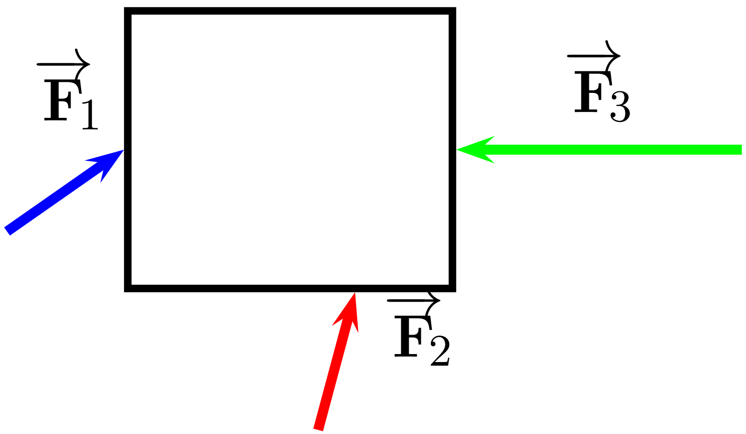

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Example: What is the total work down by forces $\overrightarrow{\mathbf{F}}_1 = 50\,N$ at $23°$ and $\overrightarrow{\mathbf{F}}_2 = 75\,N$ at $140°$ if $\overrightarrow{\mathbf{s}} = 5\,m$ at $195°$?

Work-Energy Theorem

Work-Energy Theorem - Allows us to calculate the physical effect that work has on an object. Generally, it says that work causes a change in speed.

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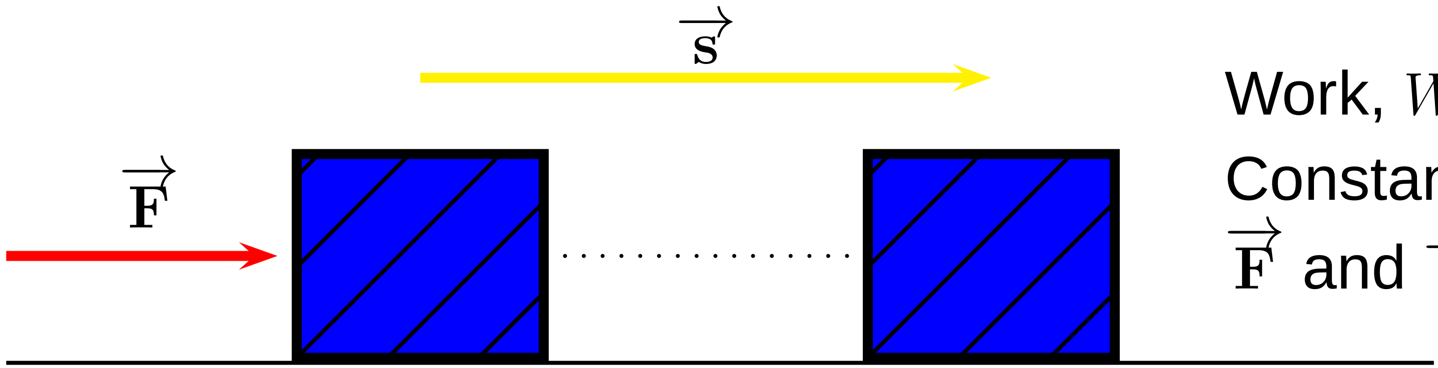

Work, $W = Fs$
Constant F
$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel

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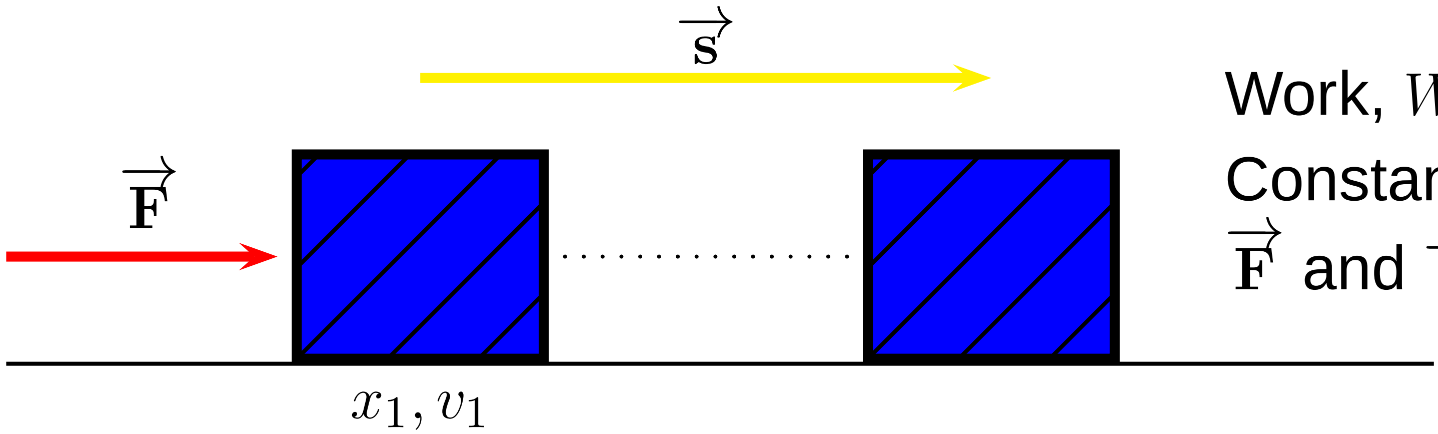

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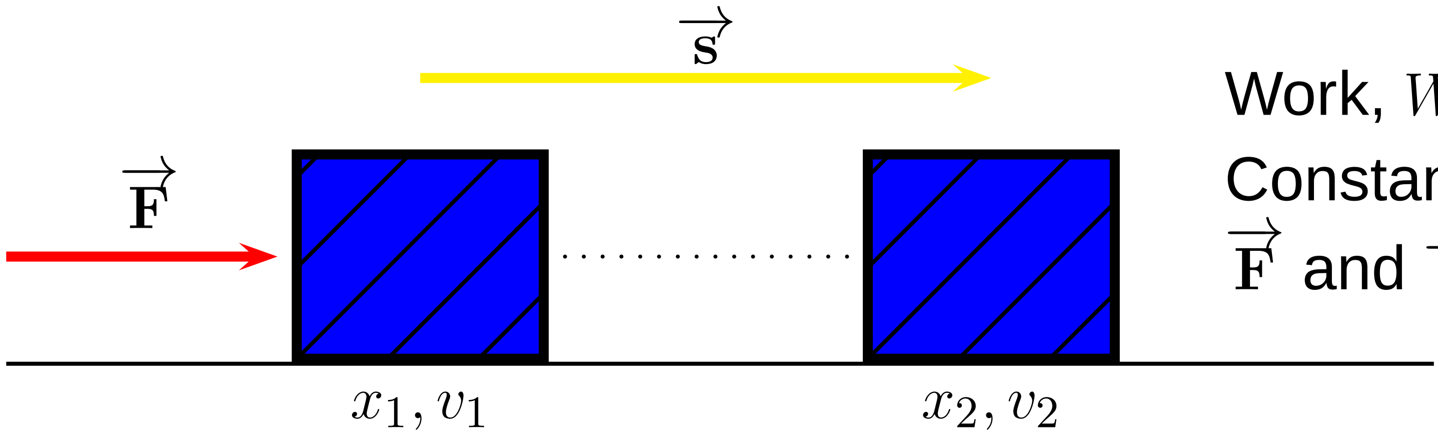

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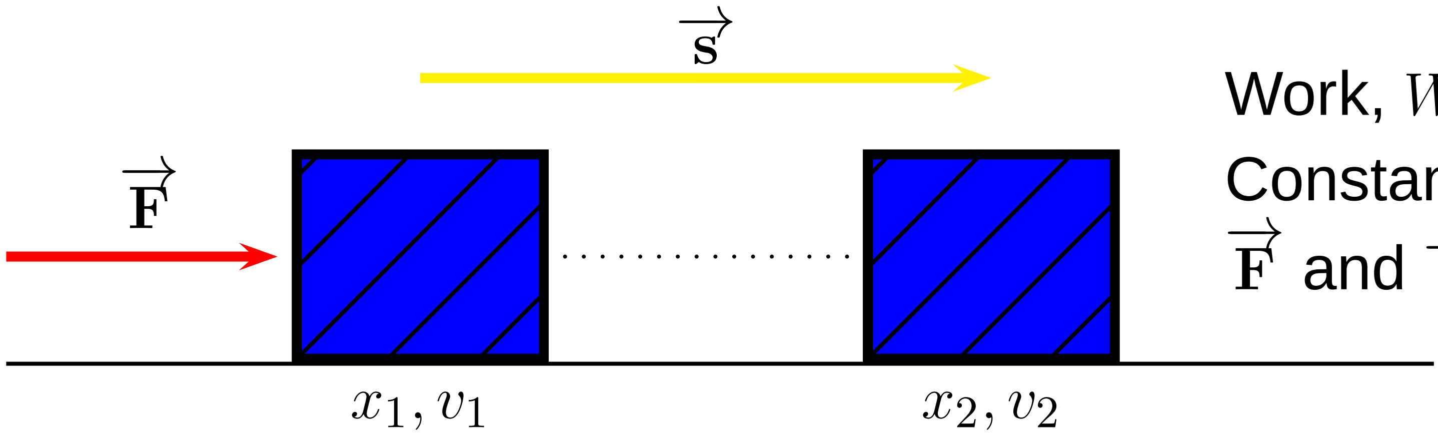

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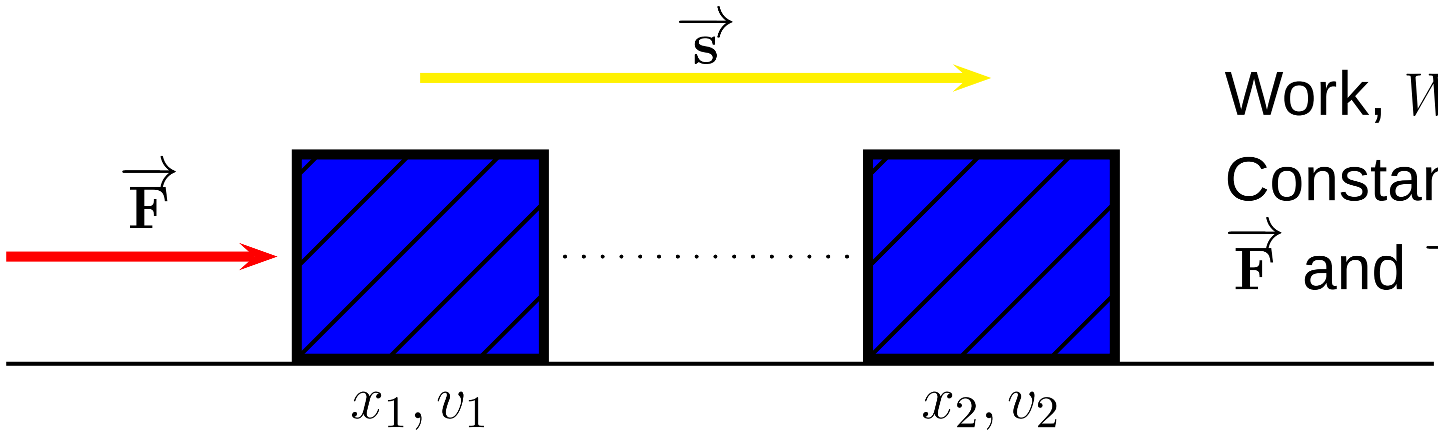

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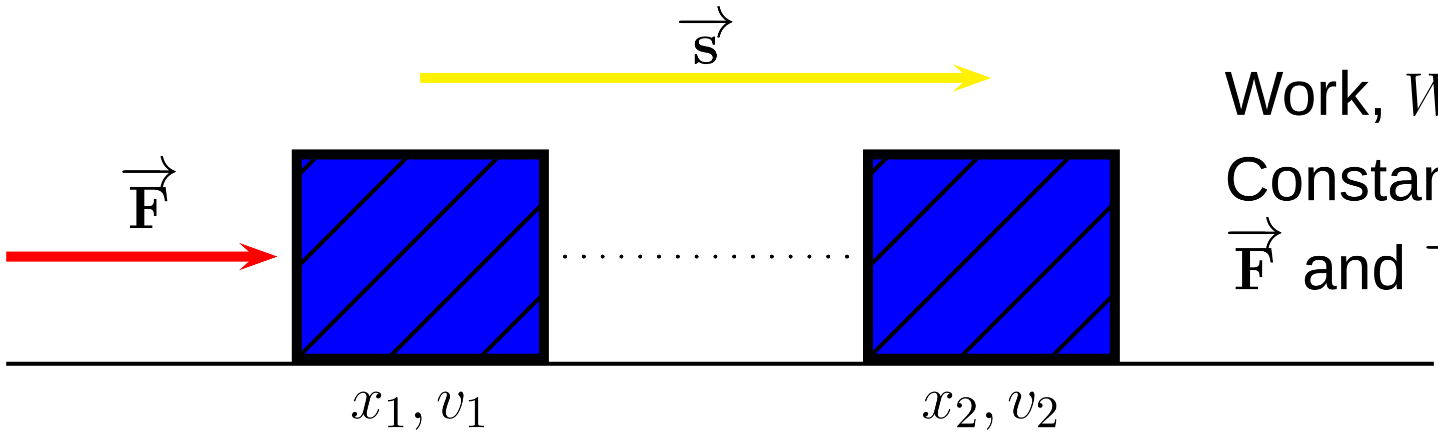

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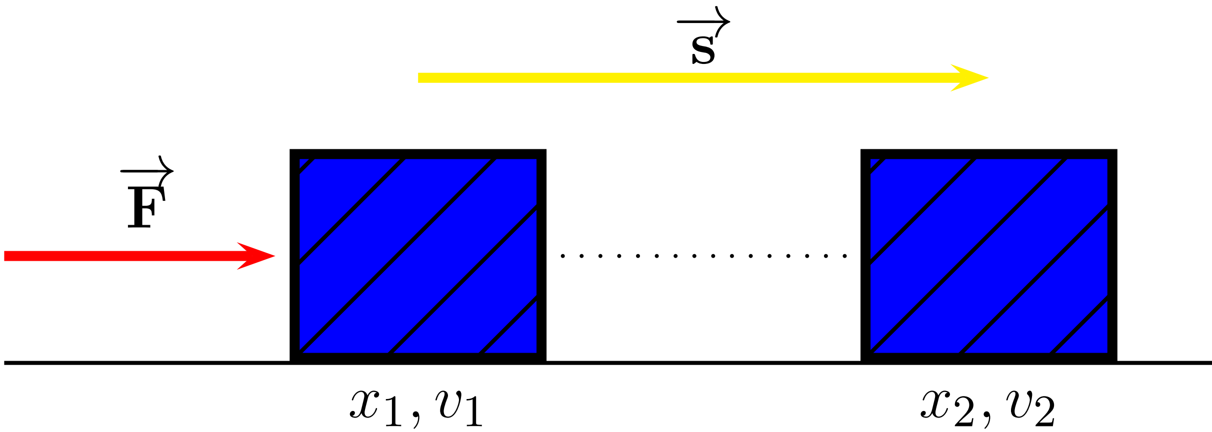

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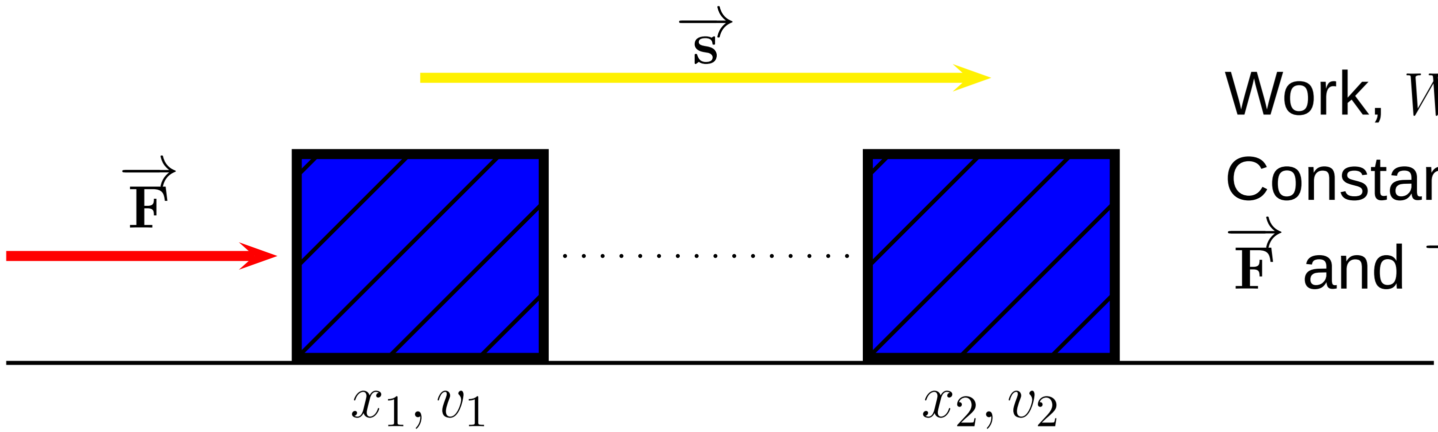

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K = Kinetic Energy = Energy of motion. *Note:* $v^2 = v_x^2 + v_y^2$ in two-dimensions.

Example

For multiple forces:

$$W_{total} = \Delta K = \frac{1}{2}Mv_2^2 - \frac{1}{2}Mv_1^2$$

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Example: A $5\,kg$ mass is moving with $\vec{\mathbf{v}}_1 = 7\,m/s$ at $180°$. If a total of $-33\,J$ of work is done by forces acting on it, what is its speed and direction after?

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To find the work done by a changing force requires integration.

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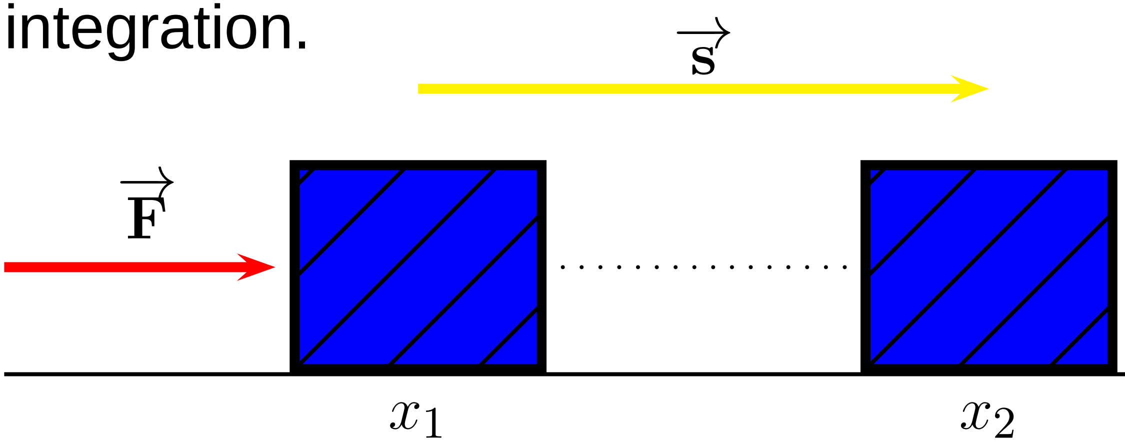

$$W = Fs = F(x_2 - x_1)$$

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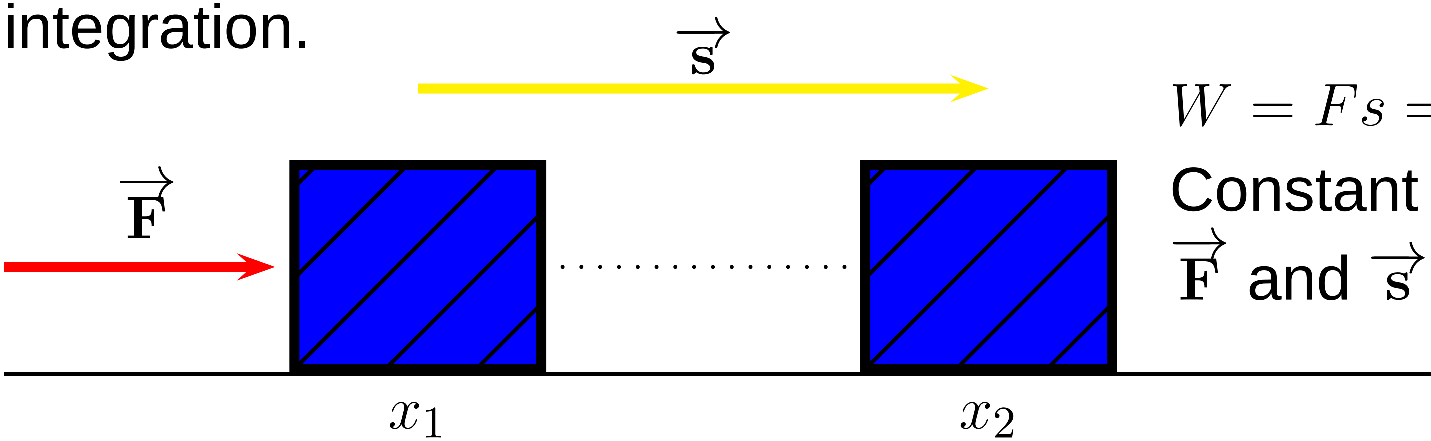

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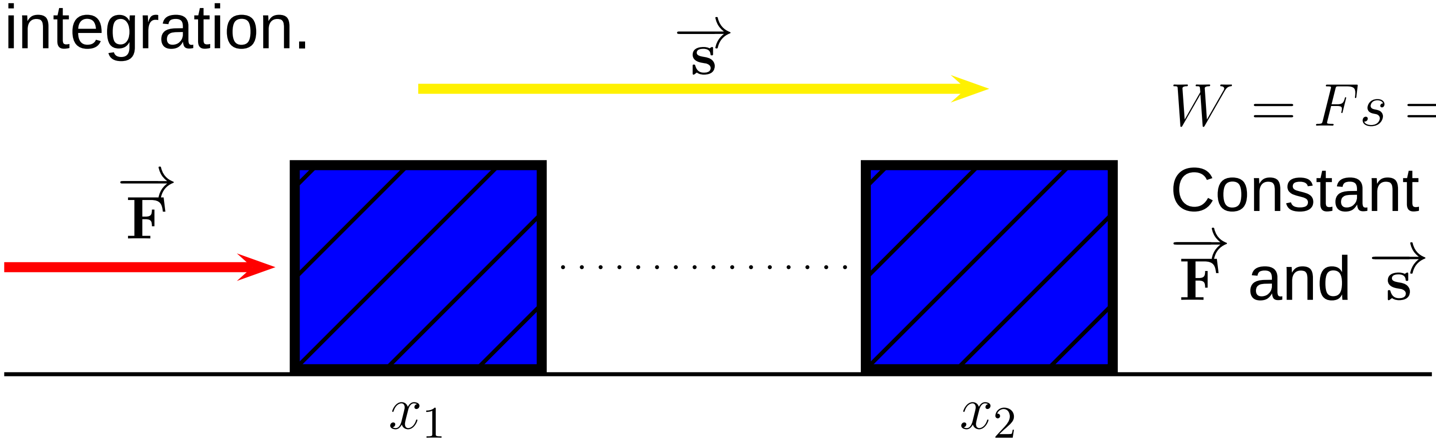

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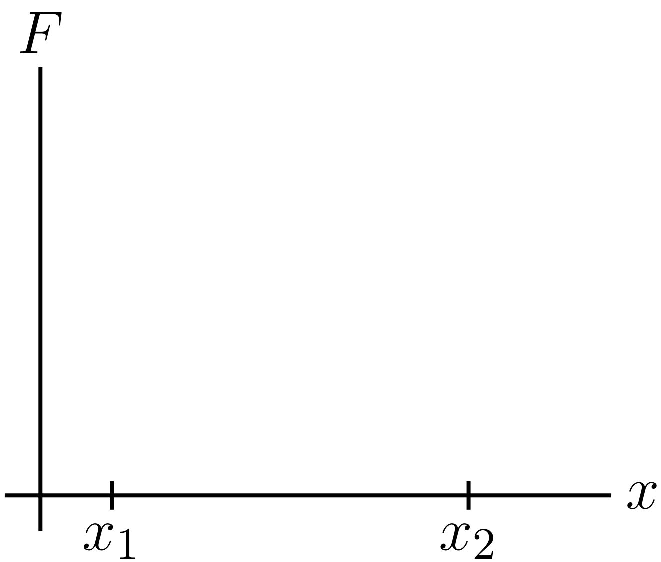

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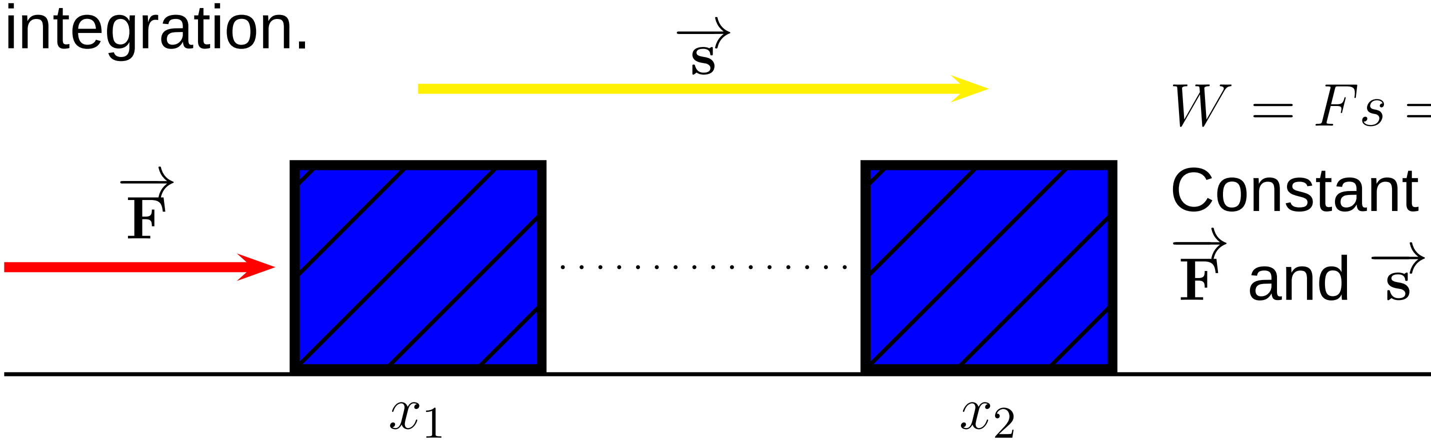

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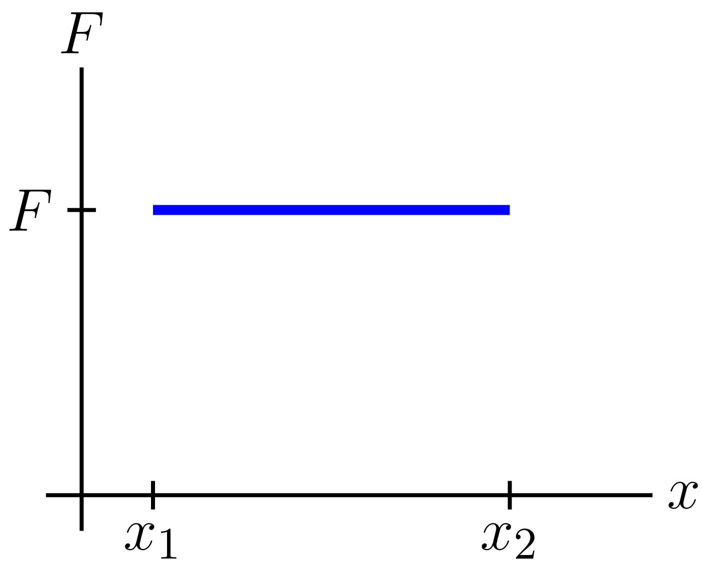

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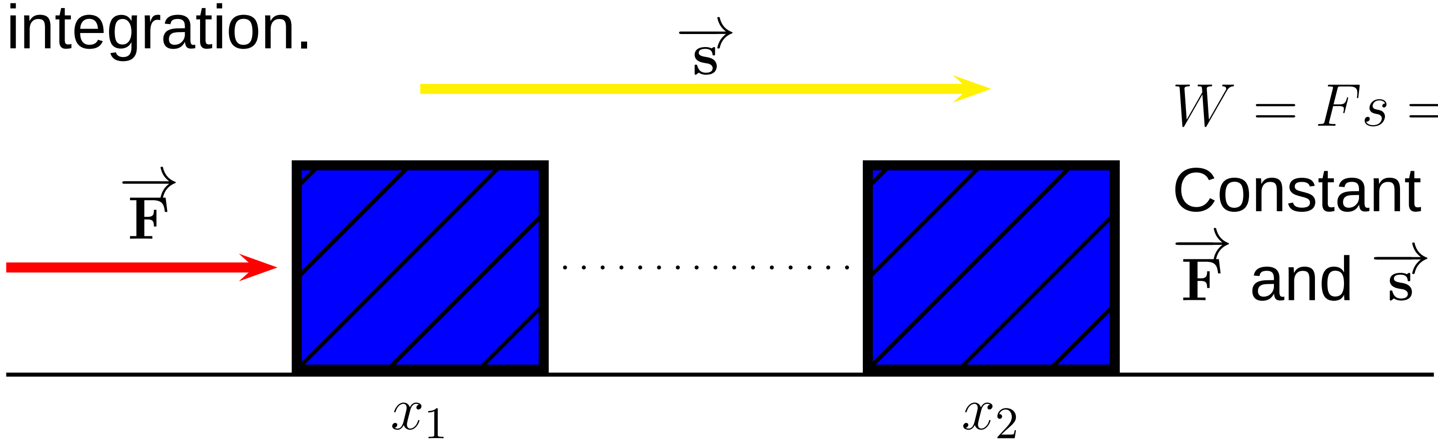

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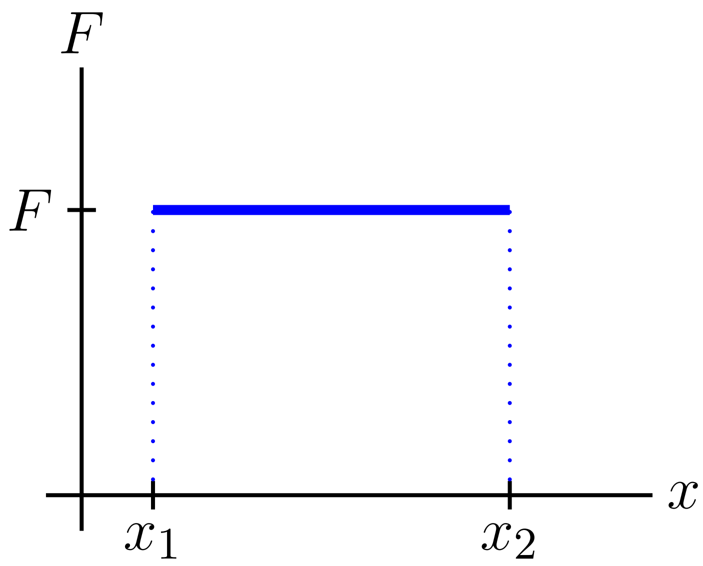

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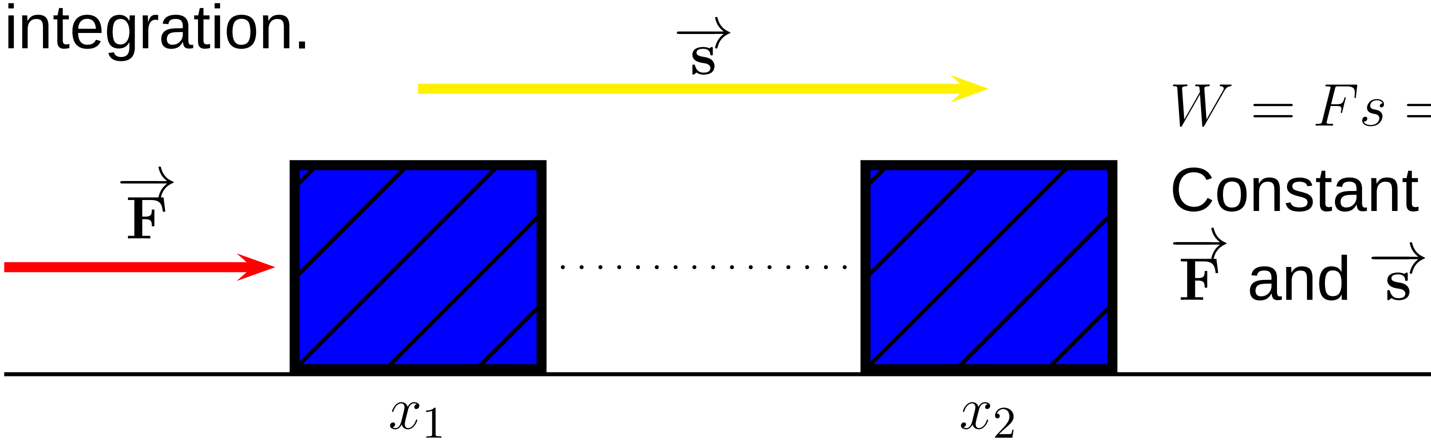

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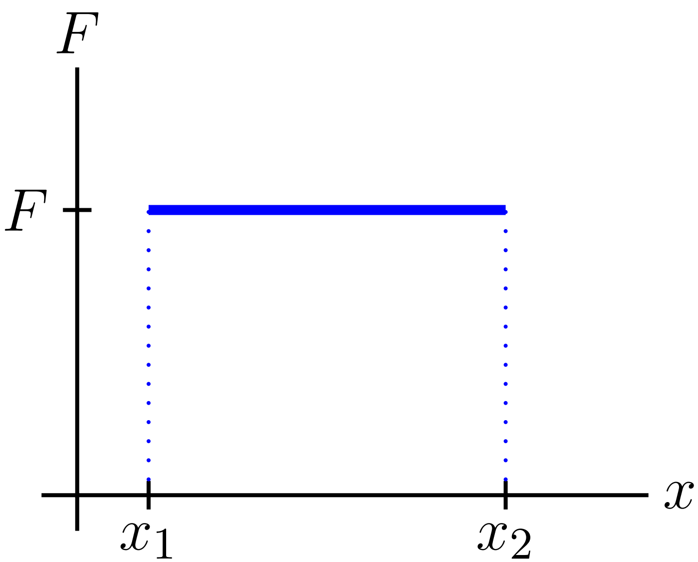

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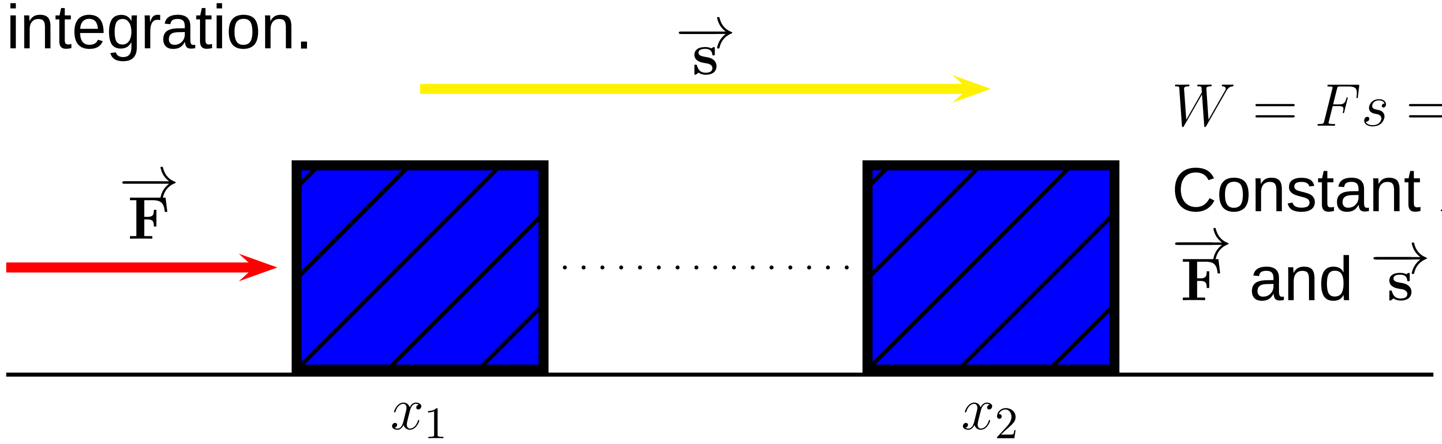

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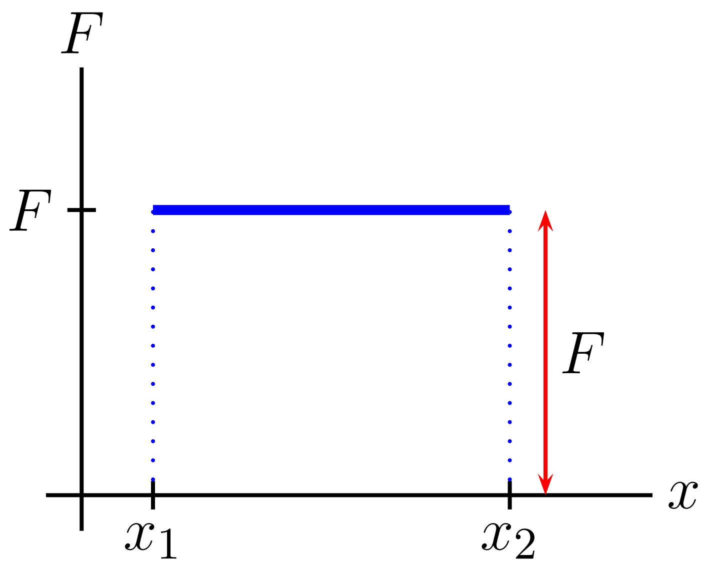

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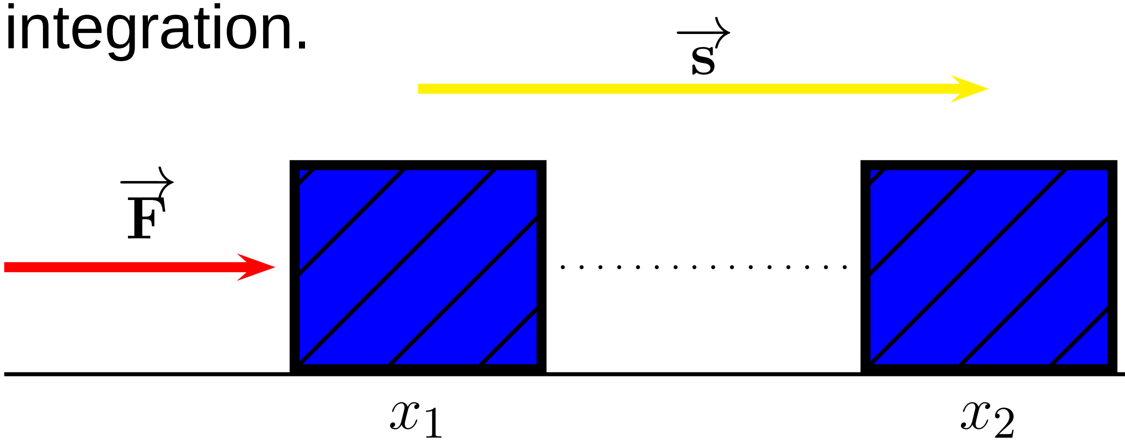

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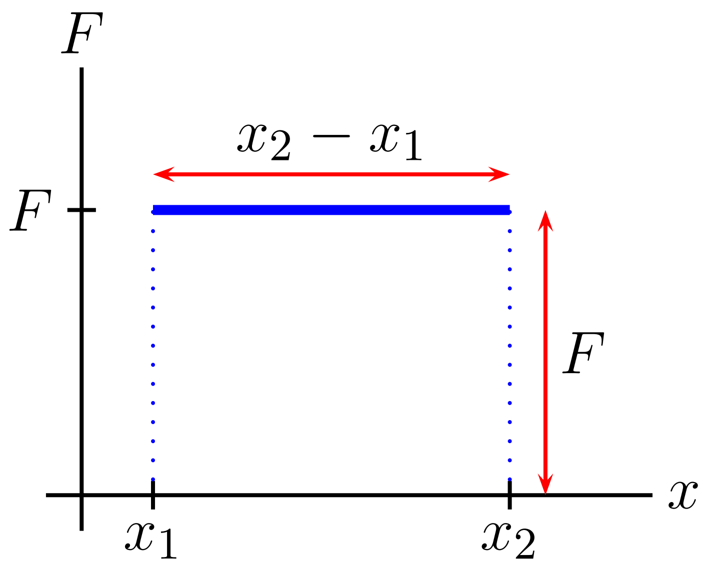

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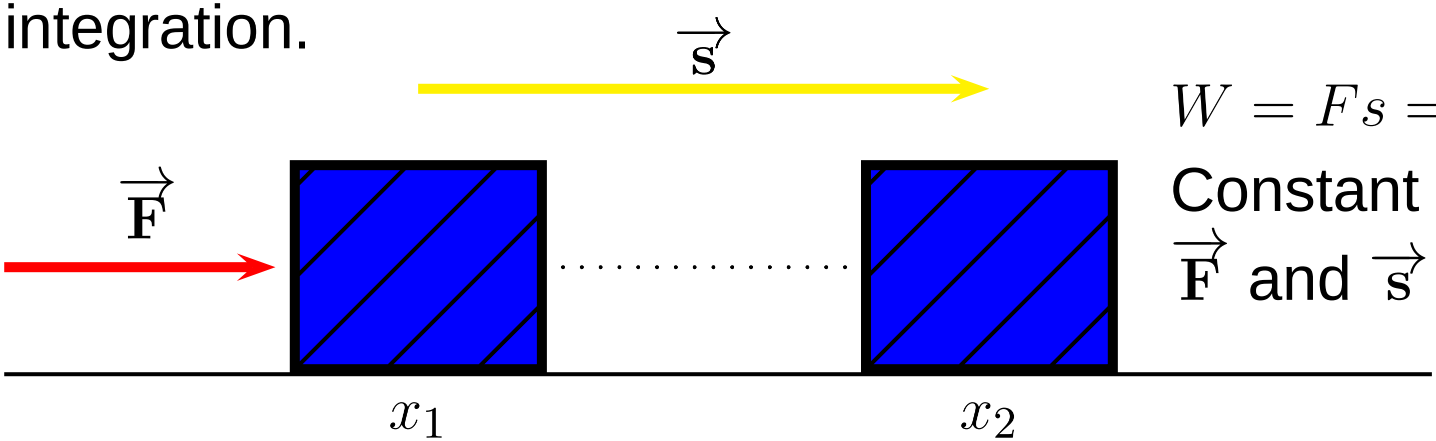

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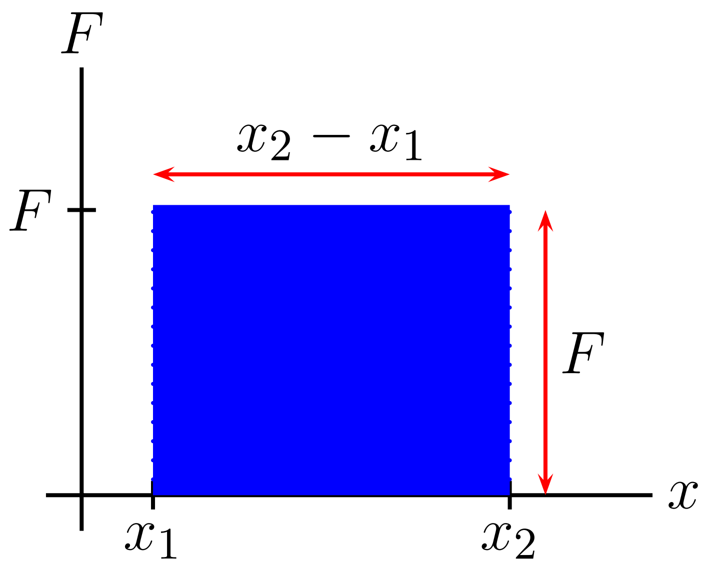

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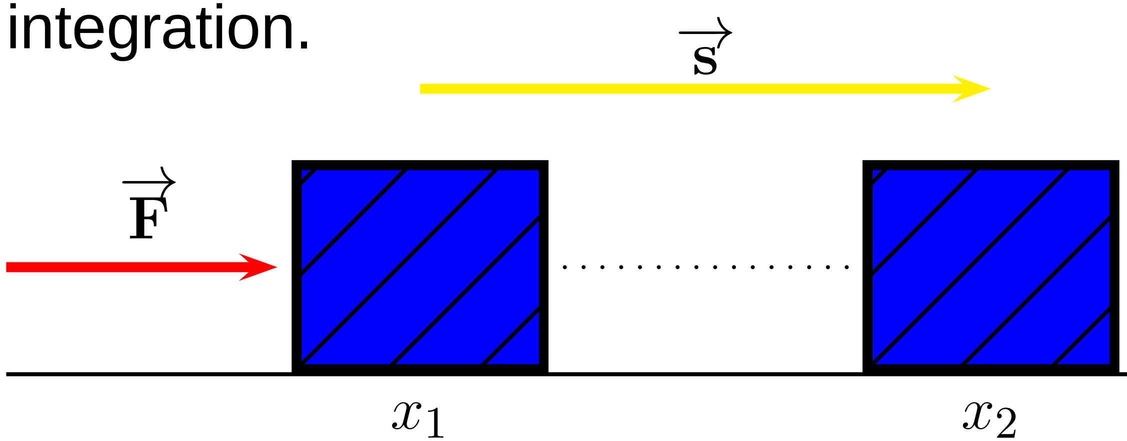

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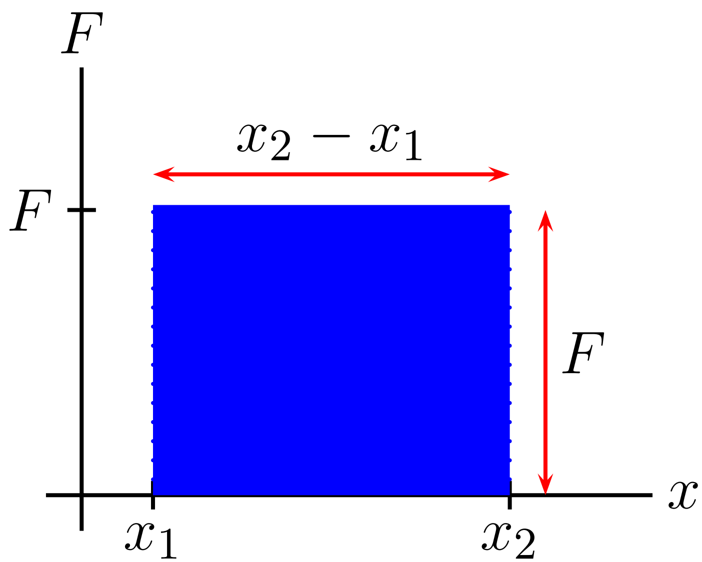

$$W = F\,(x_2 - x_1)$$

Work is the area under the curve

Variable Forces II

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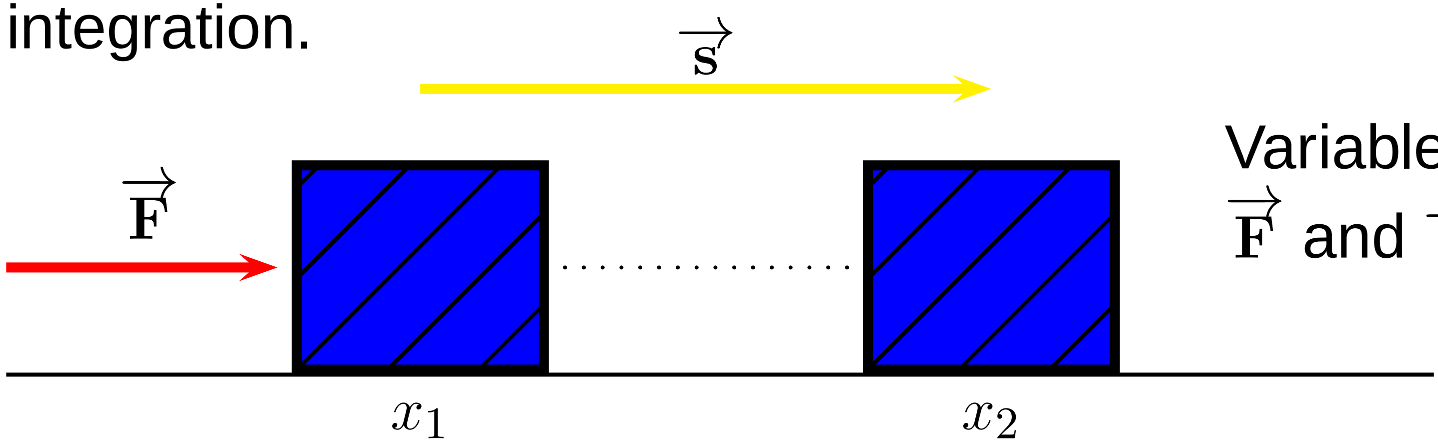

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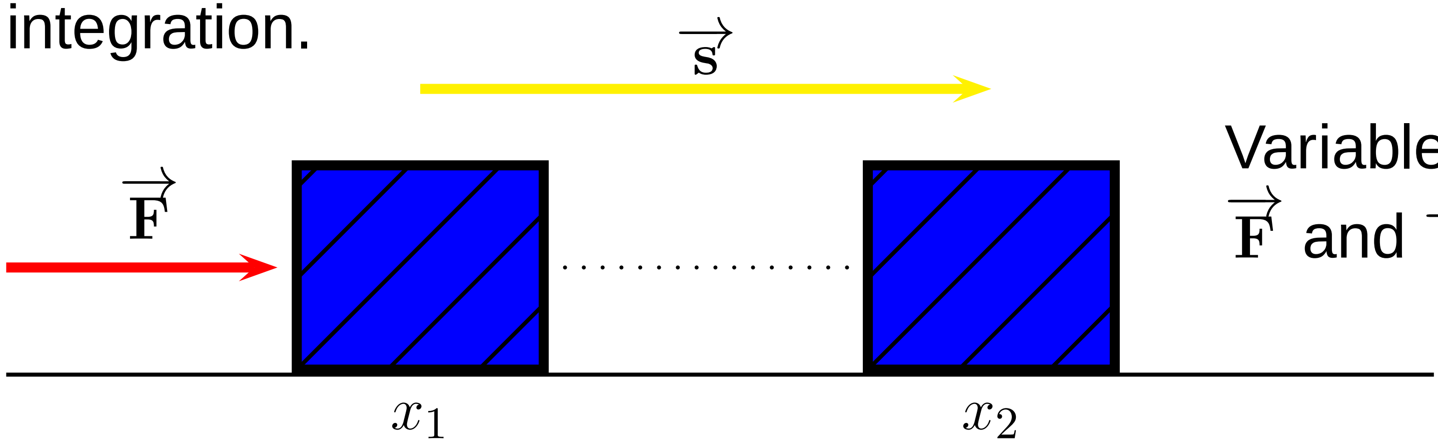

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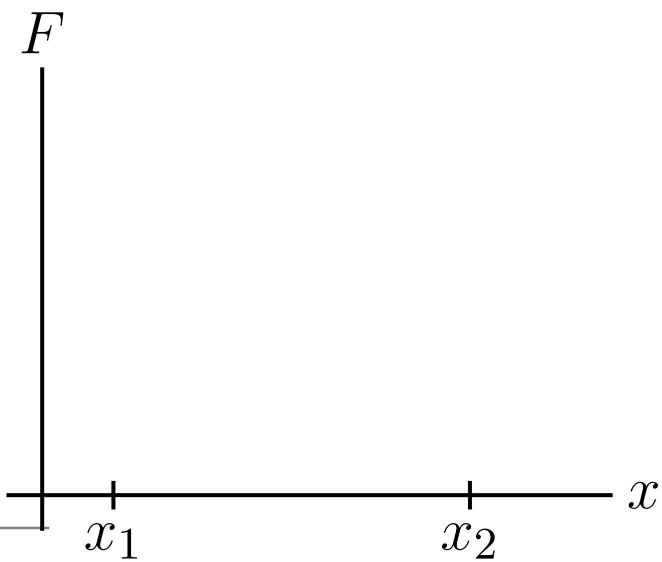

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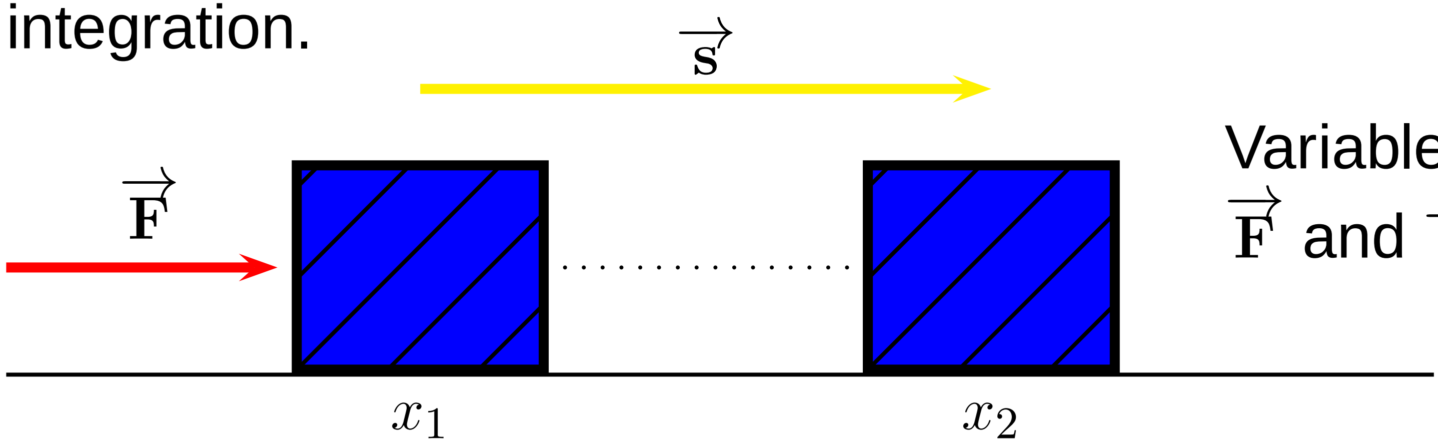

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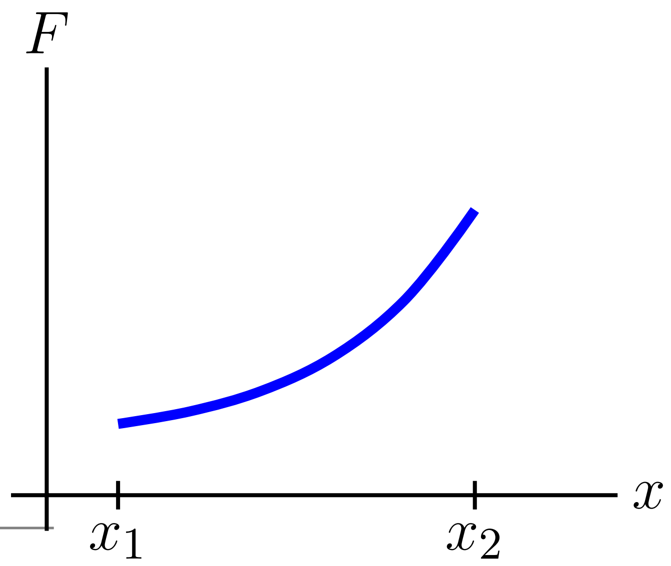
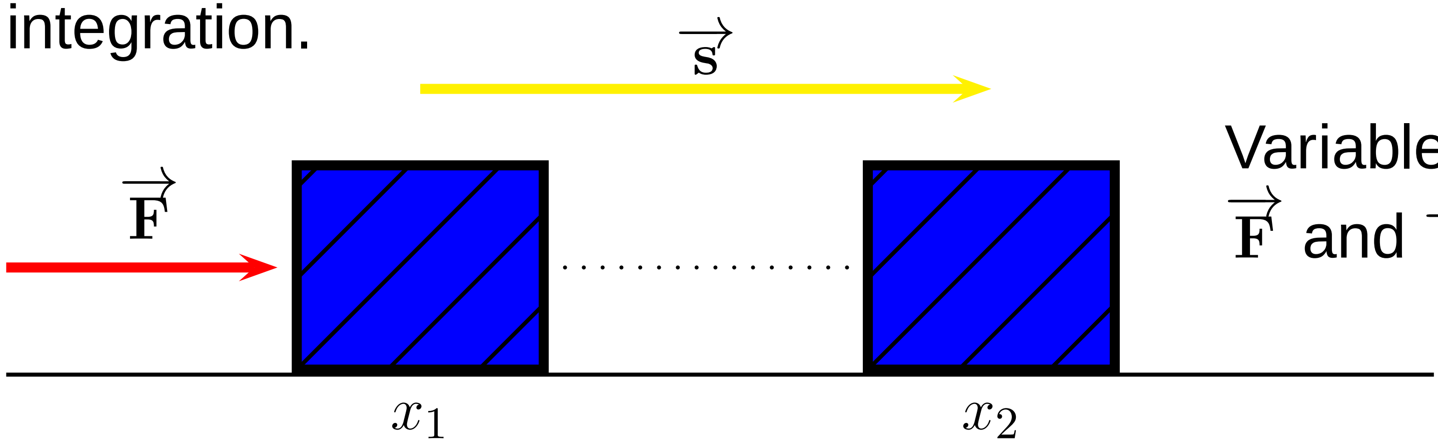


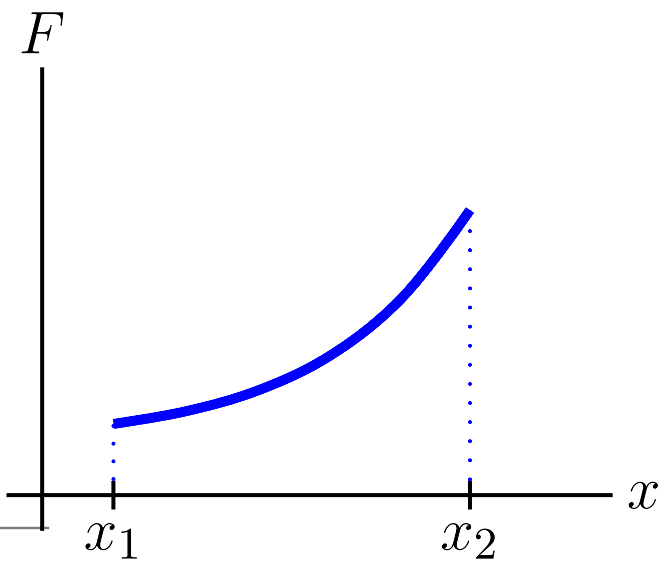
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

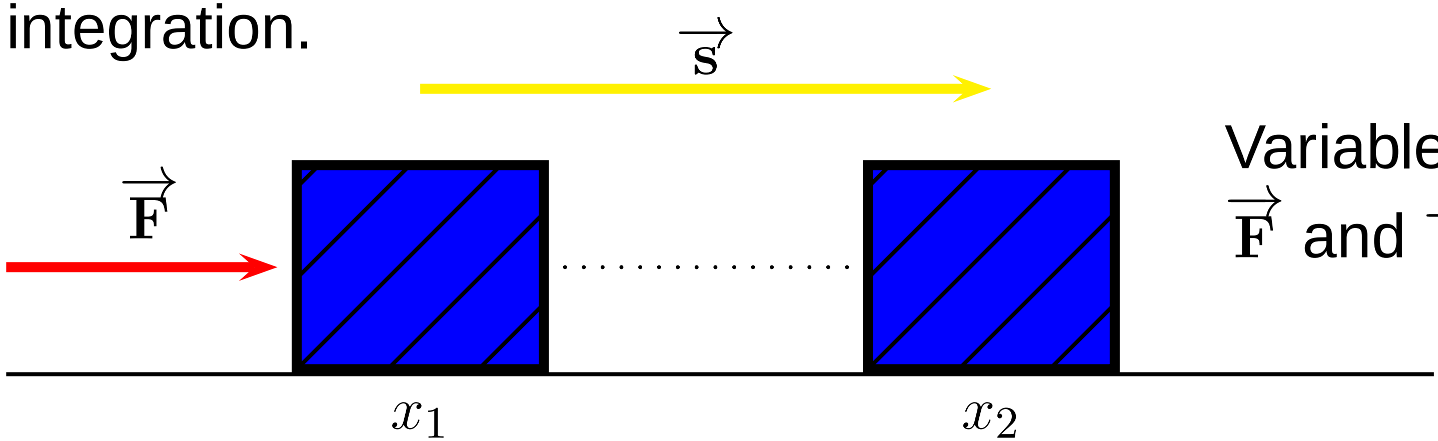
$\vec{F}$ and $\vec{s}$ parallel



Work is the area under this curve

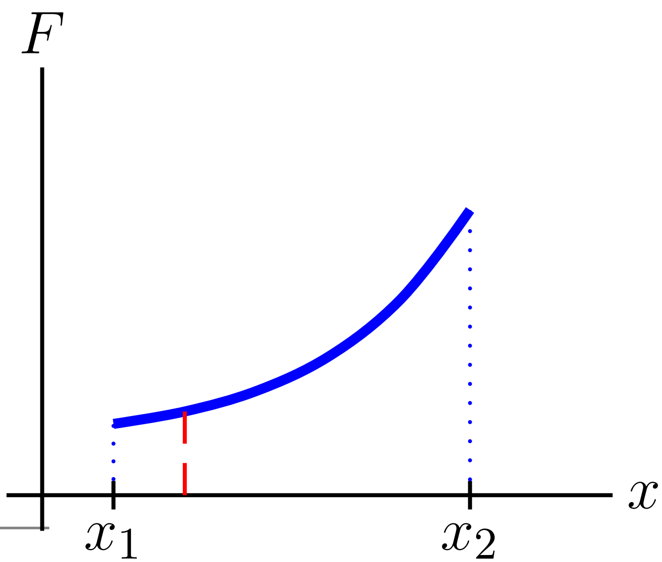
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{F}$ and $\vec{s}$ parallel

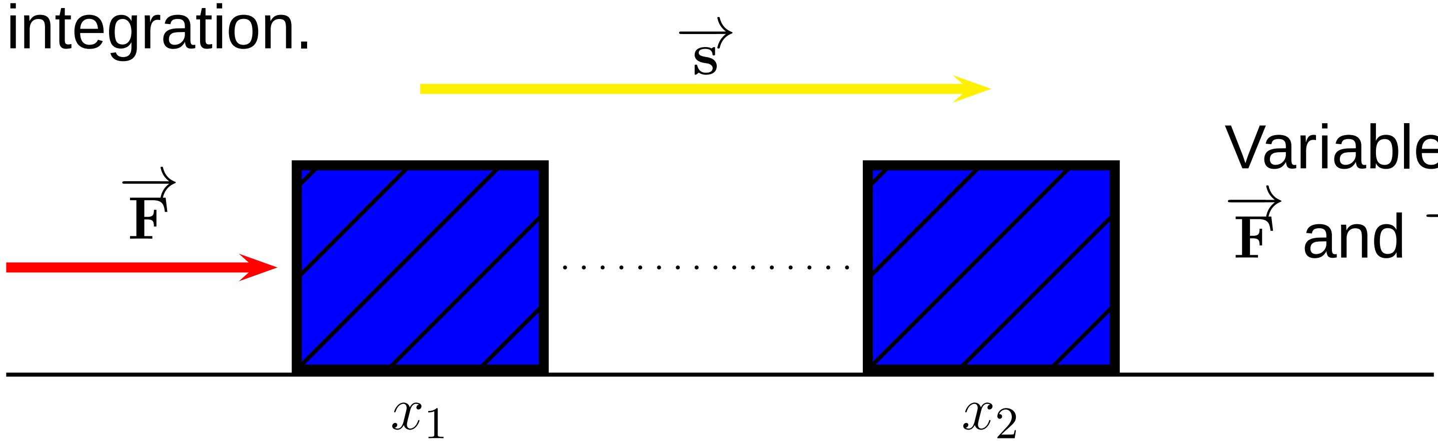


Work is the area under this curve

Split region into many small rectangles.

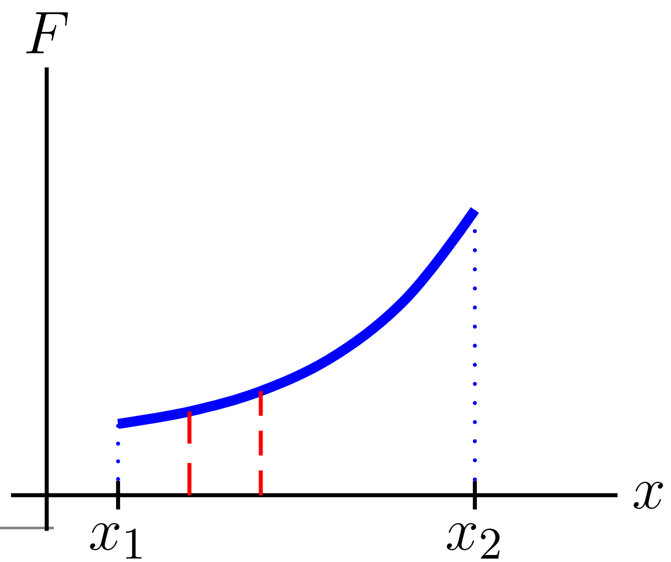
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



Work is the area under this curve

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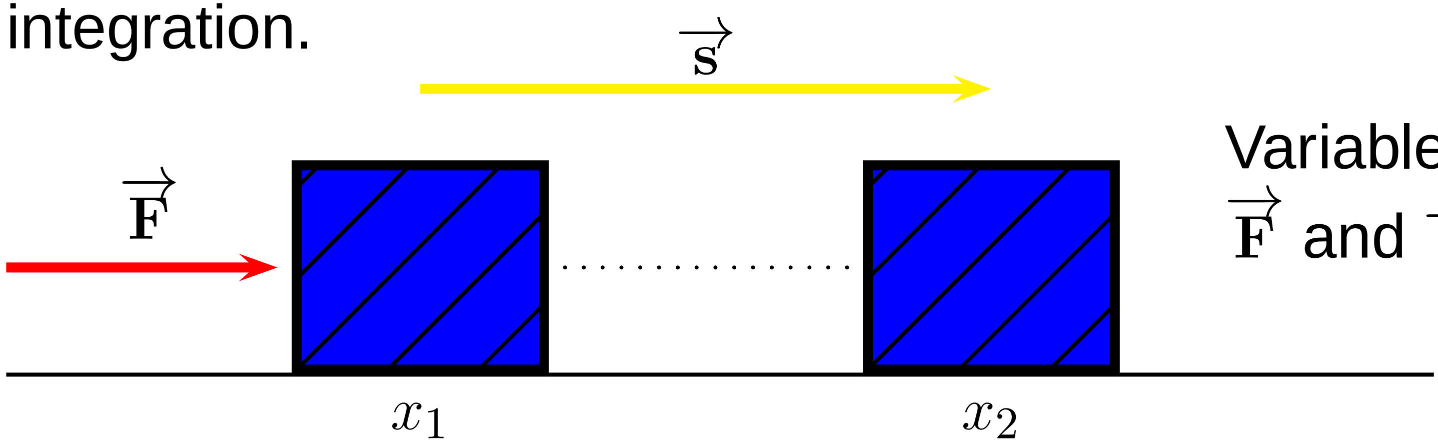
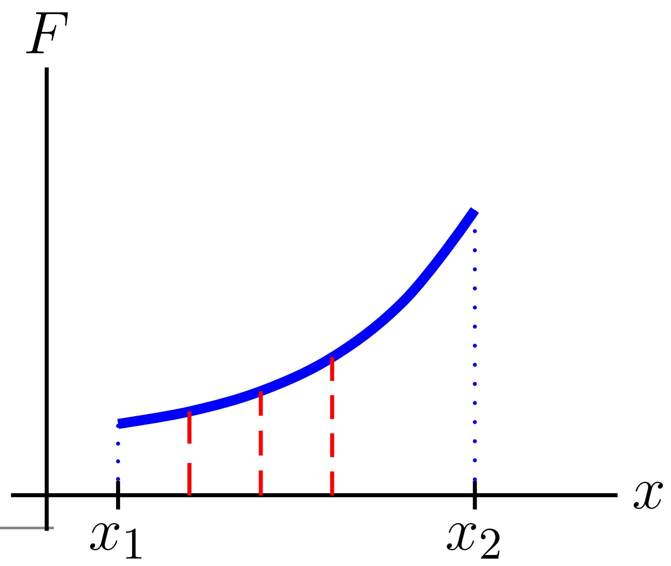
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{F}$ and $\vec{s}$ parallel

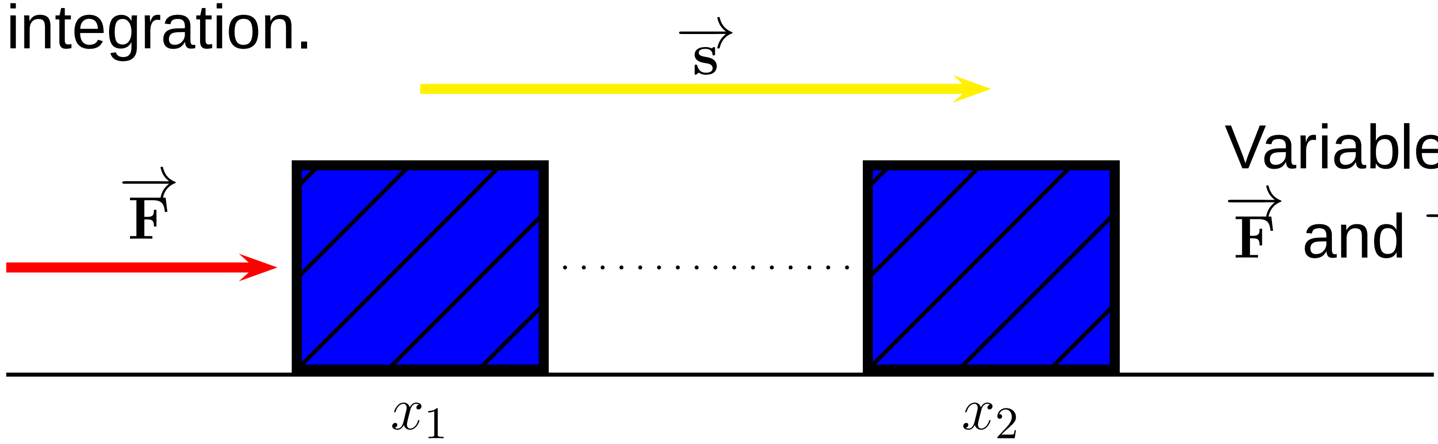


Work is the area under this curve

Split region into many small rectangles.

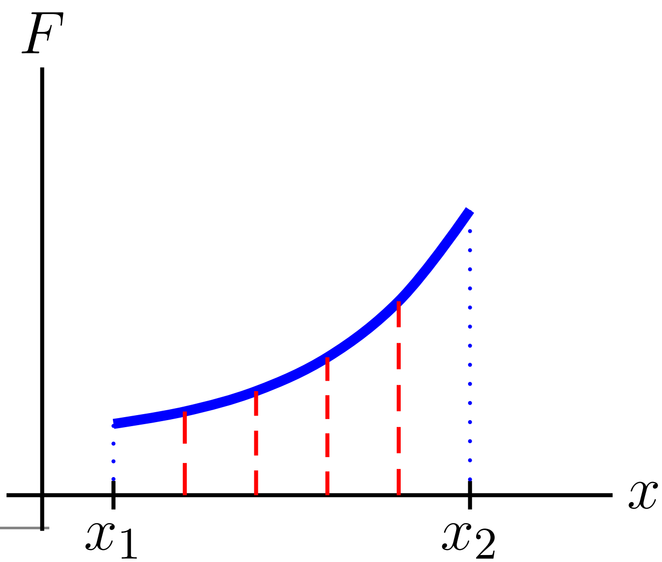
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{F}$ and $\vec{s}$ parallel

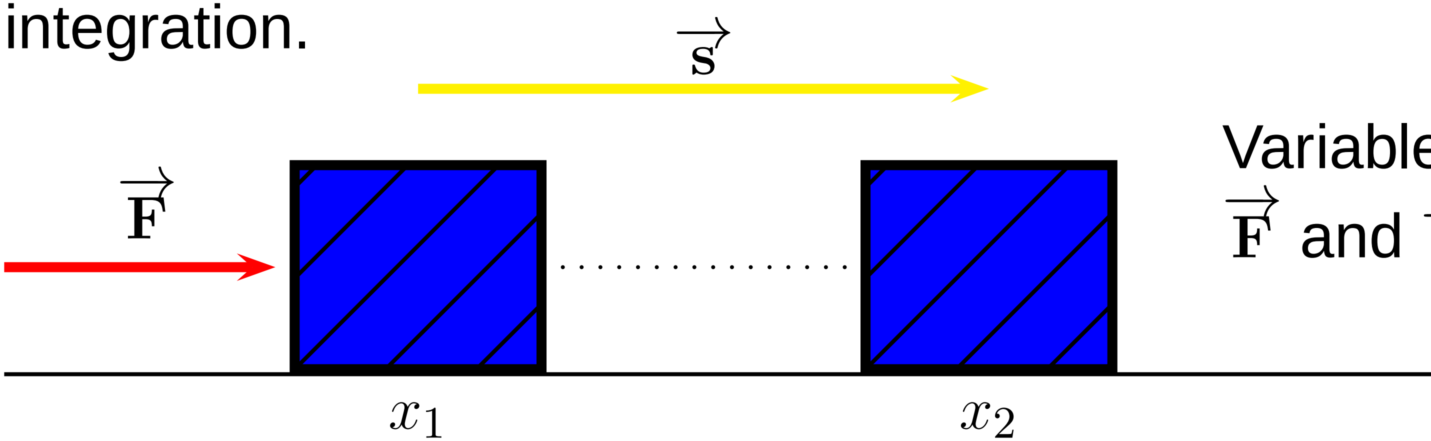


Work is the area under this curve

Split region into many small rectangles.

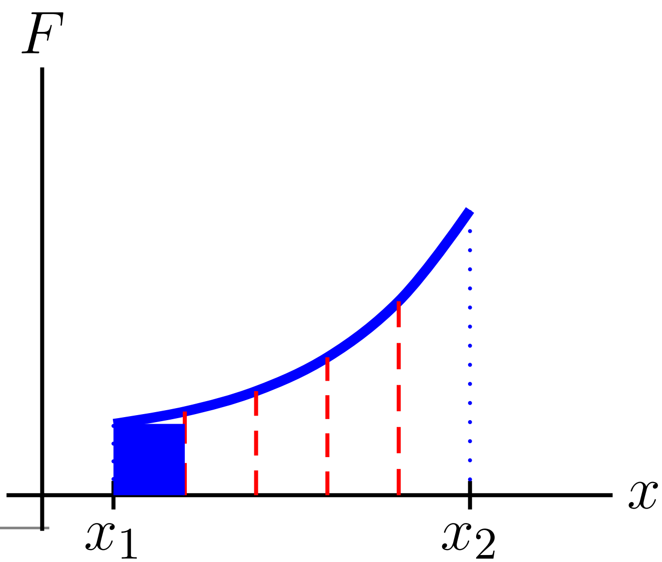
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



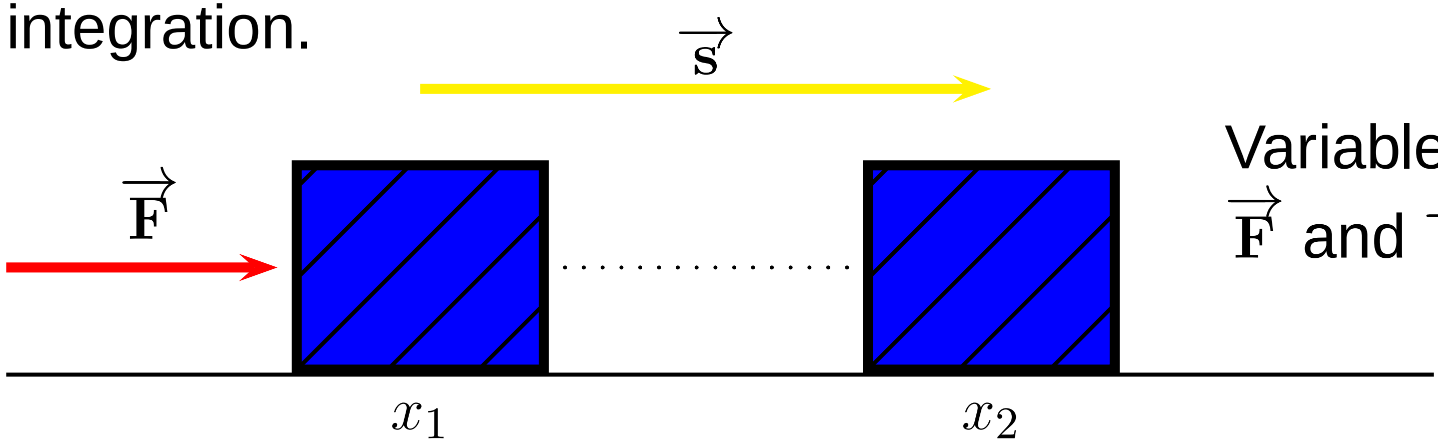
Work is the area under this curve

Split region into many small rectangles.

Find area of each rectangle and add.

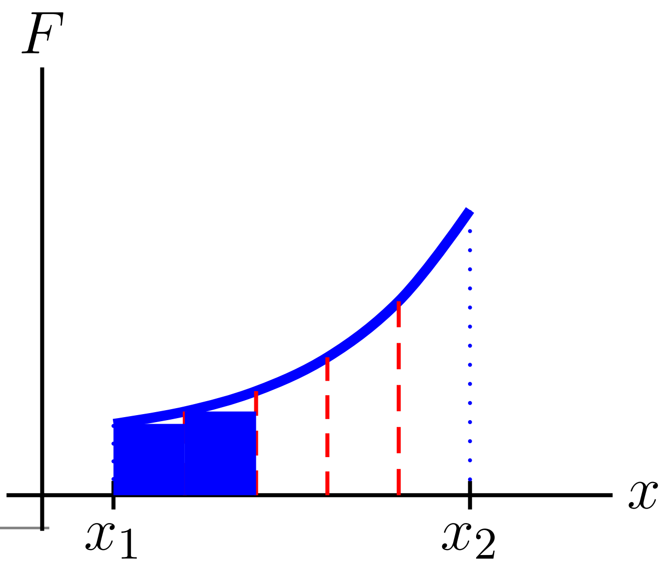
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



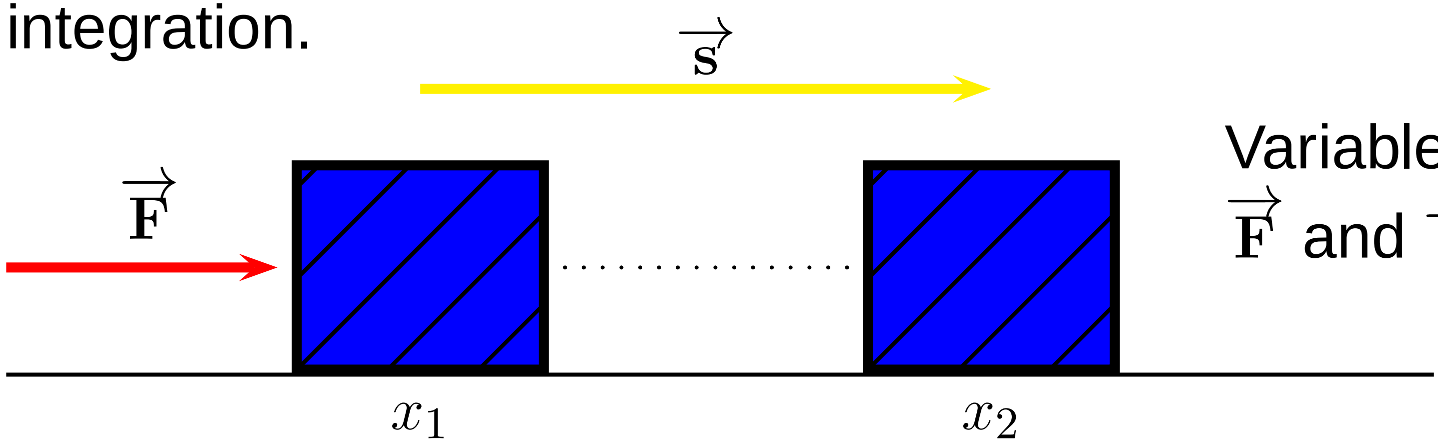
Work is the area under this curve

Split region into many small rectangles.

Find area of each rectangle and add.

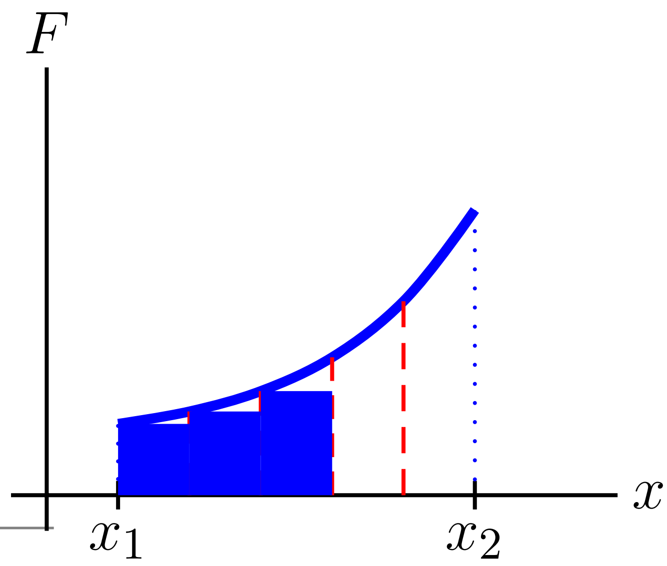
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



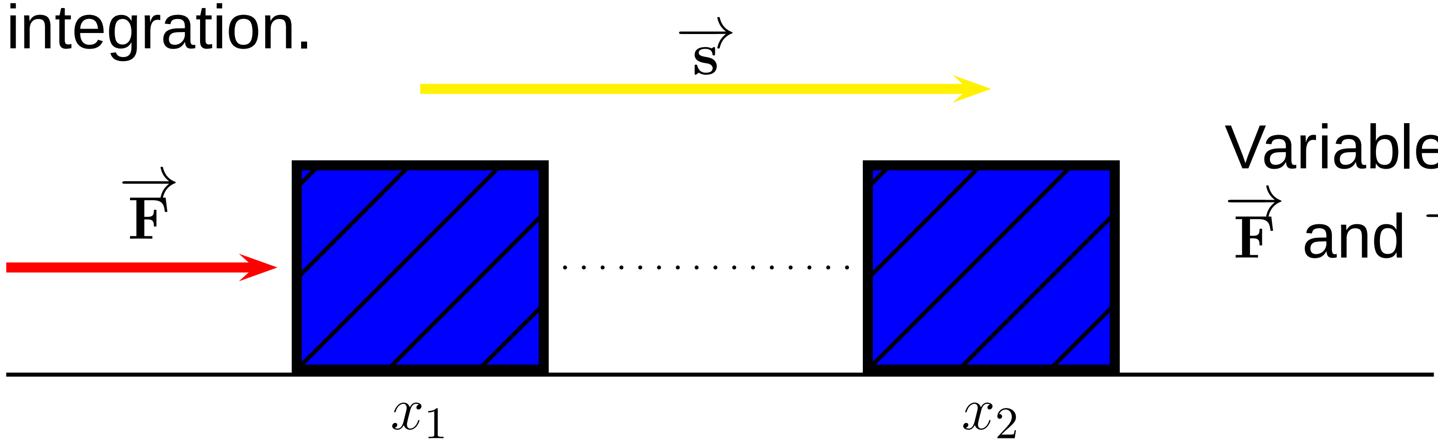
Work is the area under this curve

Split region into many small rectangles.

Find area of each rectangle and add.

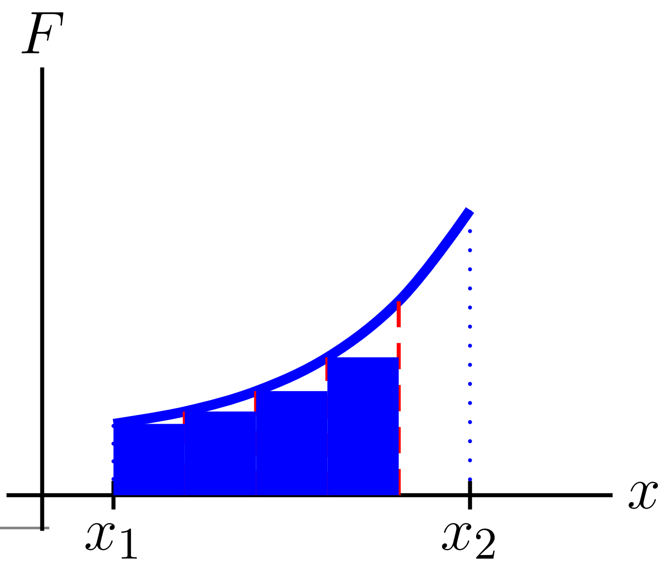
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



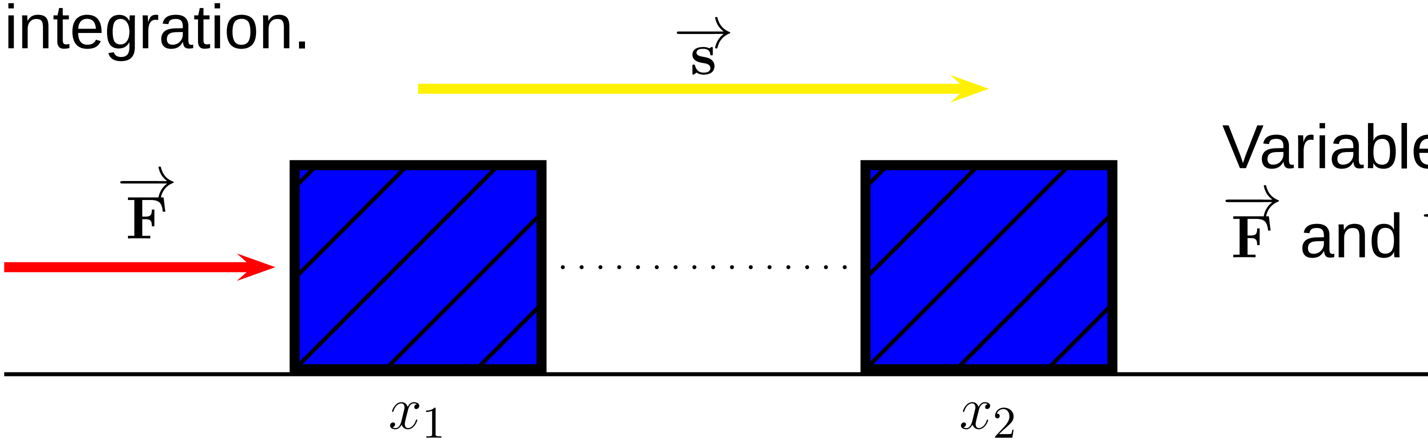
Work is the area under this curve

Split region into many small rectangles.

Find area of each rectangle and add.

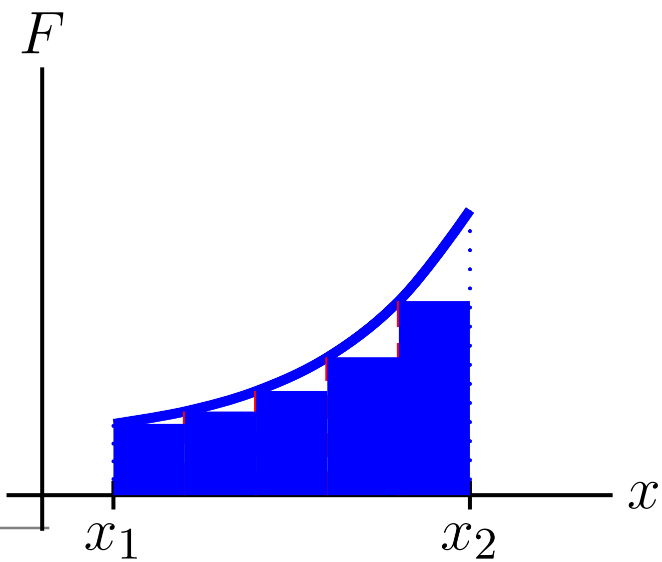
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{F}$ and $\vec{s}$ parallel



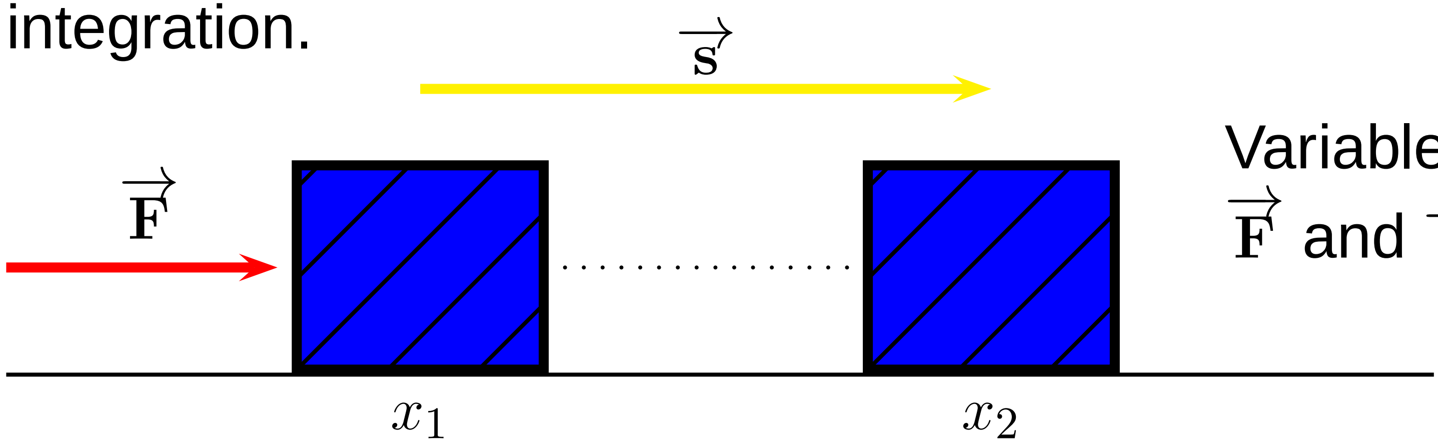
Work is the area under this curve

Split region into many small rectangles.

Find area of each rectangle and add.

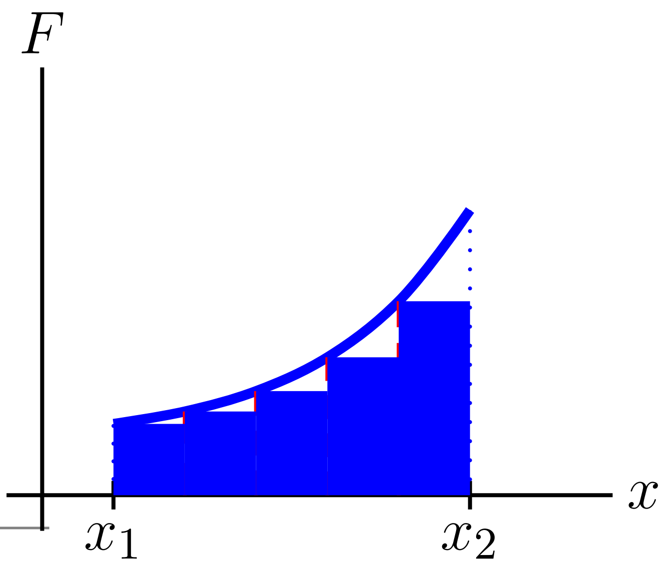
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



Work is the area under this curve

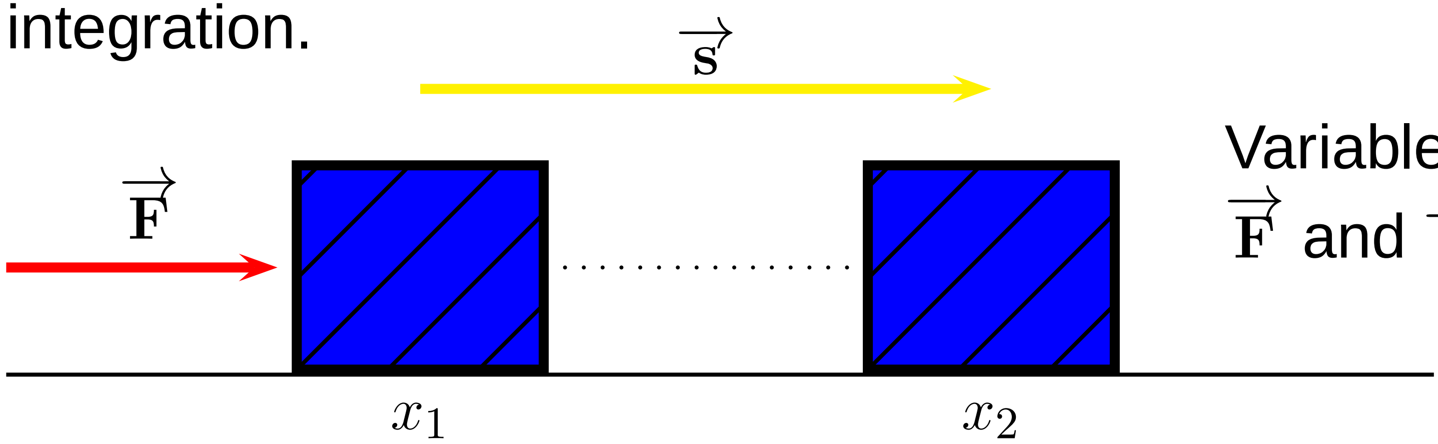
Split region into many small rectangles.

Find area of each rectangle and add.

Take a limit to find the exact area.

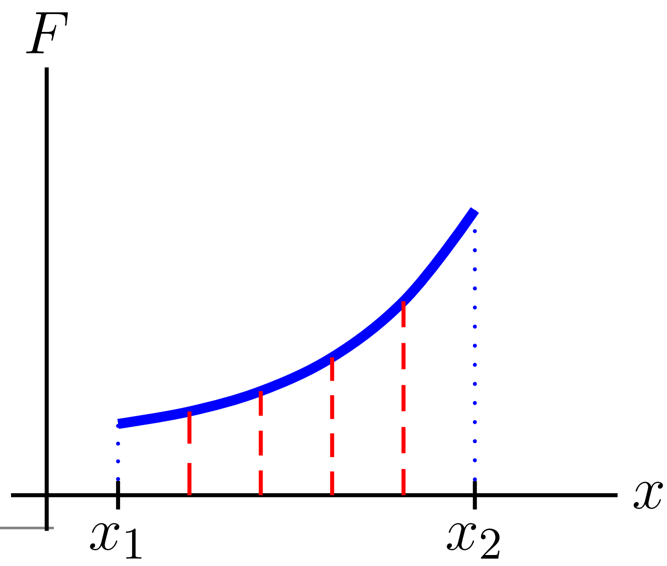
Variable Forces II

To find the work done by a changing force requires integration.



Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



Variable Forces II

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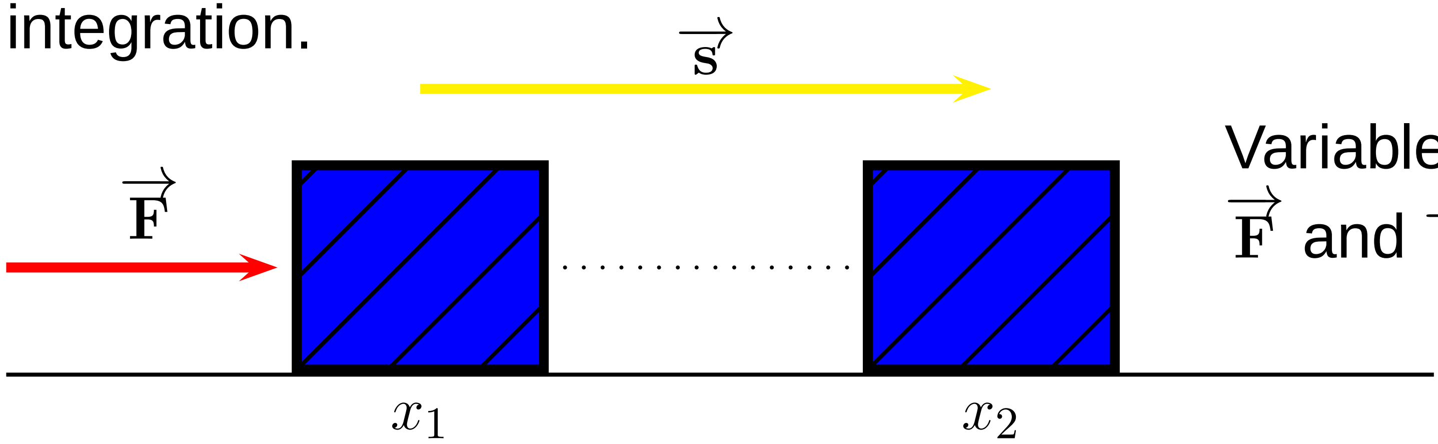

Variable F

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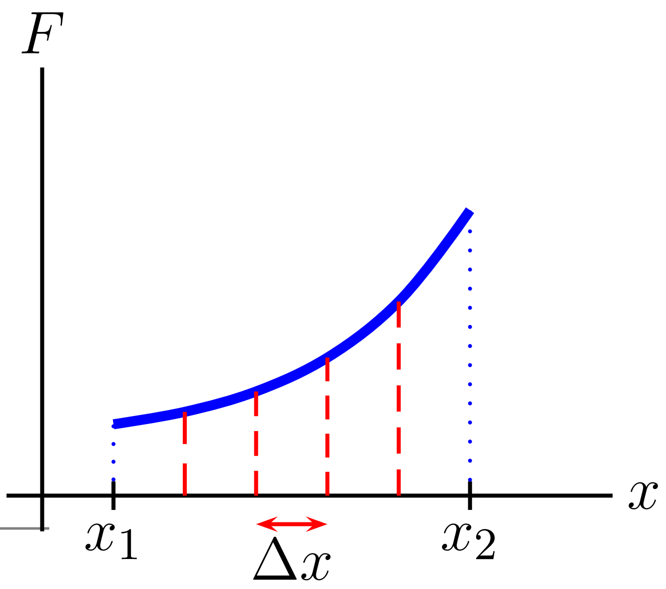

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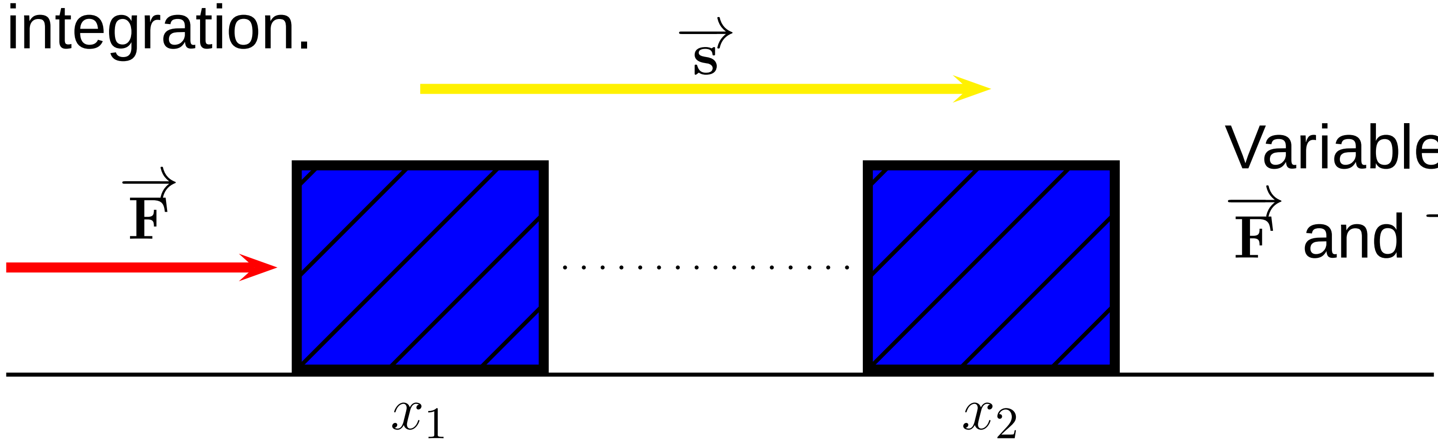

Variable F

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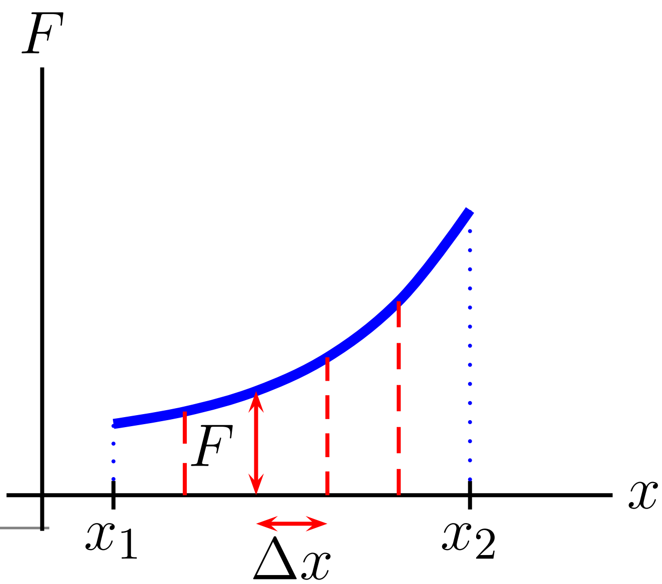

Variable Forces II

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Variable F

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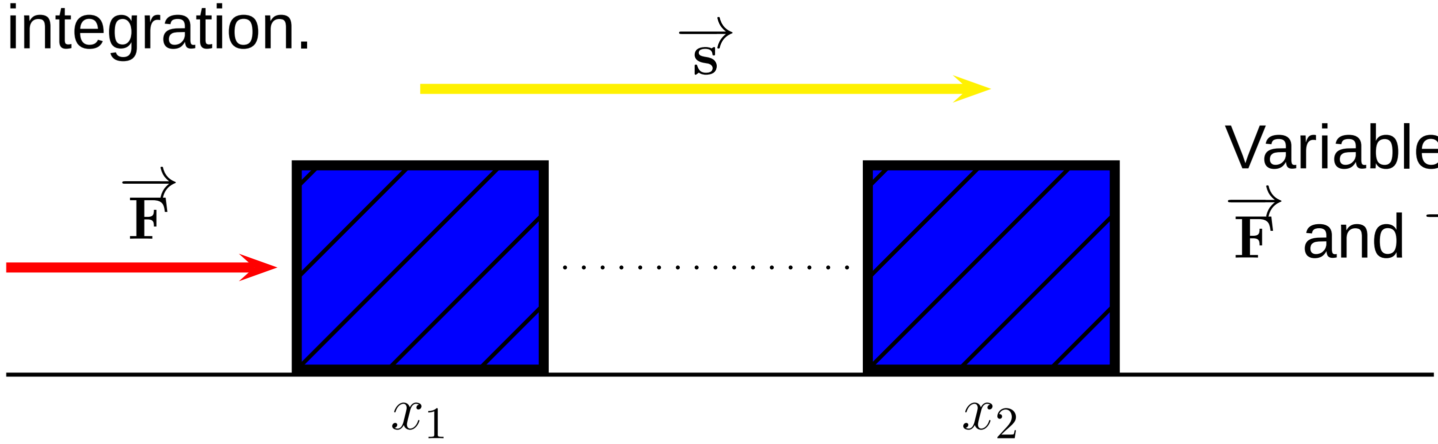

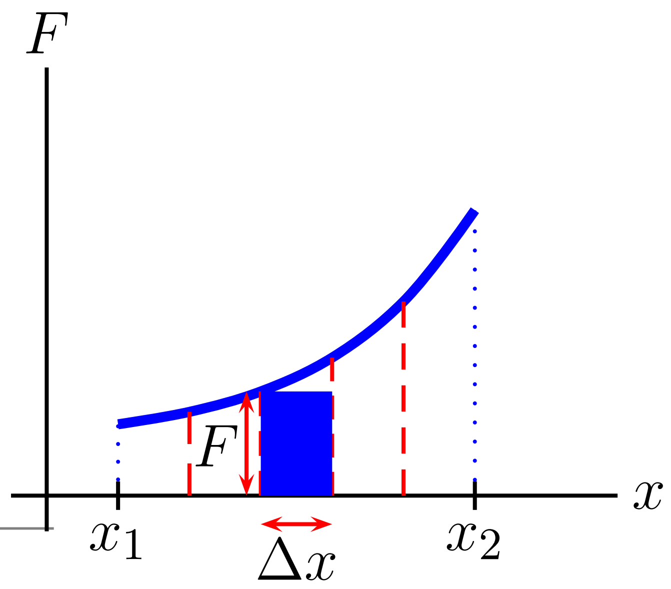

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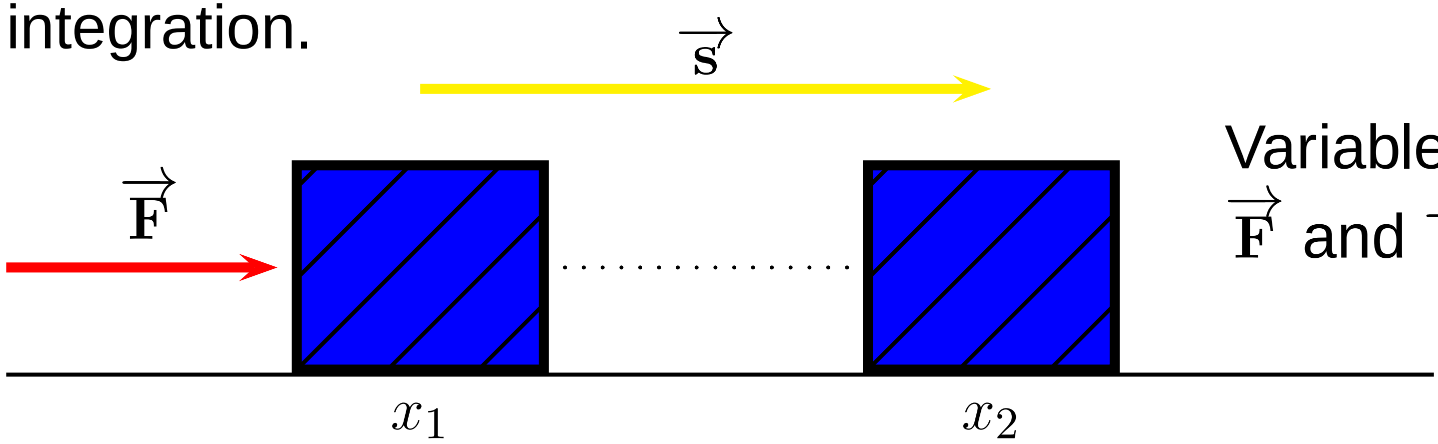

Variable F

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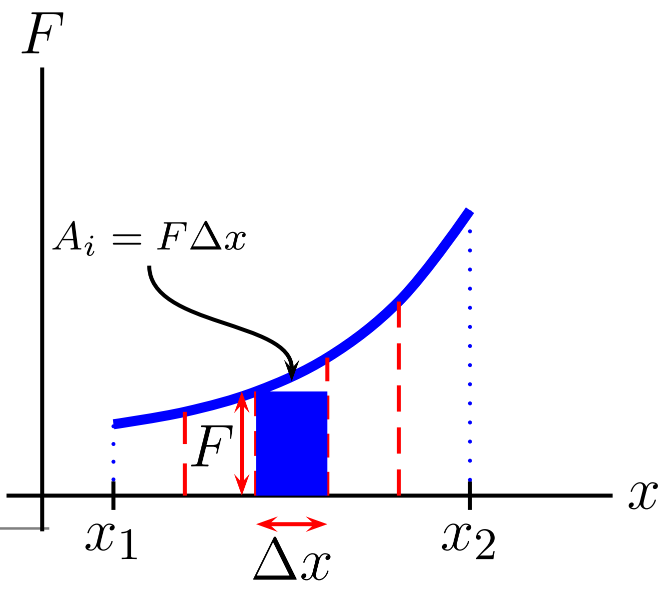

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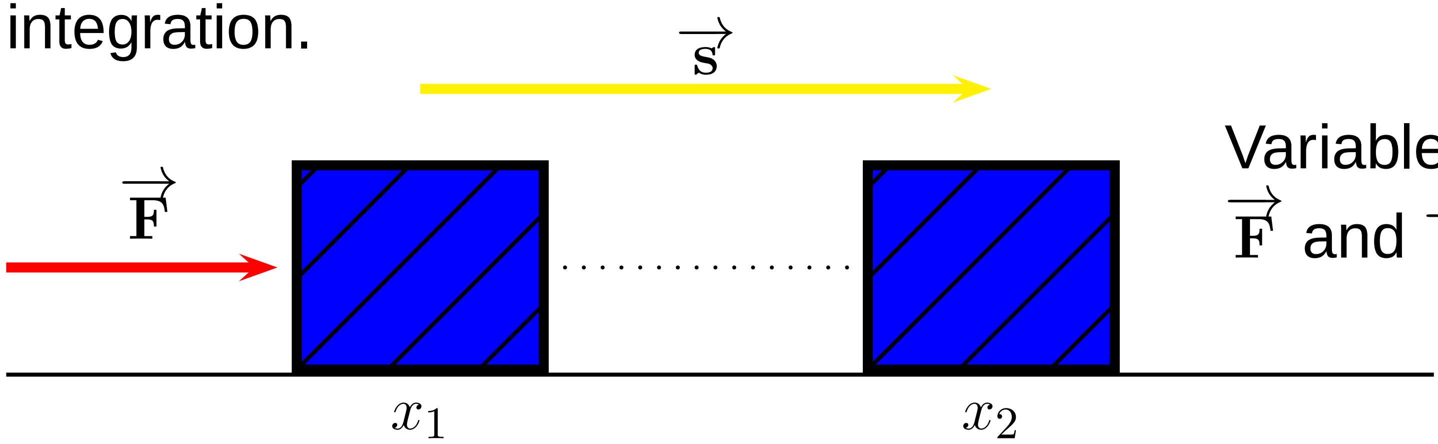

Variable F

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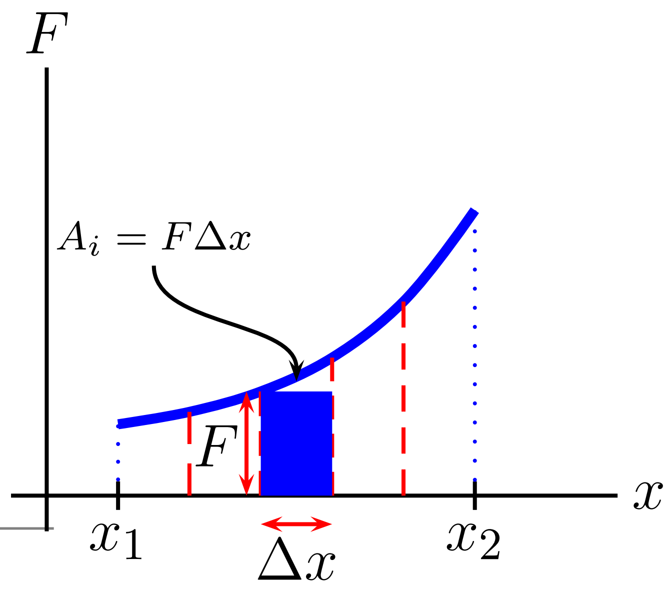

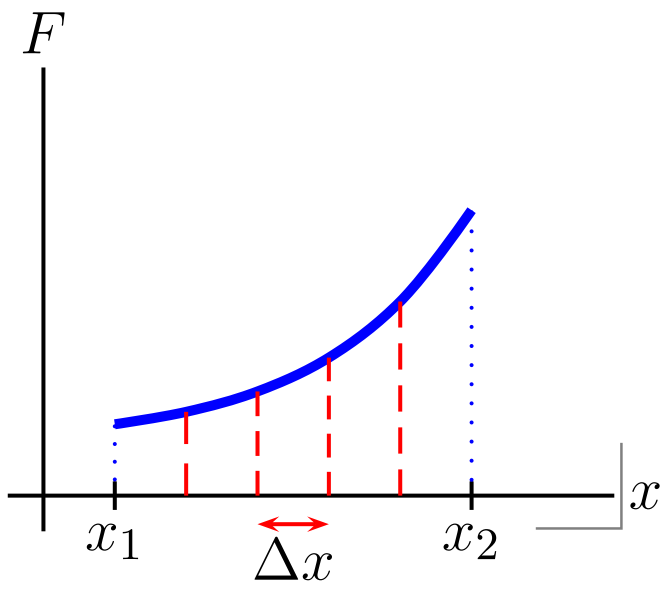

Variable Forces II

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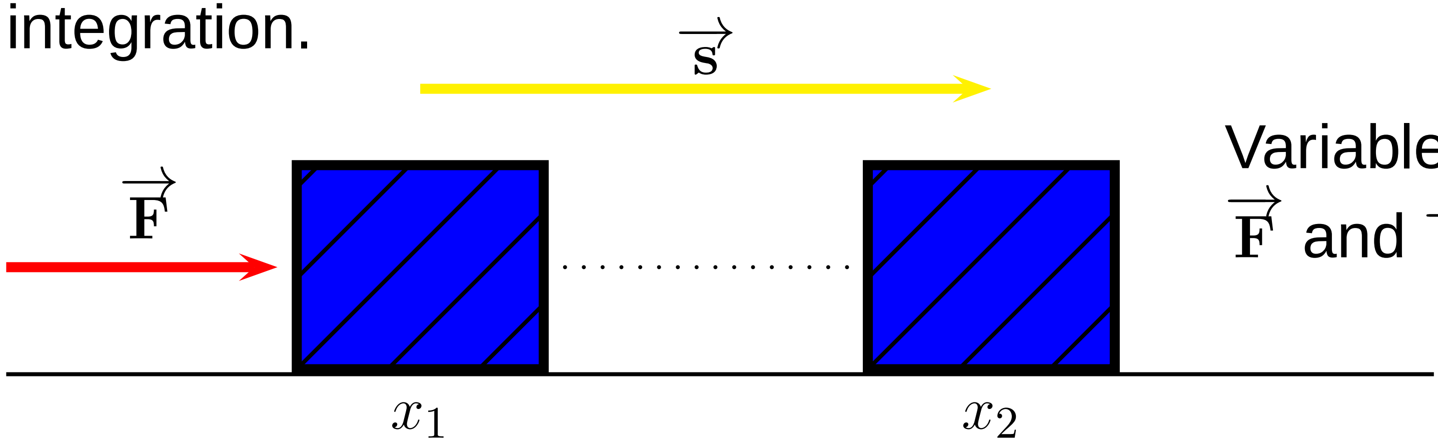

Variable F
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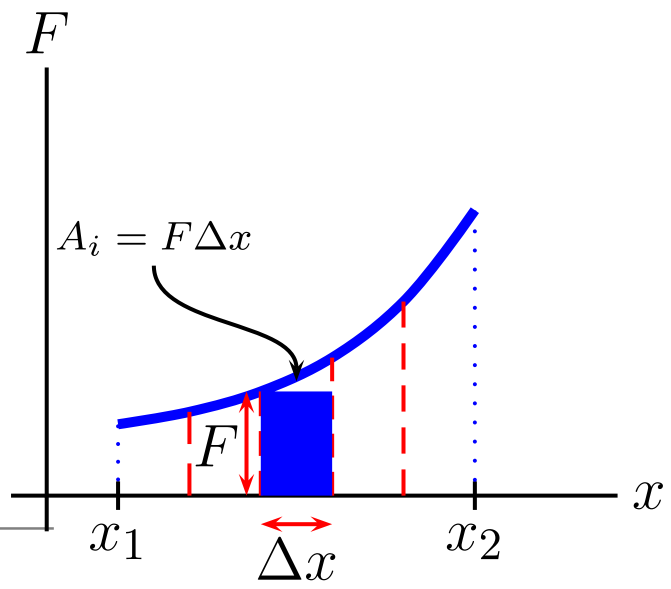

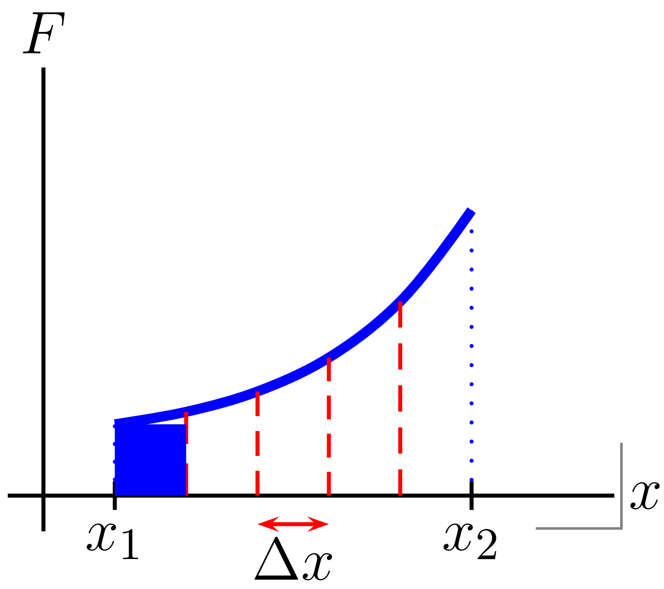

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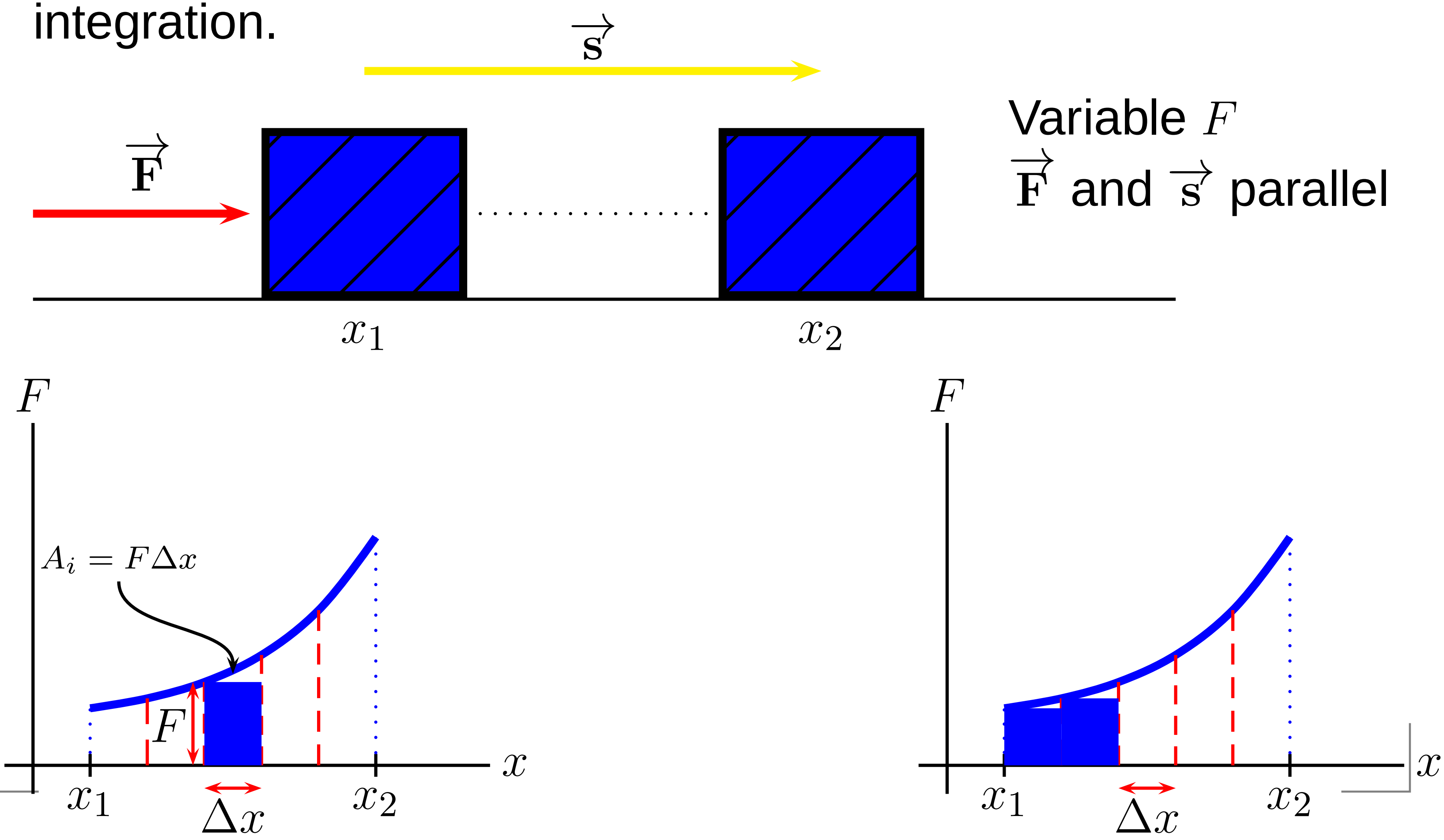

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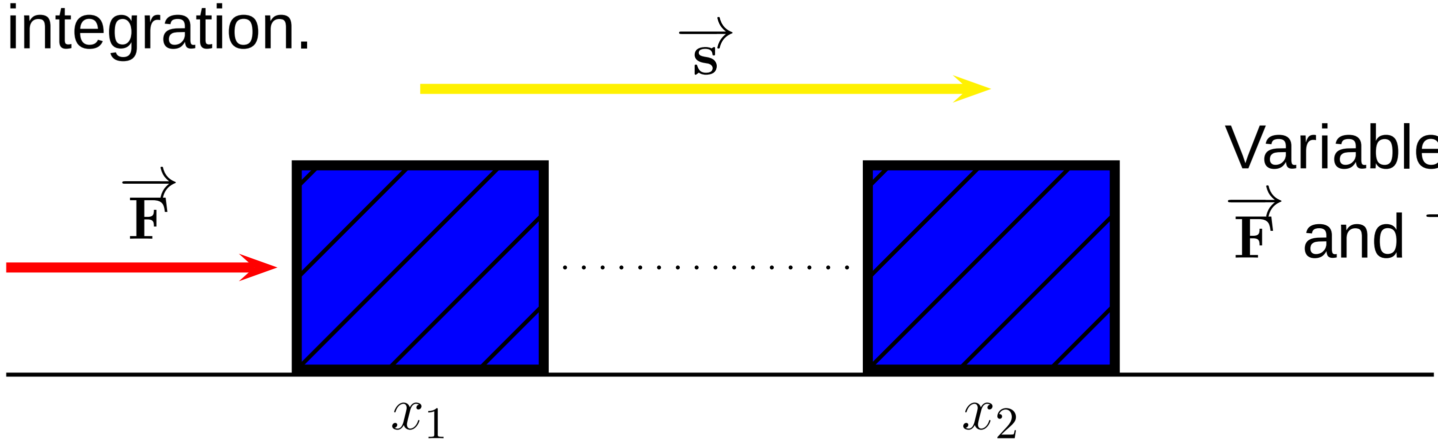
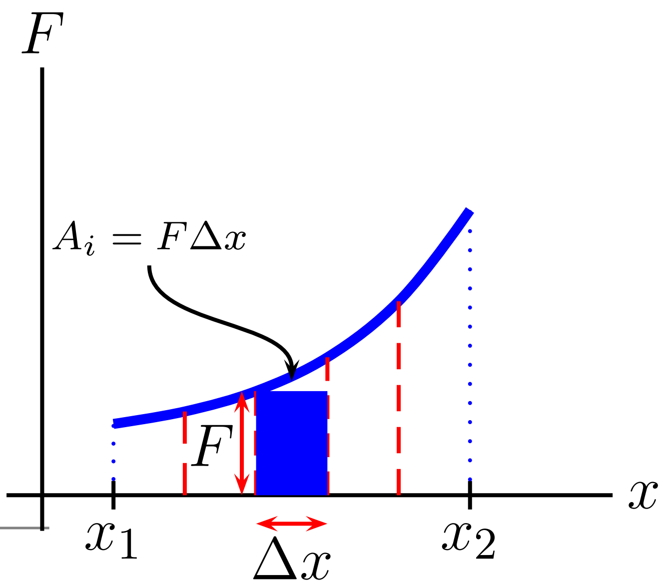
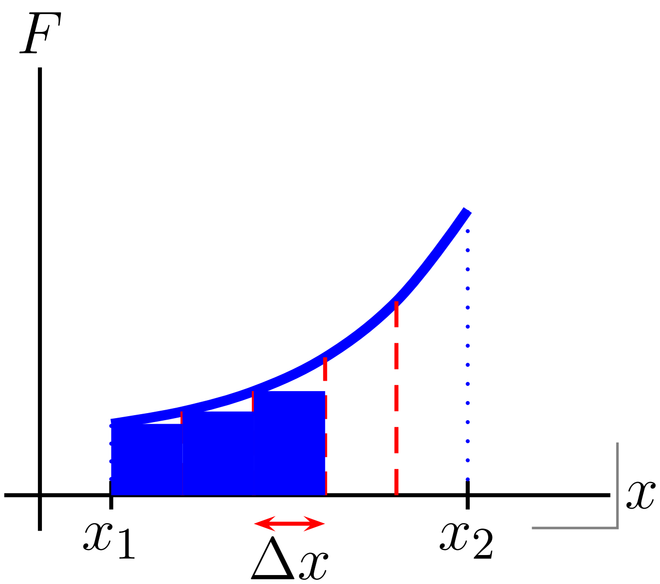


Variable F

$\vec{F}$ and $\vec{s}$ parallel





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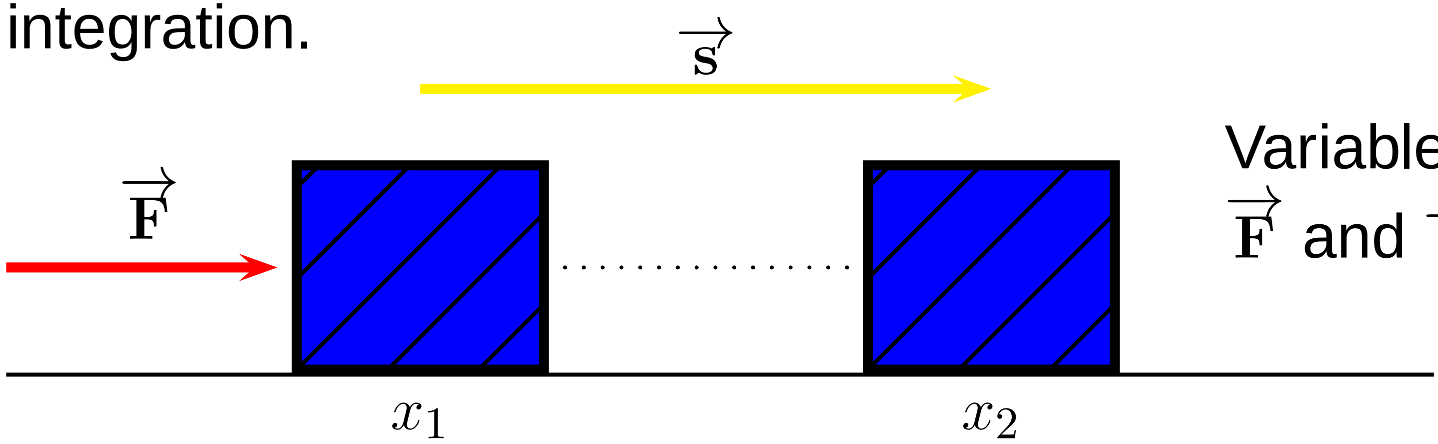
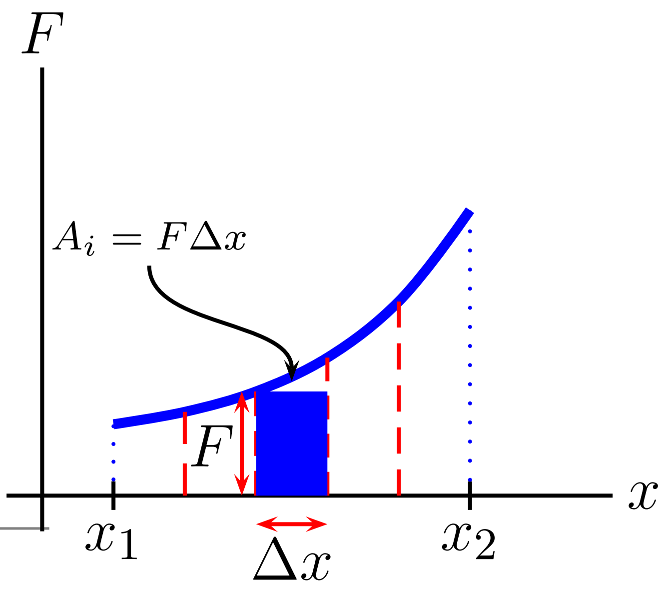
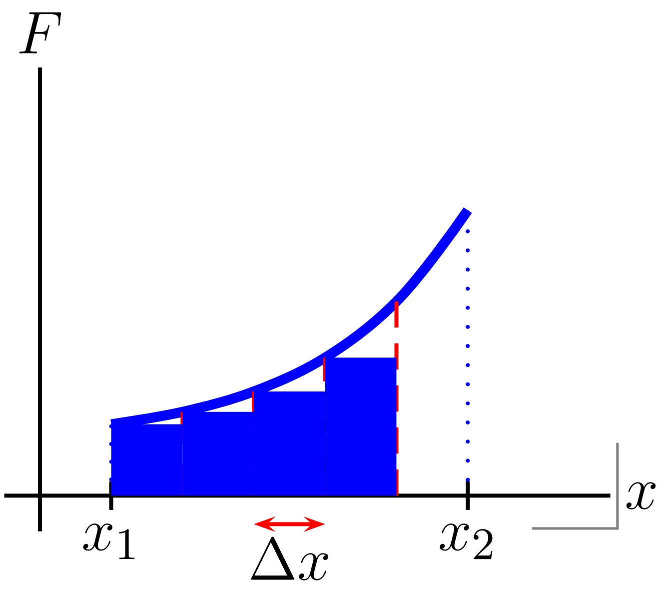


Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel





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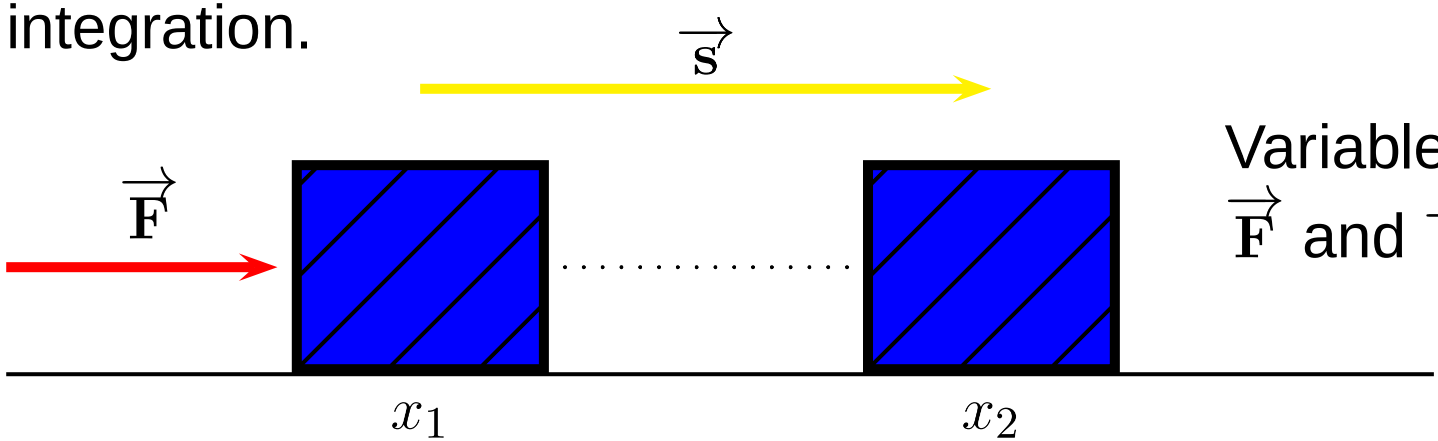
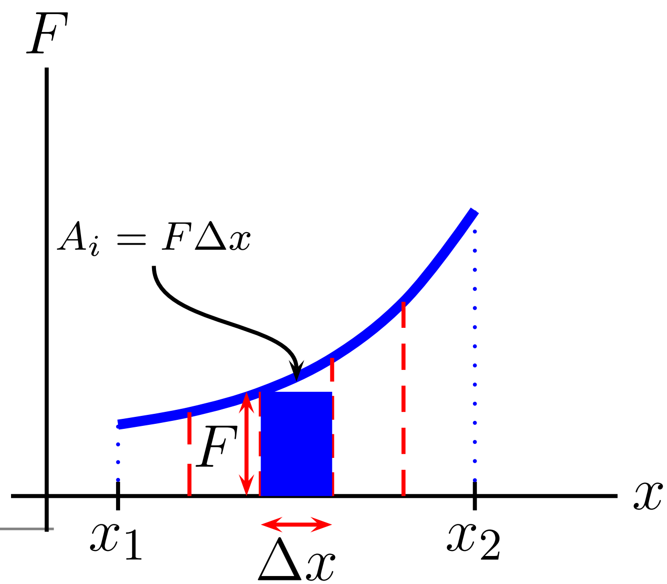
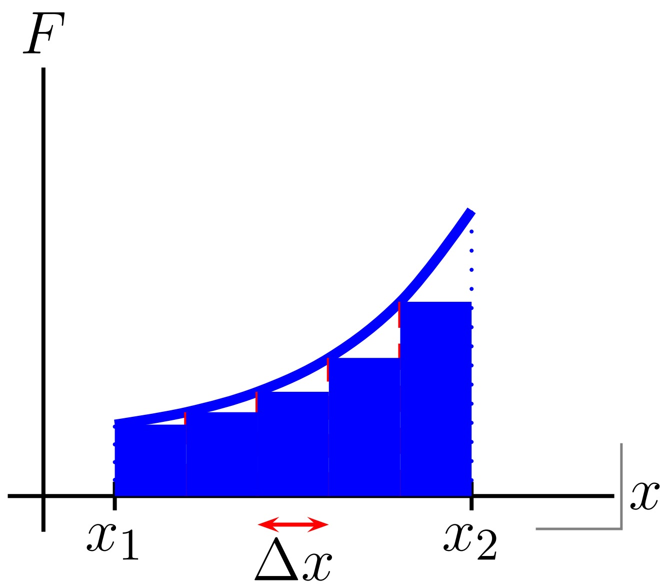


Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel





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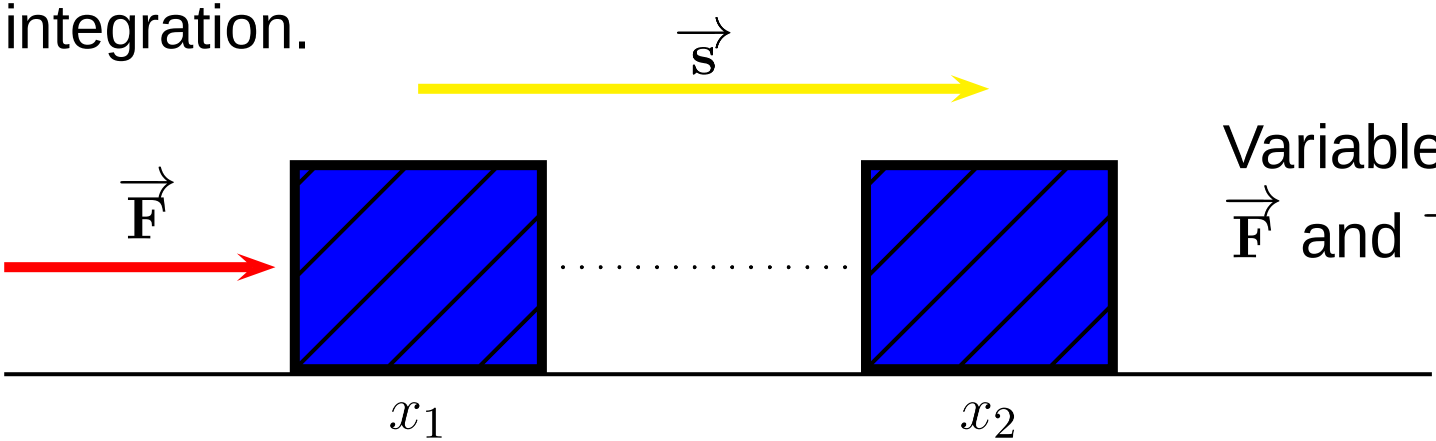

Variable F

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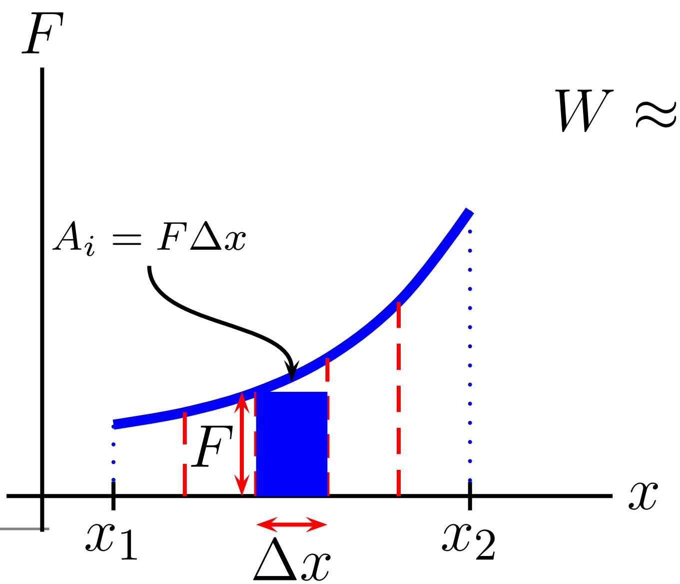
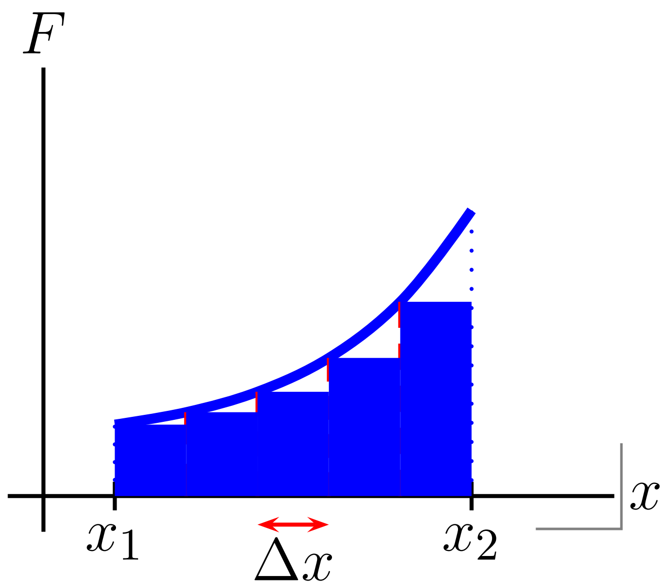


$$W \approx \sum_i A_i$$



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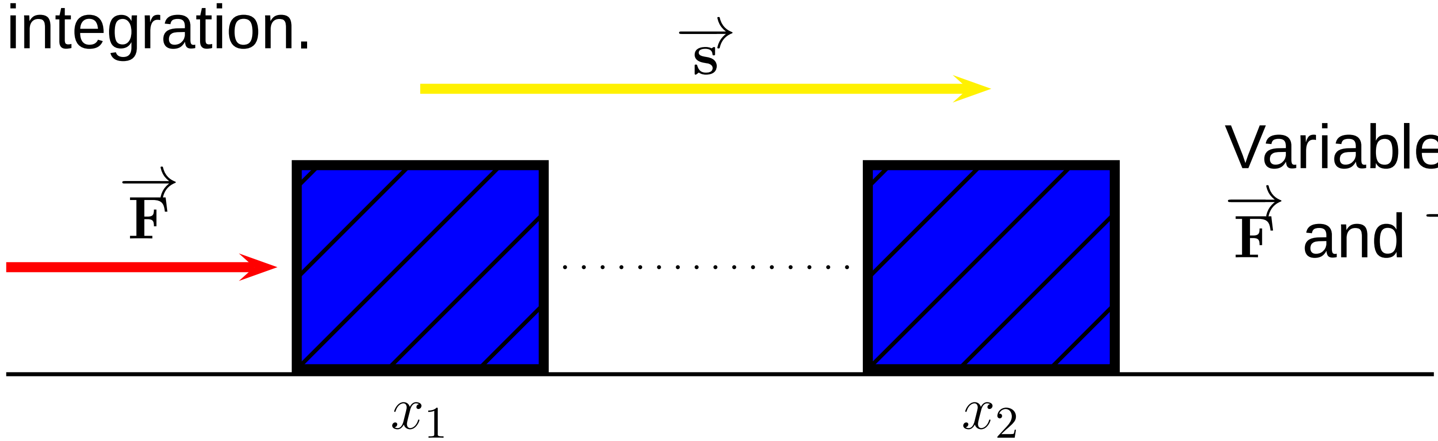
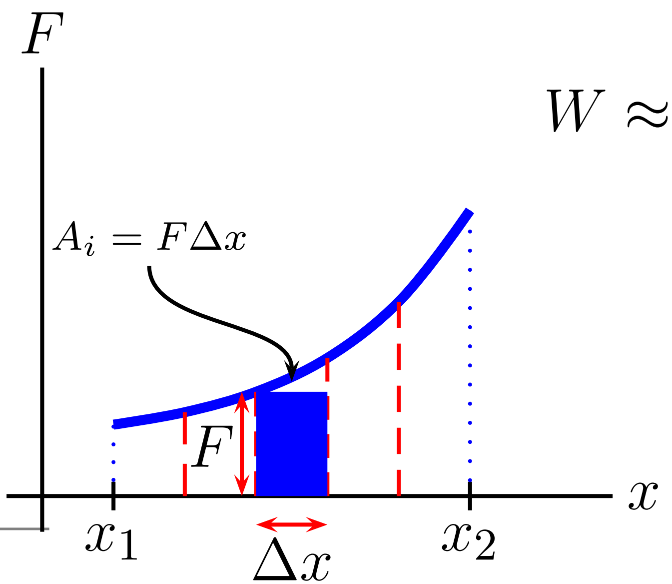
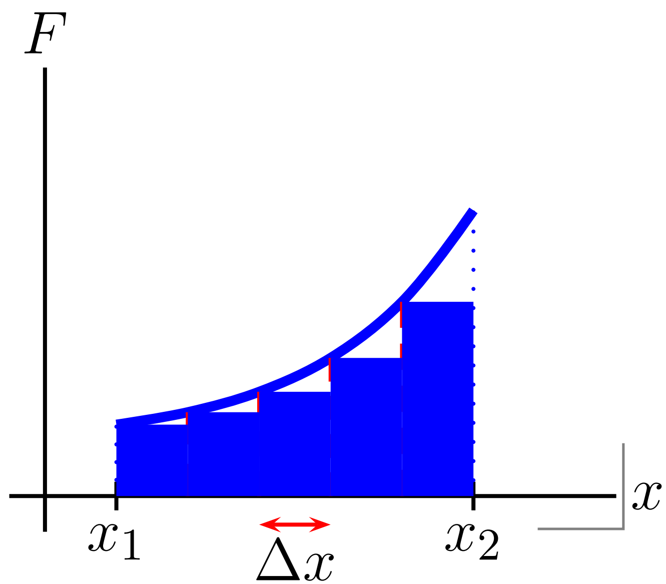


Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel

$$W \approx \sum_i A_i = \sum_i F \Delta x$$





Variable Forces II

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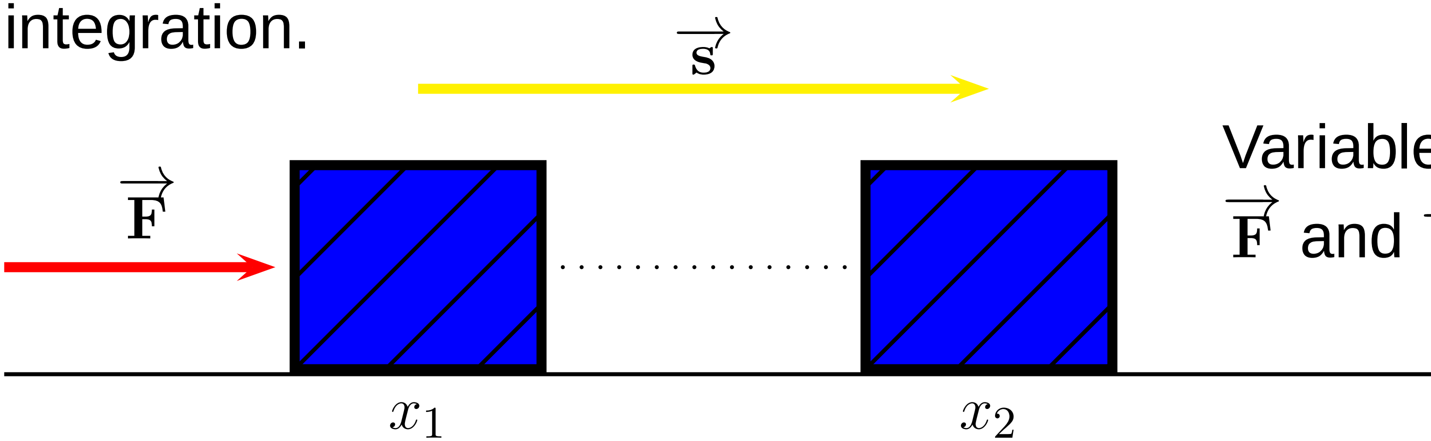
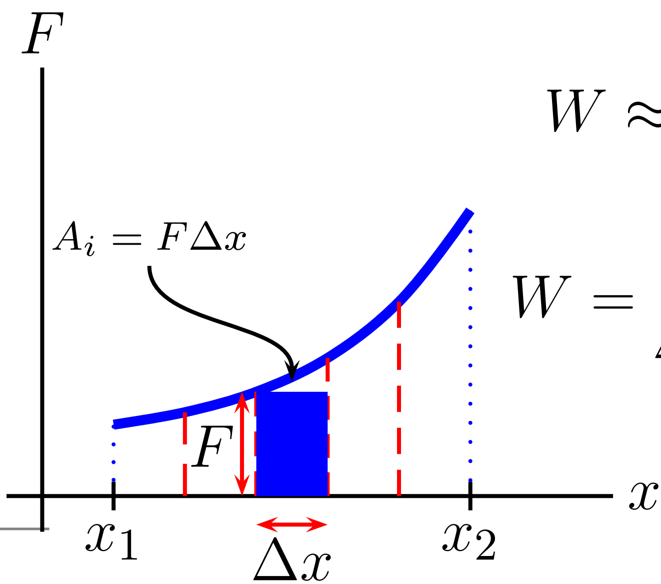


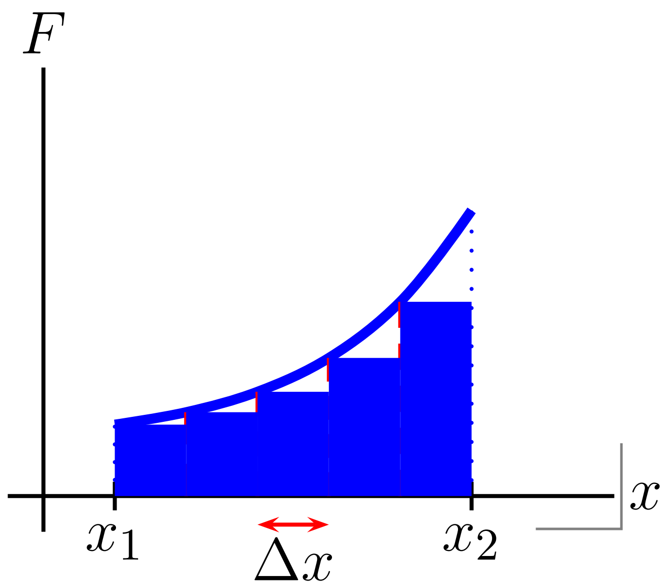
Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



$$W \approx \sum_i A_i = \sum_i F\Delta x$$

$$W = \lim_{\Delta x \to 0} \sum F\Delta x$$



Variable Forces II

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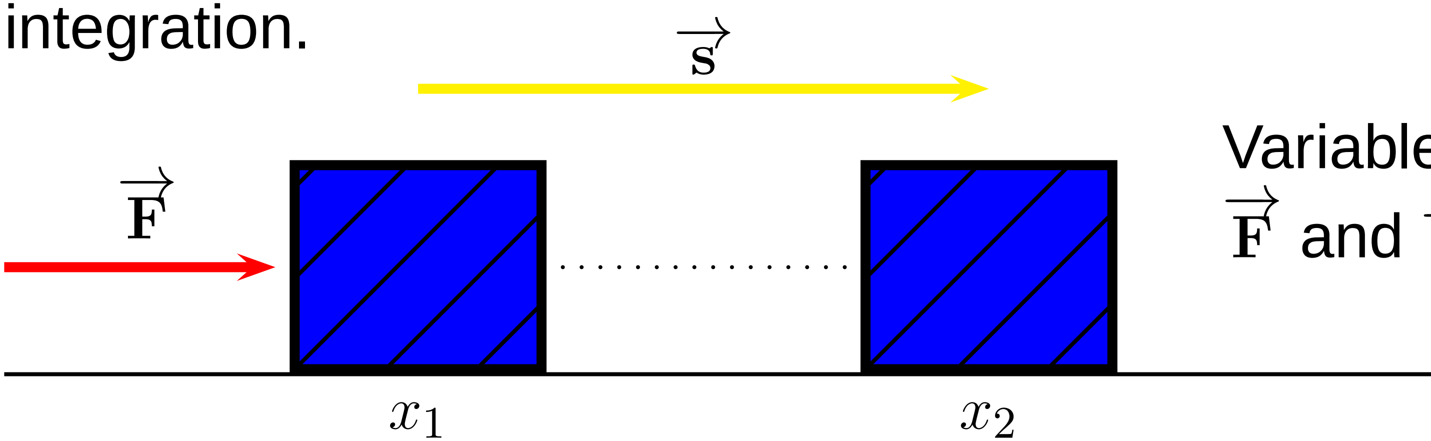
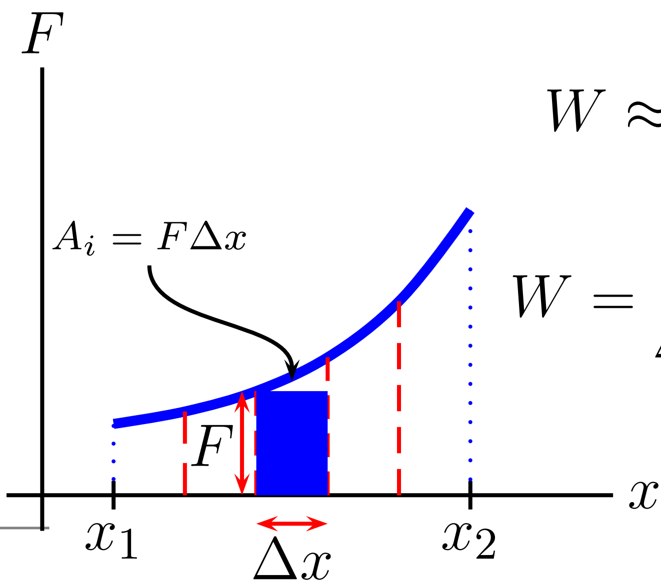


Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



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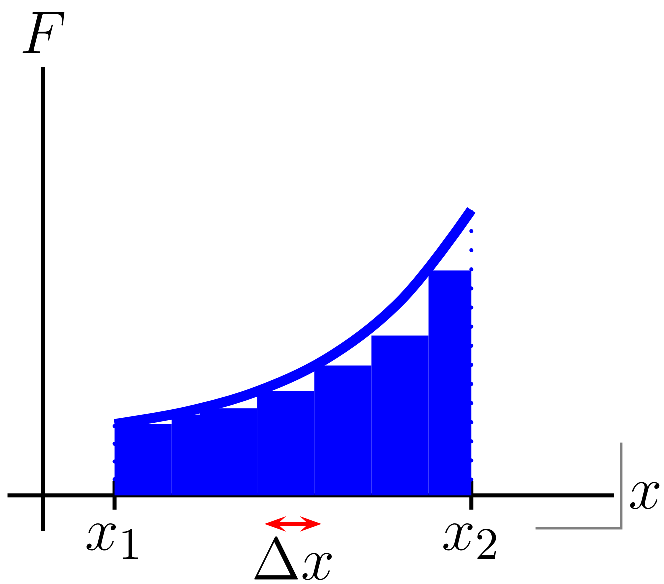

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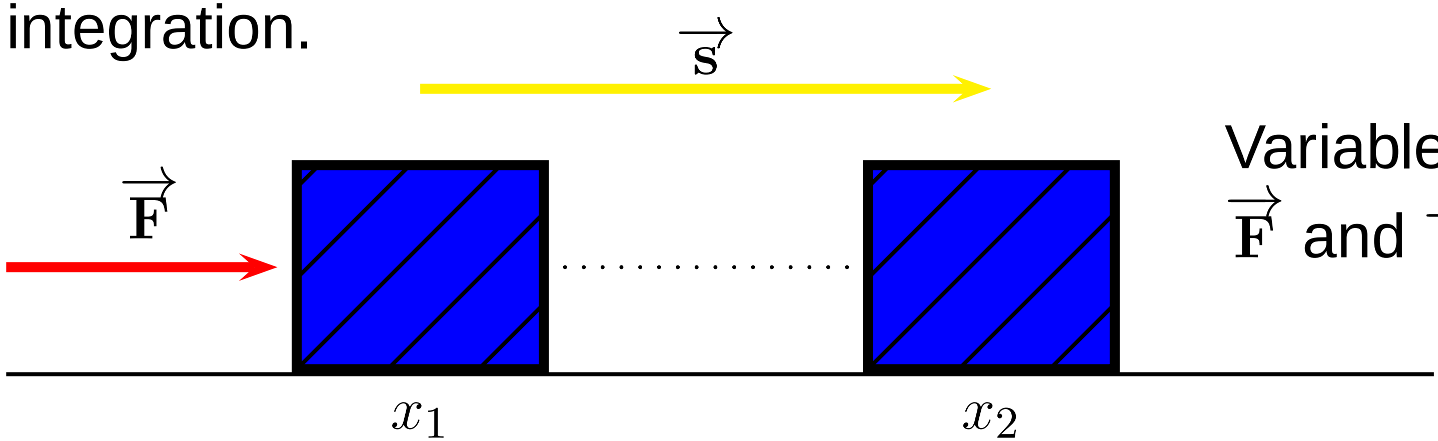
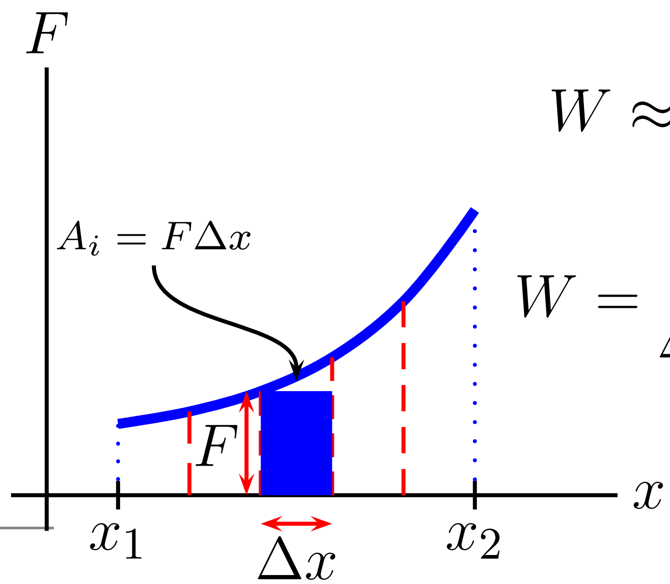


Variable F
$\vec{F}$ and $\vec{s}$ parallel



$$W \approx \sum_i A_i = \sum_i F\Delta x$$

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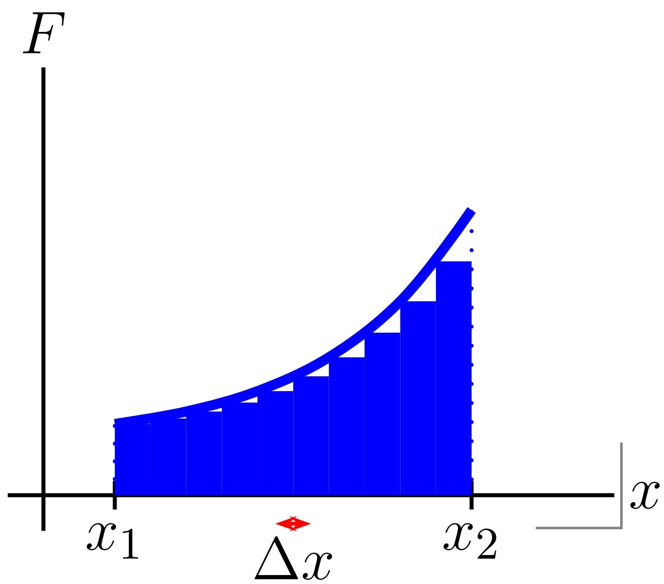

Variable Forces II

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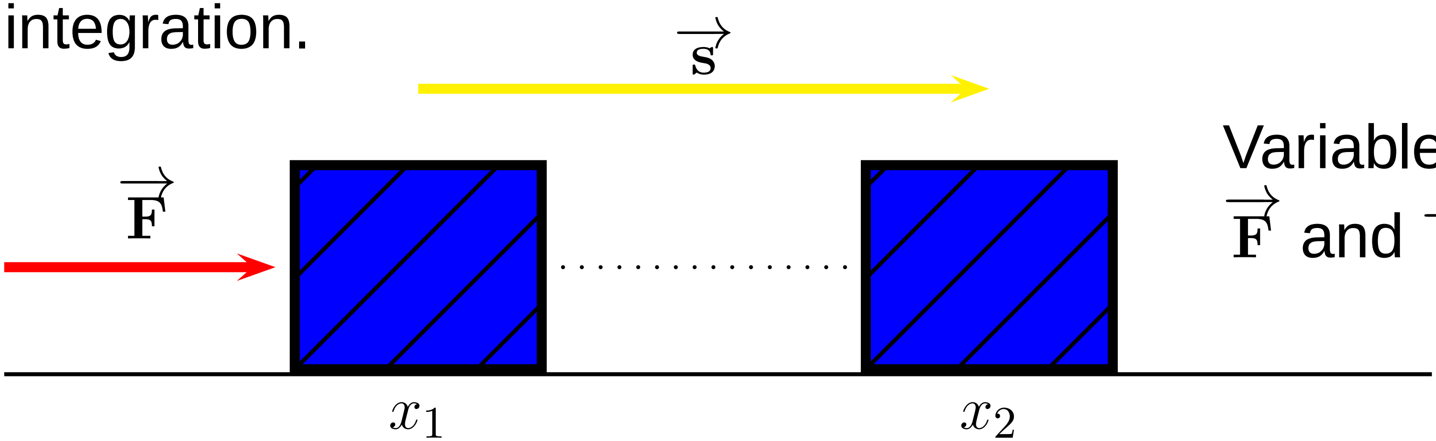
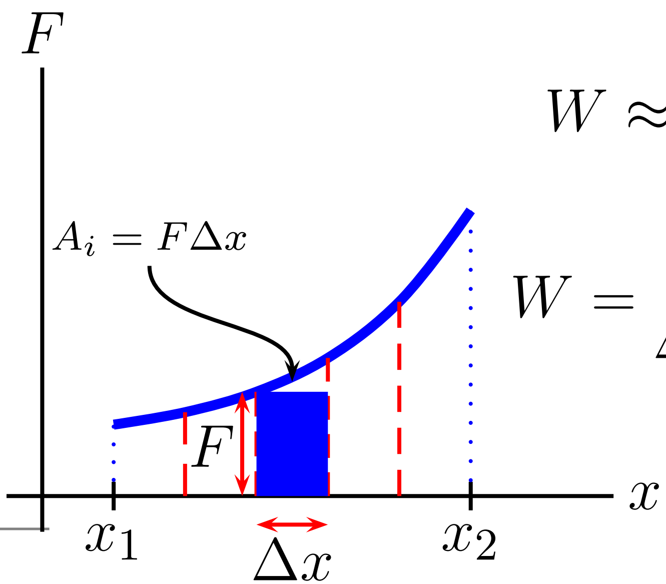


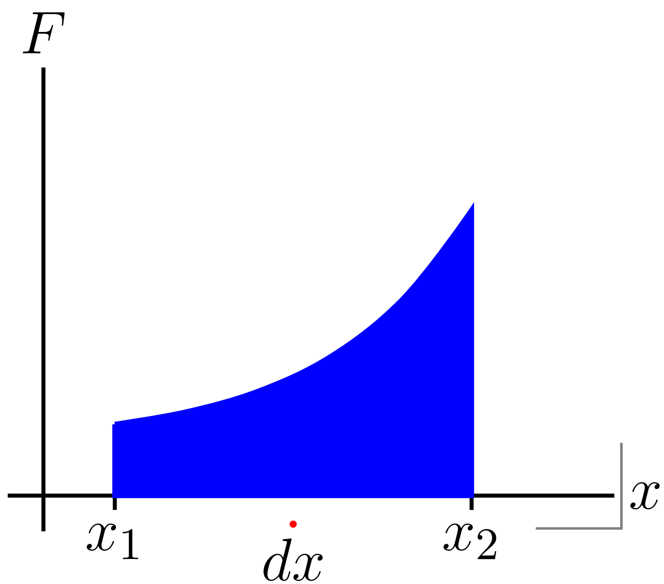
Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel



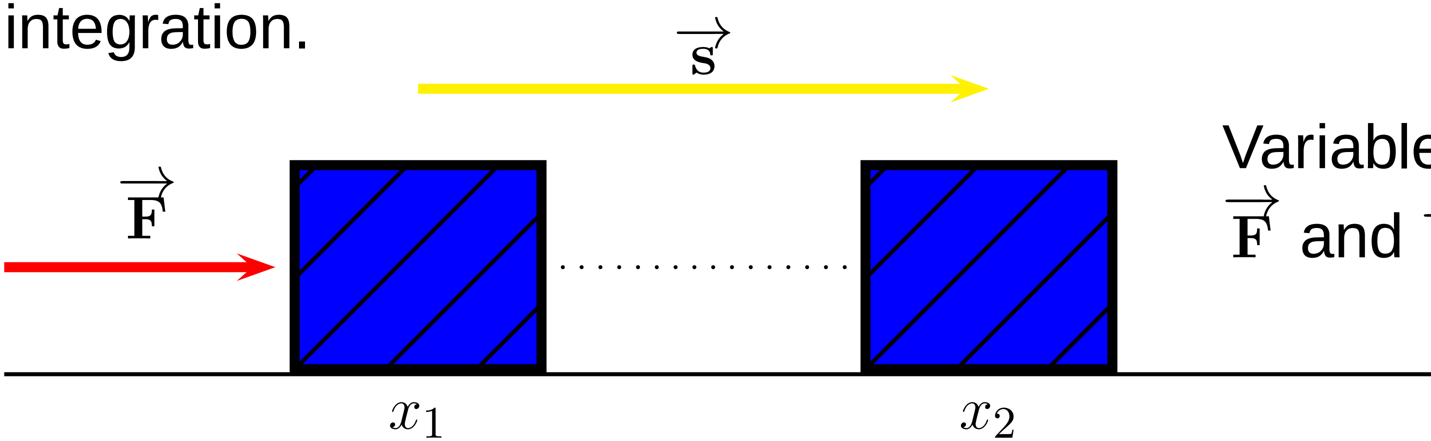
$$W \approx \sum_i A_i = \sum_i F\Delta x$$

$$W = \lim_{\Delta x \to 0} \sum F\Delta x$$



Variable Forces II

To find the work done by a changing force requires integration.



Variable F

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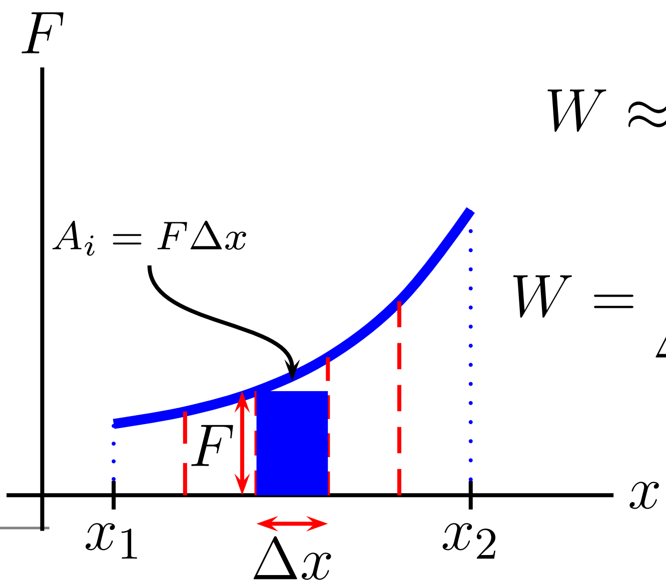
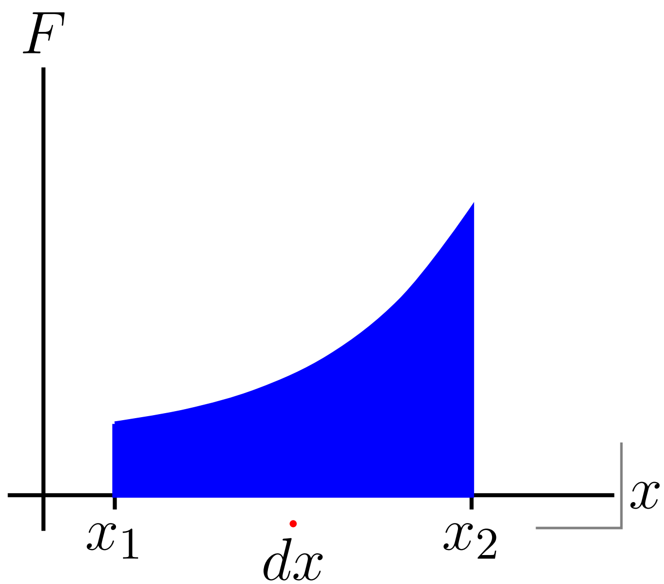


$$W \approx \sum_i A_i = \sum_i F\Delta x$$

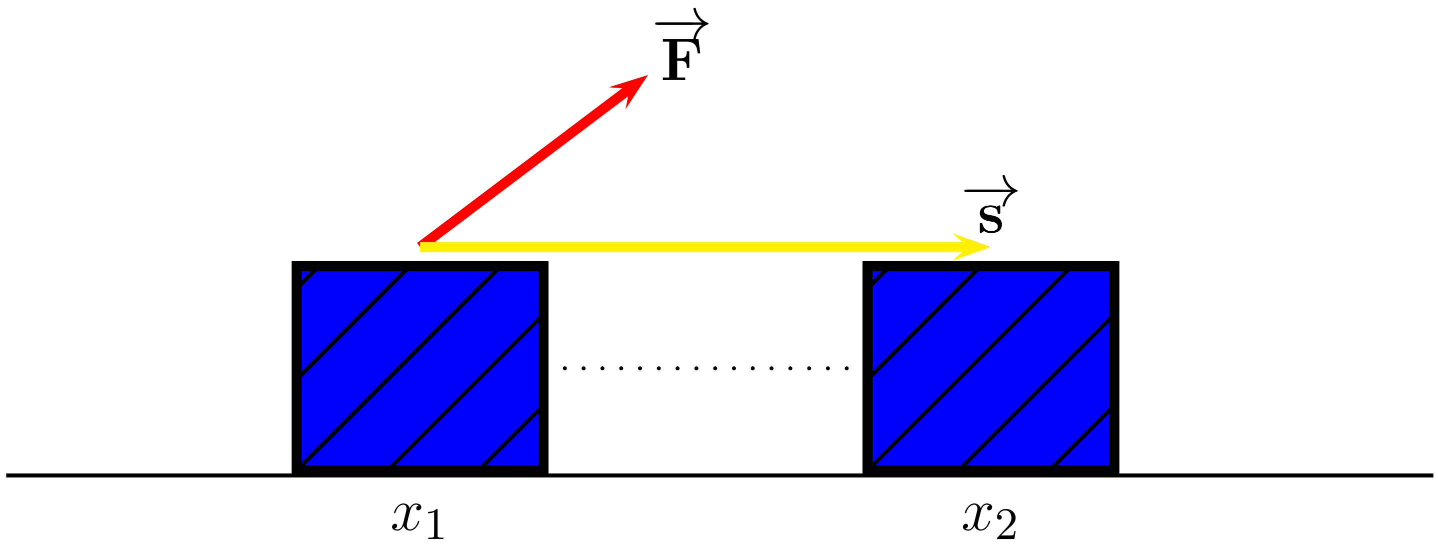
$$W = \lim_{\Delta x \to 0} \sum F\Delta x$$

$$= \int_{x_1}^{x_2} F\,dx$$



Variable Force Arbitrary Direction

Still only the component of the force parallel to the displacement does work.



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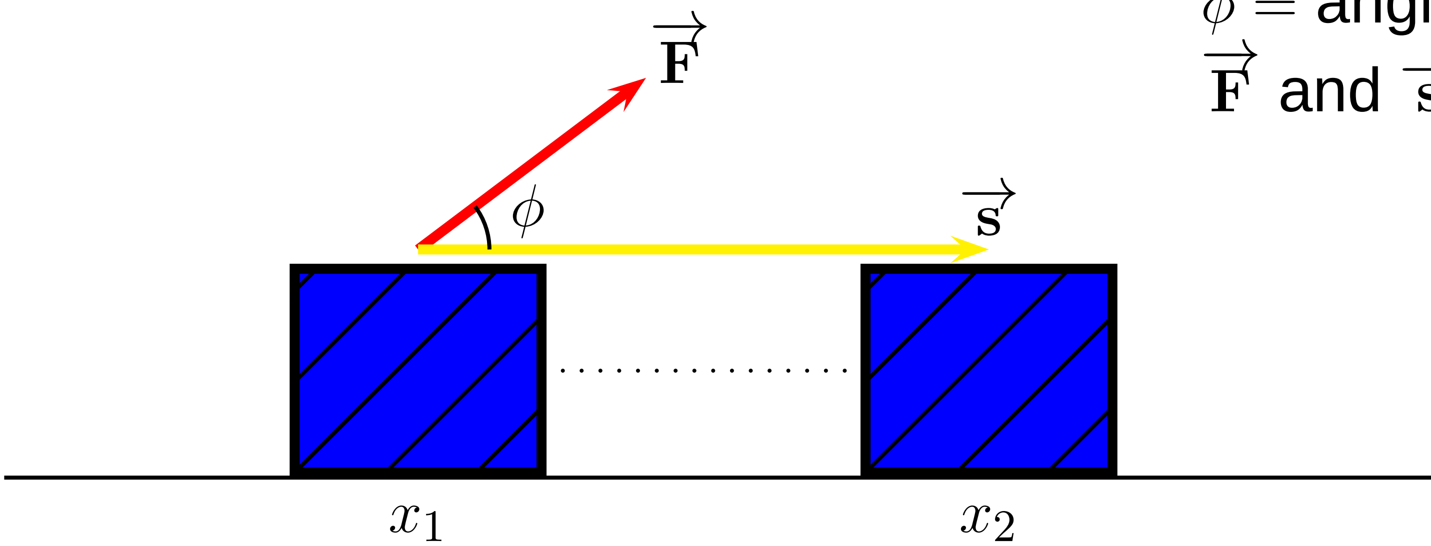

$\phi =$ angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$

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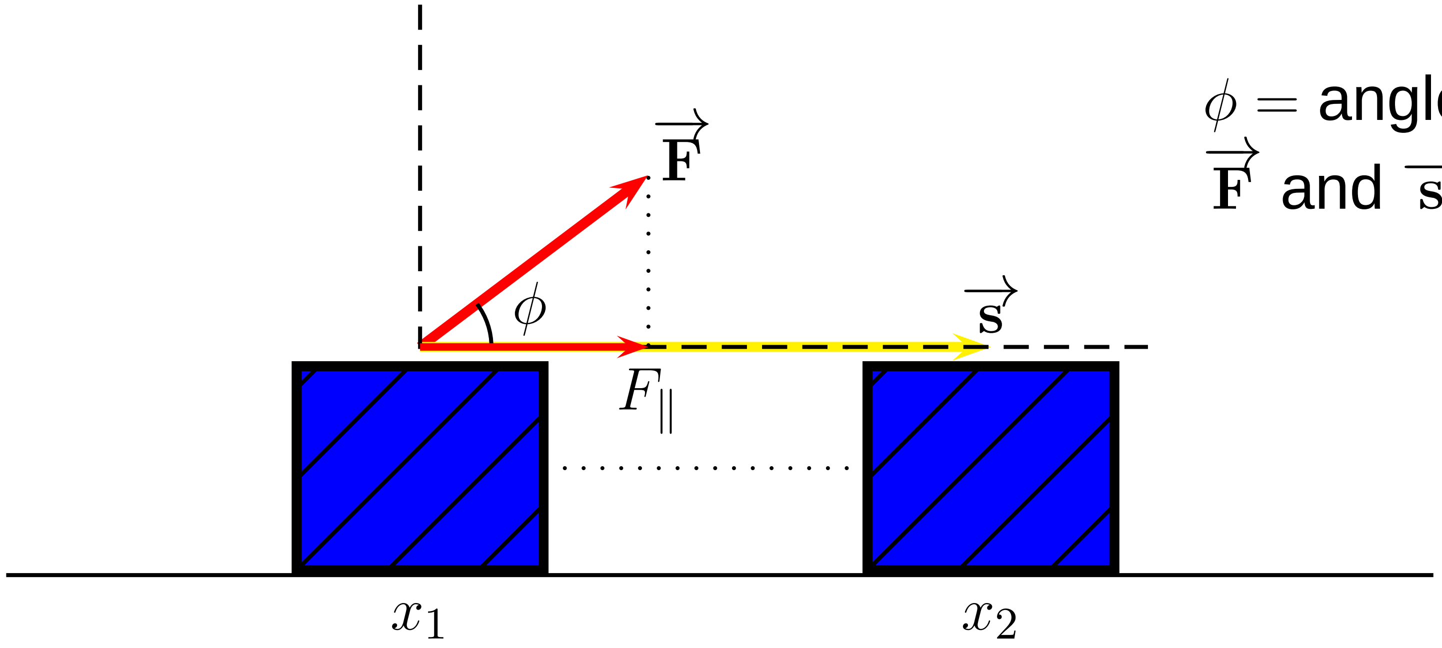

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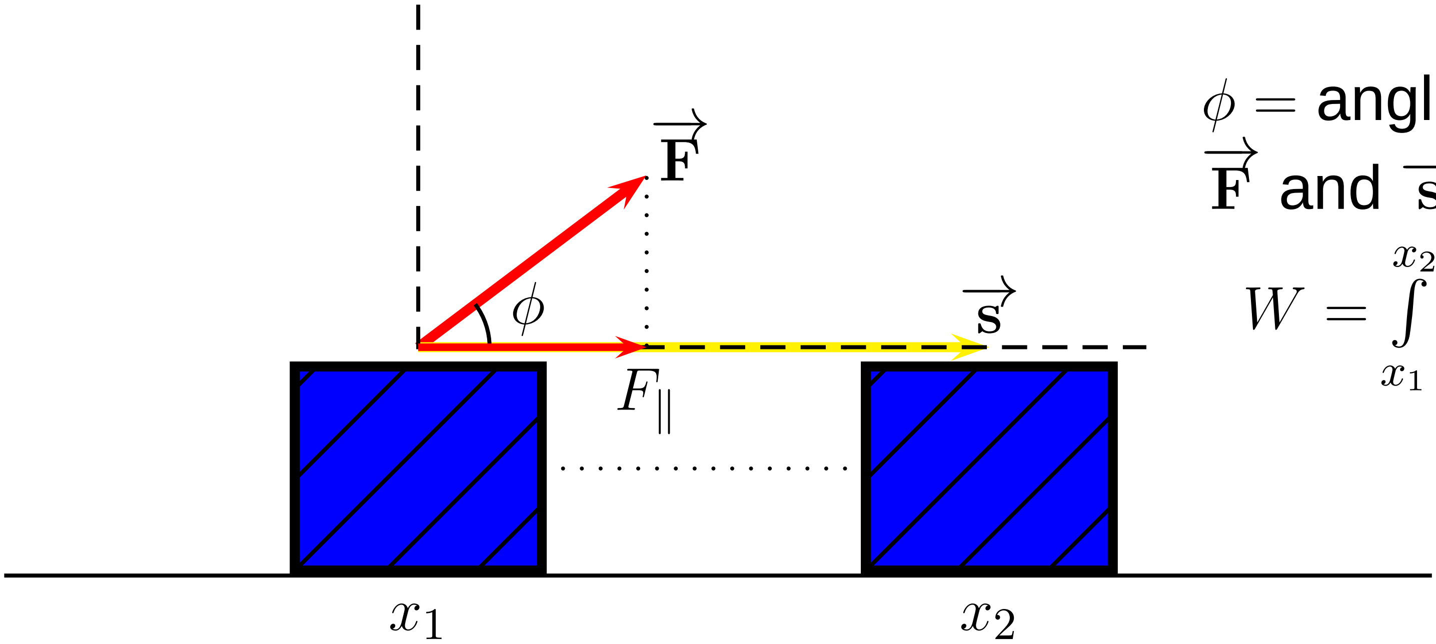

ϕ = angle between $\overrightarrow{\mathbf{F}}$ and $\overrightarrow{\mathbf{s}}$

$$W = \int_{x_1}^{x_2} F_{\|}\, dx$$

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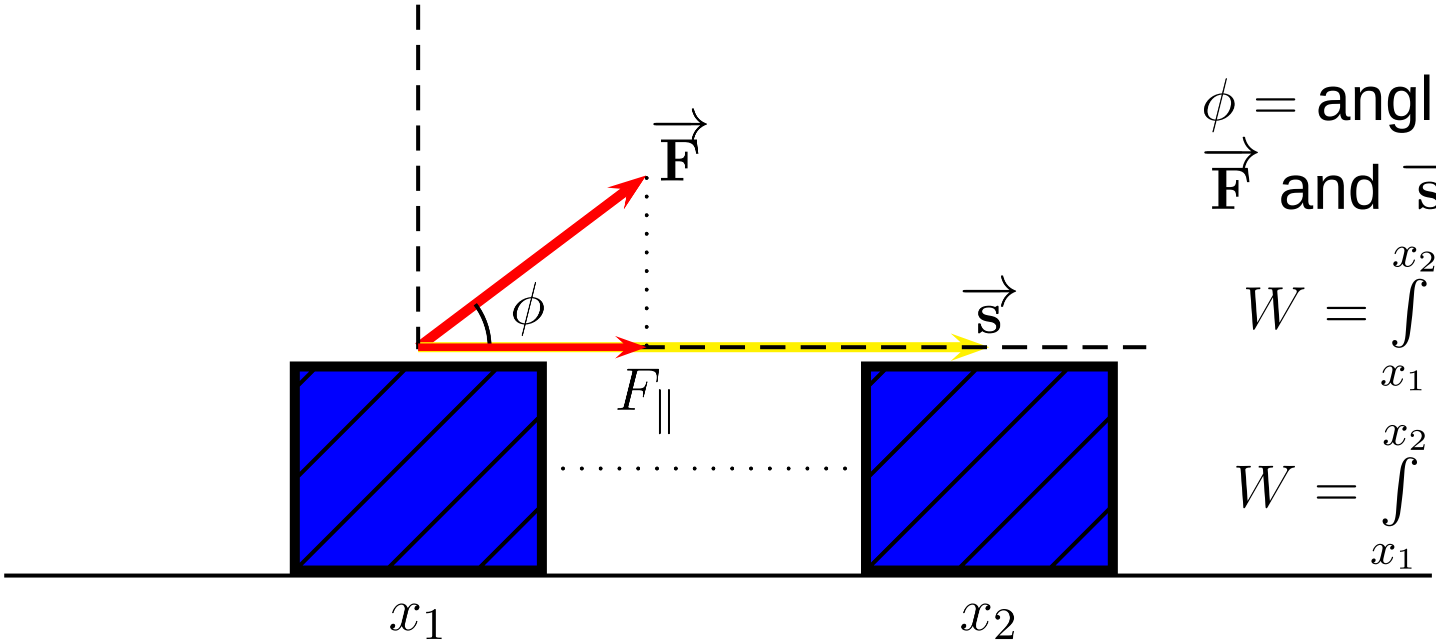

$\phi =$ angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$

$$W = \int_{x_1}^{x_2} F_{||}\, dx$$

$$W = \int_{x_1}^{x_2} F \cos\phi \, dx$$

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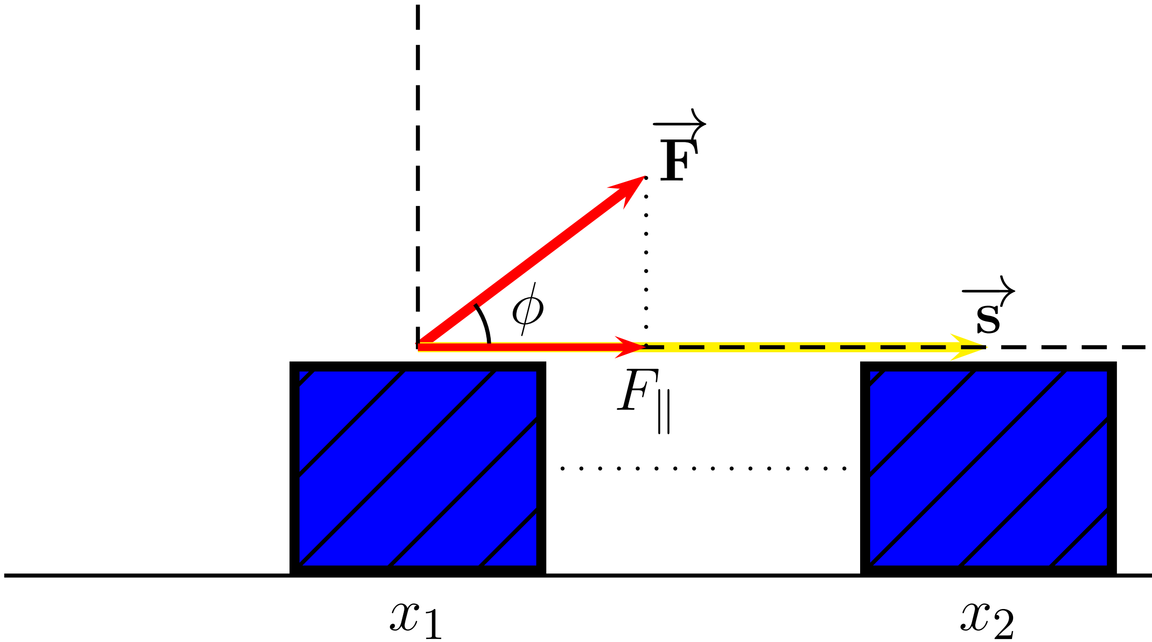

ϕ = angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$

$$W = \int_{x_1}^{x_2} F_{\parallel}\, dx$$

$$W = \int_{x_1}^{x_2} F \cos\phi\, dx$$

$$\boxed{W = \int_{x_1}^{x_2} F \cos\phi\, dx}$$

Straight-line displacement

Work-Energy Theorem (Again)

The work-energy theorem holds for variable forces!!

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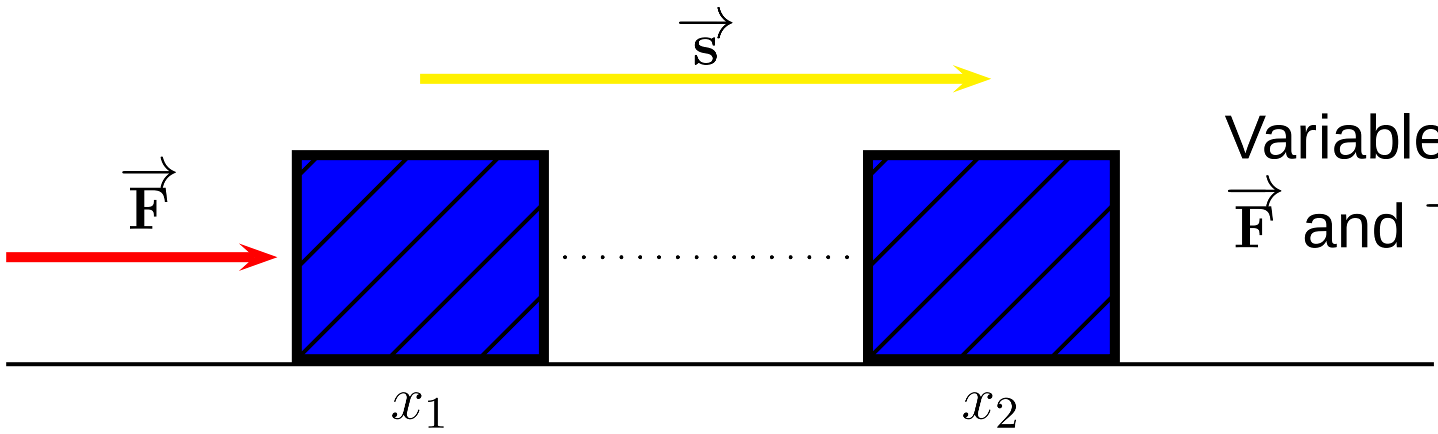

Variable F

$\vec{\mathbf{F}}$ and $\vec{\mathbf{s}}$ parallel

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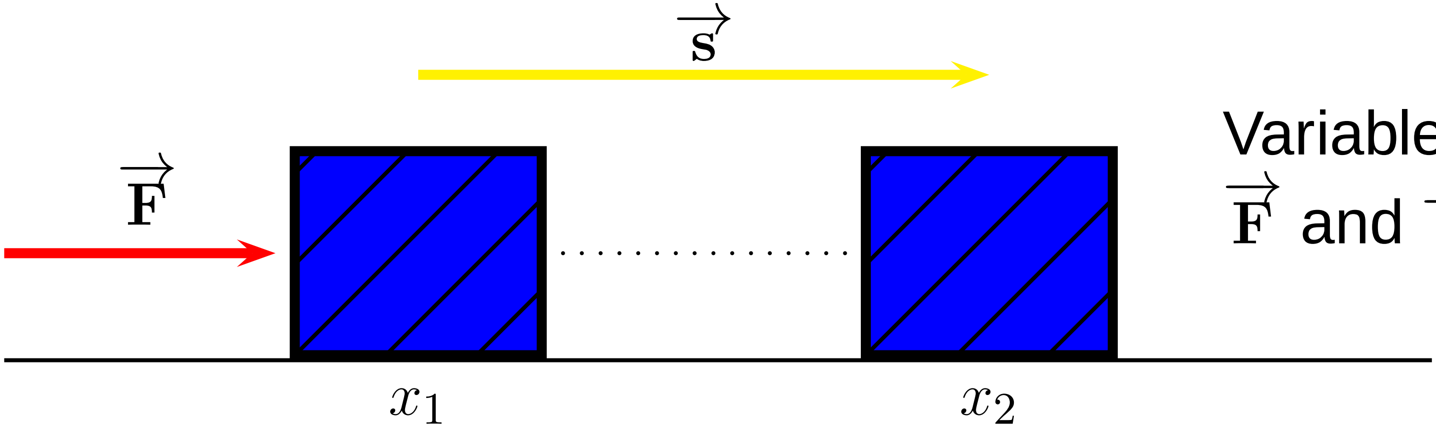

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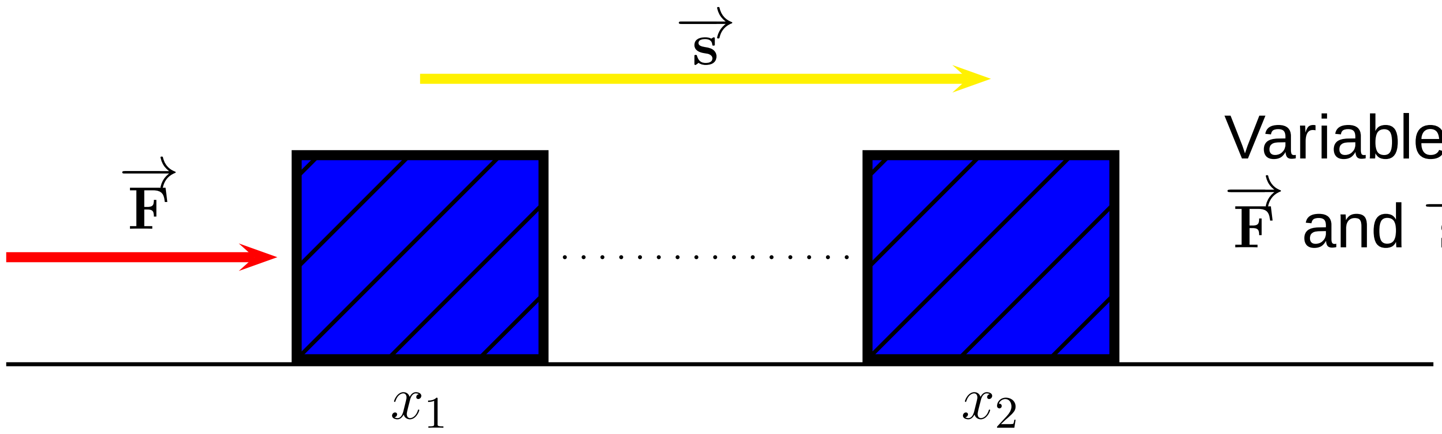

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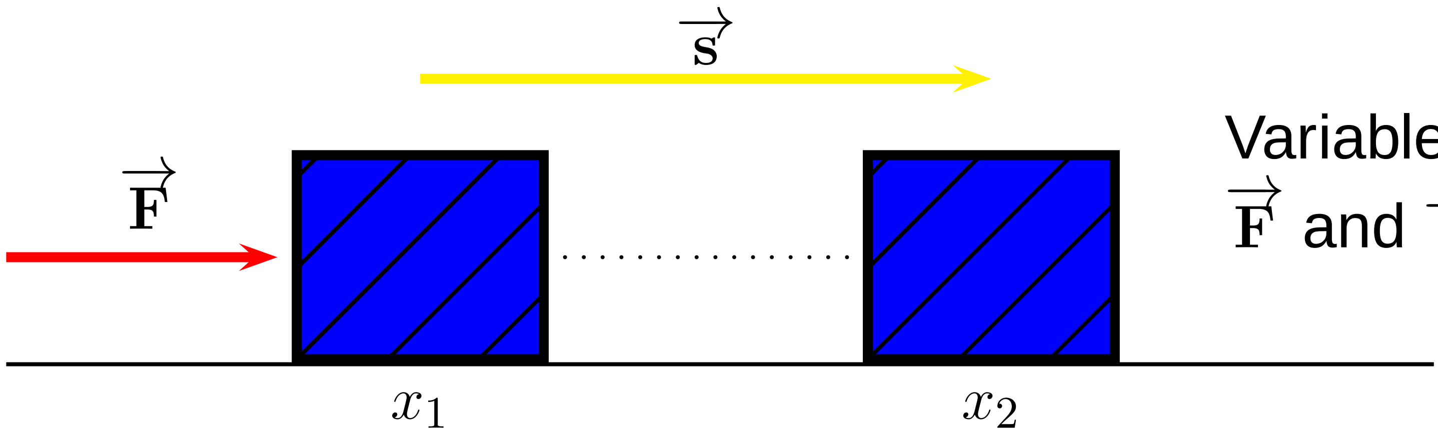

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$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} M a \, dx = M \int_{x_1}^{x_2} \left(\frac{dv}{dt} \right) dx$$

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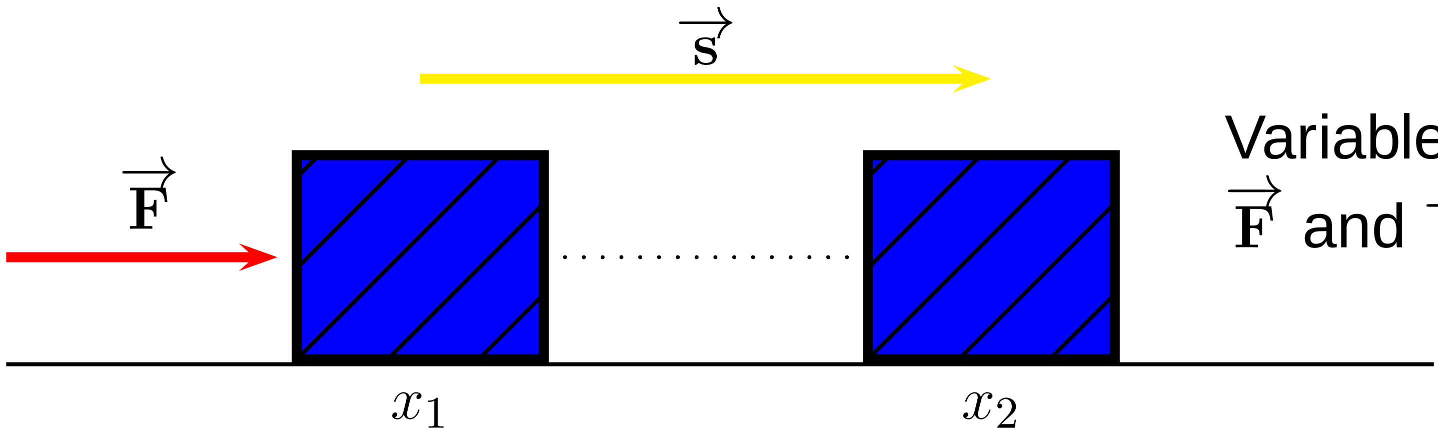

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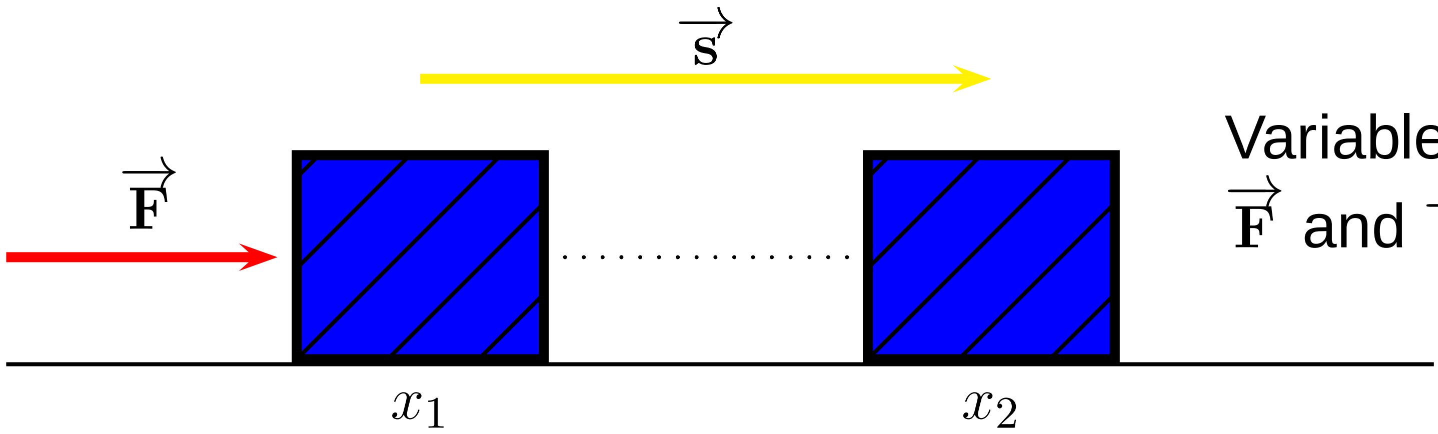

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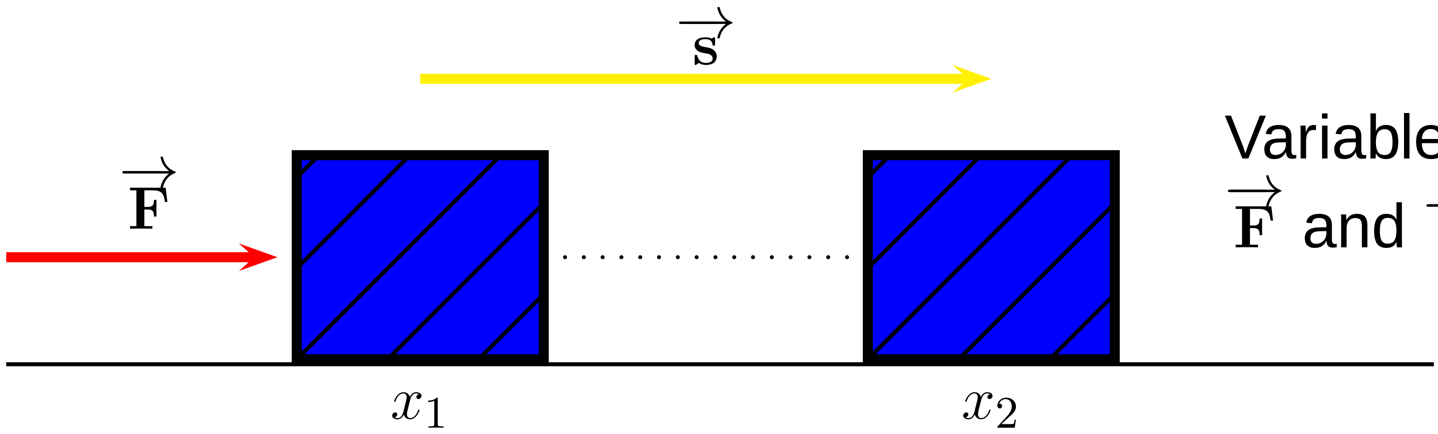

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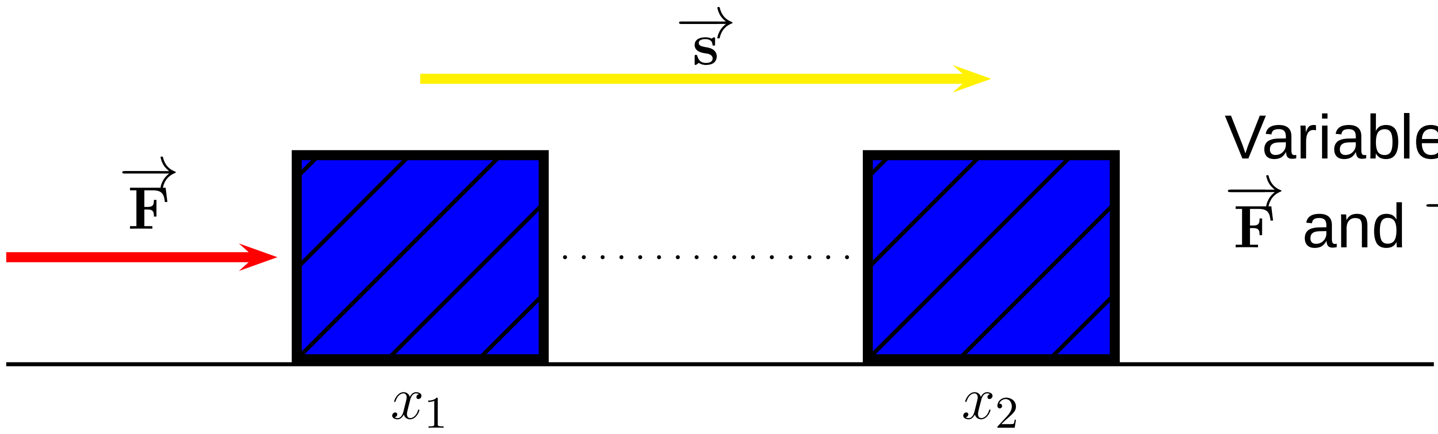

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$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} M a\, dx = M \int_{x_1}^{x_2} \left(\frac{dv}{dt}\right) \frac{dx}{dt} = M \int dv \left(\frac{dx}{dt}\right)$$

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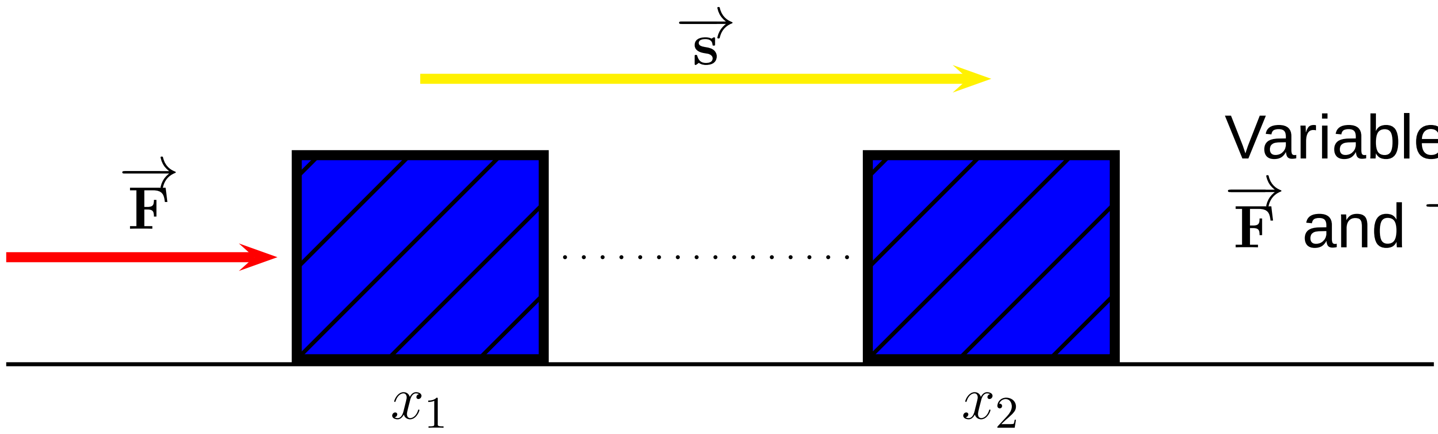

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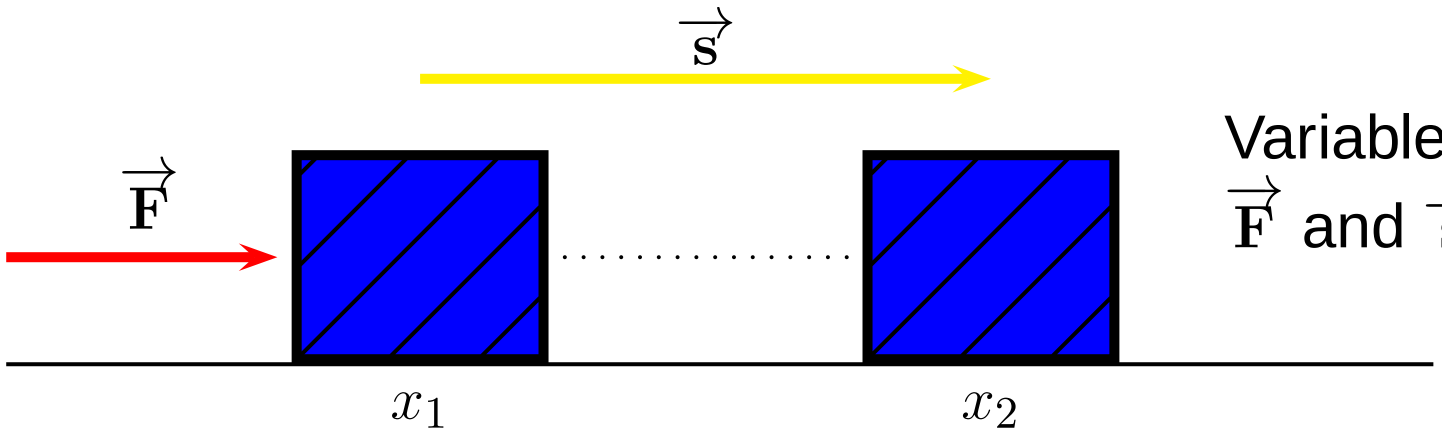

Variable F

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$$W = M \int dv \left(\frac{dx}{dt} \right) = M \int_{v_1}^{v_2} dv \ (v)$$

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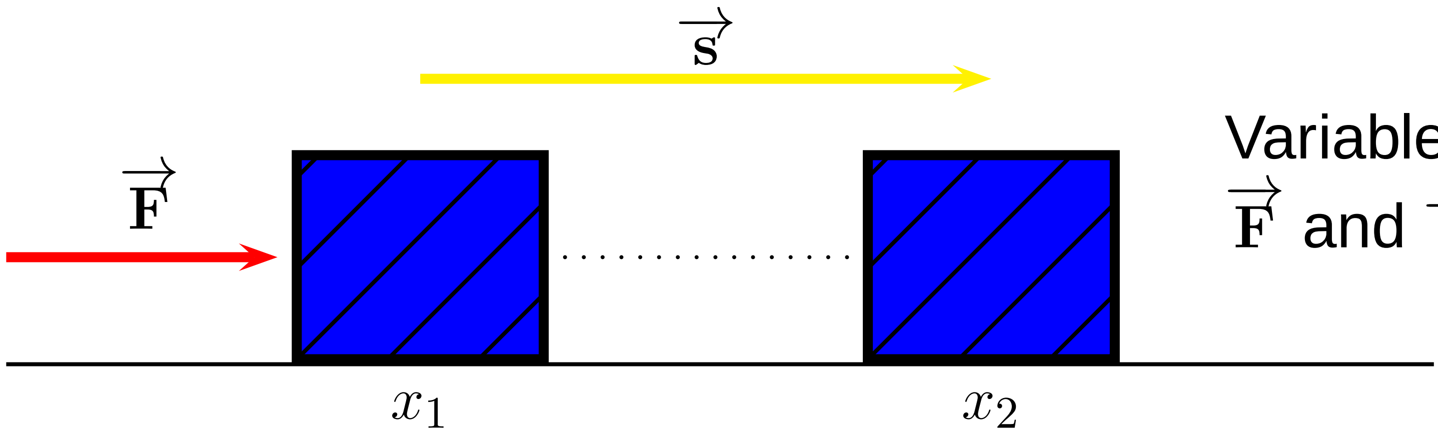

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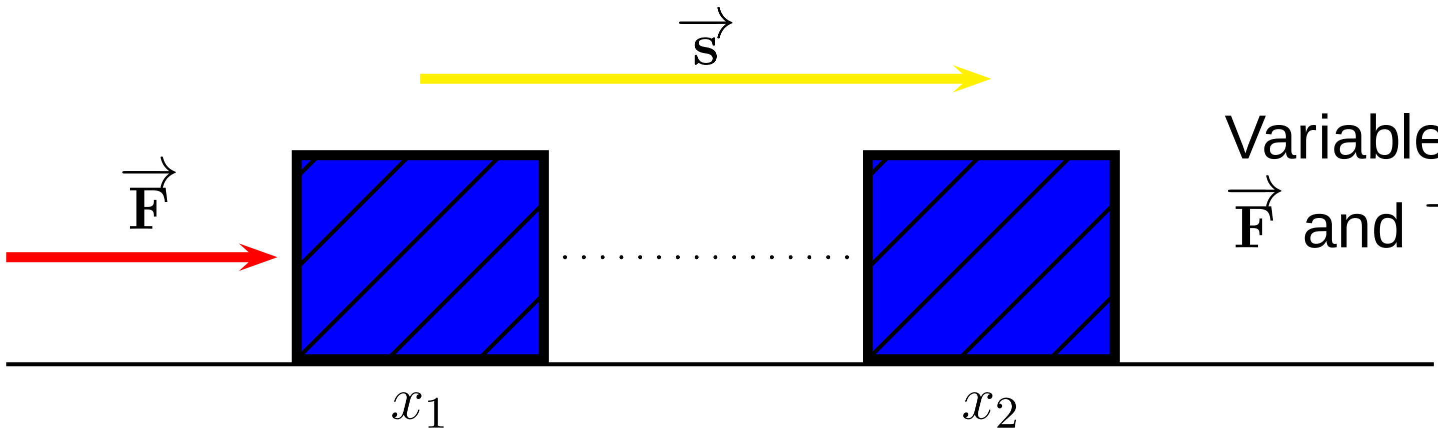

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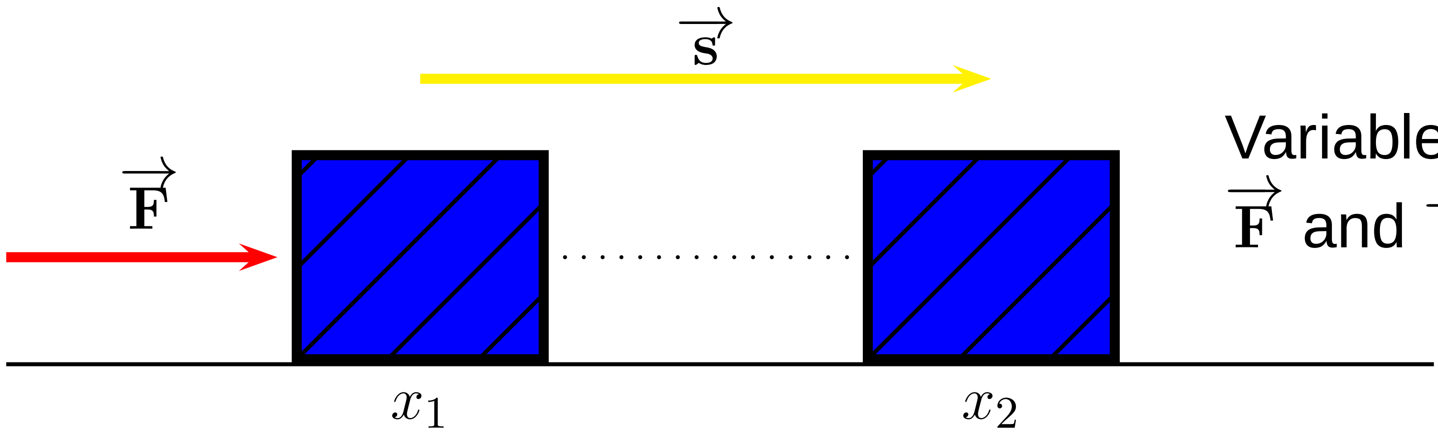

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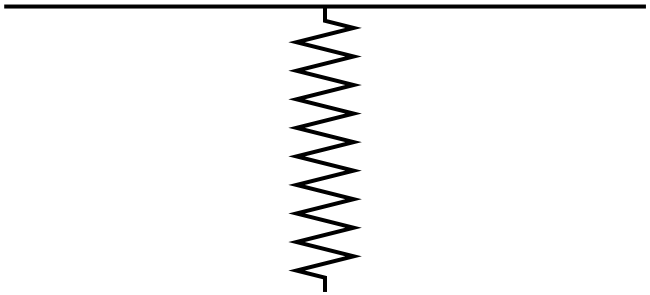
$$W = \frac{1}{2} M v_2^2 - \frac{1}{2} M v_1^2$$

Hooke's Law

A simple example of a variable force is the force needed to stretch a spring.



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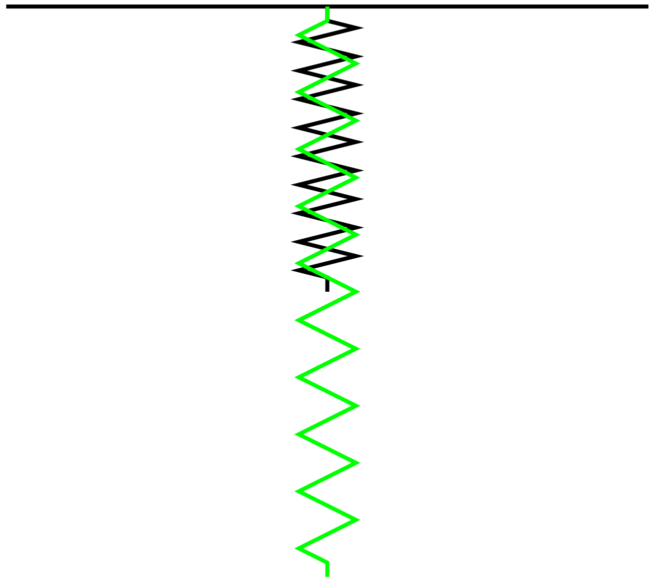

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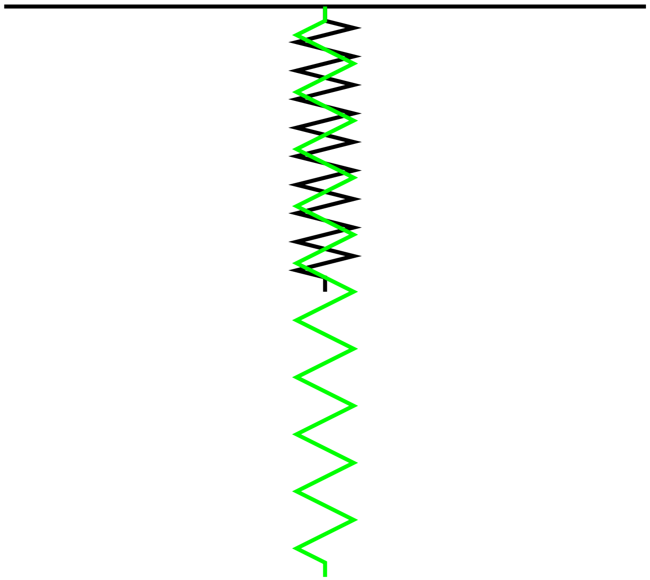

Hooke’s Law - The force needed to stretch or compress a spring increases linearly with stretching distance

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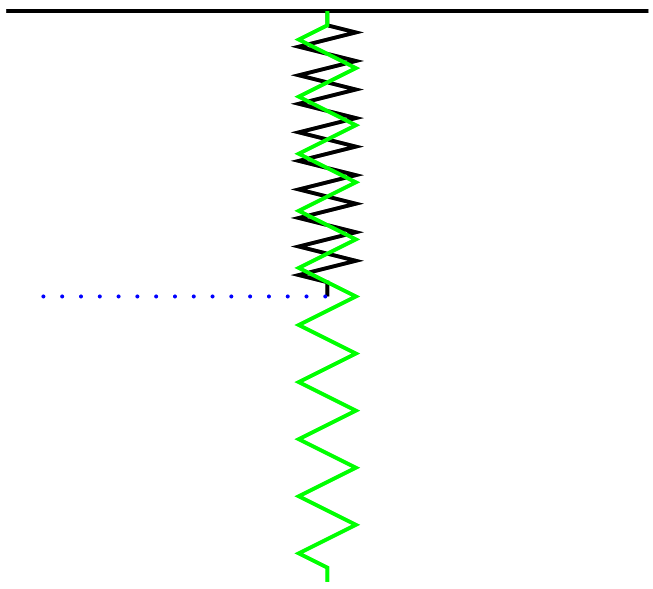

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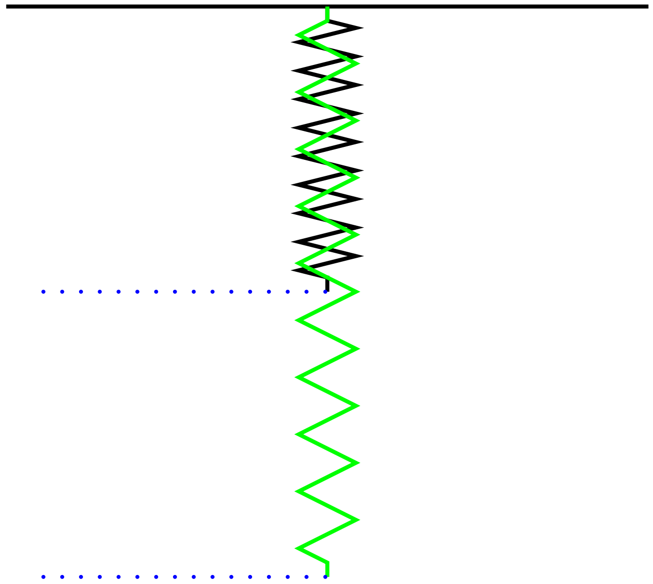

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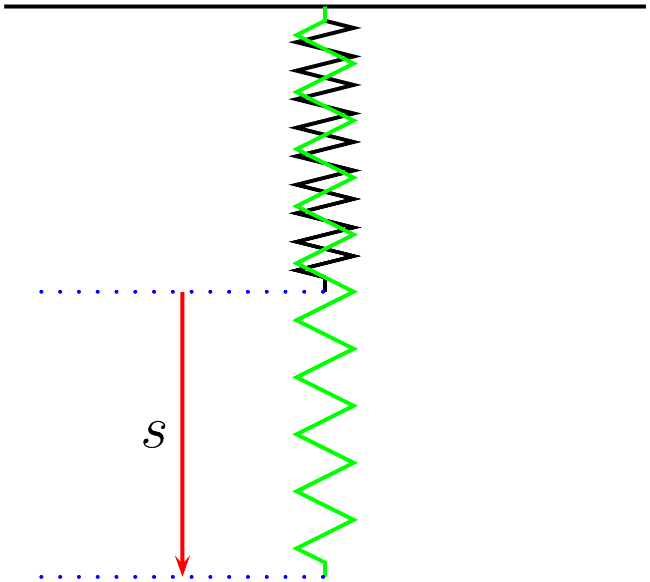

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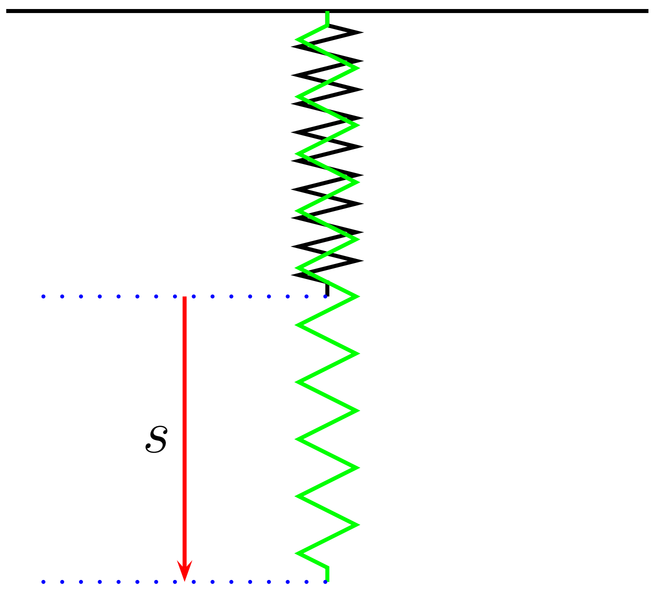

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$$\boxed{F = ks}$$

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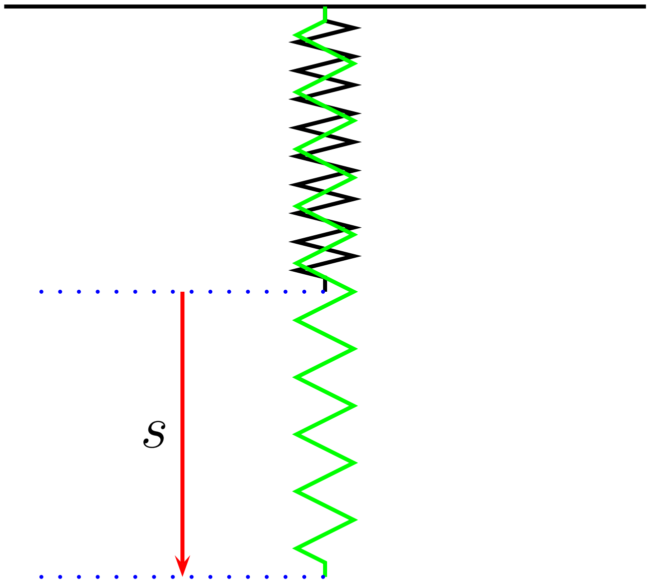

Hooke's Law - The force needed to stretch or compress a spring increases linearly with stretching distance

$$\boxed{F = ks}$$

k = spring constant
s = stretching distance

Hooke's Law

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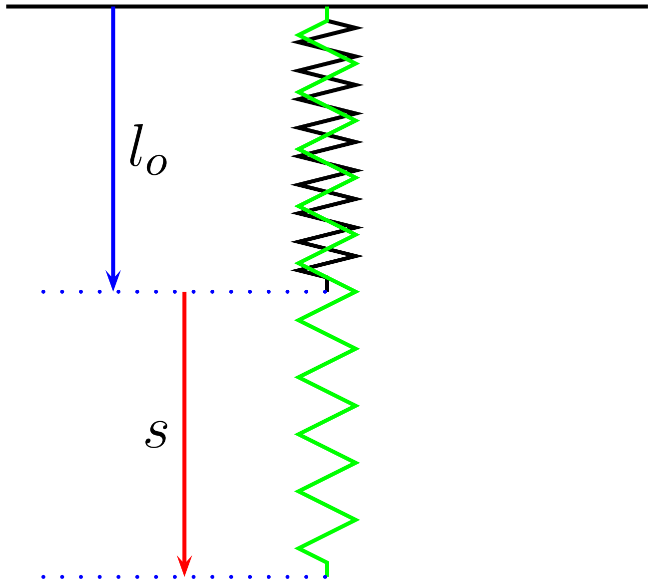

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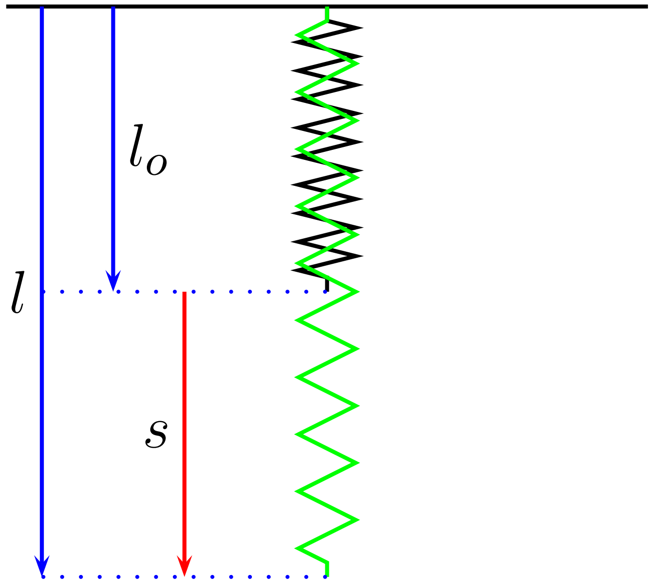

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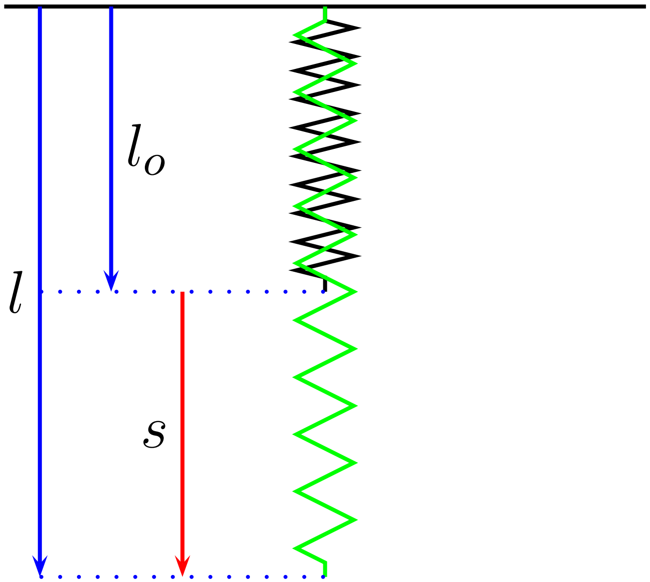

$s = l - l_o$

Hooke's Law - The force needed to stretch or compress a spring increases linearly with stretching distance

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Spring Examples

Example: A $5\,kg$ mass is hung from a spring which stretches $10\,cm$. What is the spring constant?

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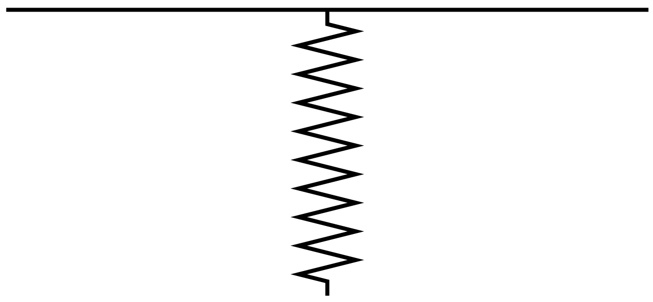
Example: How far would a spring with a constant twice as large stretch when $5\,kg$ is hung from it?

Work to Stretch a Spring



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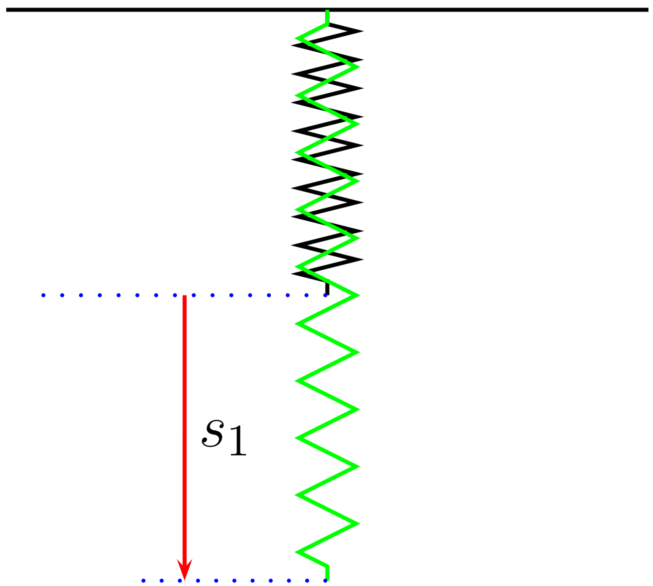

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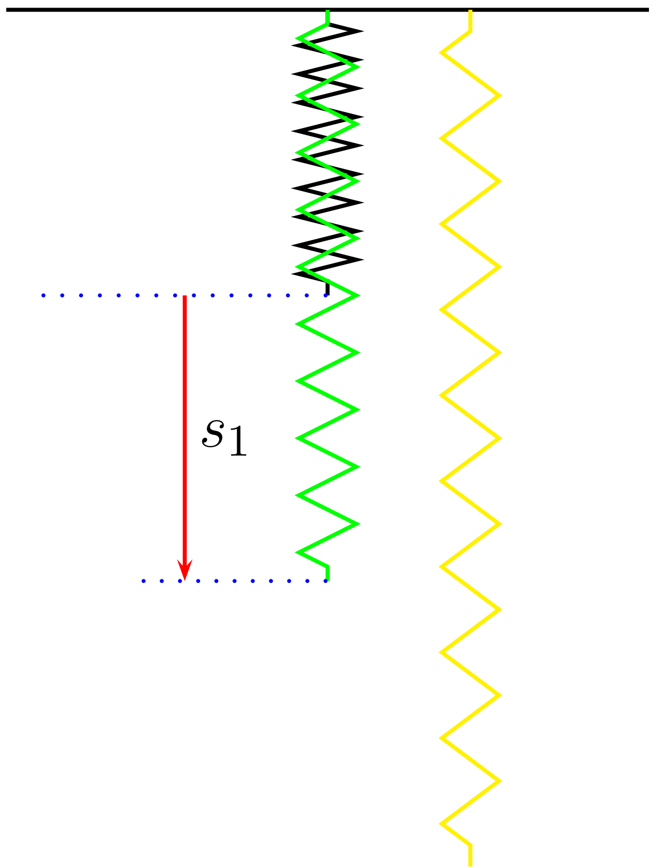

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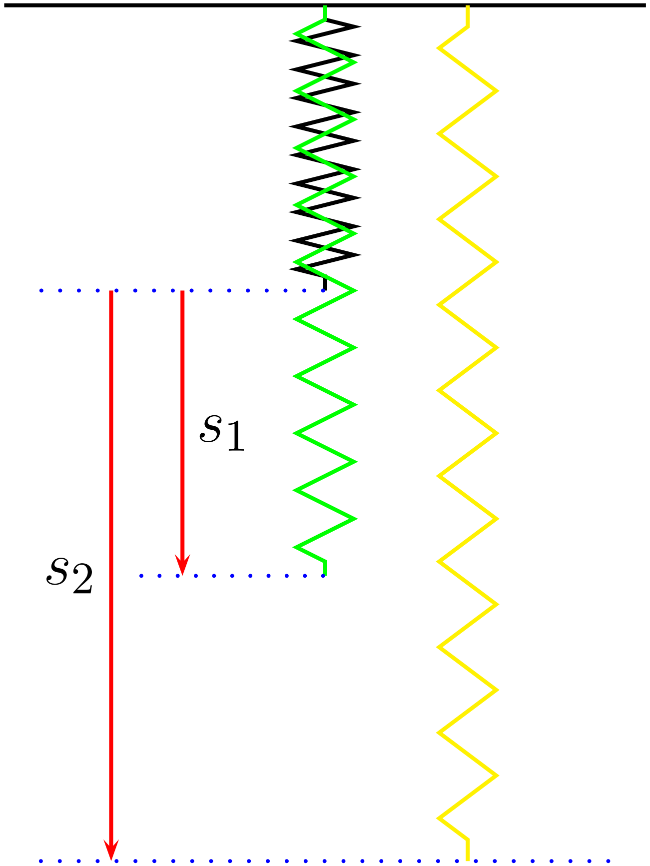

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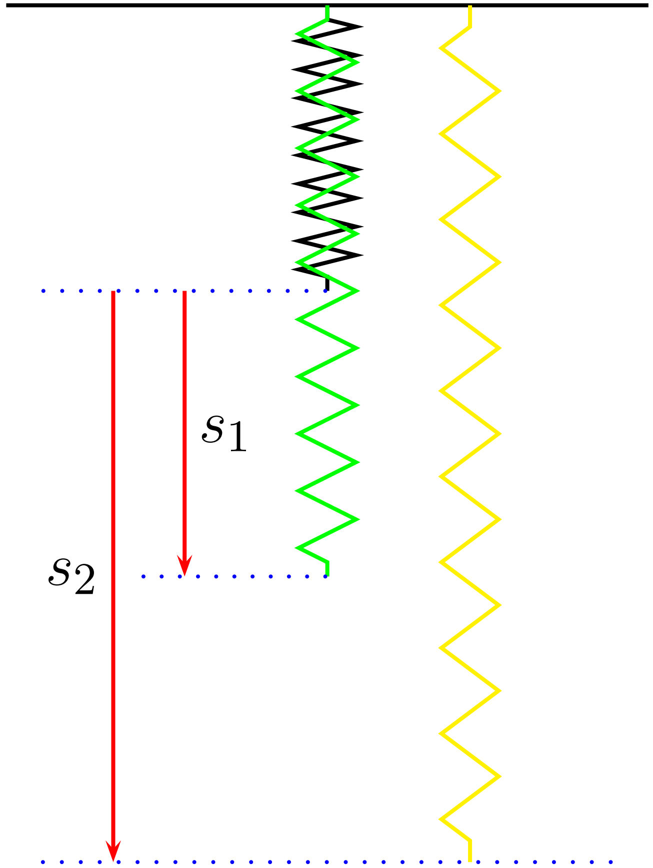

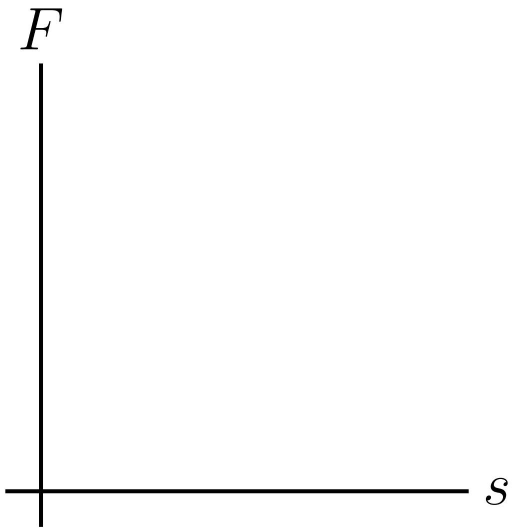

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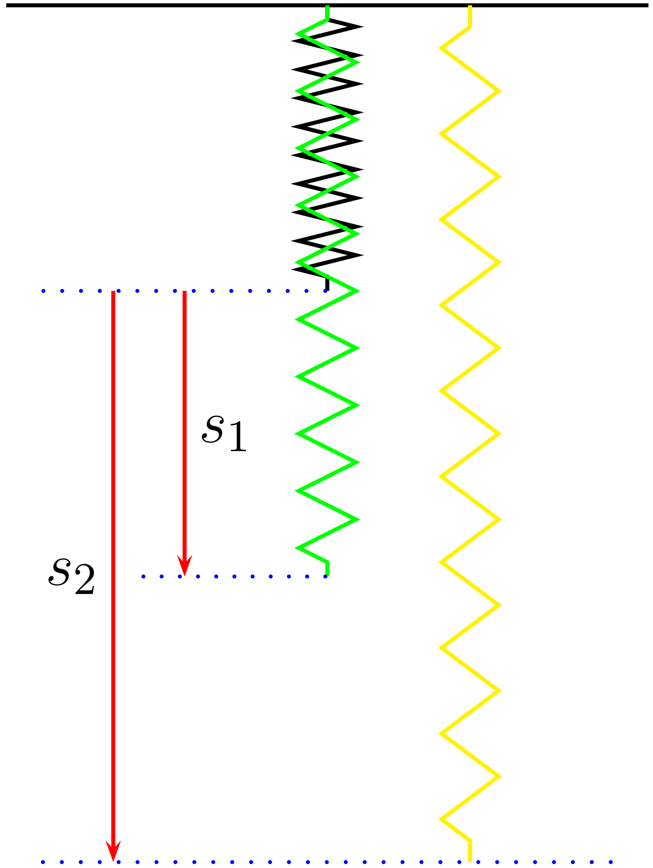
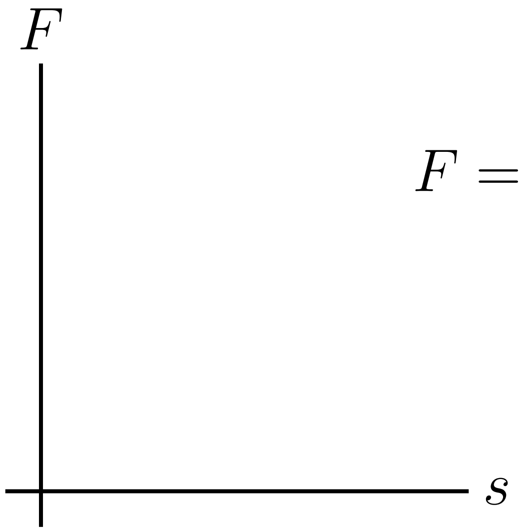
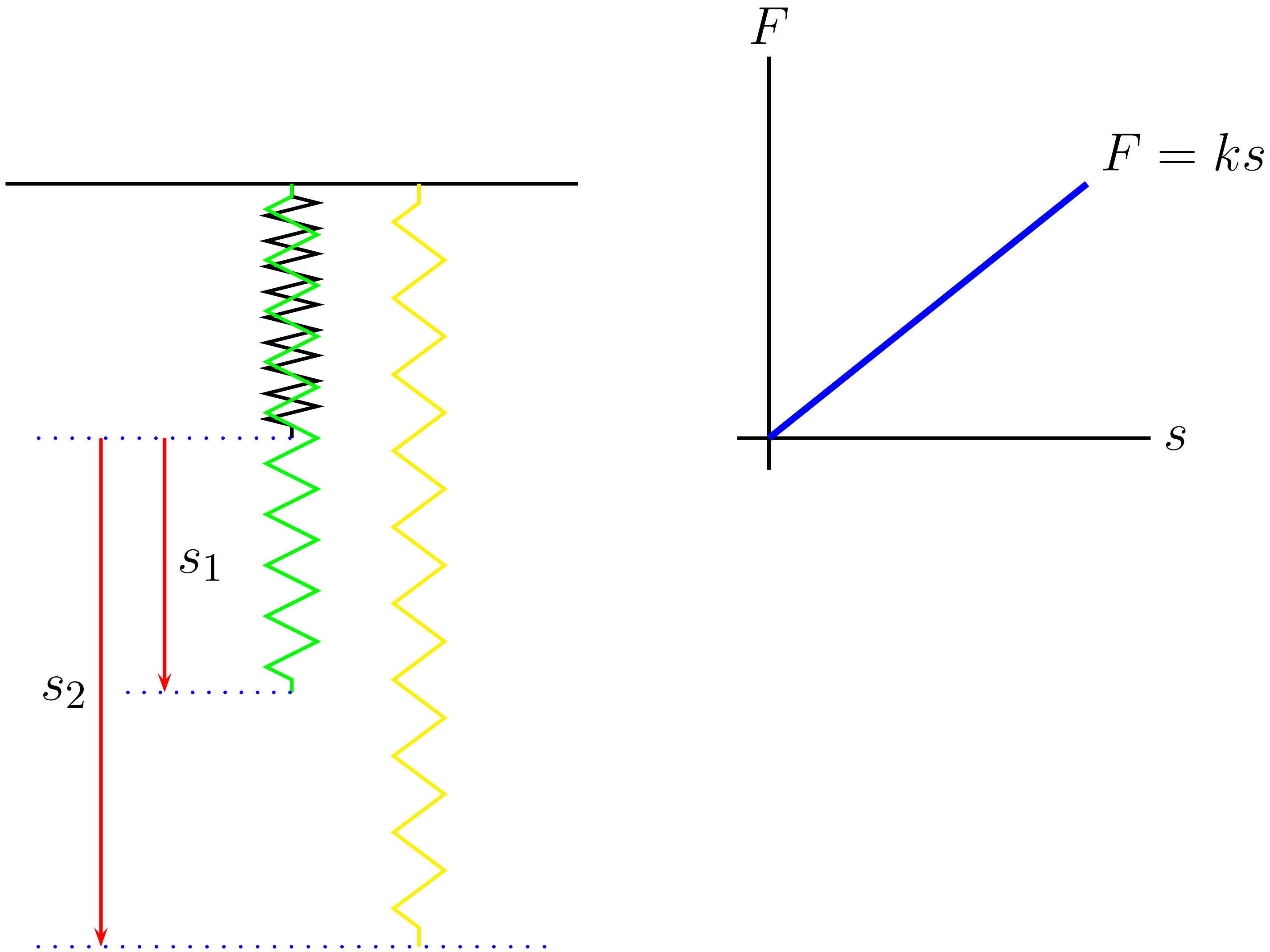




$F = ks$

Work to Stretch a Spring



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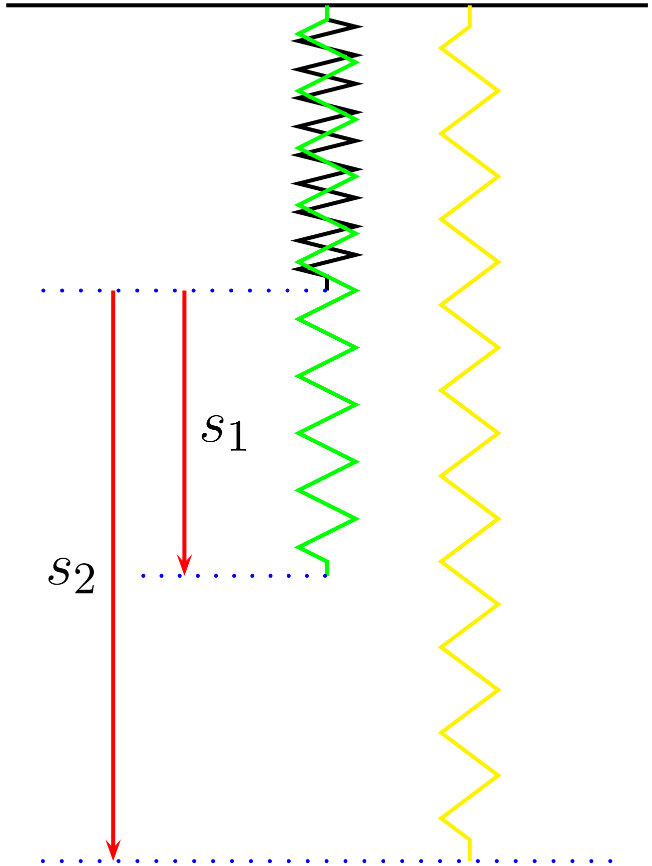

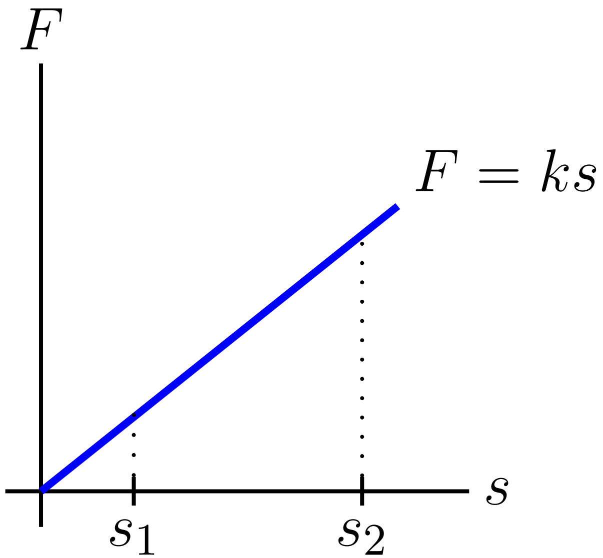

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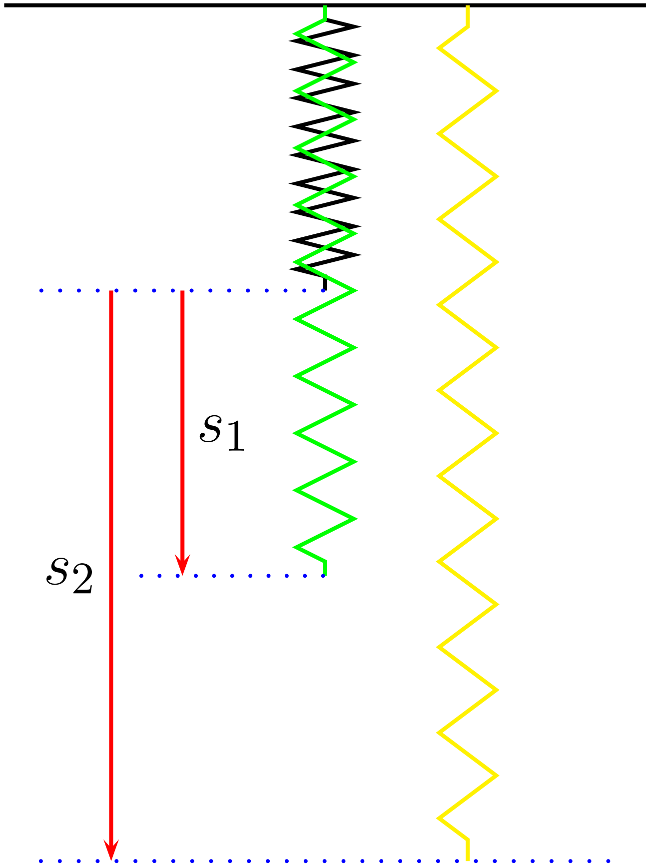

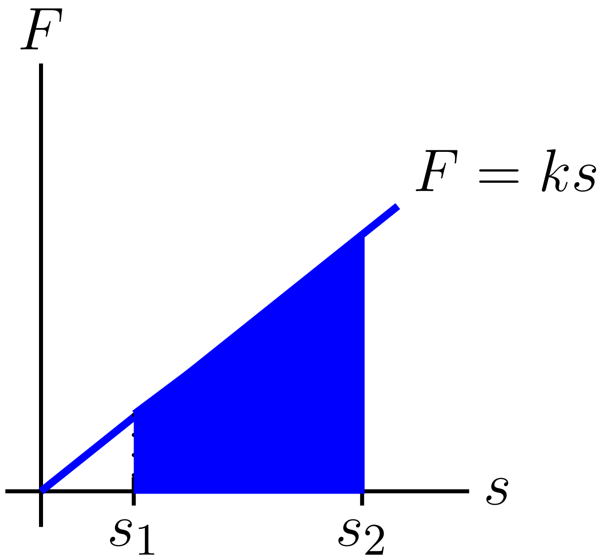

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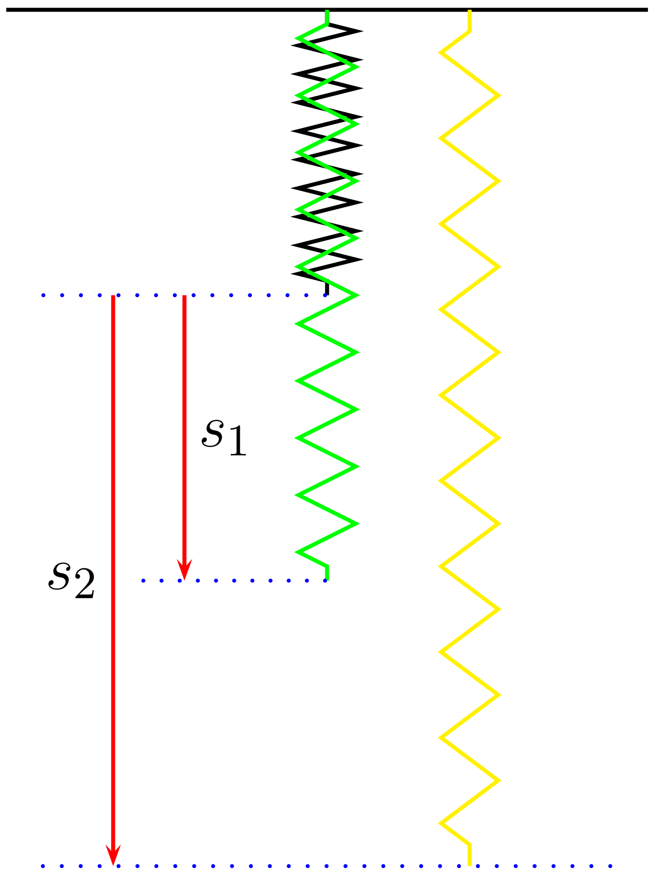
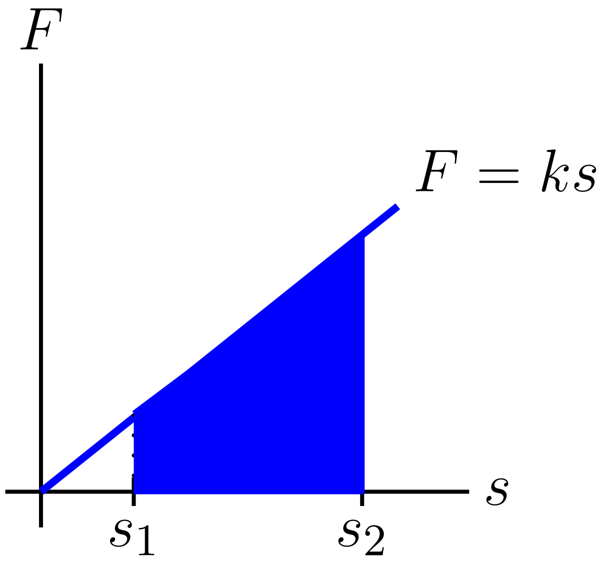
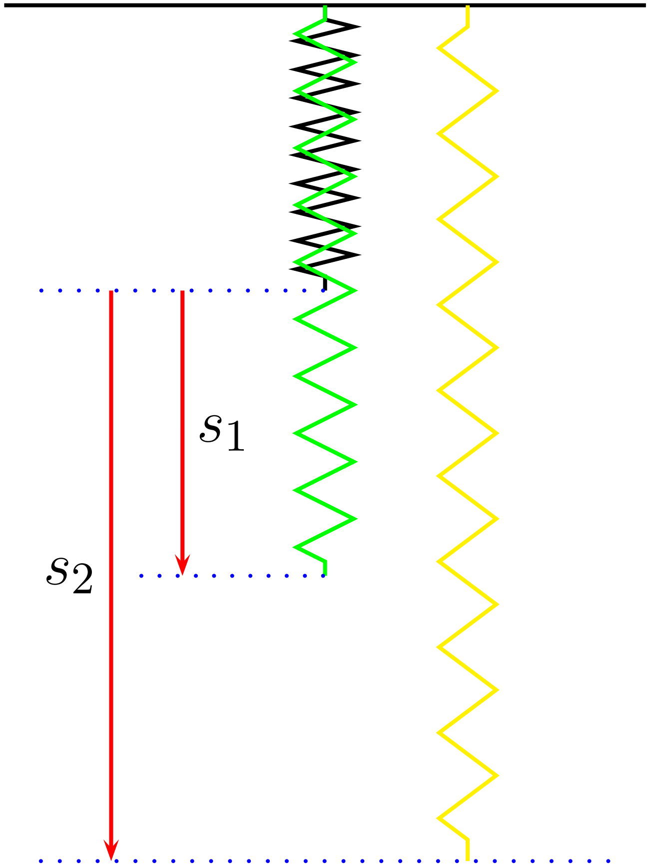
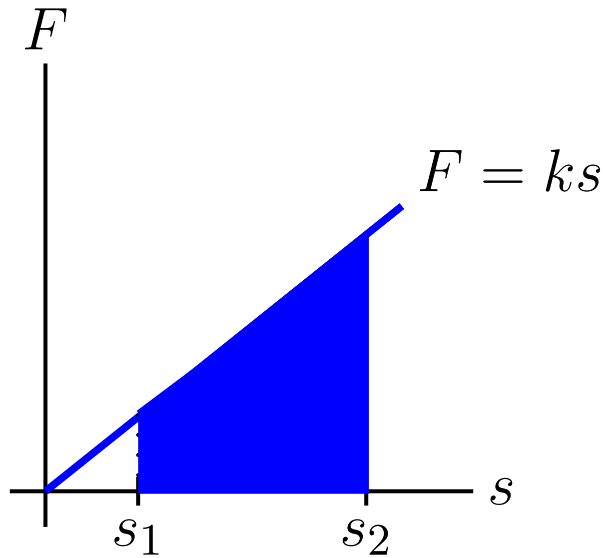




$$W = \frac{1}{2}(s_2)(F_2) - \frac{1}{2}(s_1)(F_1)$$

Work to Stretch a Spring





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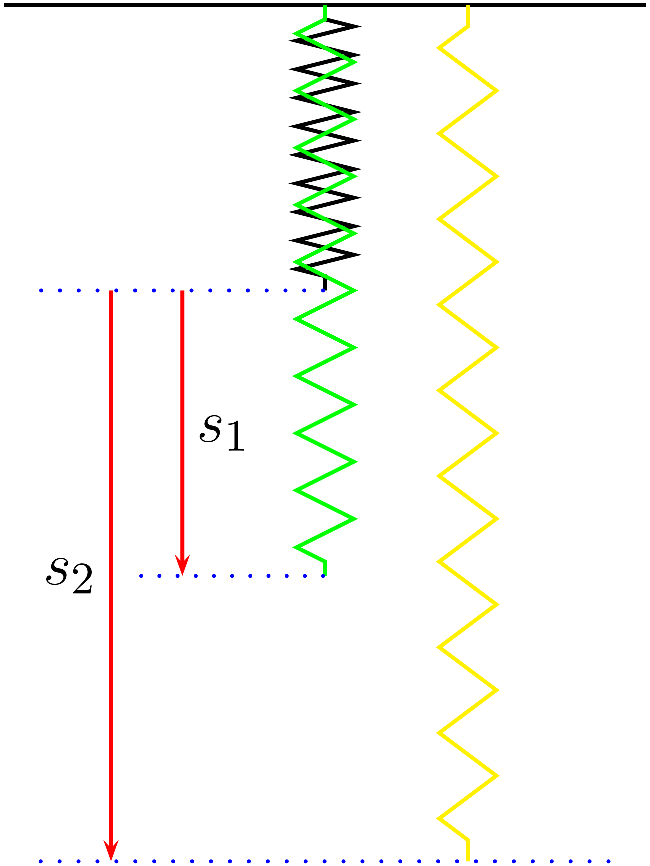
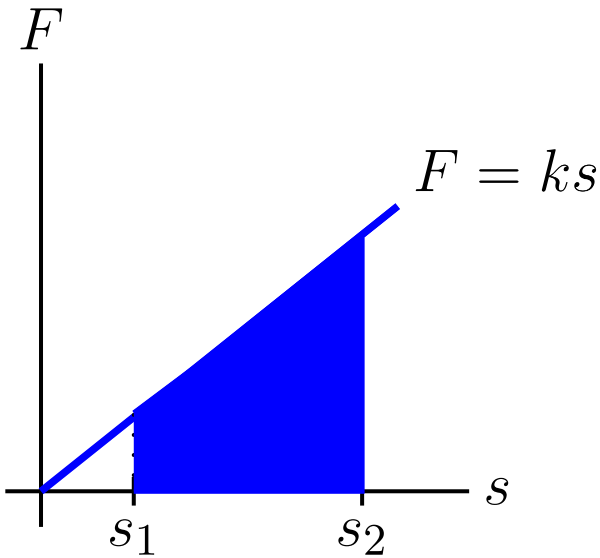
$$W = \frac{1}{2}(s_2)(ks_2) - \frac{1}{2}(s_1)(ks_1)$$

Work to Stretch a Spring





$$W = \frac{1}{2}(s_2)(F_2) - \frac{1}{2}(s_1)(F_1)$$

$$W = \frac{1}{2}(s_2)(ks_2) - \frac{1}{2}(s_1)(ks_1)$$

$$W = \frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2$$