

# February 13, Week 5

Today: Chapter 3, Finish Projectile Motion

Exam #2 should be graded by Wednesday.

Homework 4, Due February 20.

Mastering Physics: 9 problems from chapters 1 and 3.

Written Question: 3.56

# Projectile Equations

Projectile - Any object that is launched into motion and then acted on by only gravity.

$$a_x = 0 \quad a_y = -g$$

## Projectile Equations

$v_x = v_{o,x}$	$v_y = v_{o,y} - gt$
$x = x_o + v_{o,x}t$	$y = y_o + v_{o,y}t - \frac{1}{2}gt^2$
$v_{o,x} = v_o \cos \alpha$	$v_{o,y} = v_o \sin \alpha$

# Trajectory Equation

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$\tan \alpha$

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Simplifying gives:

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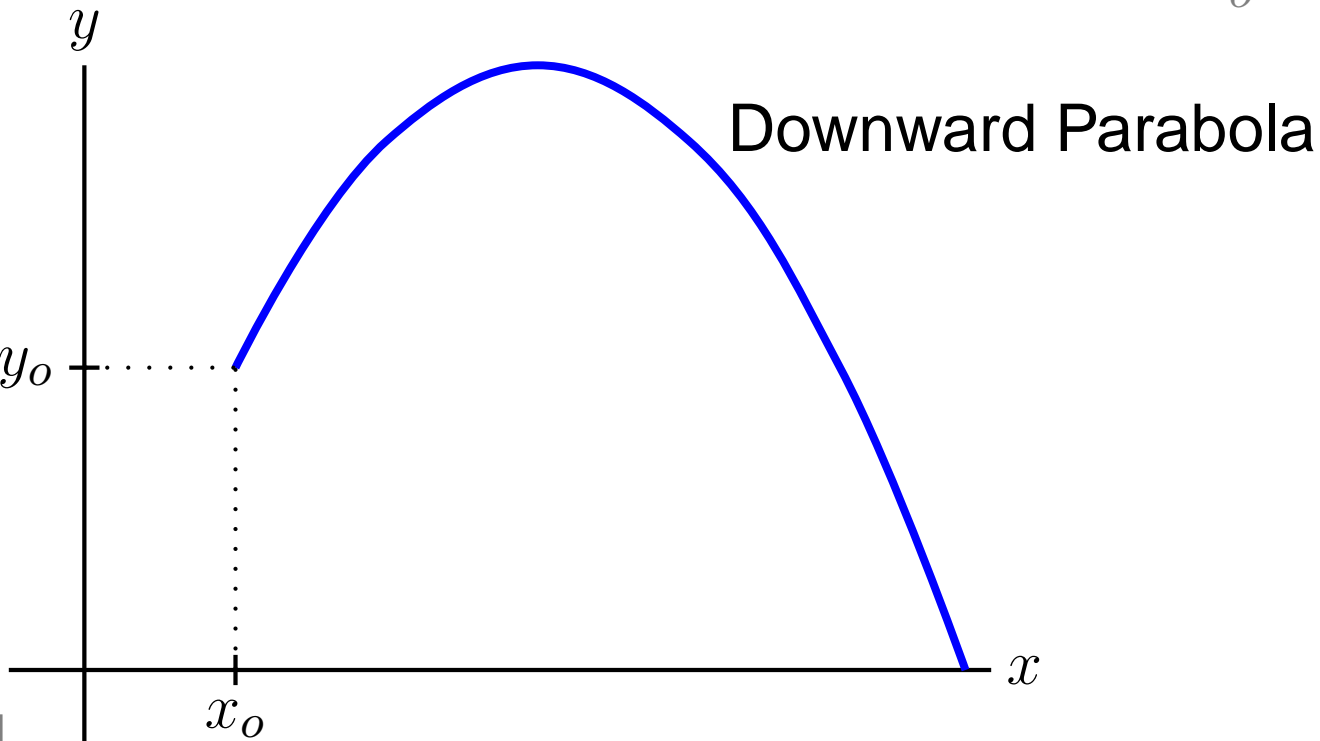
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Downward Parabola

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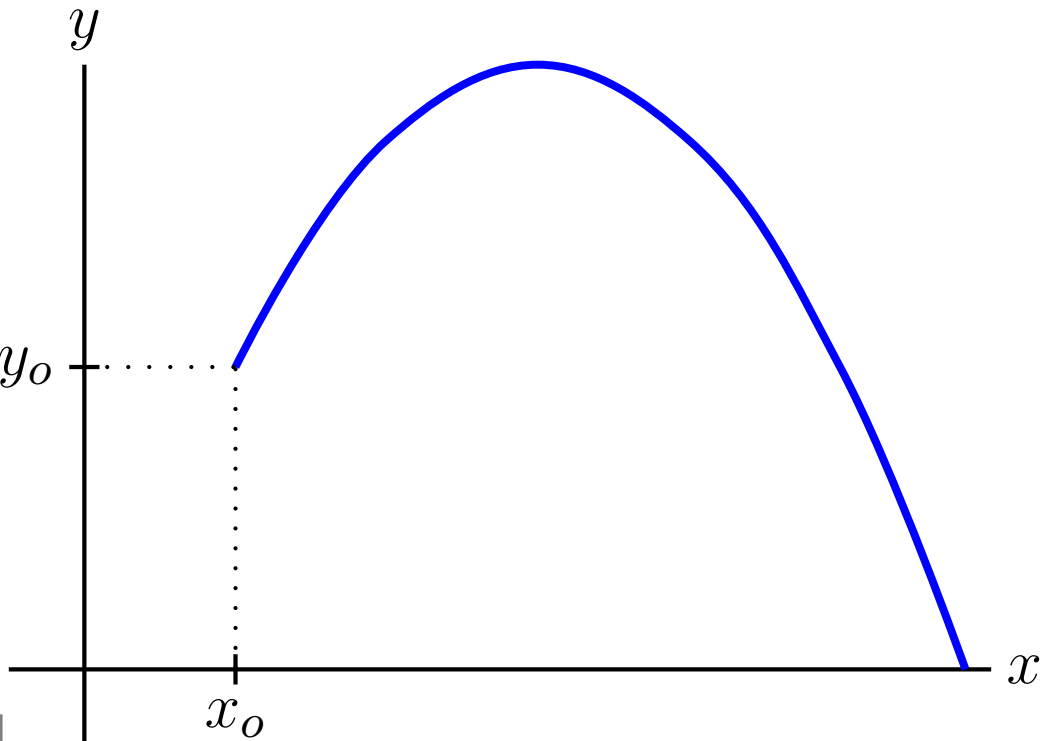
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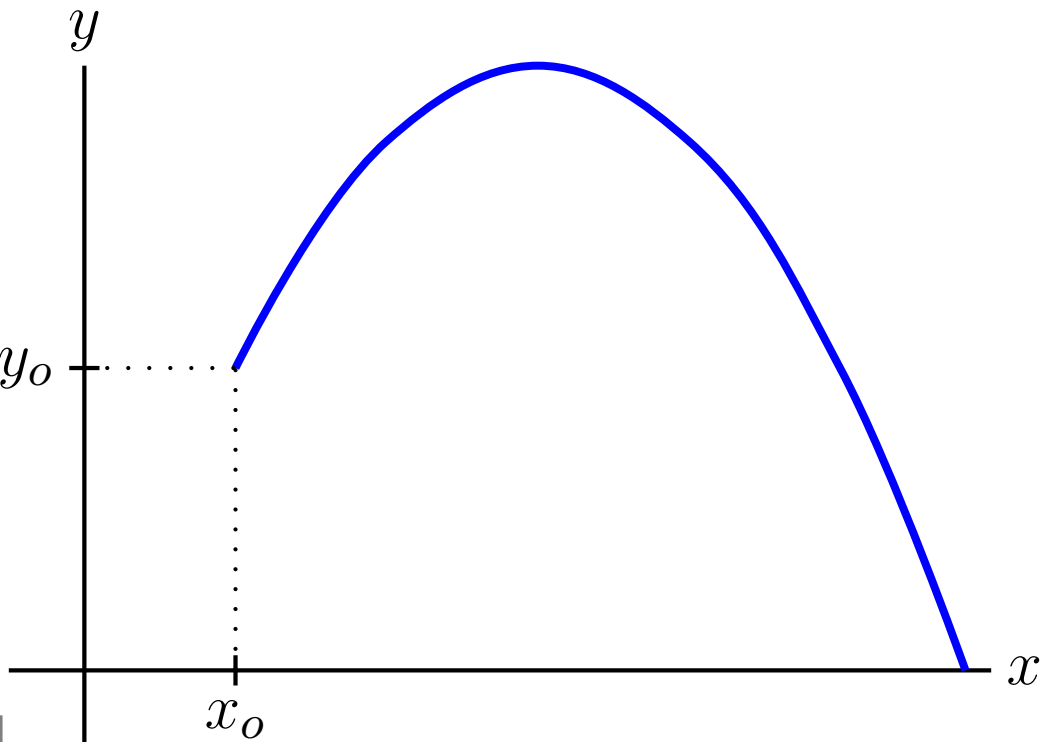
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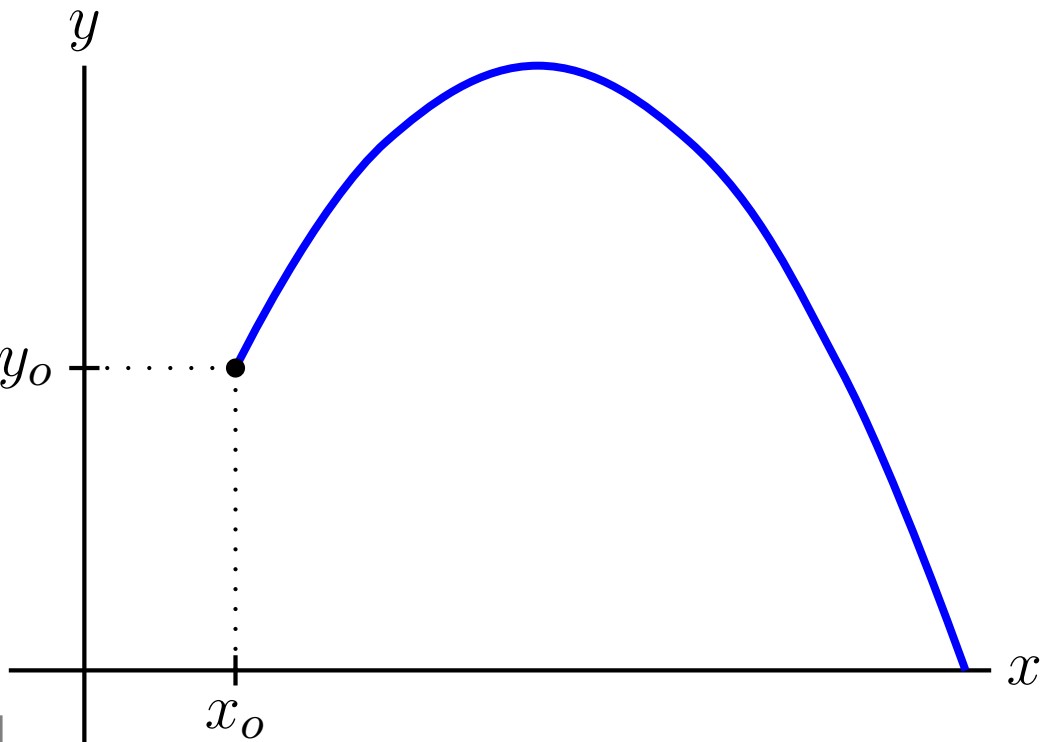
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Time of flight -  $t_f$

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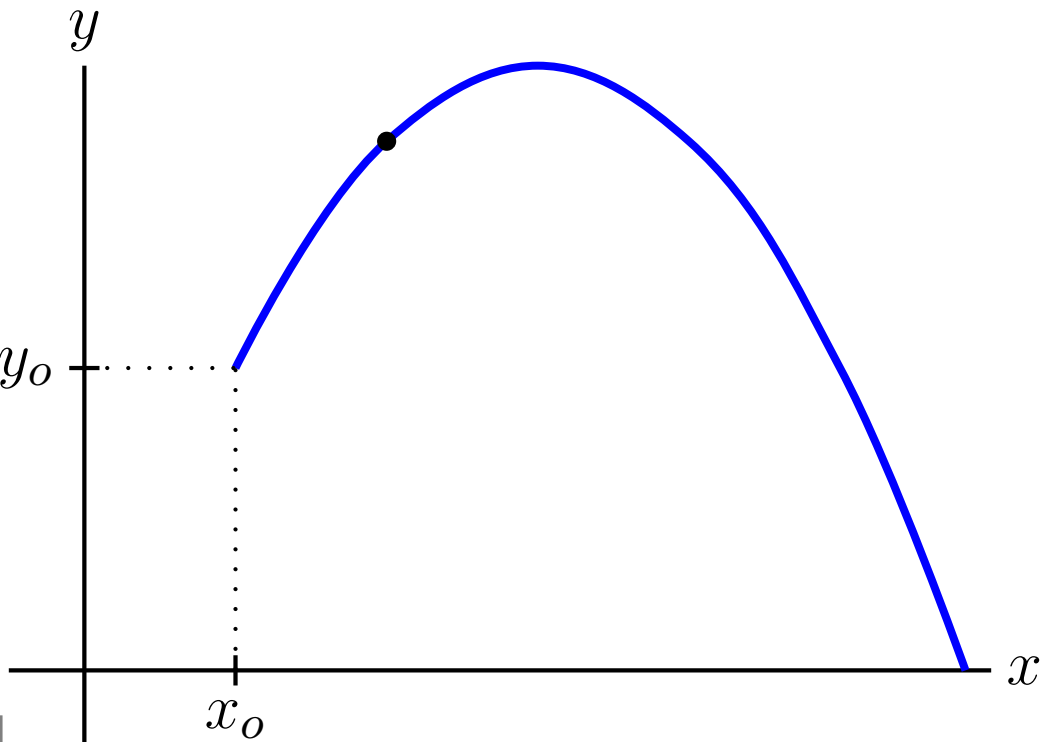
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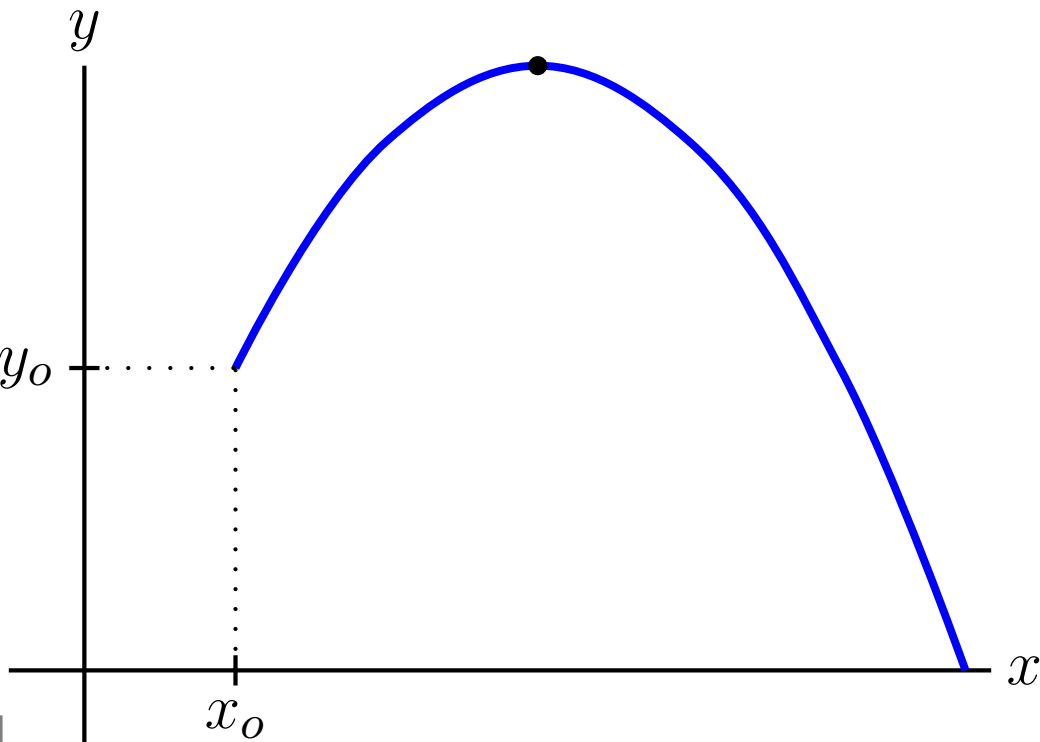
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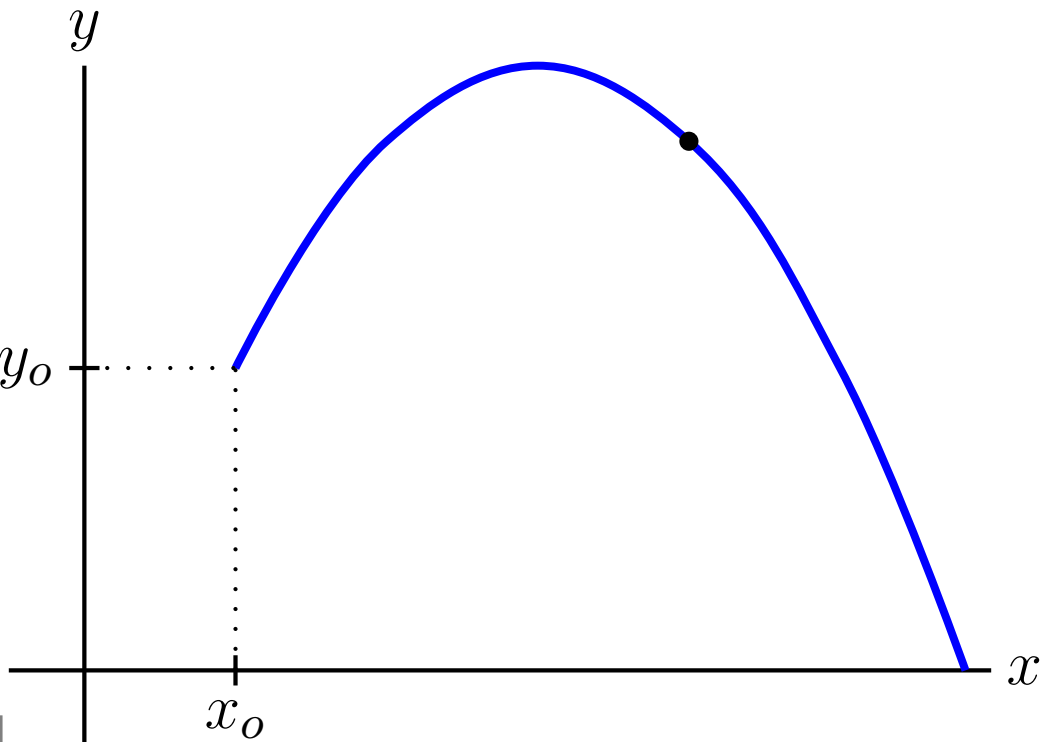
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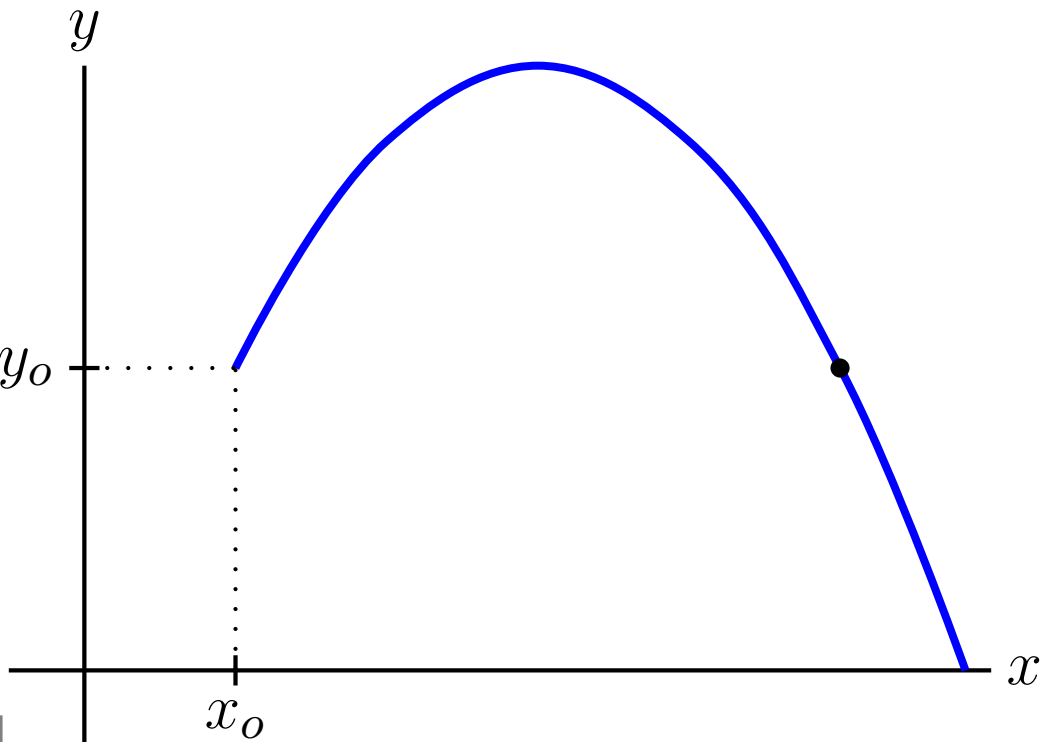
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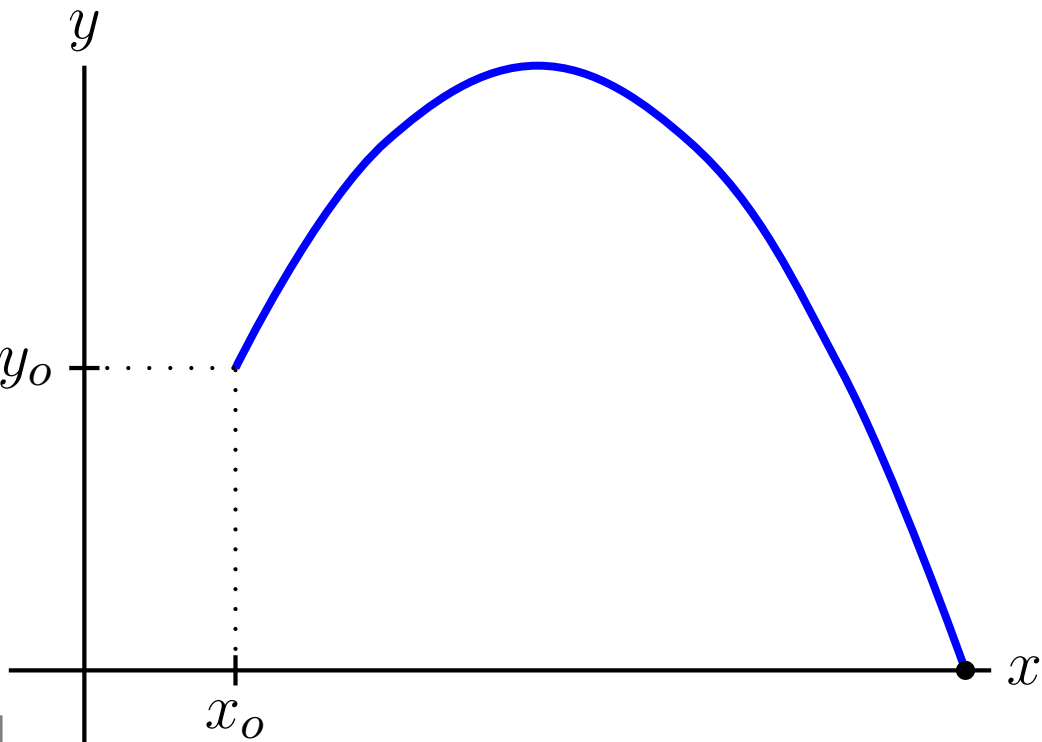
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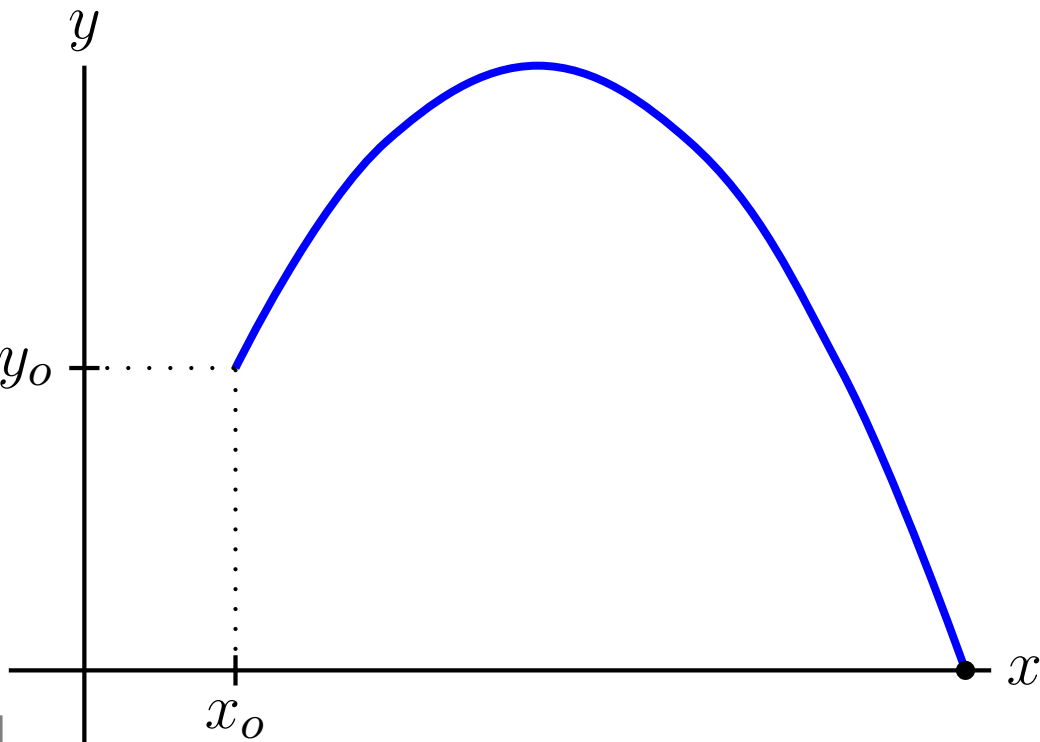
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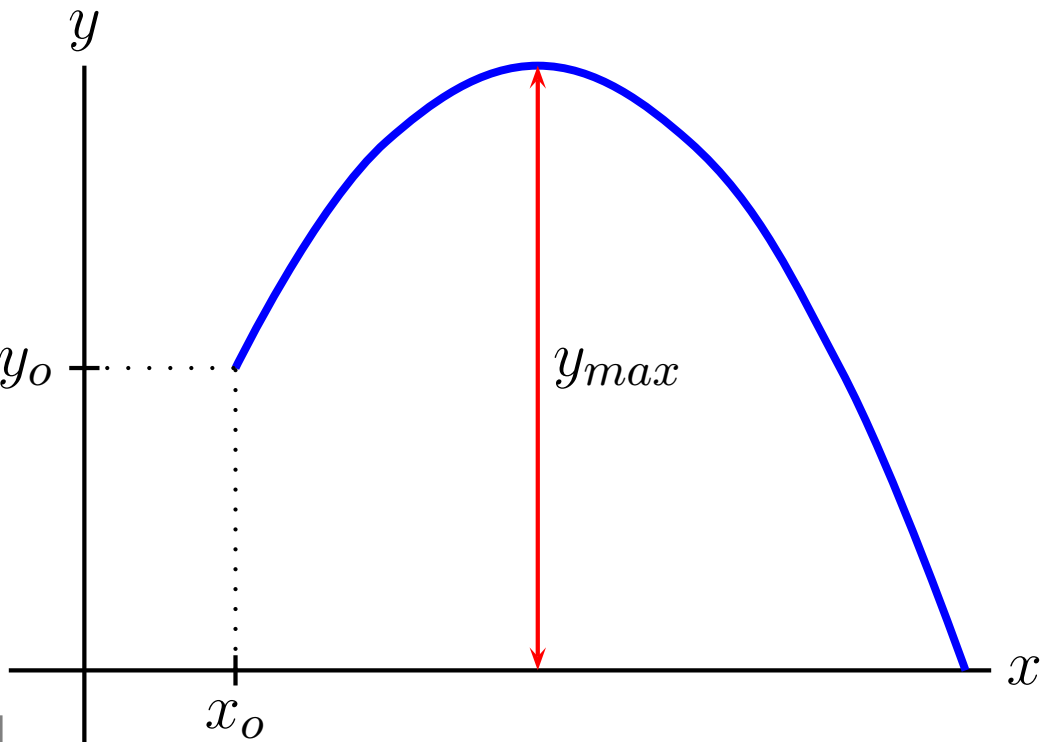
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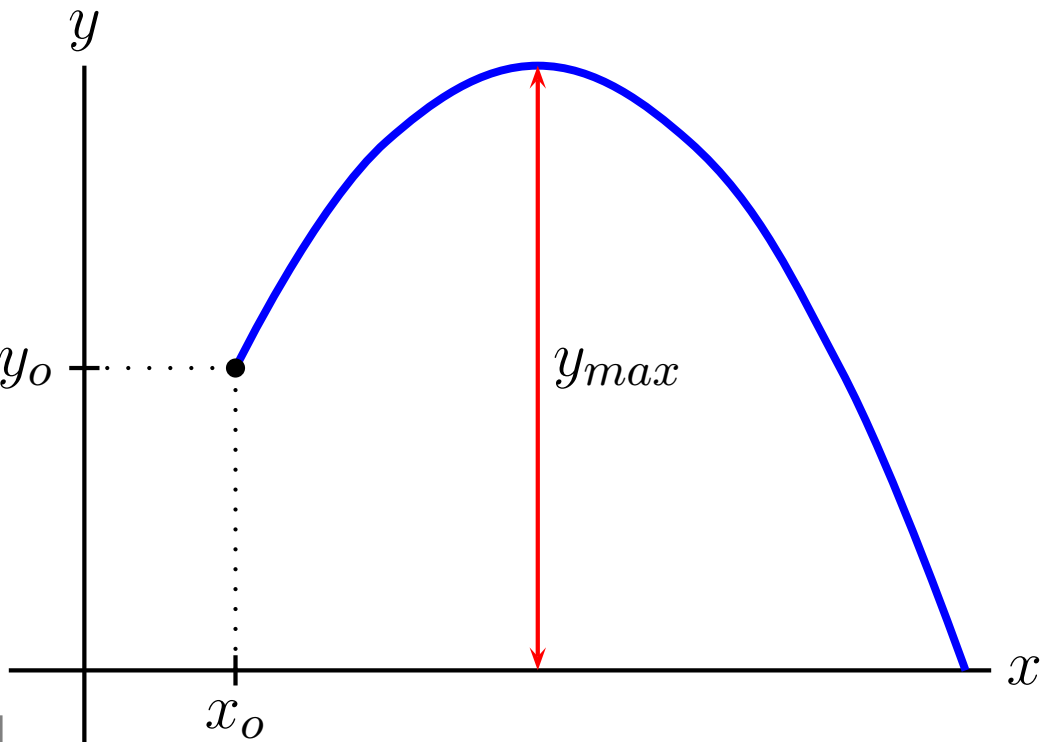
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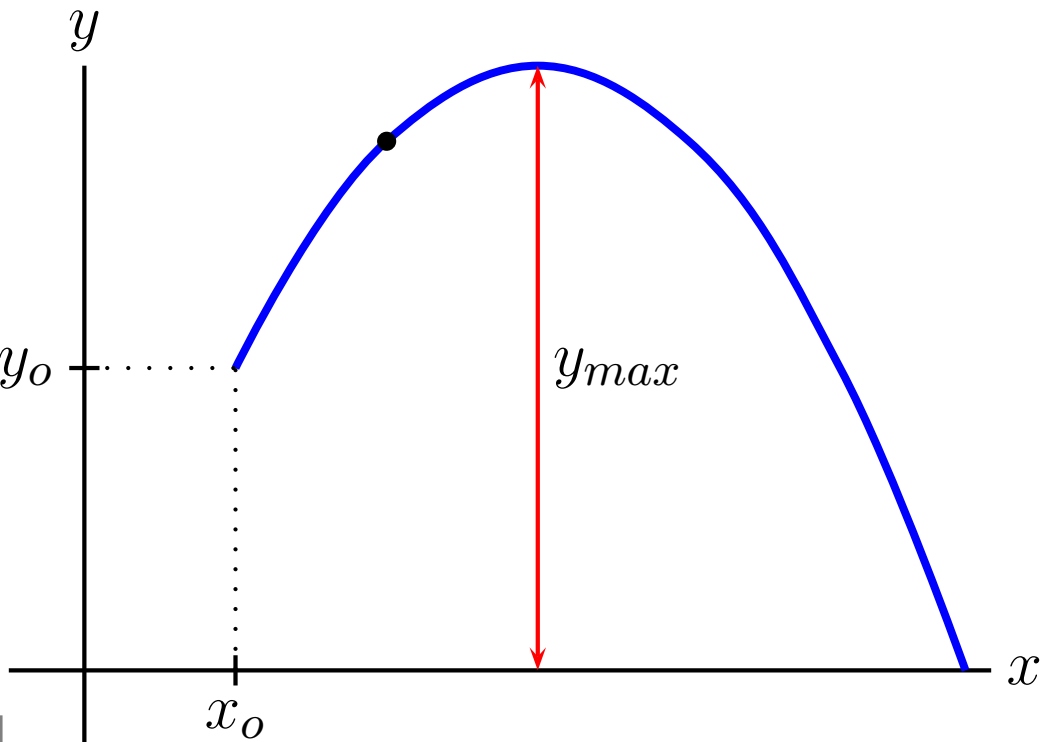
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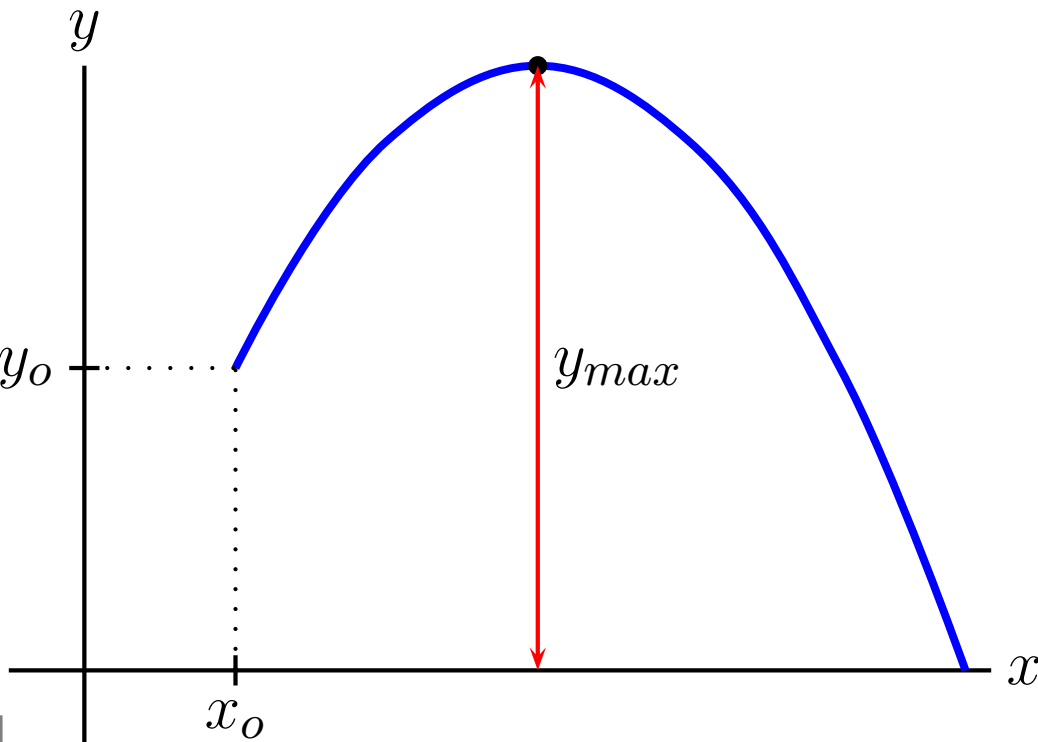
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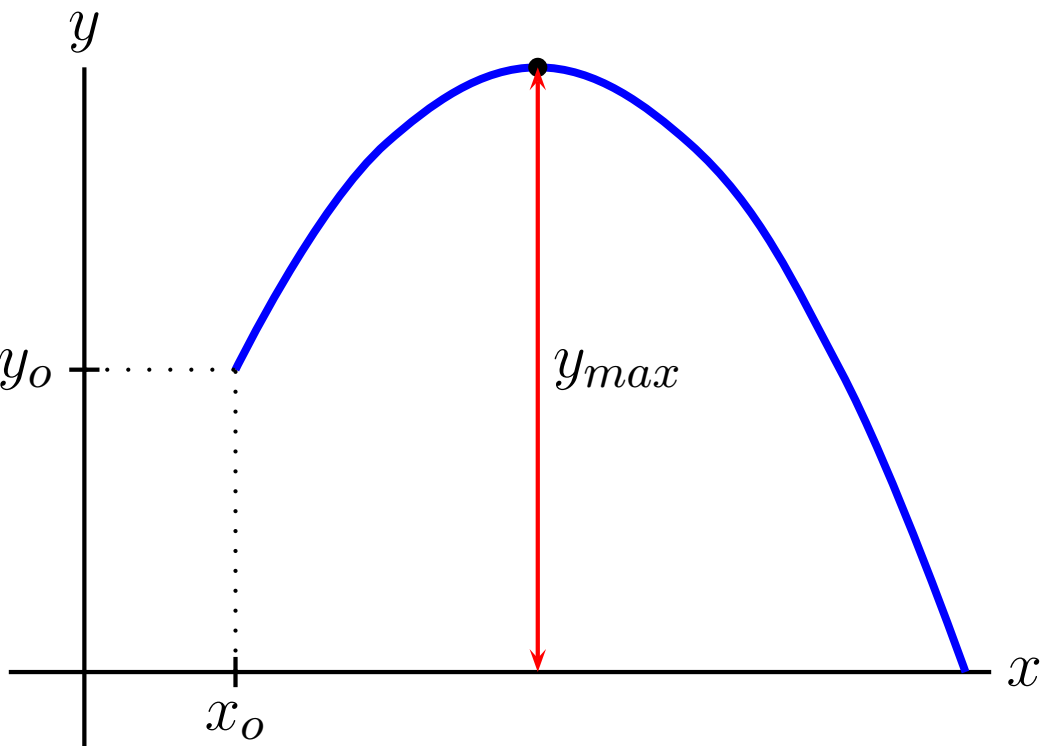
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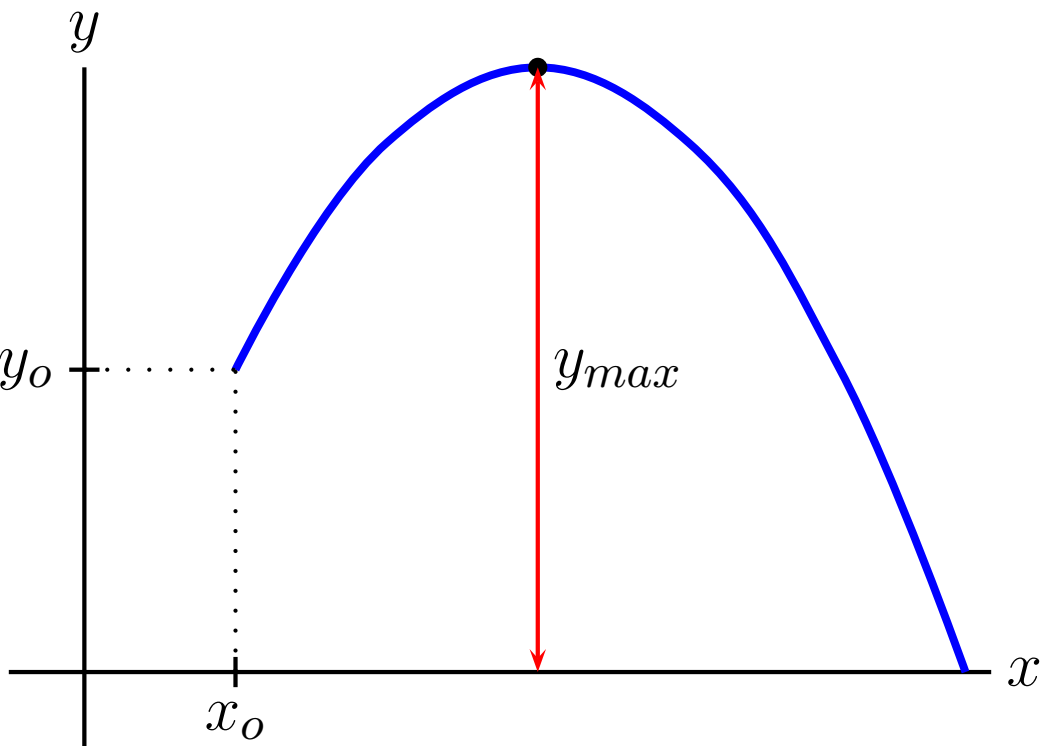
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At a projectile's maximum height, which of the following is a true statement?



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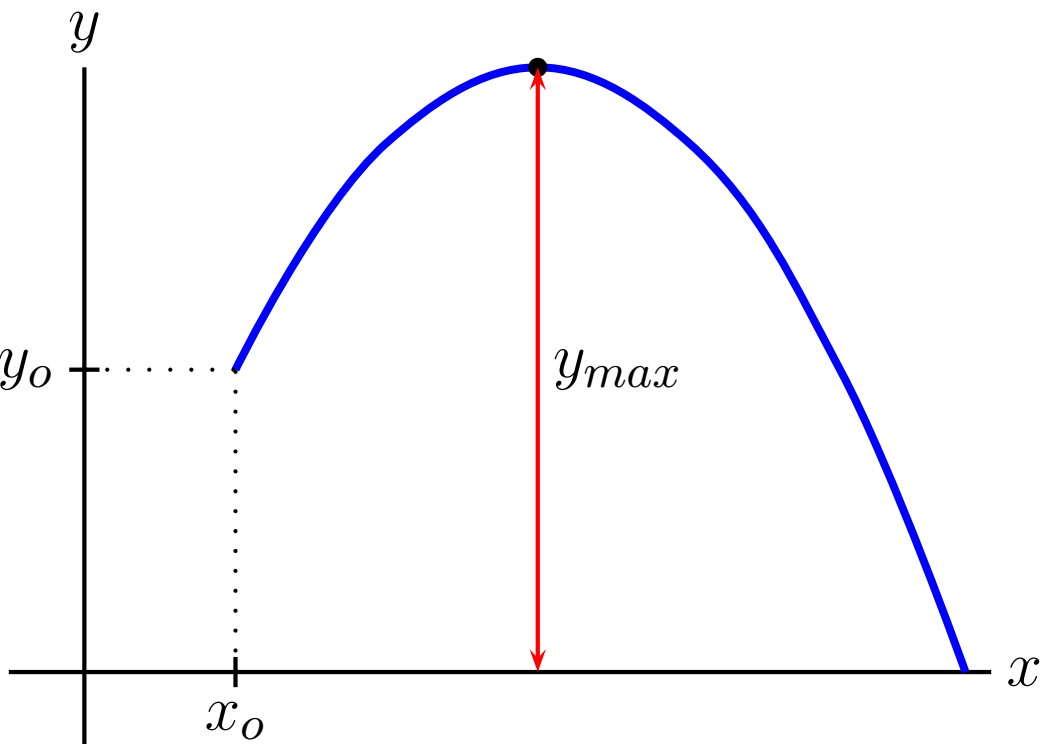
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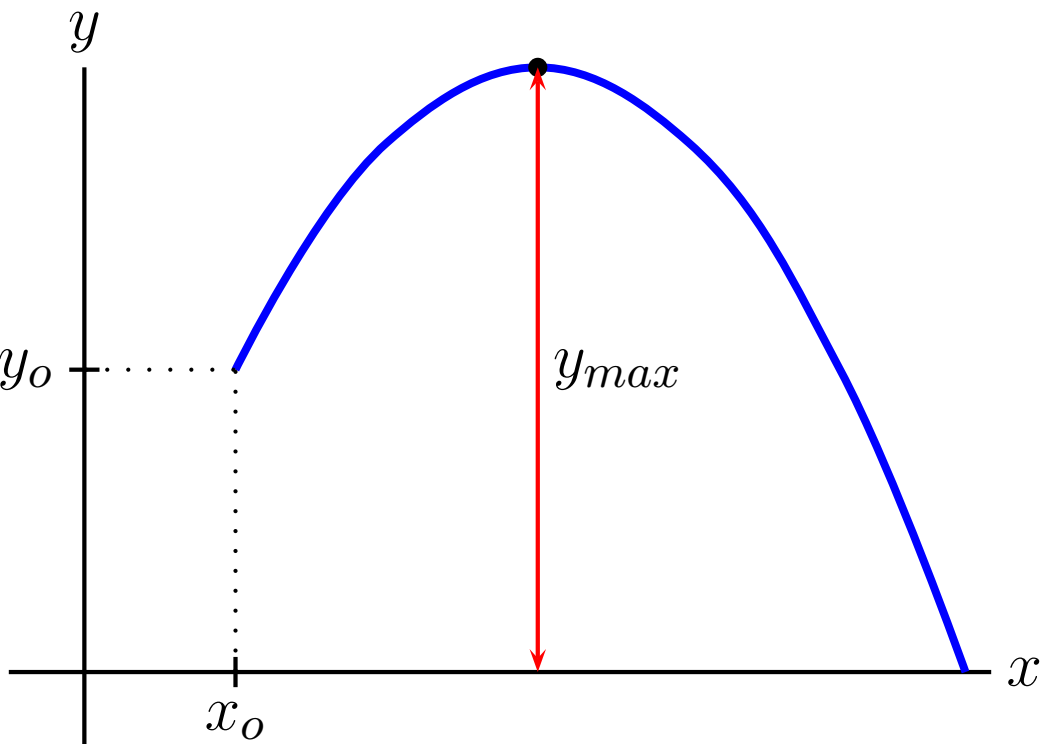


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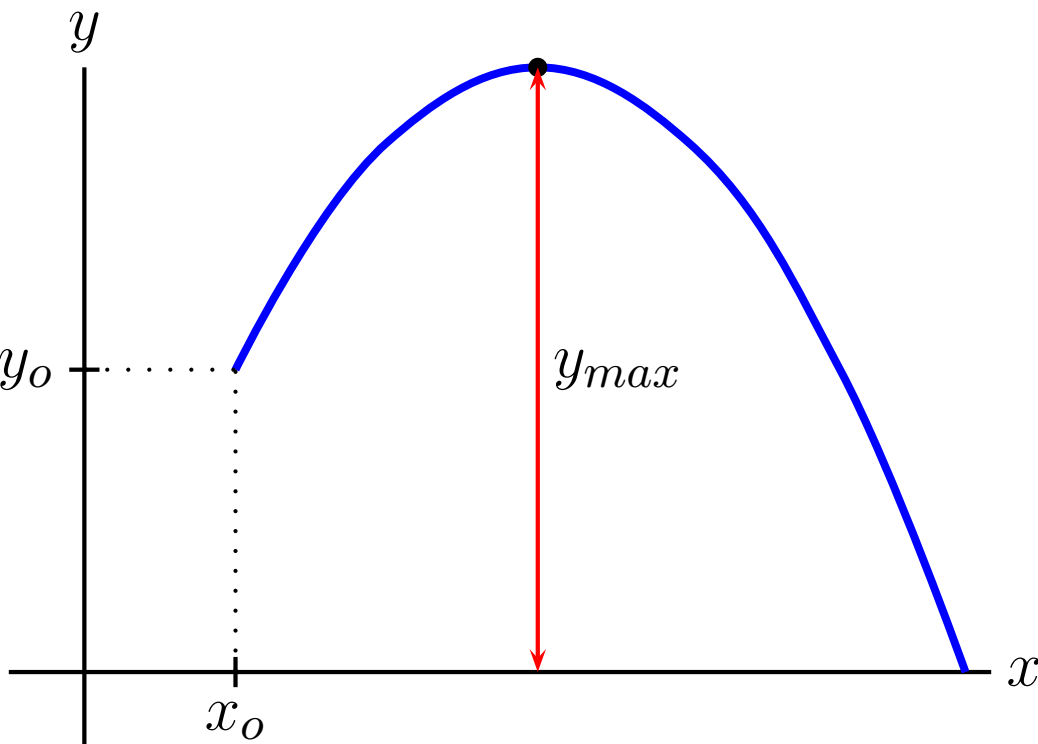
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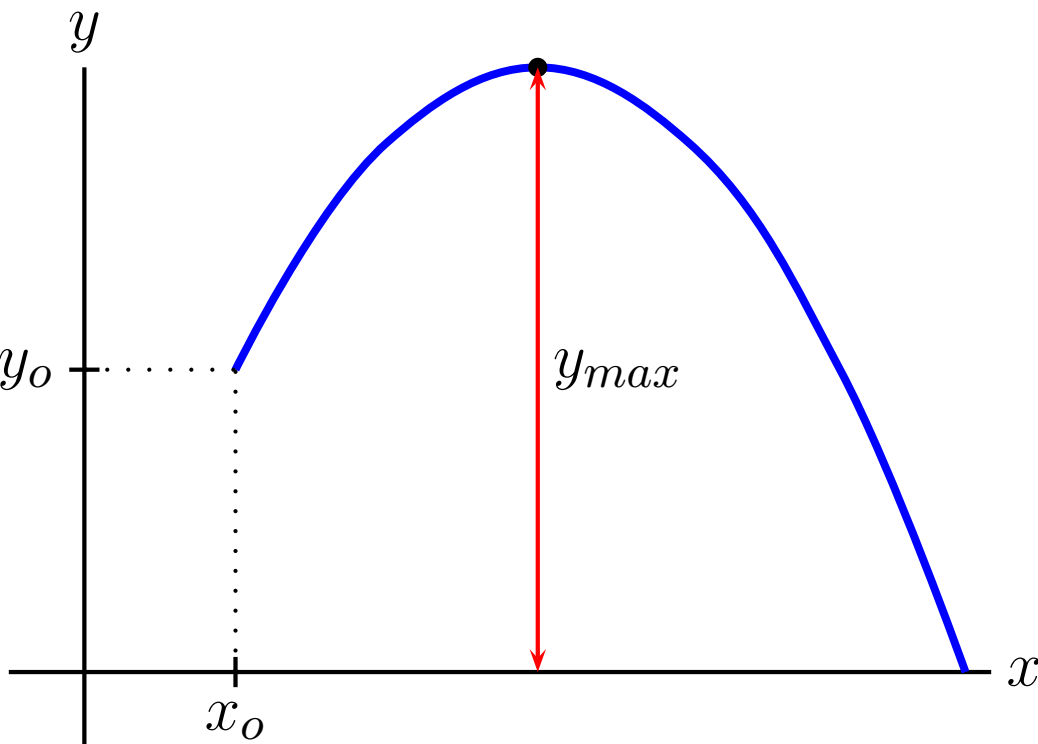
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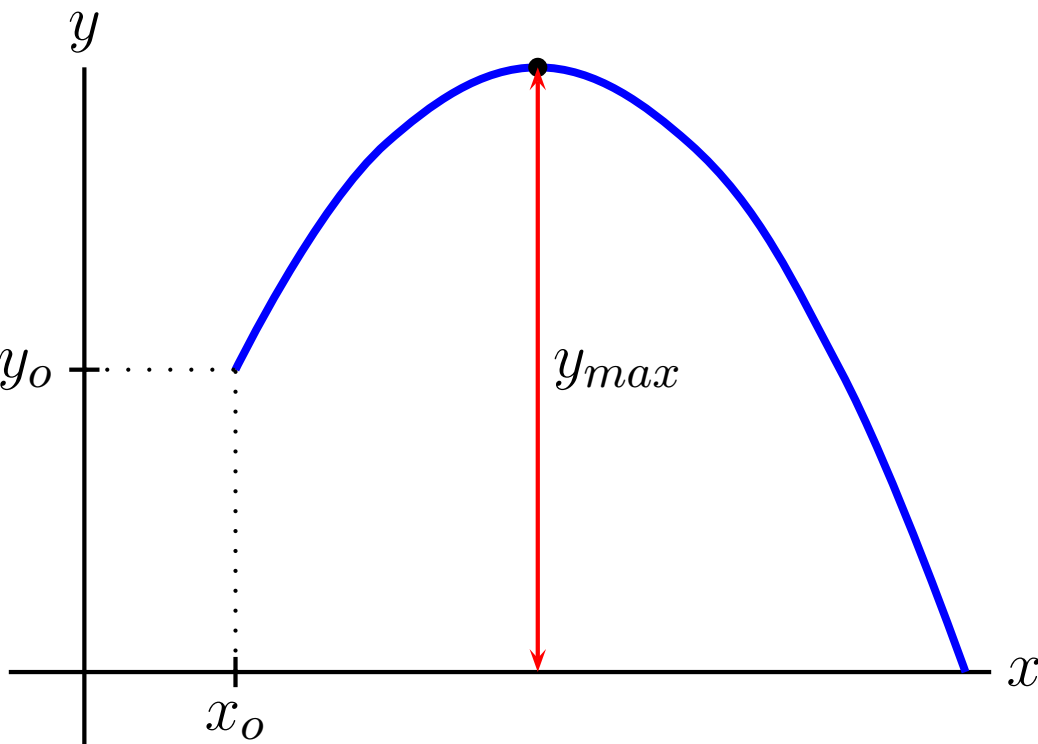
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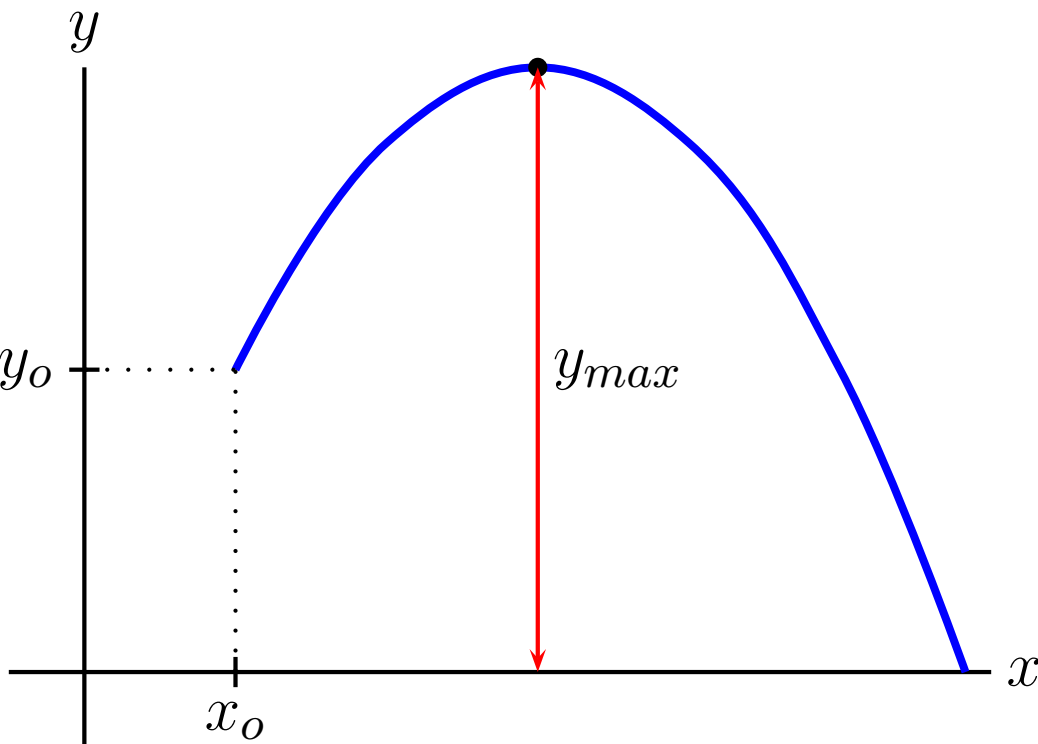
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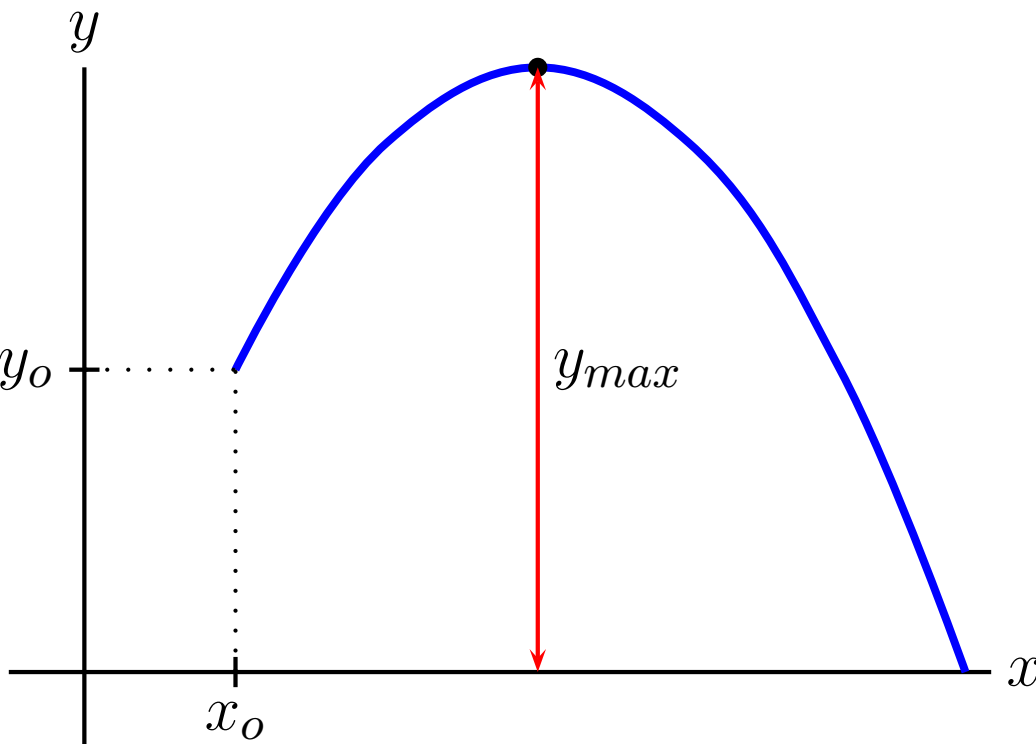
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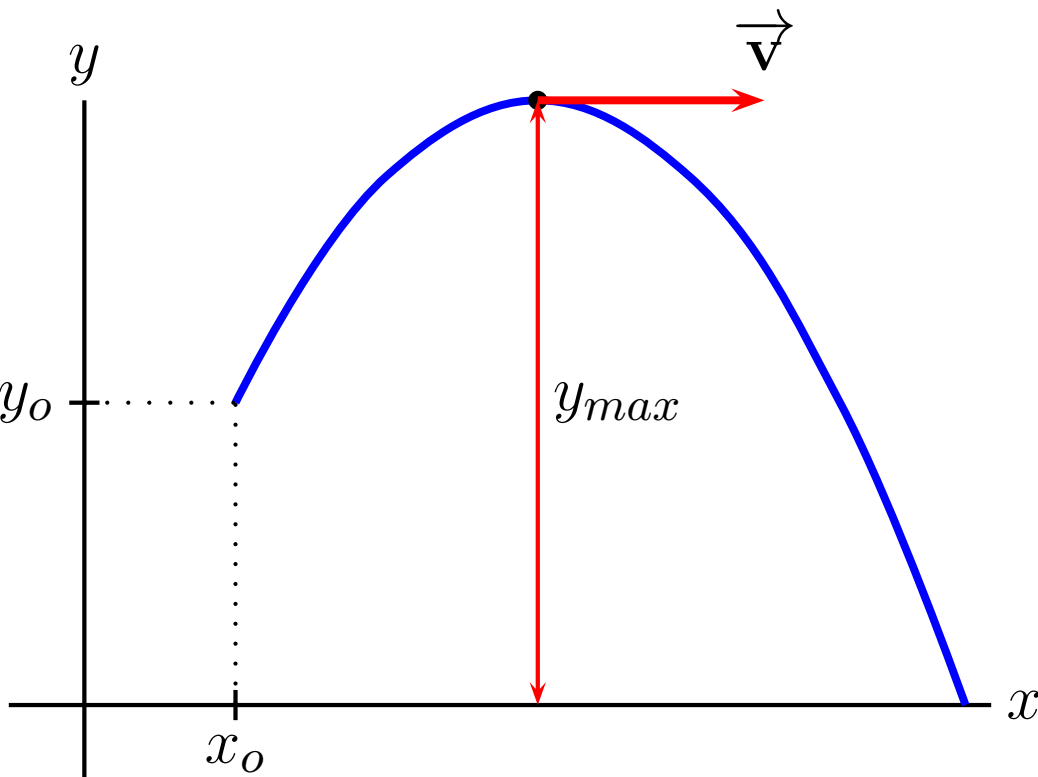
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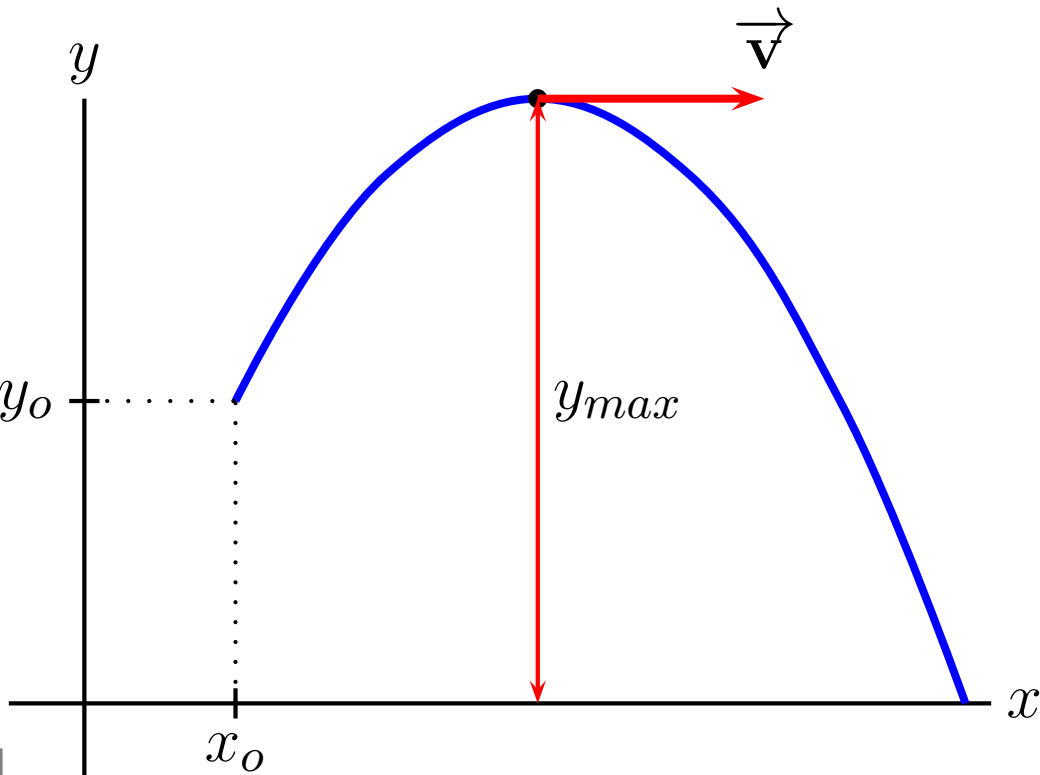
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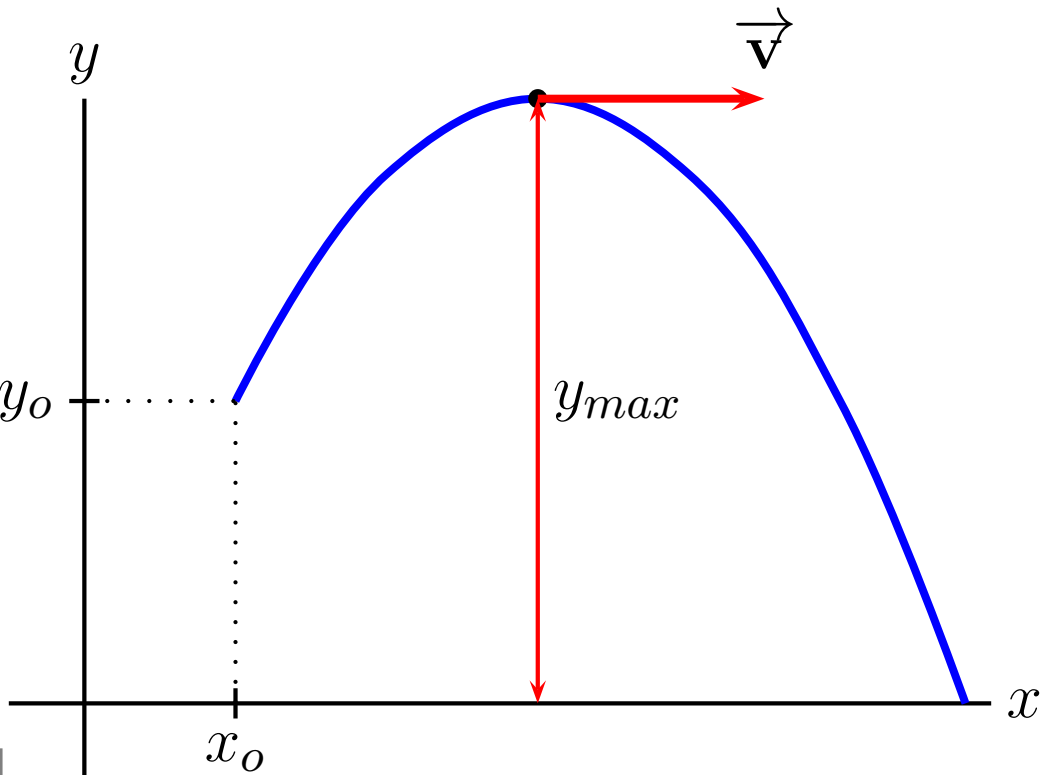
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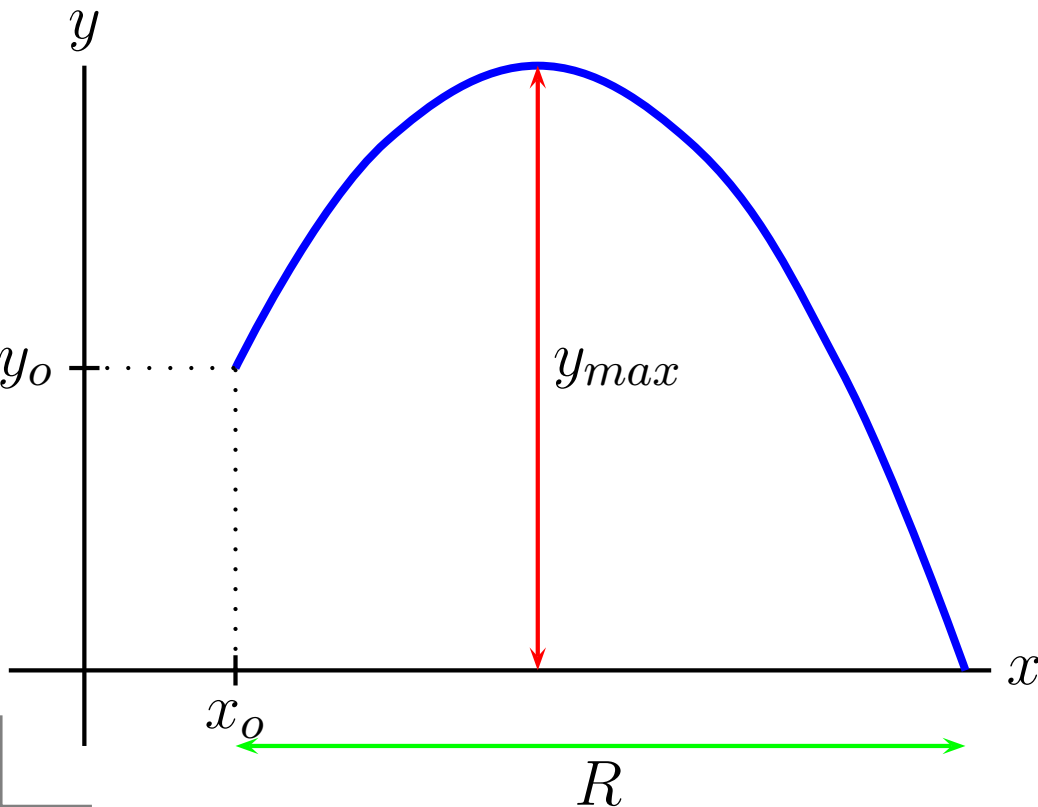
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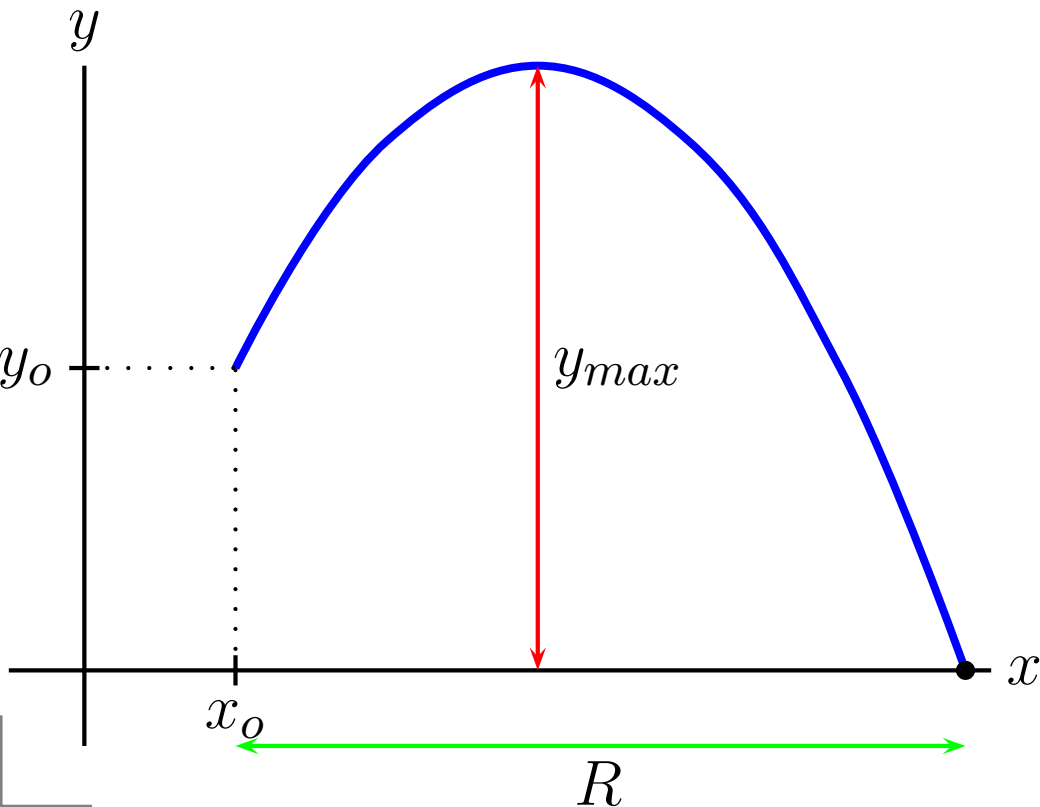
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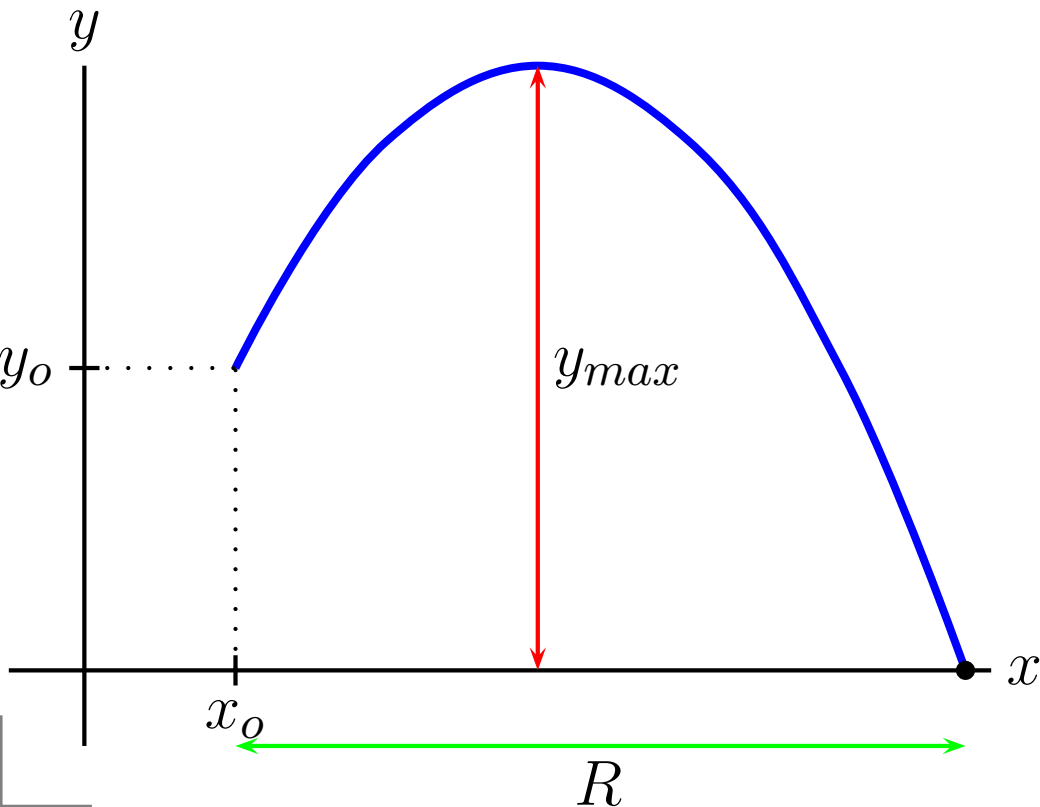
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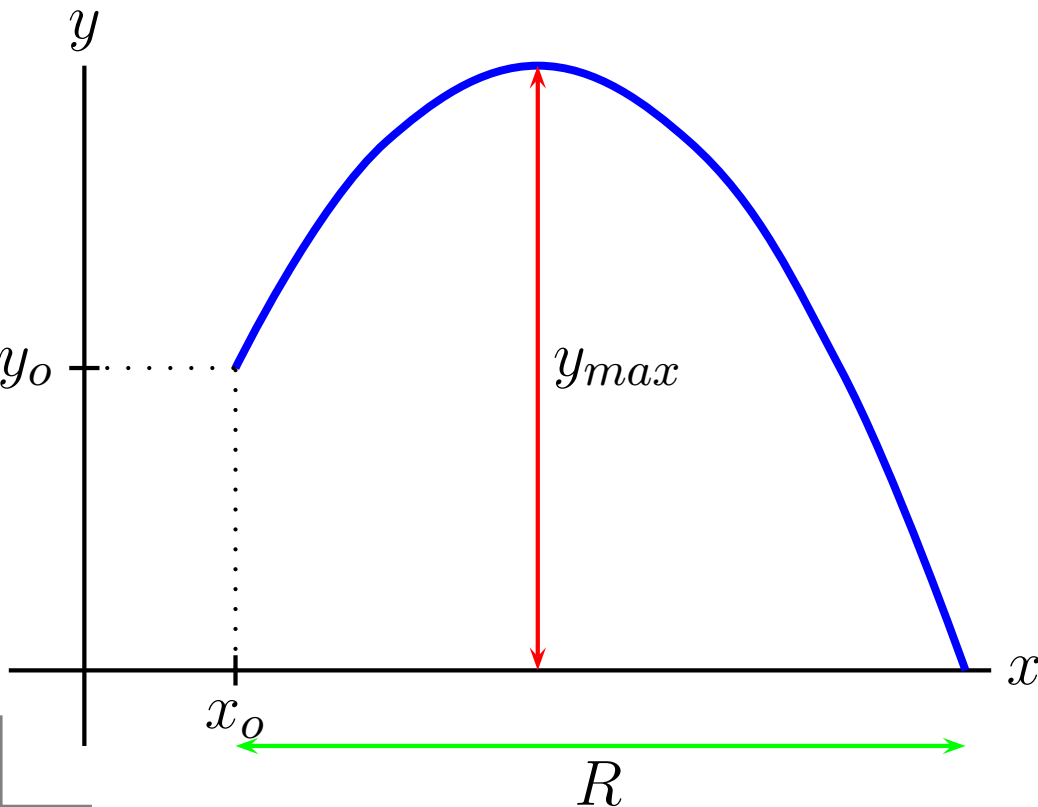
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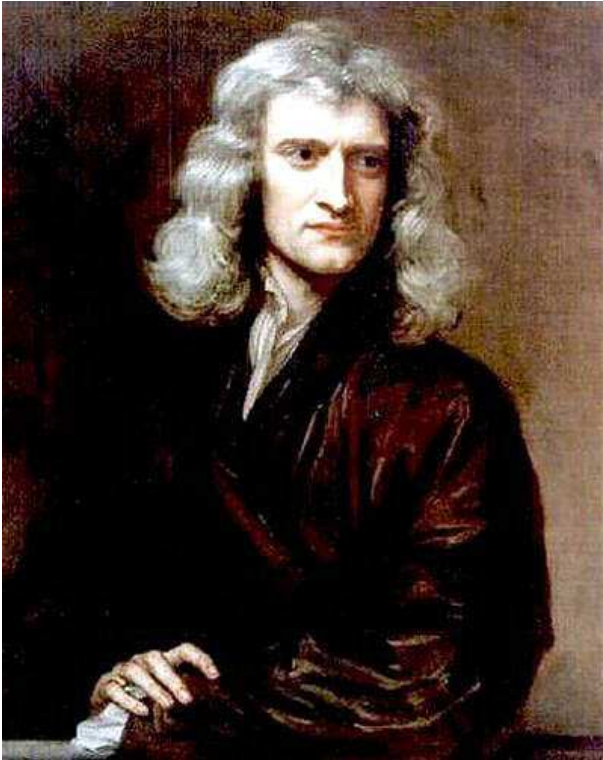
Warning: Book Equations  
only for  $y_o = y = 0$

# Dynamics

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Sir Isaac Newton (1642-1727)  
British Physicist, In 1687 he  
published the *Philosophiæ Naturalis  
Principia Mathematica*. The *Principia*  
details how all motion can be  
explained by one of three  
simple statements = Newton's  
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- When an object's velocity approaches the speed of light - Einstein's theory of relativity.
- Motion of atomic-sized objects - Quantum Mechanics (Also started by Einstein).

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# Superposition

Usually there is more than one force acting on an object.

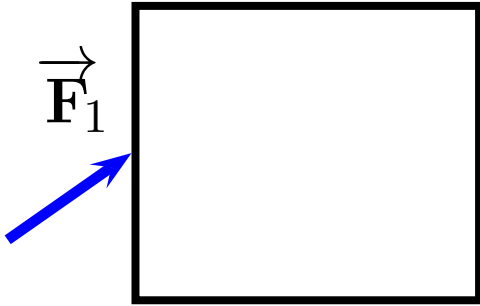
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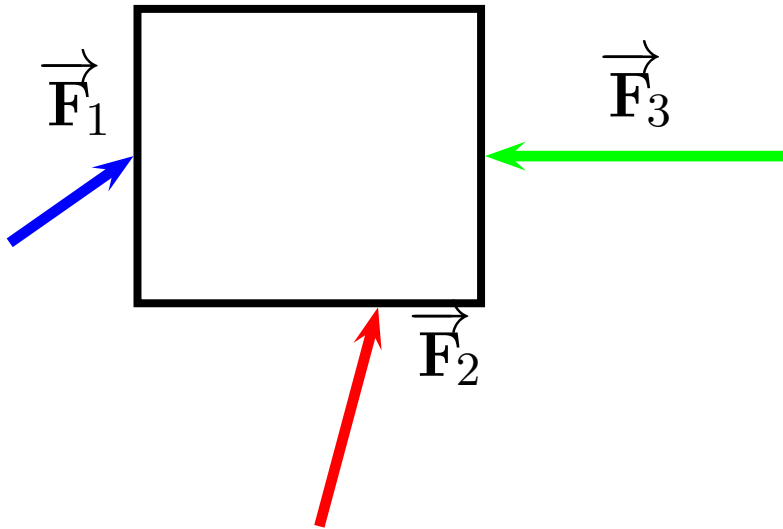
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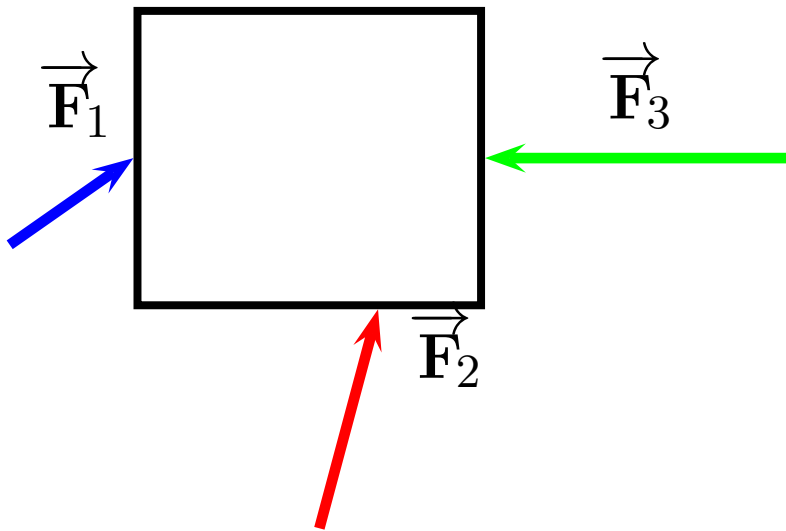
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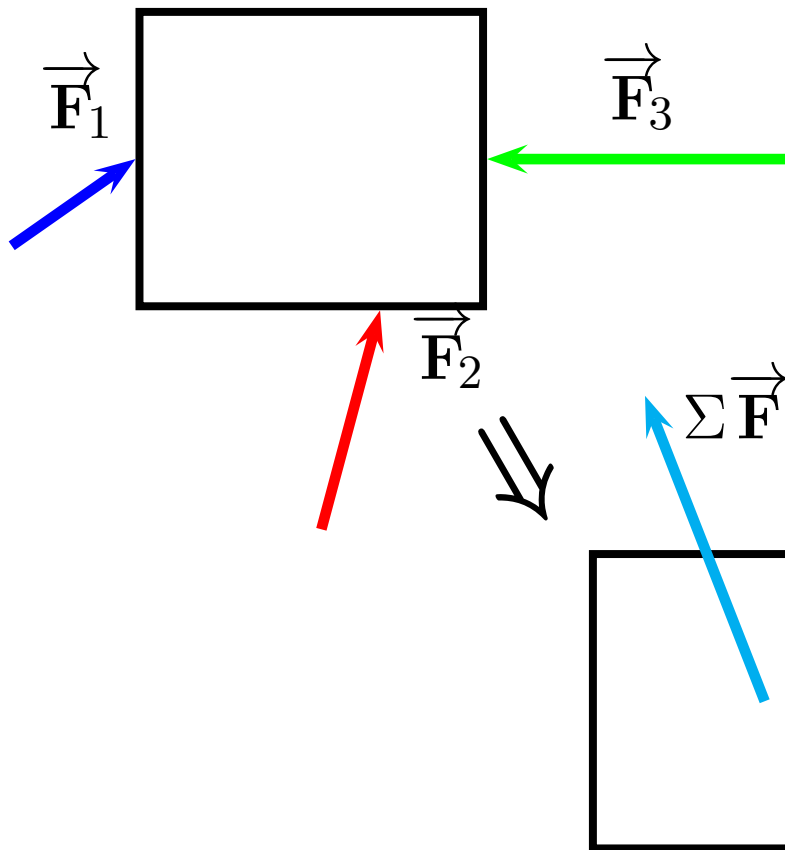


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One single force.

Applied at the center

to avoid rotation.