

# February 6, Week 4

Today: Chapter 3, Two-Dimensional Motion

Homework Assignment #3 due Tonight

Mastering Physics: 6 problems.

Written Problem: 2.88.

Homework #1 now in boxes.

No New homework assignment this week.

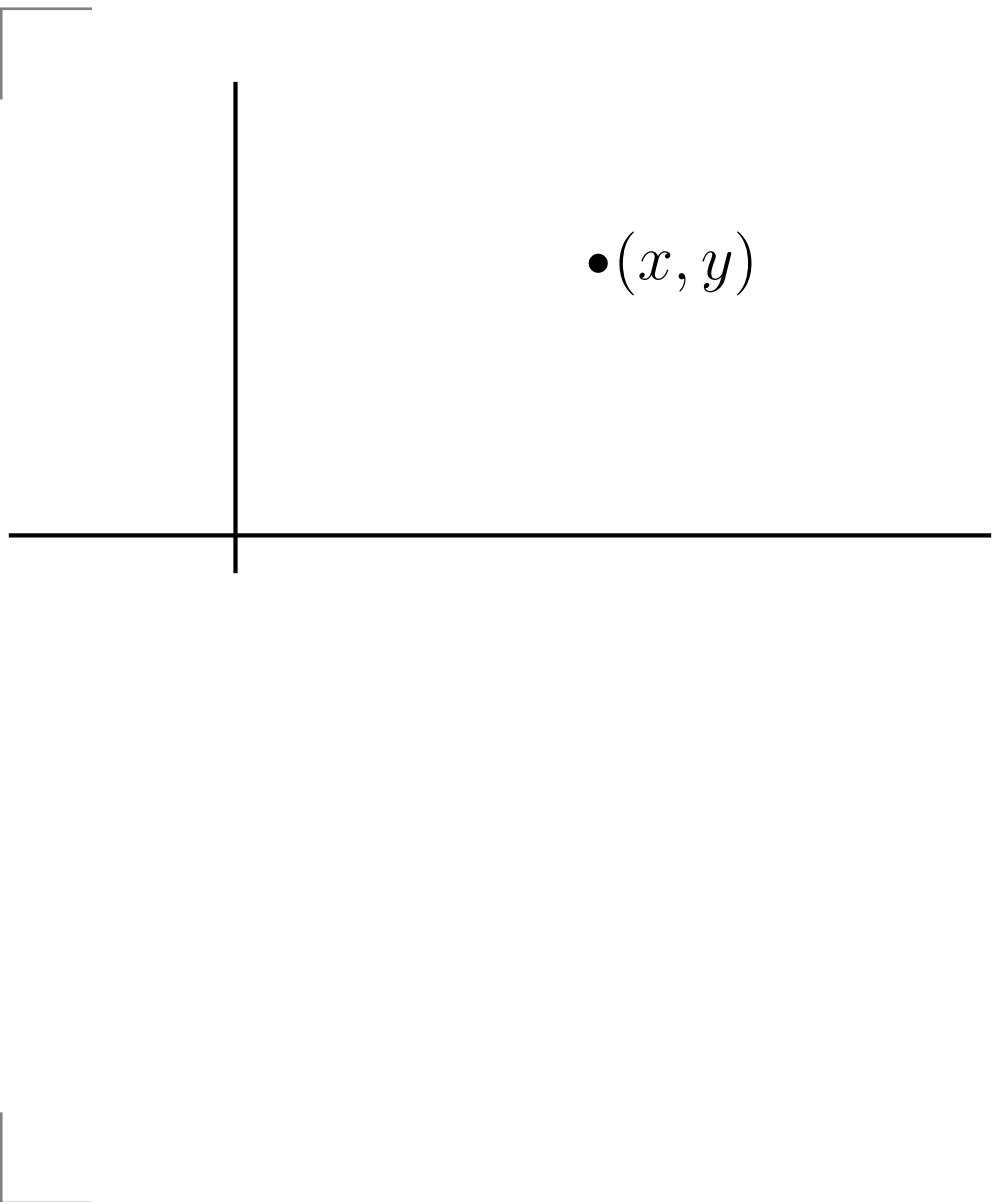
**Exam #1** Friday, February 10.

Practice Exam available on website.

Review Session, Thursday, 7:30 PM.

Chapter 2 practice problems on Mastering Physics.

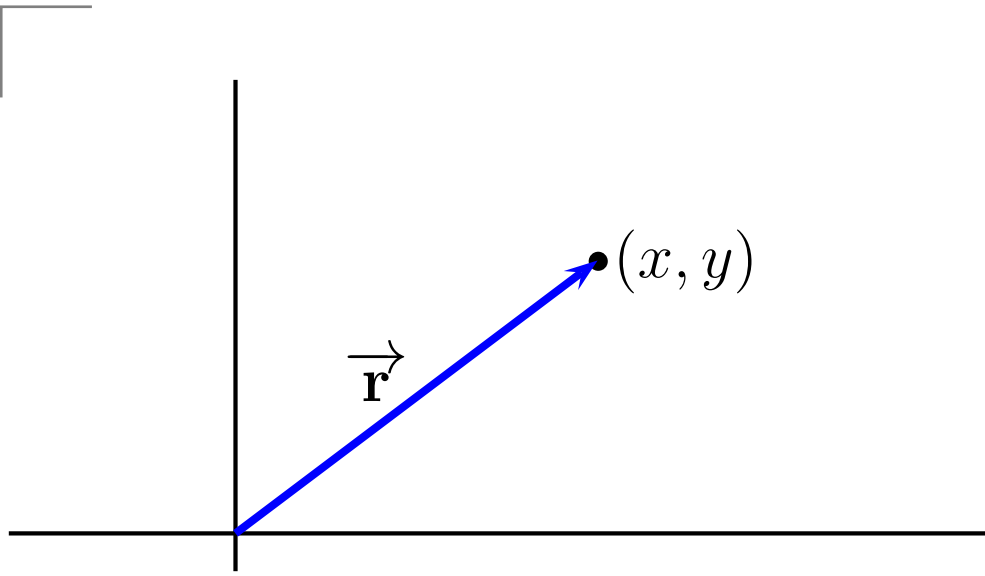
# Review



A 2D Cartesian coordinate system is shown with a horizontal x-axis and a vertical y-axis. The axes intersect at the origin. A point is plotted in the first quadrant, labeled with the coordinates  $(x, y)$ . The point is represented by a solid black dot. There are four L-shaped corner markers at the corners of the page: top-left, top-right, bottom-left, and bottom-right.

•  $(x, y)$

# Review



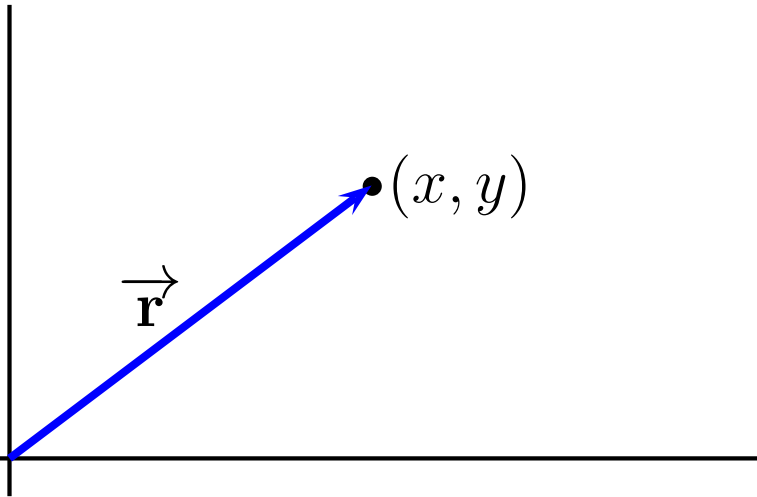
$$\vec{r} = x\hat{i} + y\hat{j}$$

$\Rightarrow$  the position vector goes from the origin to the object's location.

# Review

$$\vec{r} = x\hat{i} + y\hat{j}$$

$\Rightarrow$  the position vector goes from the origin to the object's location.



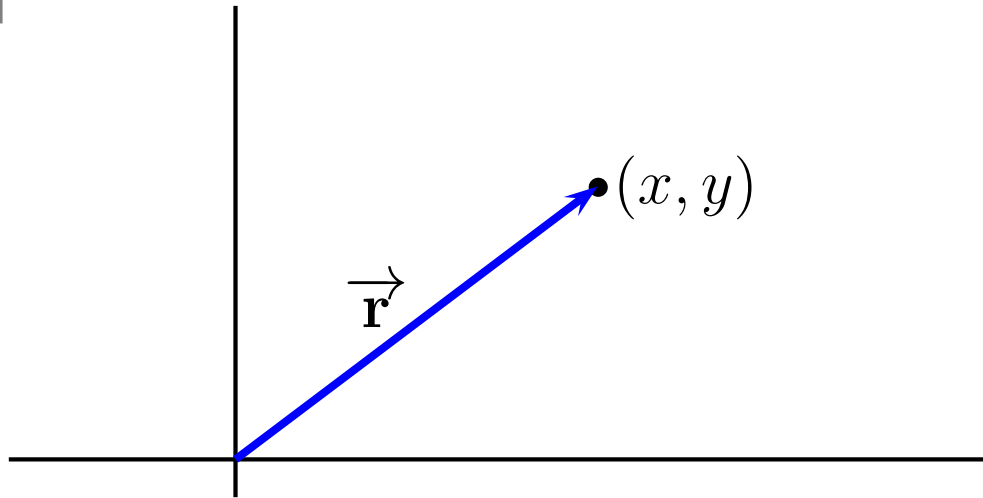
Final Position



Initial Position

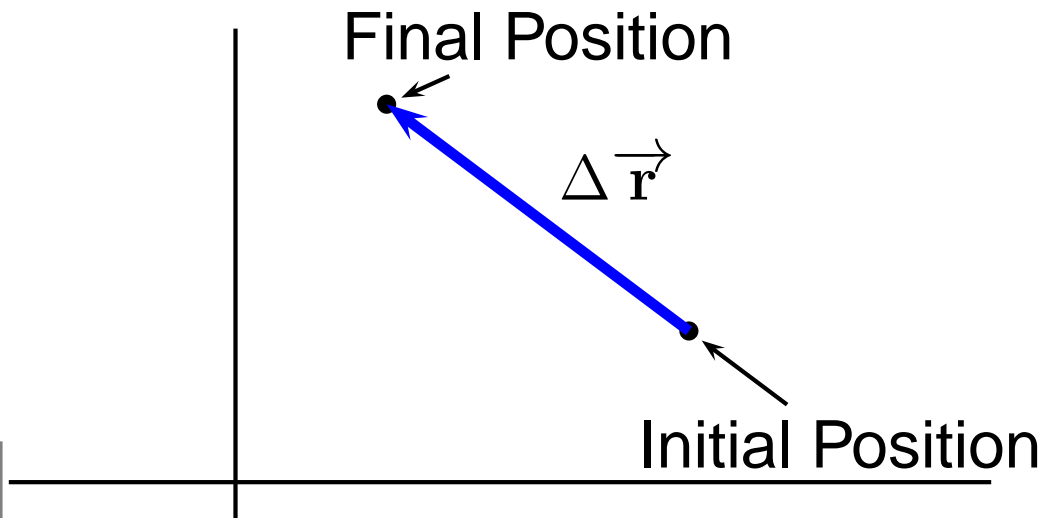


# Review



$$\vec{r} = x\hat{i} + y\hat{j}$$

$\Rightarrow$  the position vector goes from the origin to the object's location.



$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

# Review II

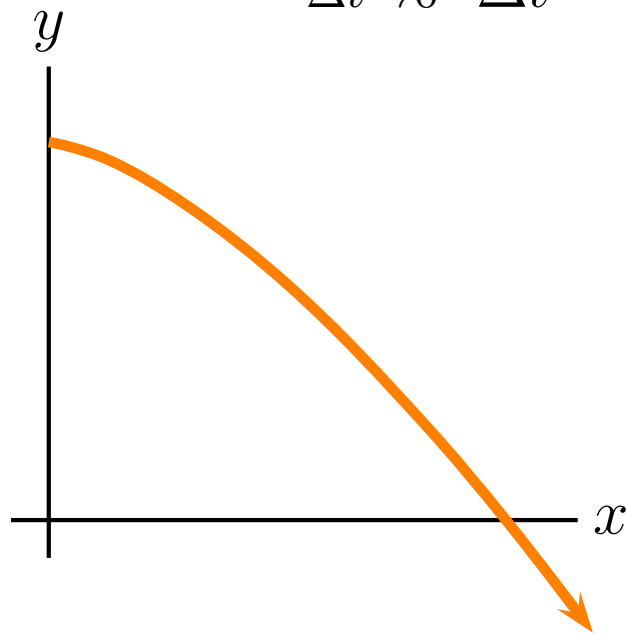
Velocity = Speed and direction.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

# Review II

Velocity = Speed and direction.

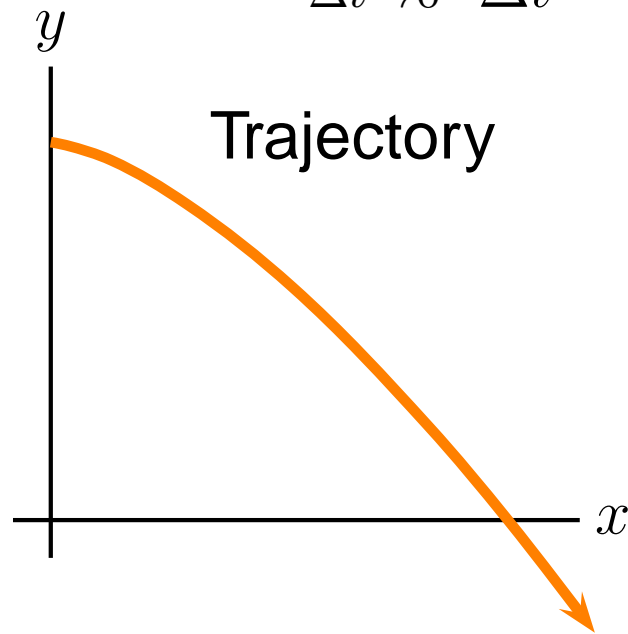
$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



# Review II

Velocity = Speed and direction.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

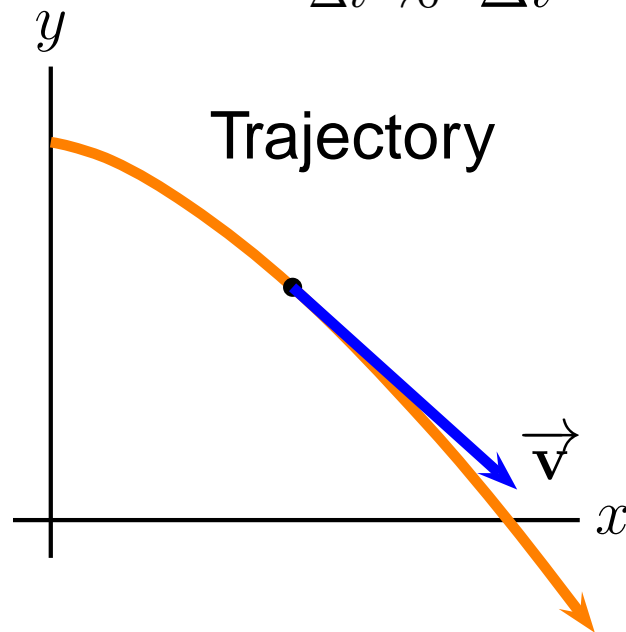




# Review II

Velocity = Speed and direction.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

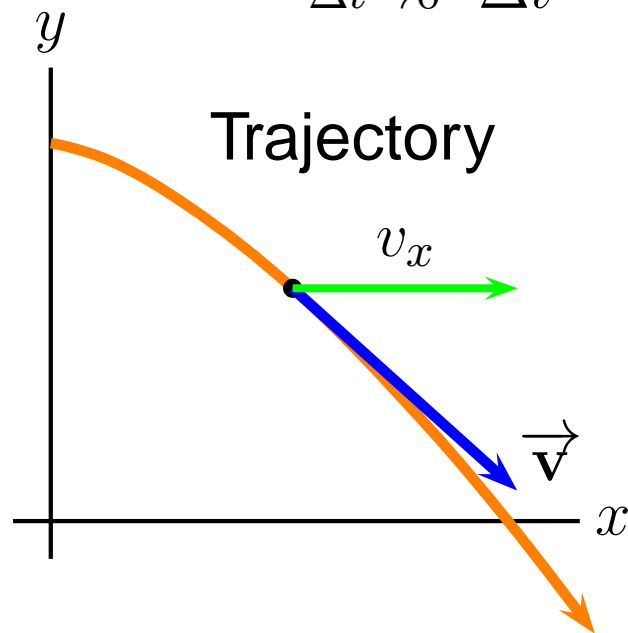


$\vec{v}$  is tangent  
to trajectory

# Review II

Velocity = Speed and direction.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



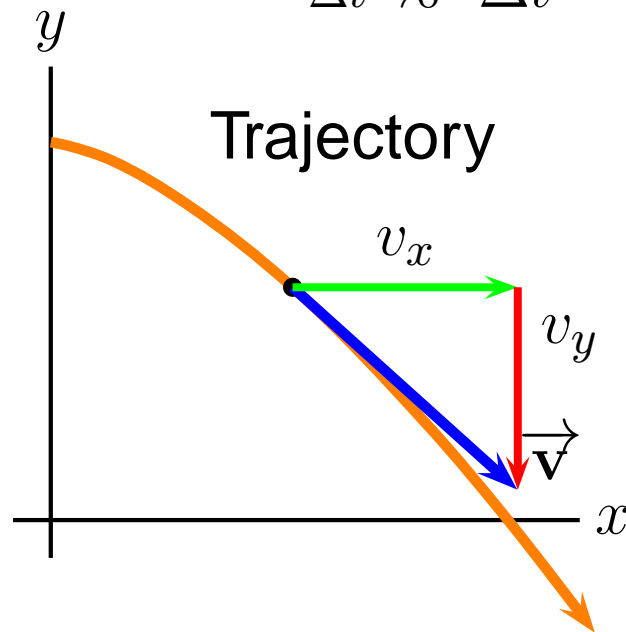
$\vec{v}$  is tangent  
to trajectory

$$v_x = \frac{dx}{dt}$$

# Review II

Velocity = Speed and direction.

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$



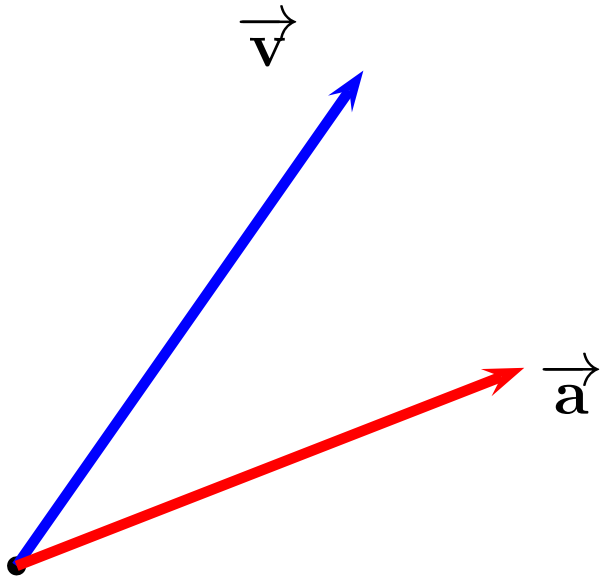
$\vec{v}$  is tangent  
to trajectory

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

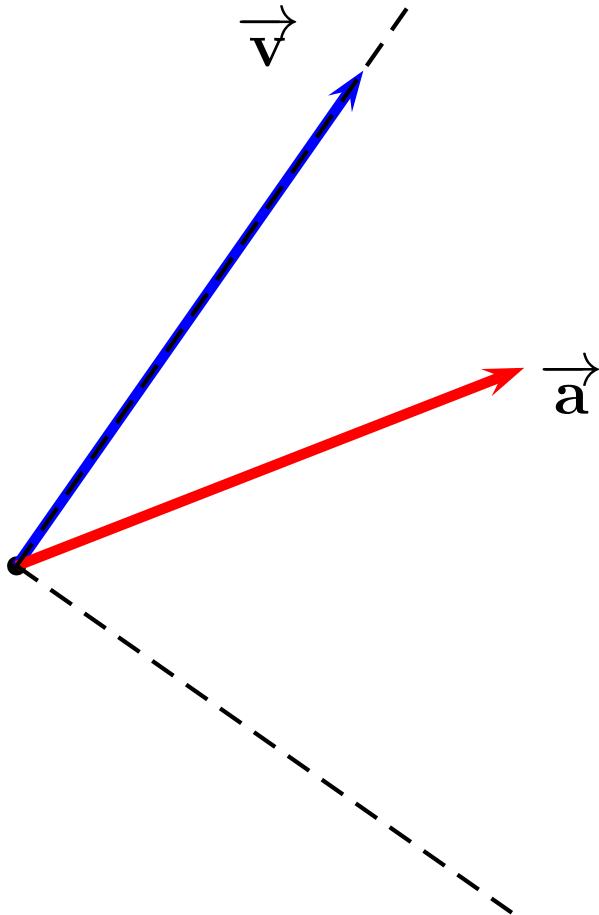
# Review III

An acceleration in an arbitrary direction will cause a change in both speed and direction.



# Review III

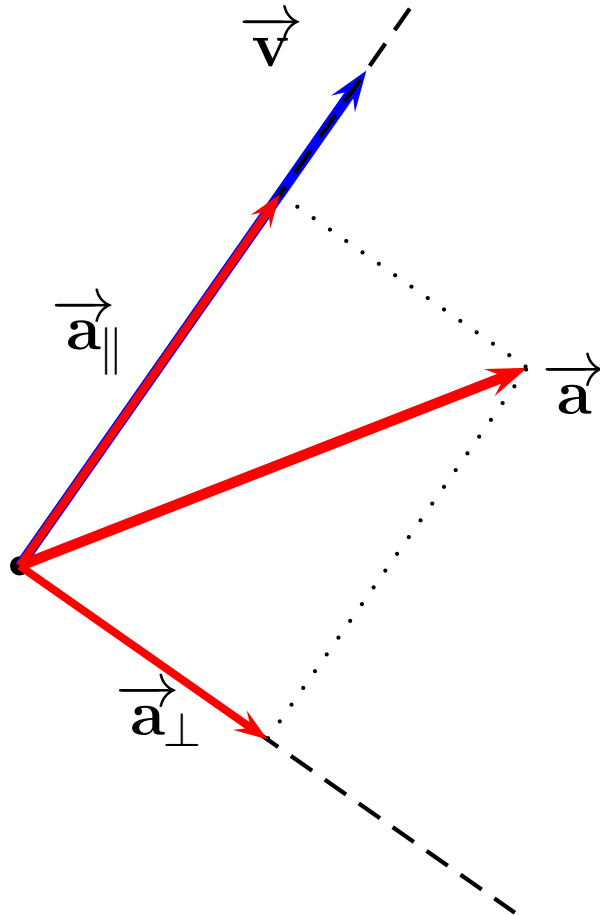
An acceleration in an arbitrary direction will cause a change in both speed and direction.



Use coordinates parallel and perpendicular to  $\vec{v}$

# Review III

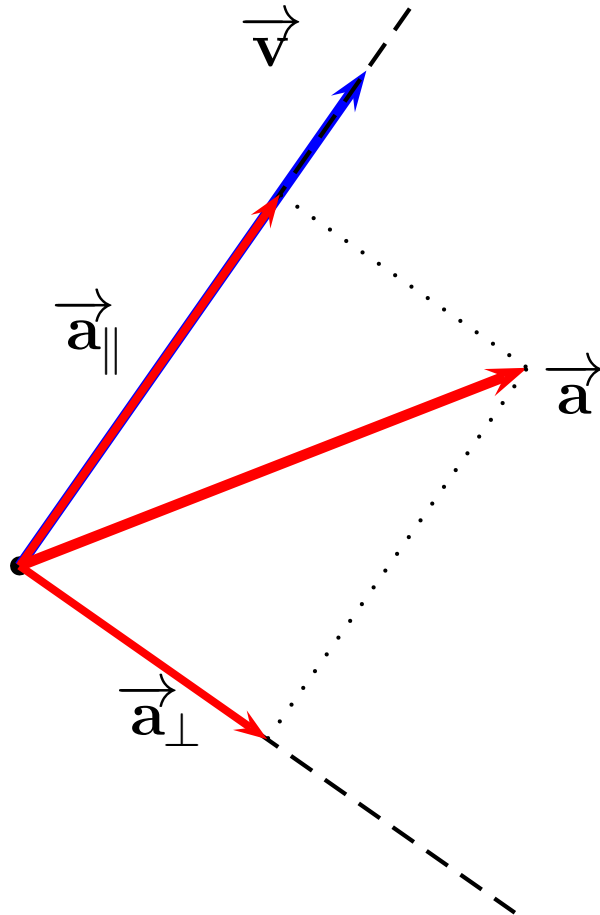
An acceleration in an arbitrary direction will cause a change in both speed and direction.



Split into components  
parallel and  
perpendicular to  $\vec{v}$

# Review III

An acceleration in an arbitrary direction will cause a change in both speed and direction.

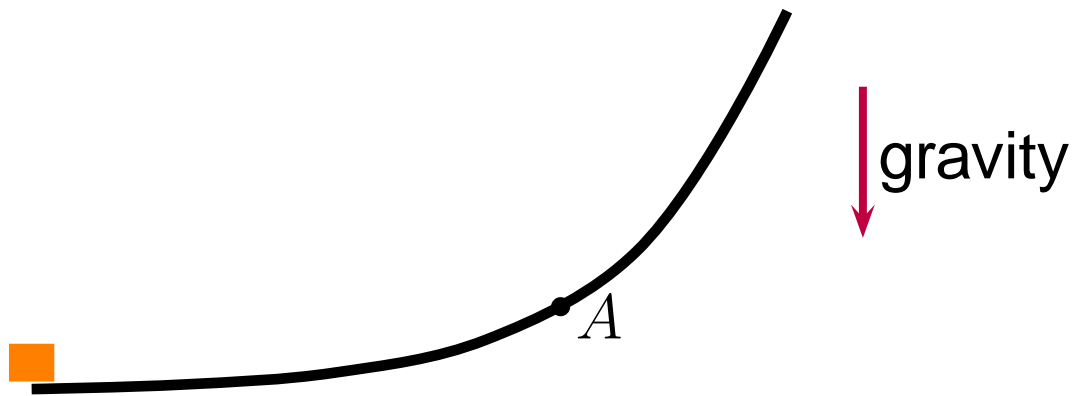


Split into components  
parallel and  
perpendicular to  $\vec{v}$

$\vec{a}_{\parallel}$  changes speed  
 $\vec{a}_{\perp}$  changes direction

# Clicker Quiz

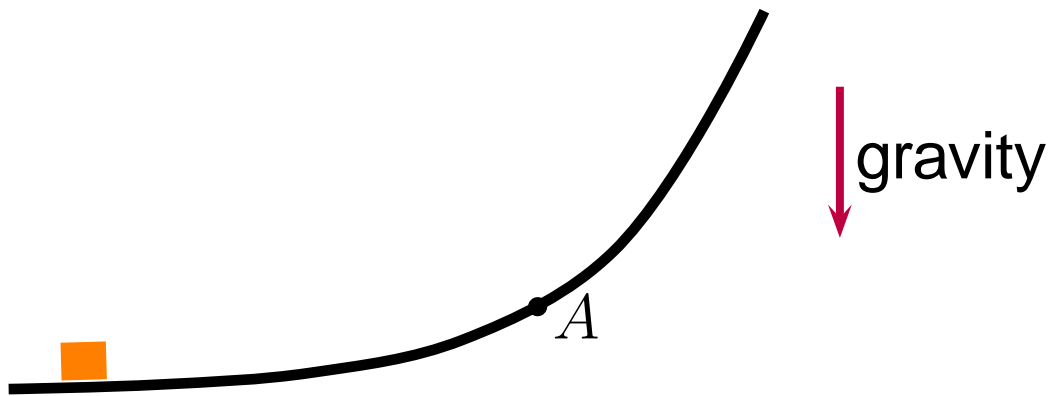
A mass slides up a ramp (against gravity) as shown.





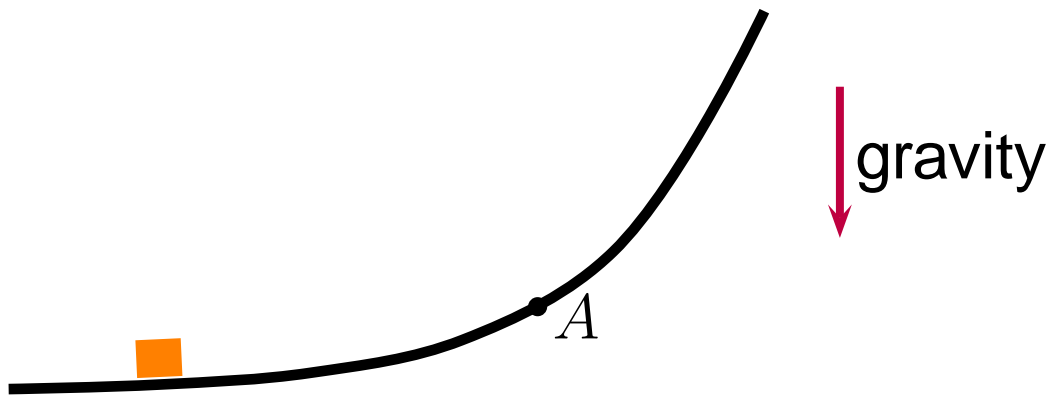
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



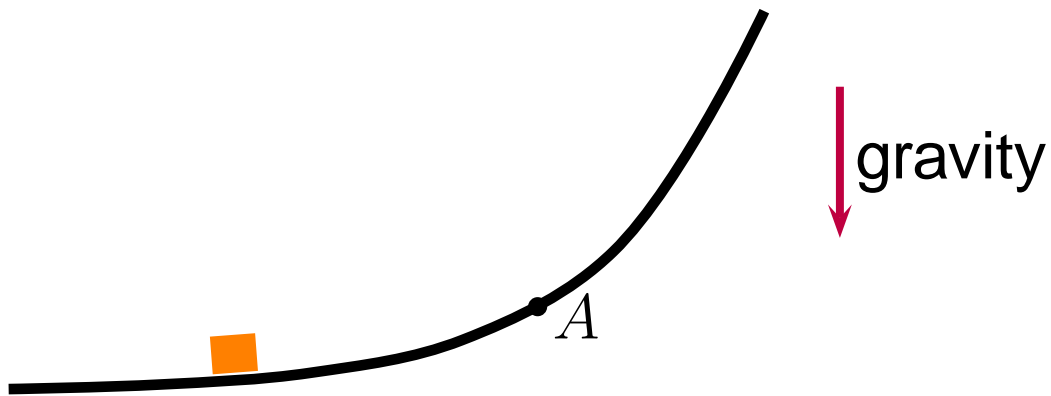
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



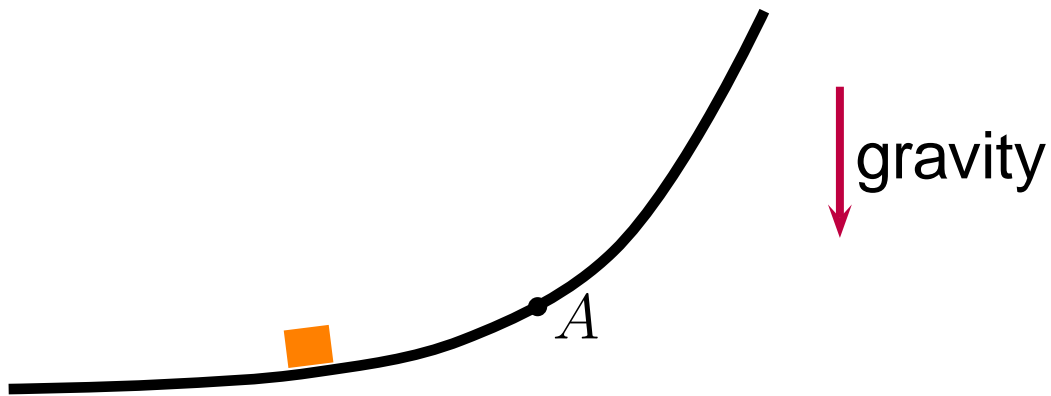
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



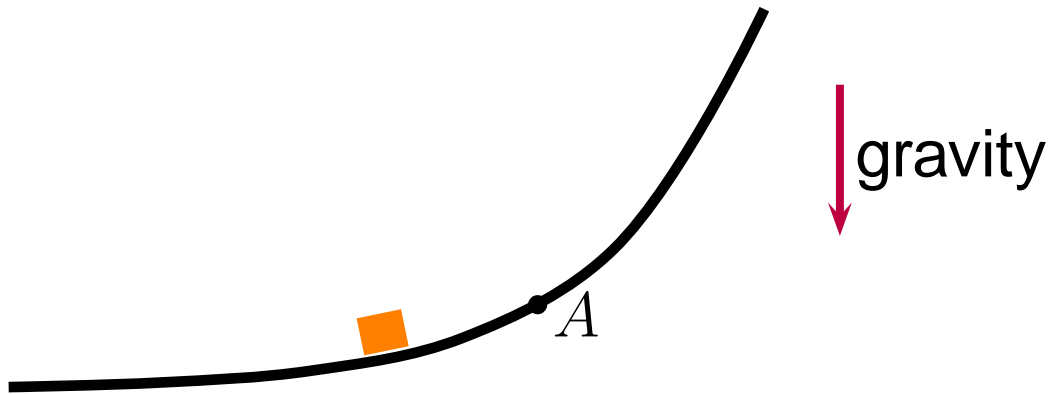
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



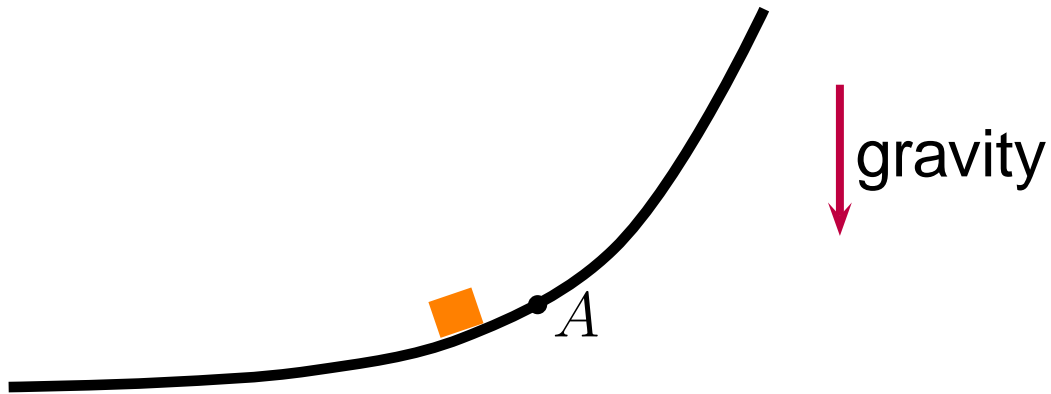
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



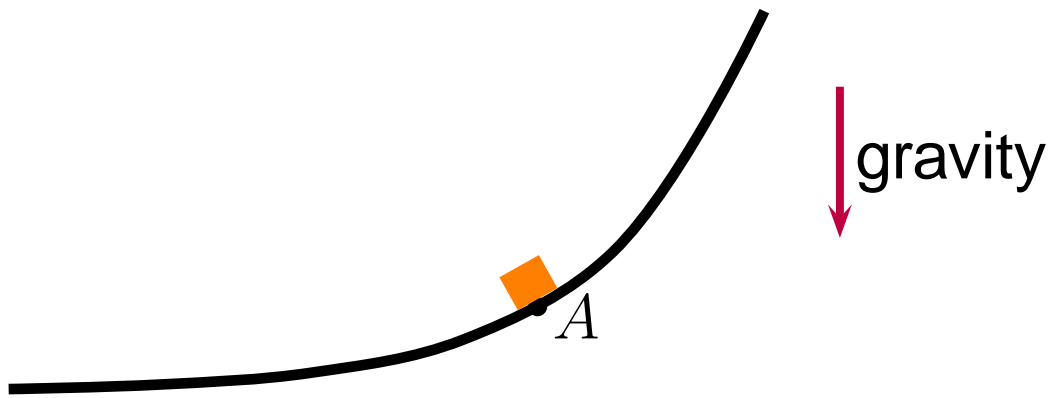
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



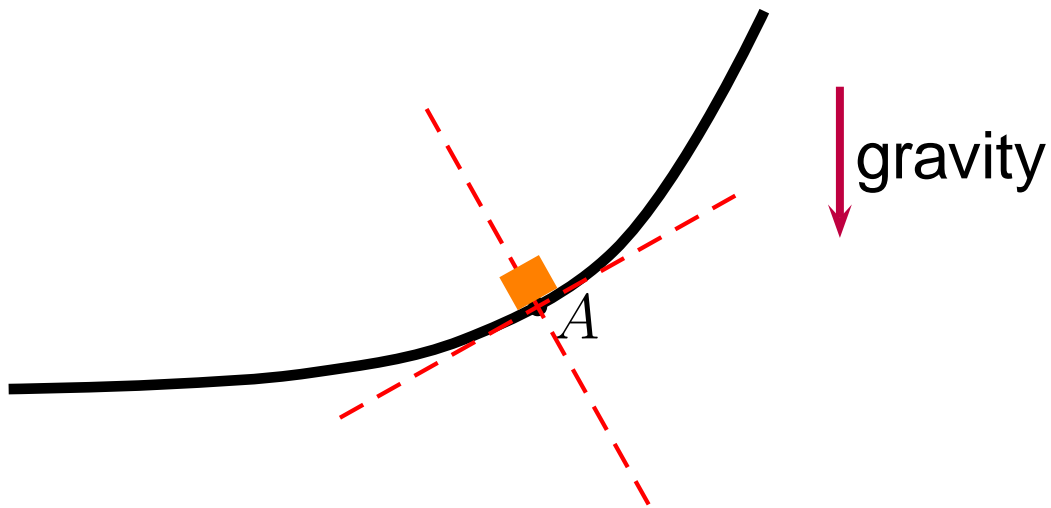
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown.



# Clicker Quiz

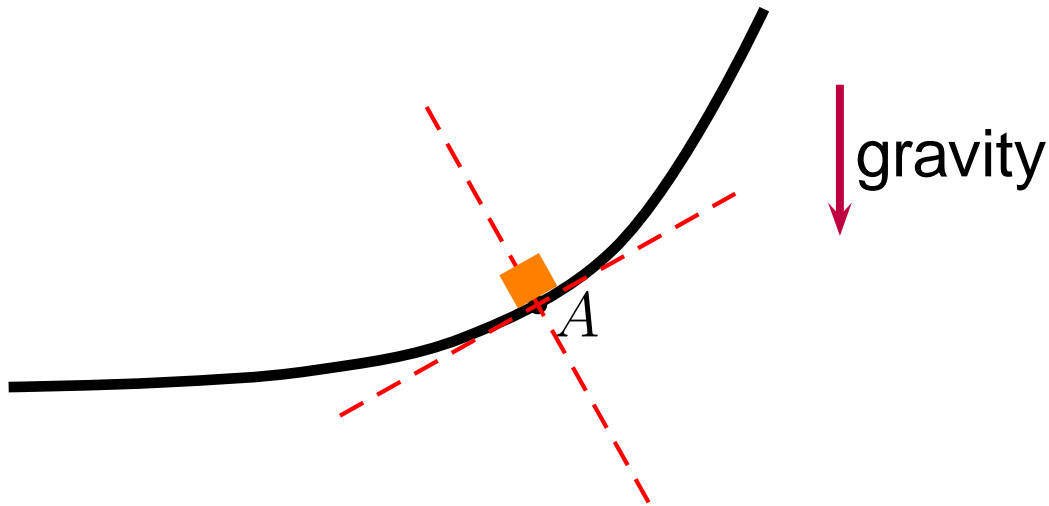
A mass slides up a ramp (against gravity) as shown.





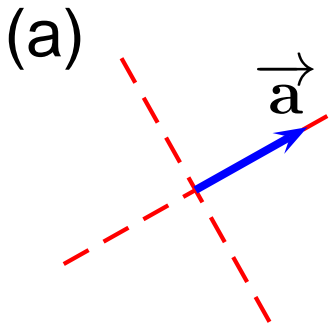
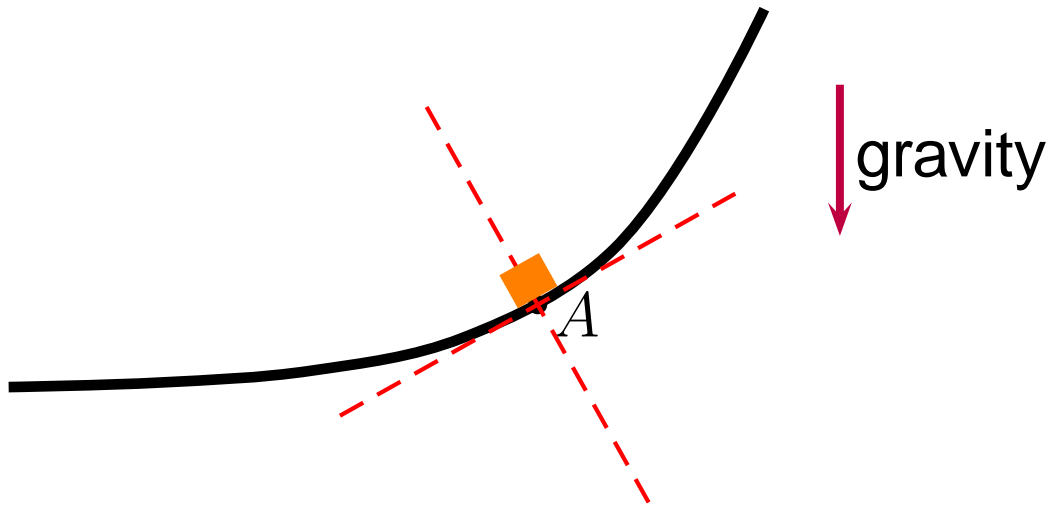
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



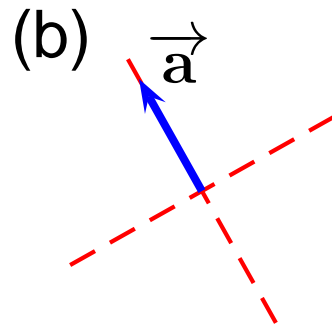
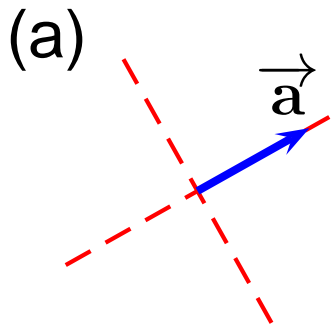
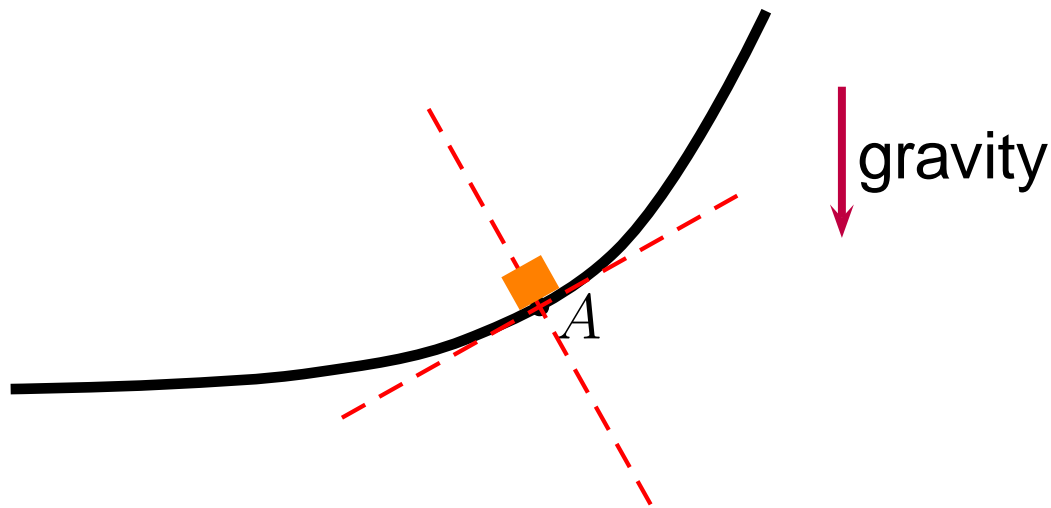
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



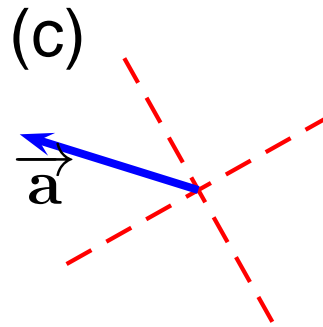
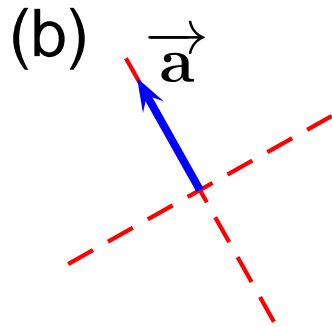
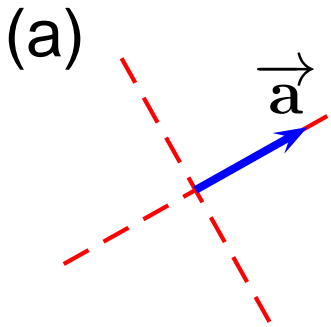
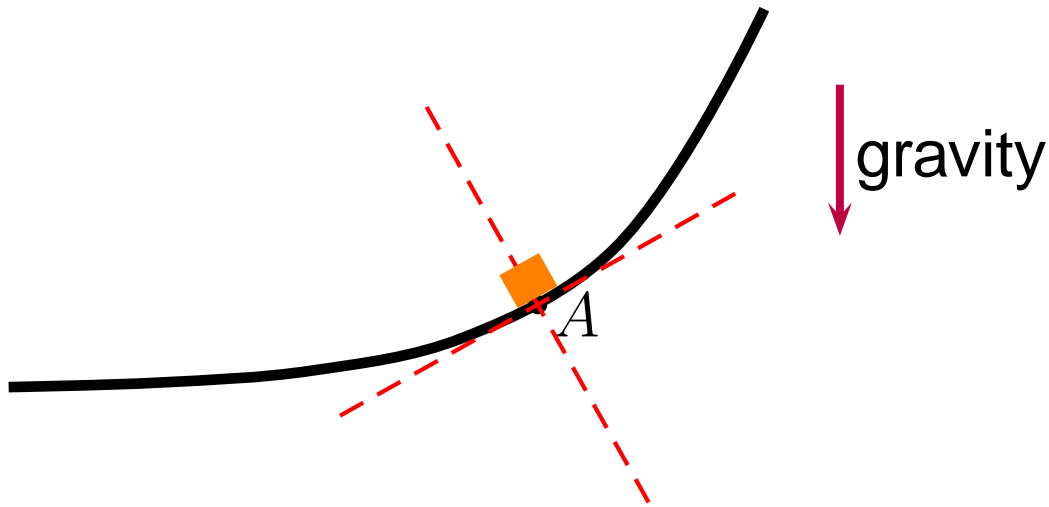
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



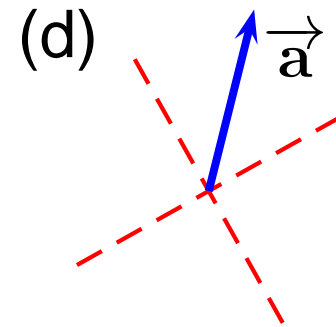
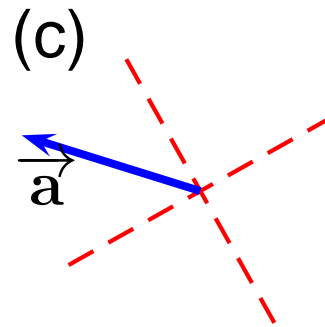
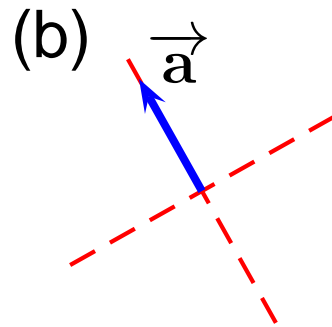
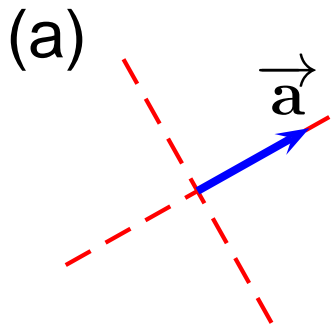
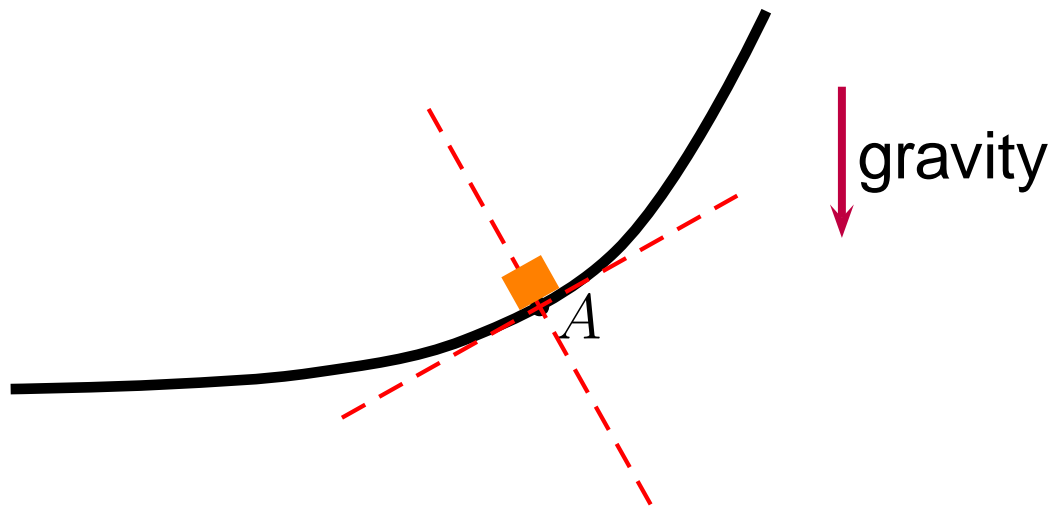
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



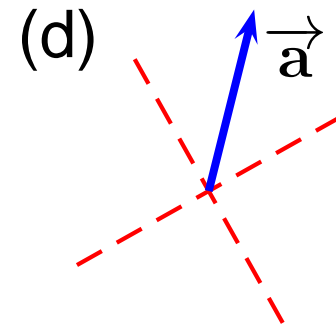
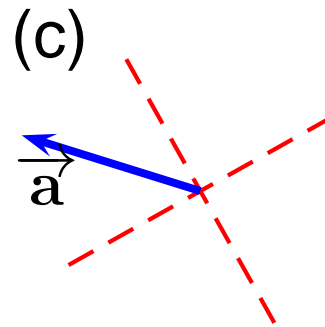
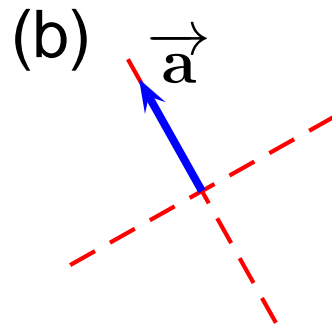
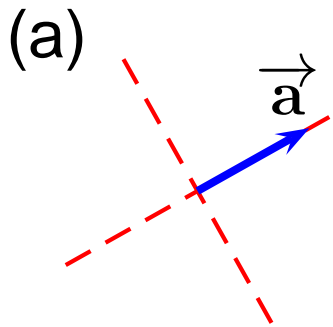
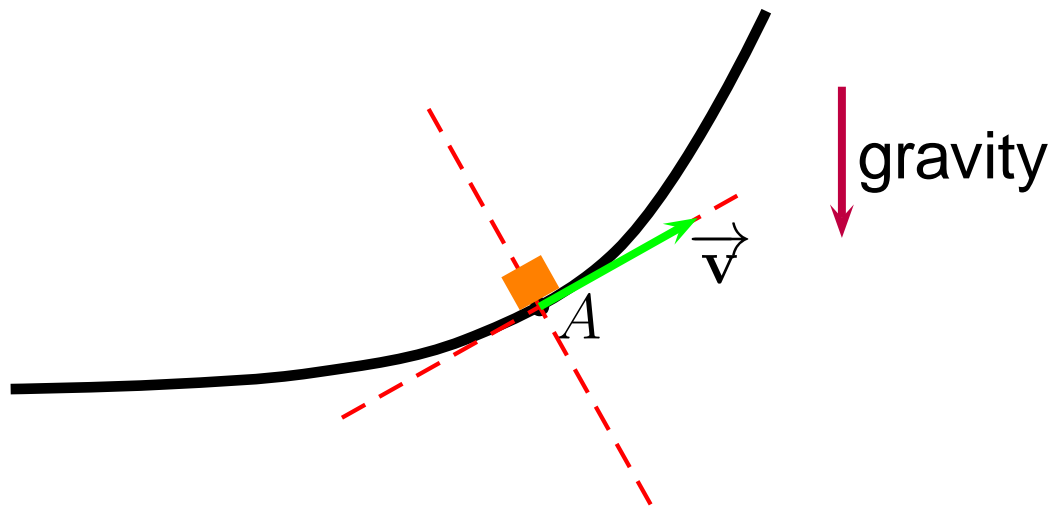
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



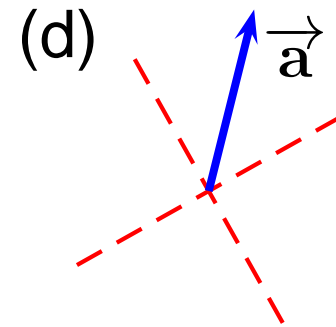
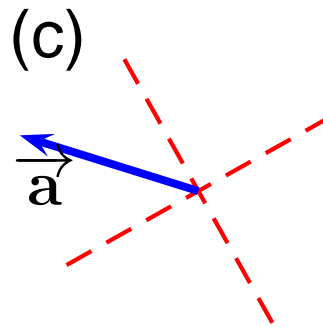
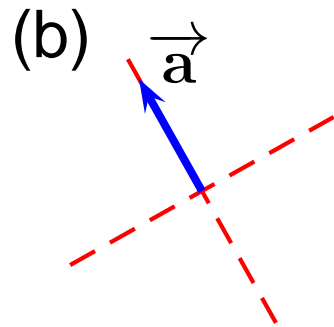
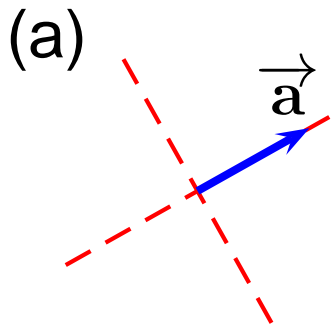
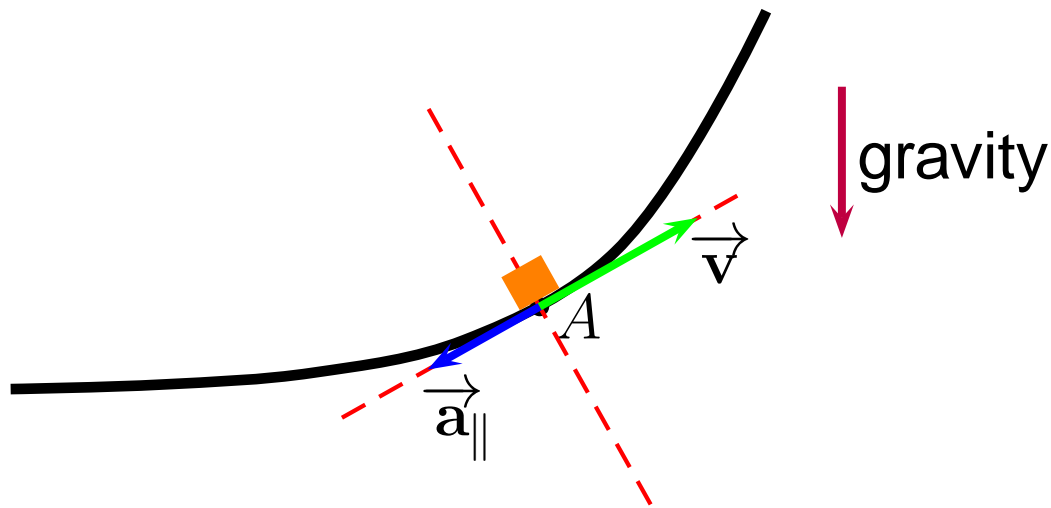
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



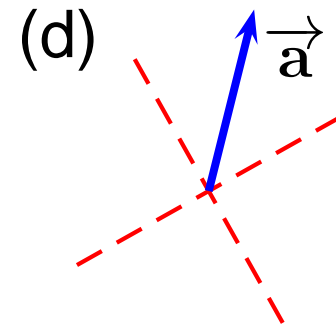
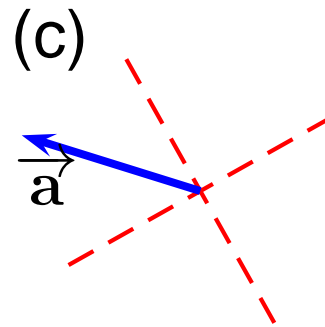
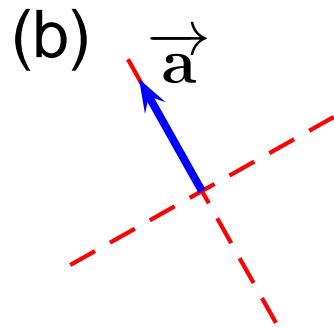
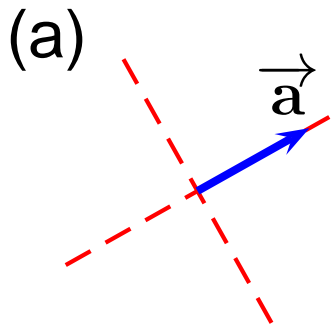
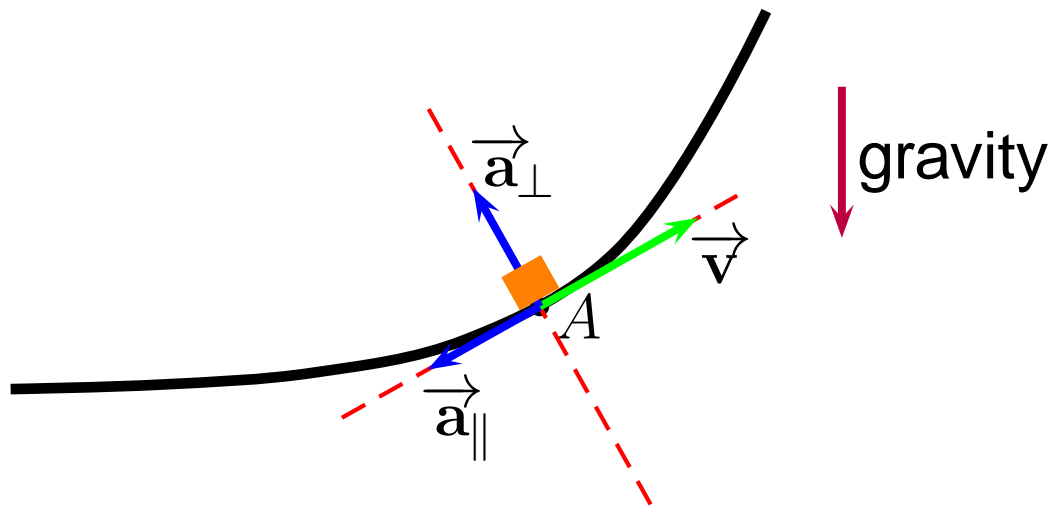
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



# Clicker Quiz

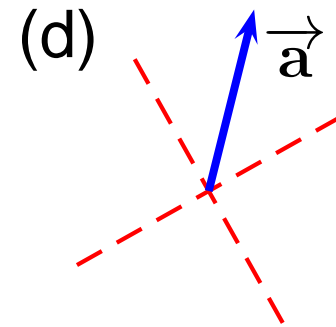
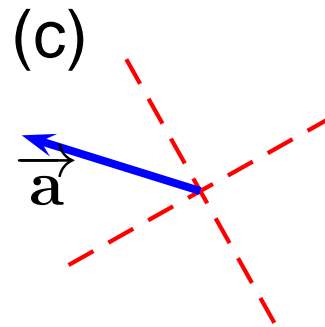
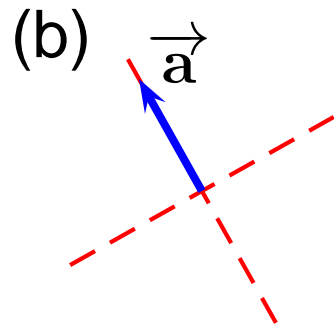
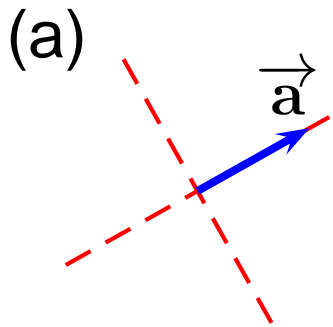
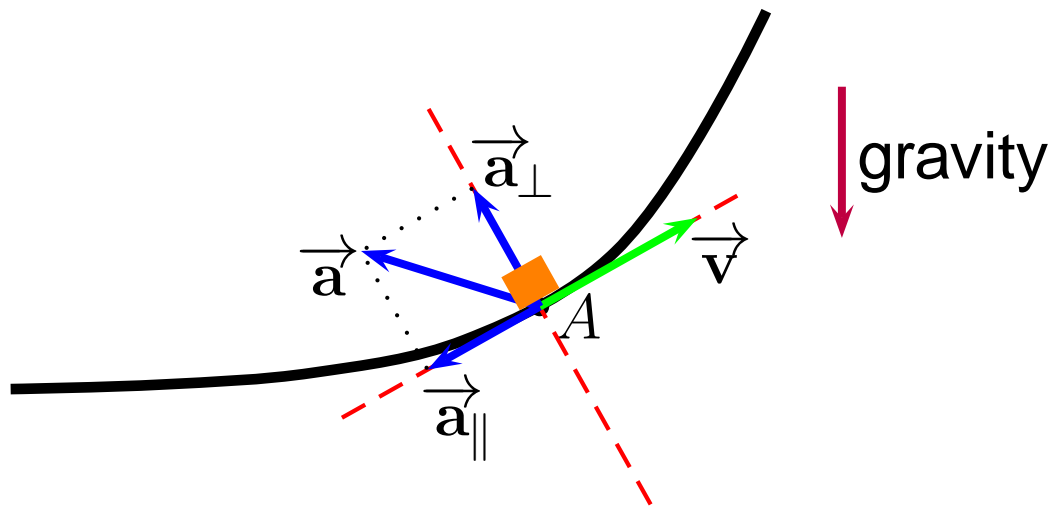
A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?





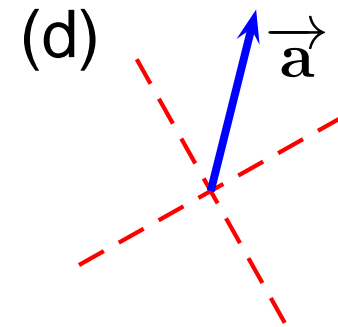
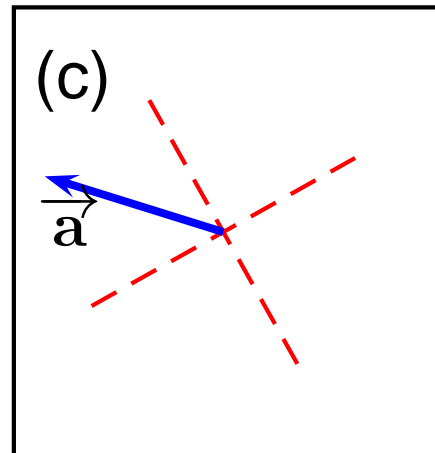
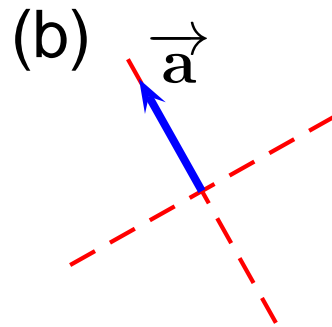
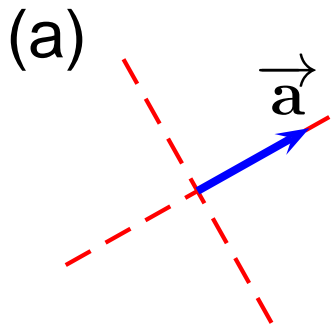
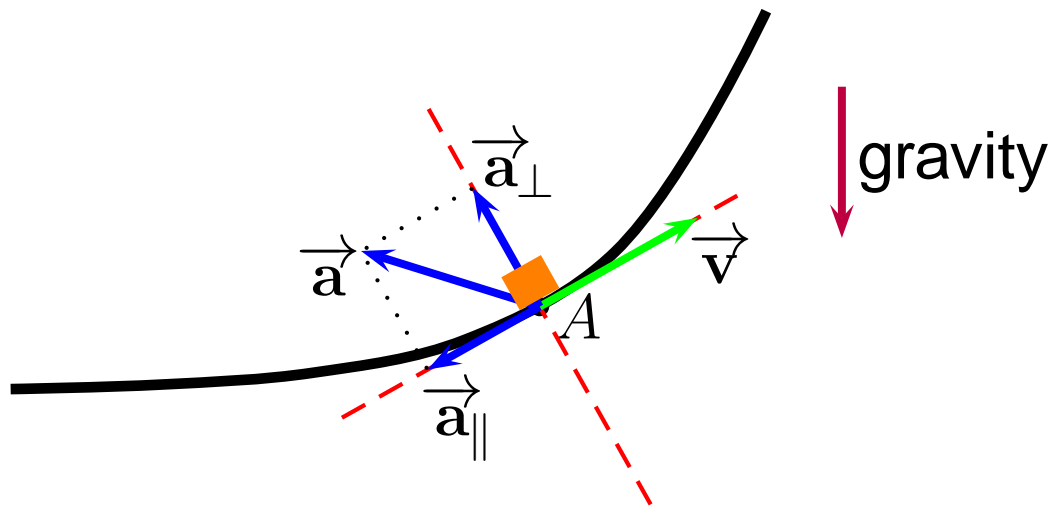
# Clicker Quiz

A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



# Clicker Quiz

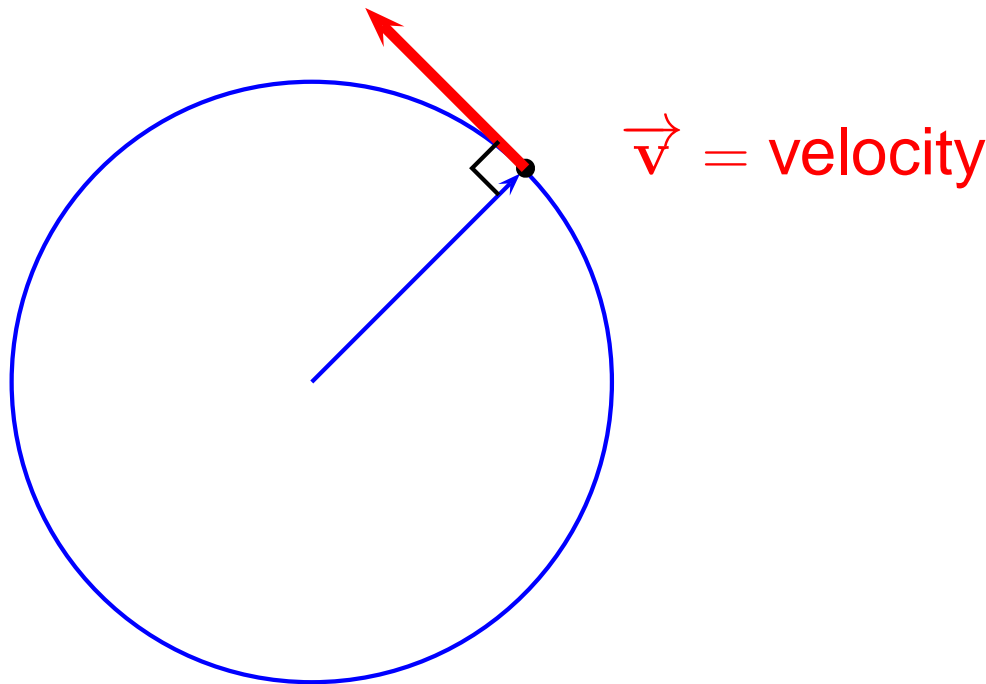
A mass slides up a ramp (against gravity) as shown. At the point labeled  $A$  which of the following choices is a possible acceleration vector?



# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

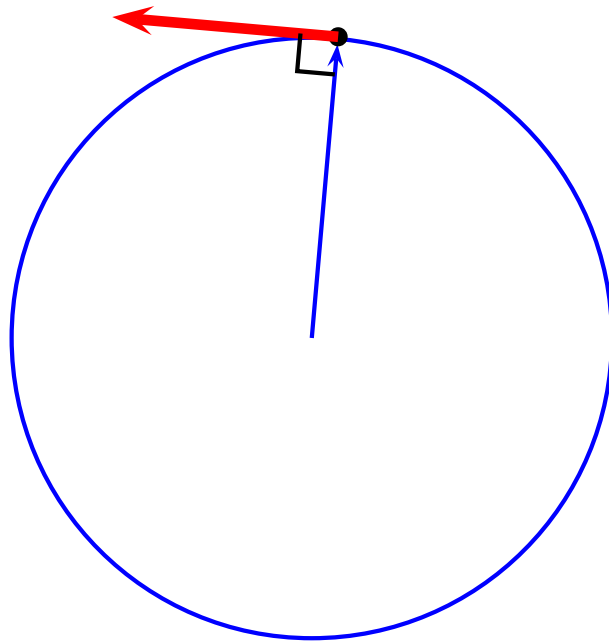
For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



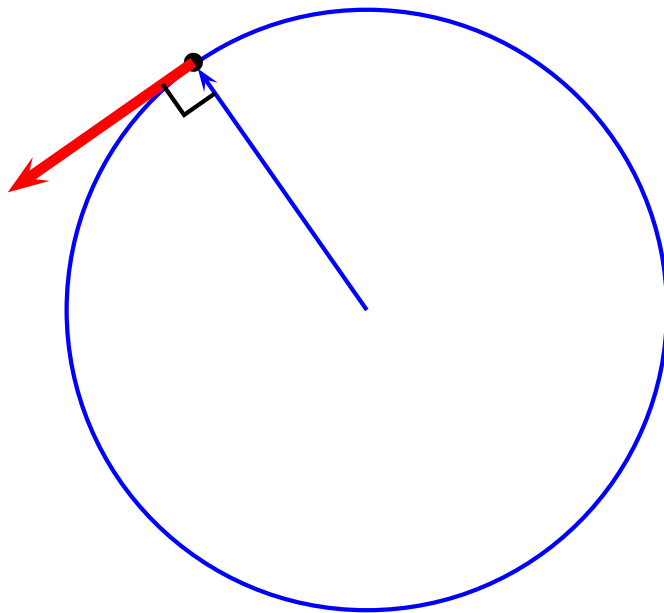
$\vec{v}$  = velocity

As the object  
goes around  
the circle  
its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



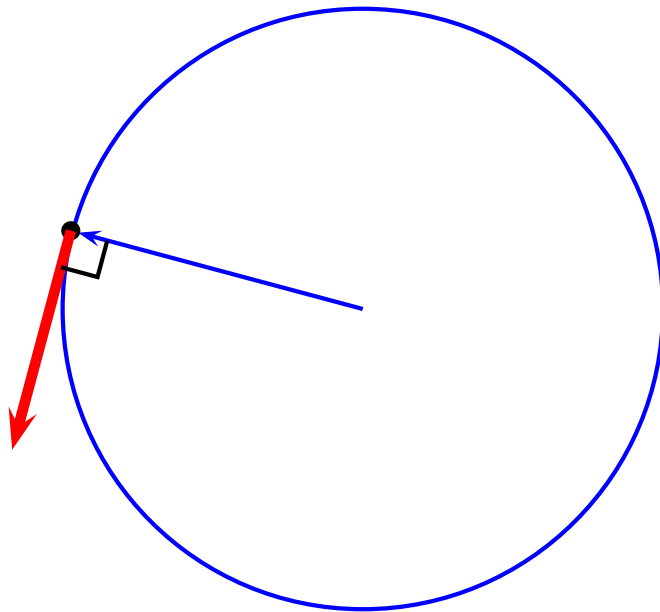
$\vec{v}$  = velocity

As the object  
goes around  
the circle  
its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



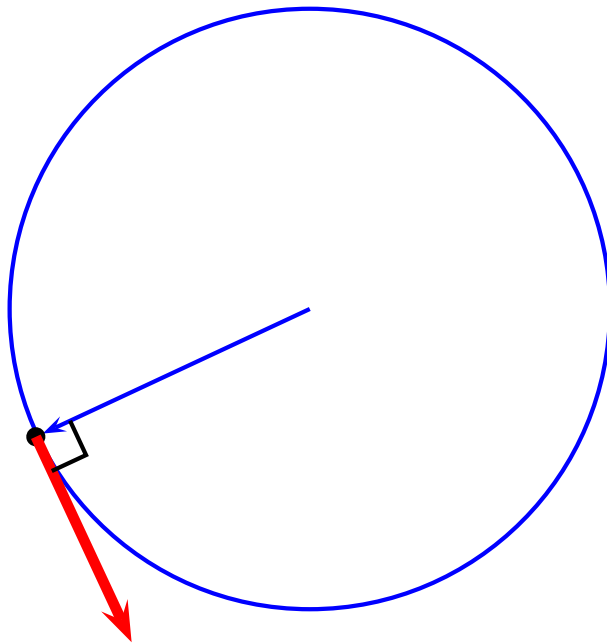
$\vec{v}$  = velocity

As the object  
goes around  
the circle  
its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



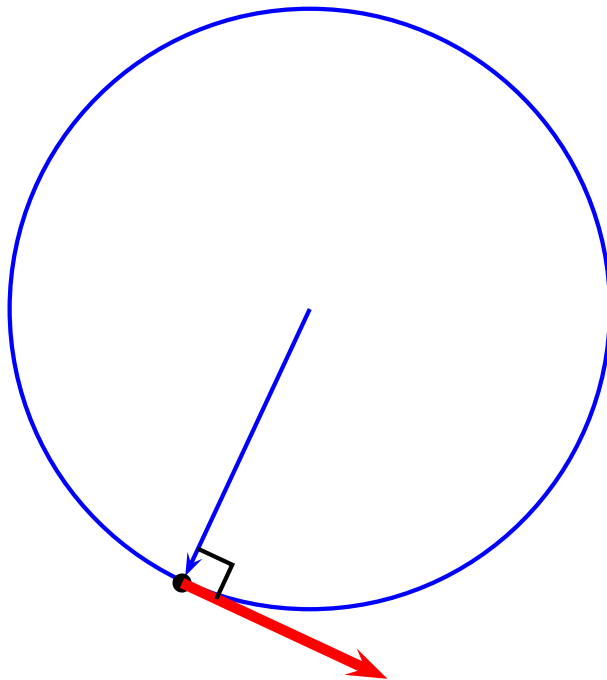
$\vec{v}$  = velocity

As the object  
goes around  
the circle  
its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



$\vec{v}$  = velocity

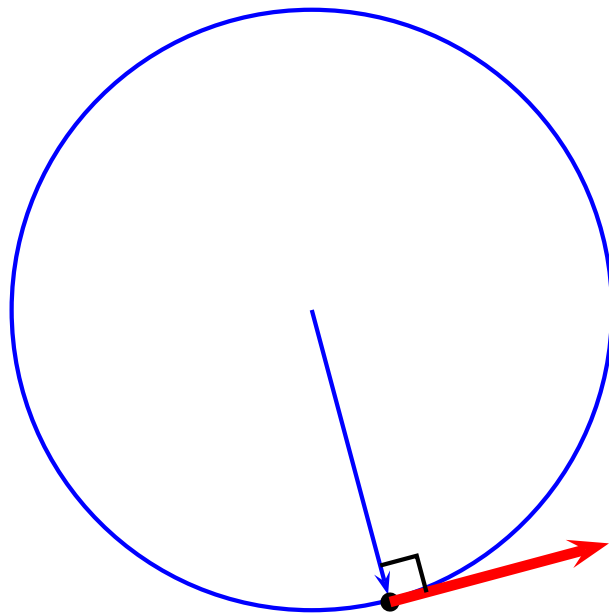
As the object  
goes around  
the circle  
its direction changes



# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



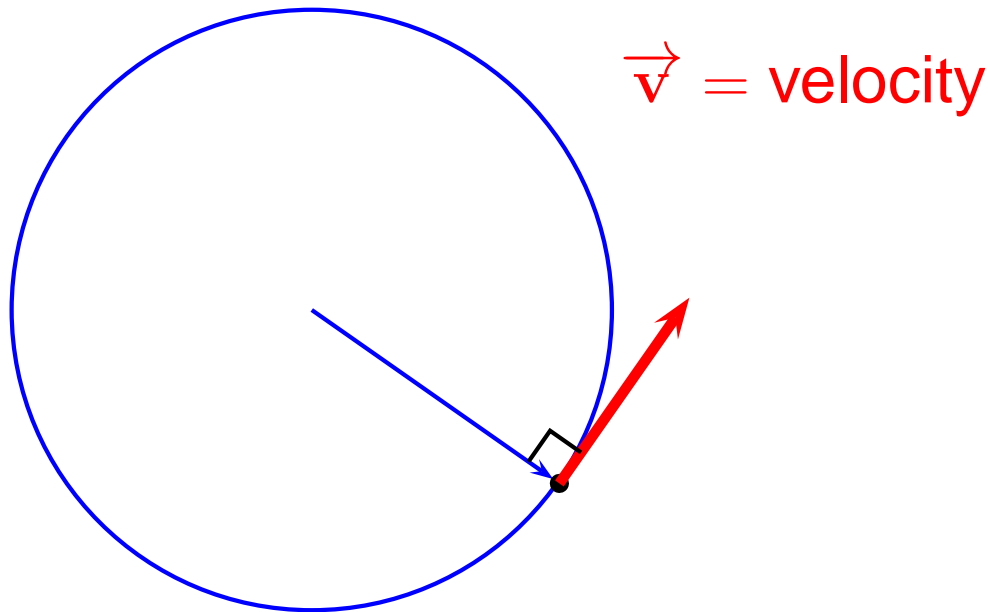
$\vec{v}$  = velocity

As the object goes around the circle its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.

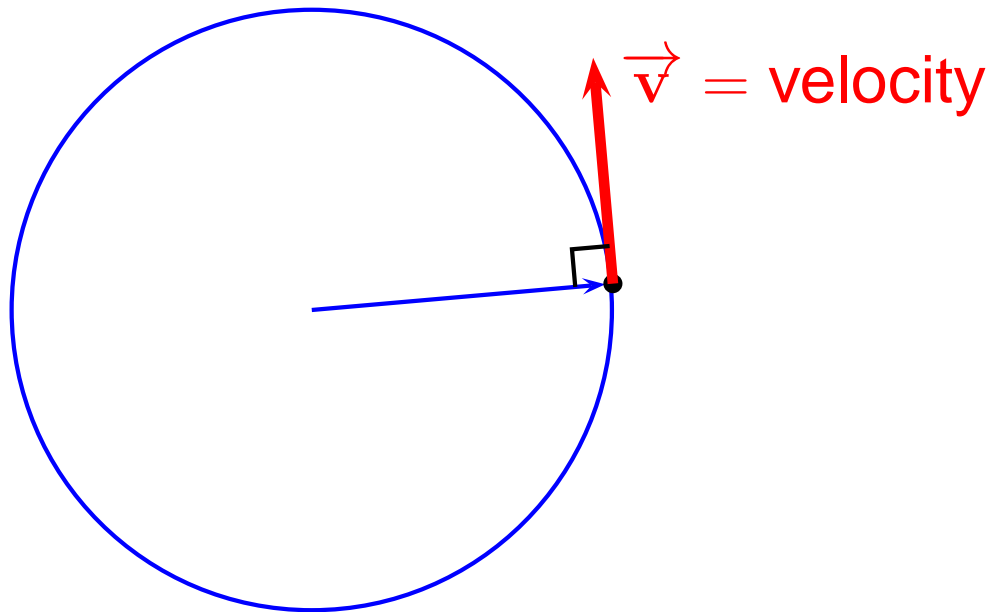


As the object goes around the circle its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.

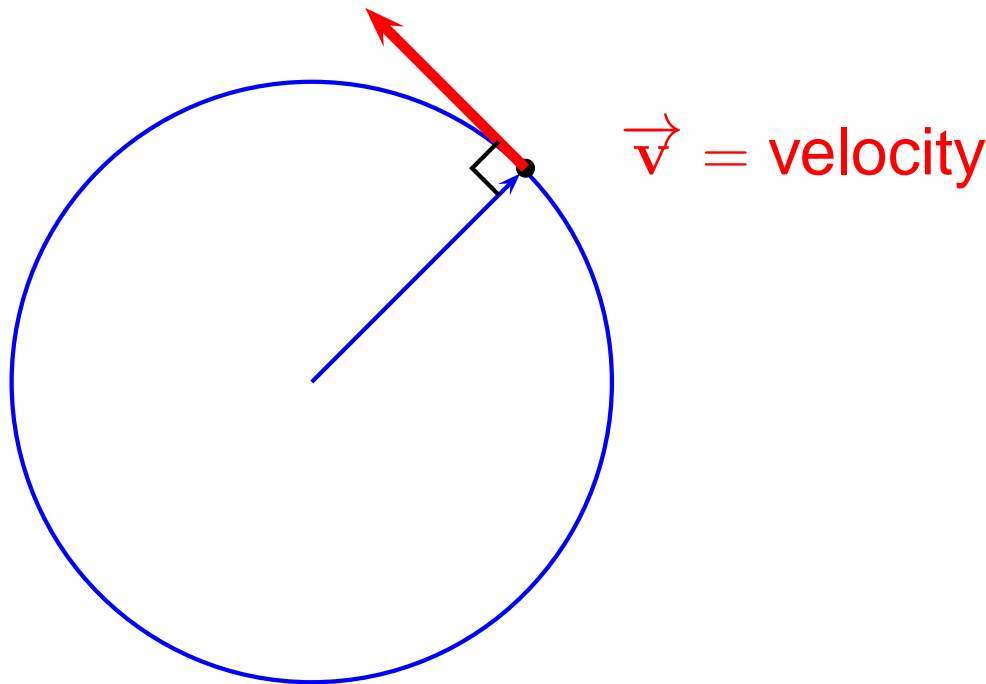


As the object  
goes around  
the circle  
its direction changes

# Uniform Circular Motion

An application of this idea is the problem of an object going around a circle with constant speed.

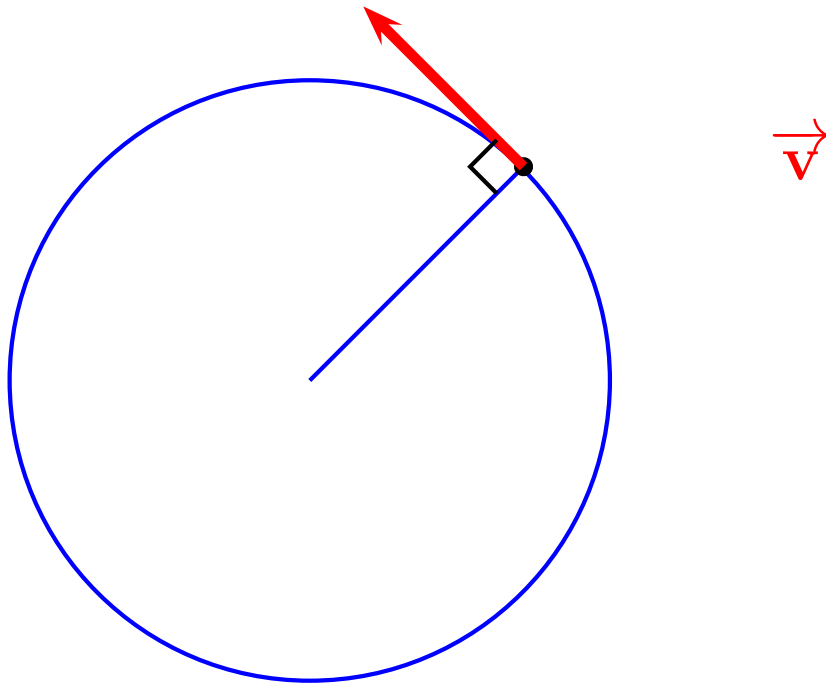
For circular motion, the velocity is tangent to the circle  
 $\Rightarrow 90^\circ$  to the circle's radius.



As the object  
goes around  
the circle  
its direction changes

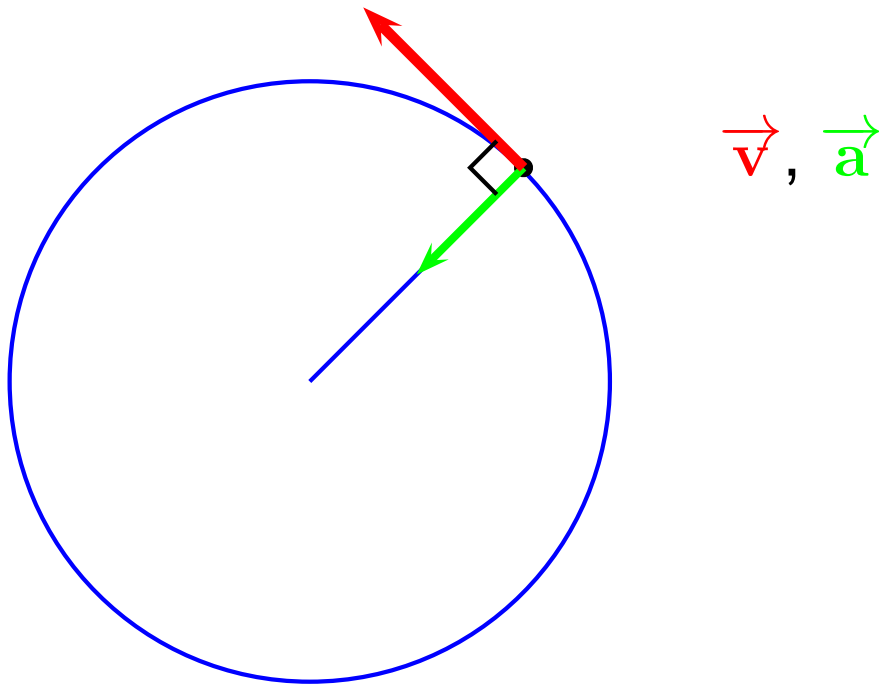
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



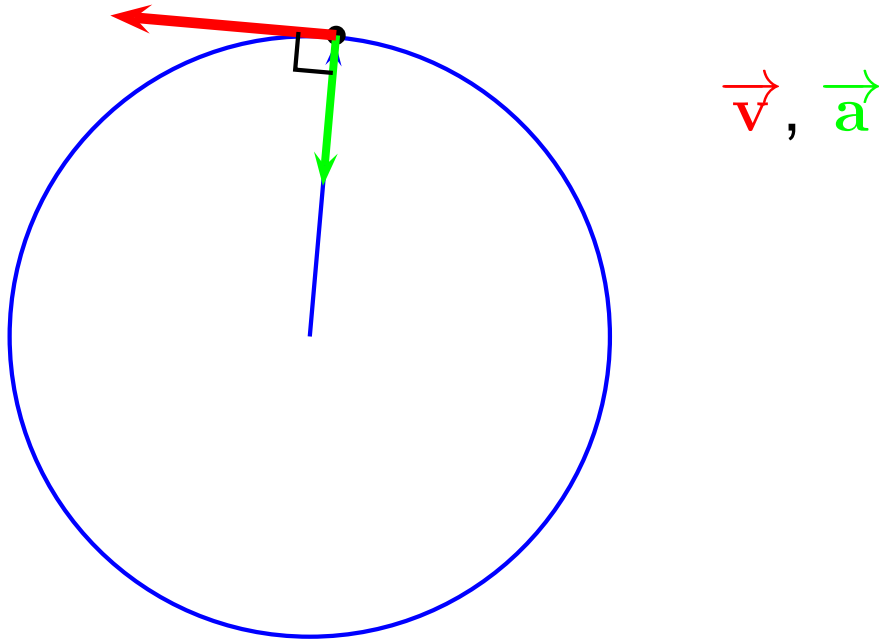
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



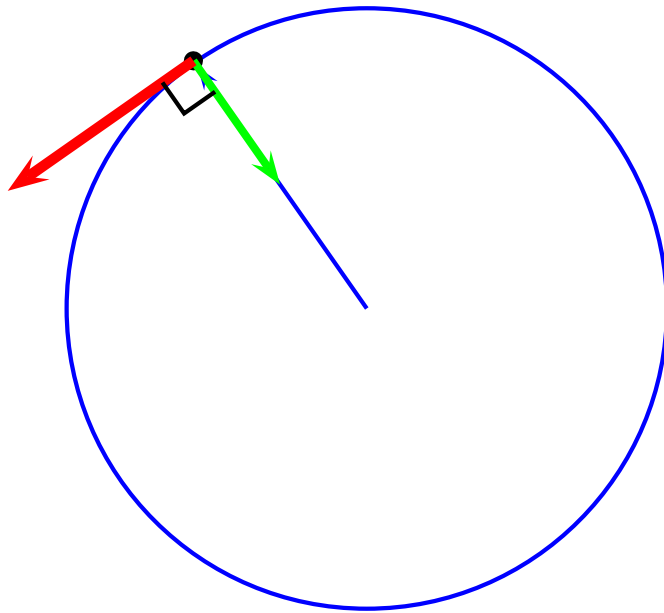
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.

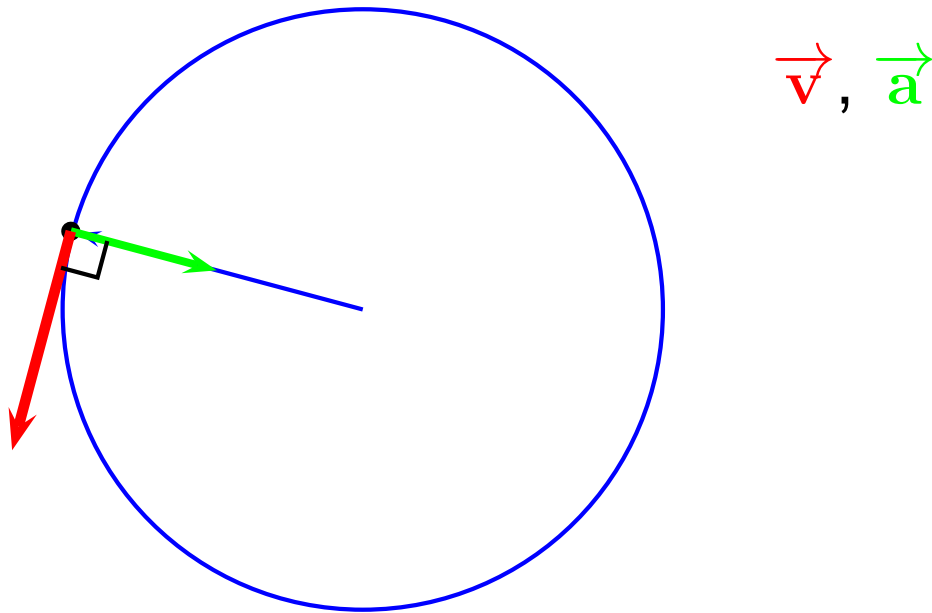


$\vec{v}$ ,  $\vec{a}$



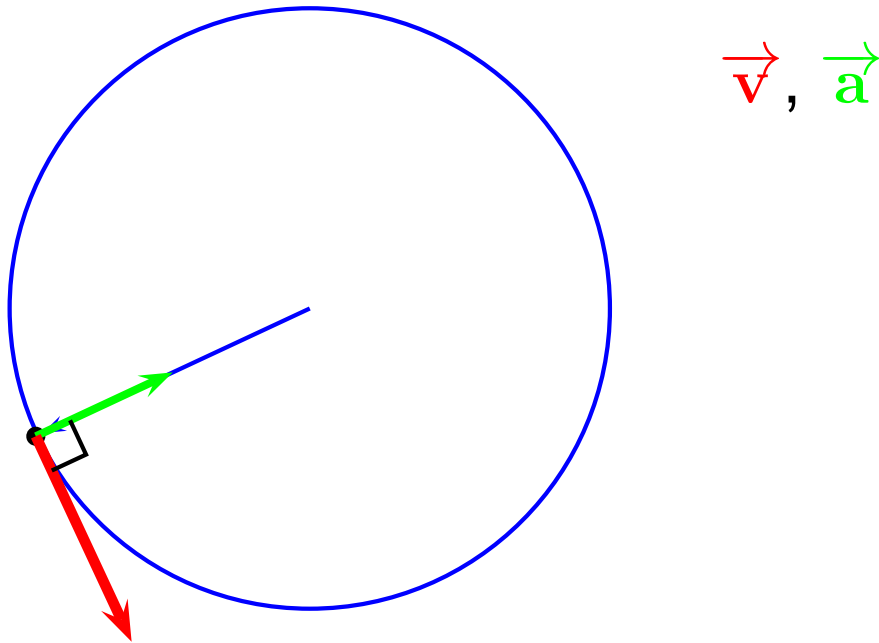
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



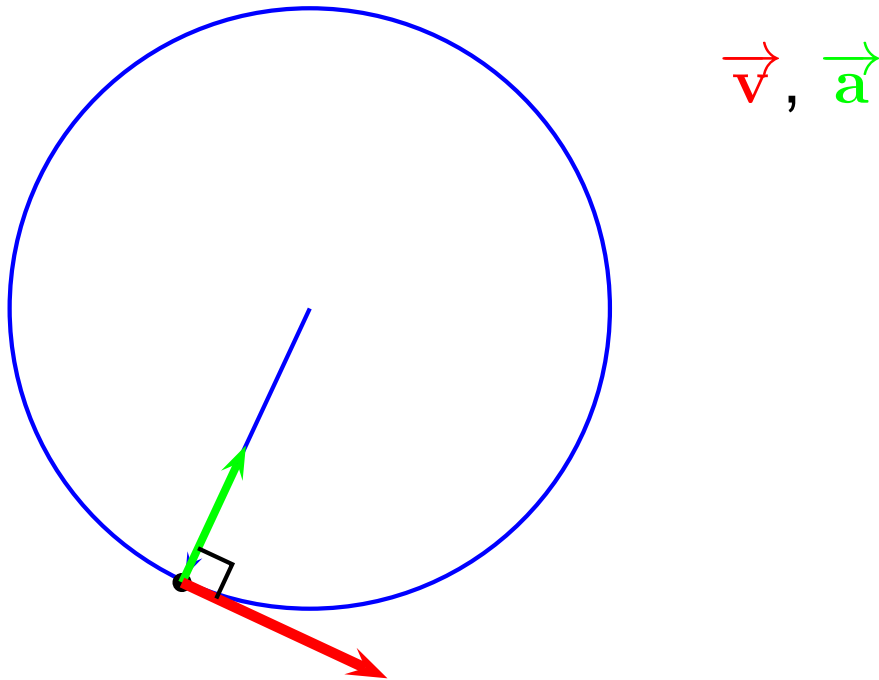
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



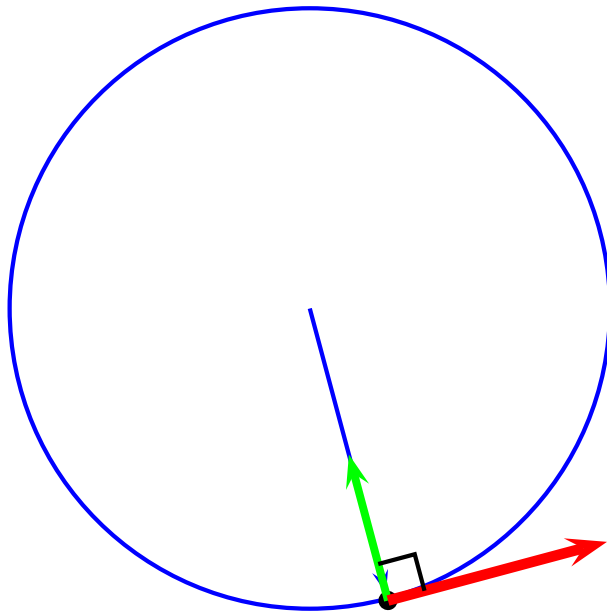
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



# Centripetal Acceleration

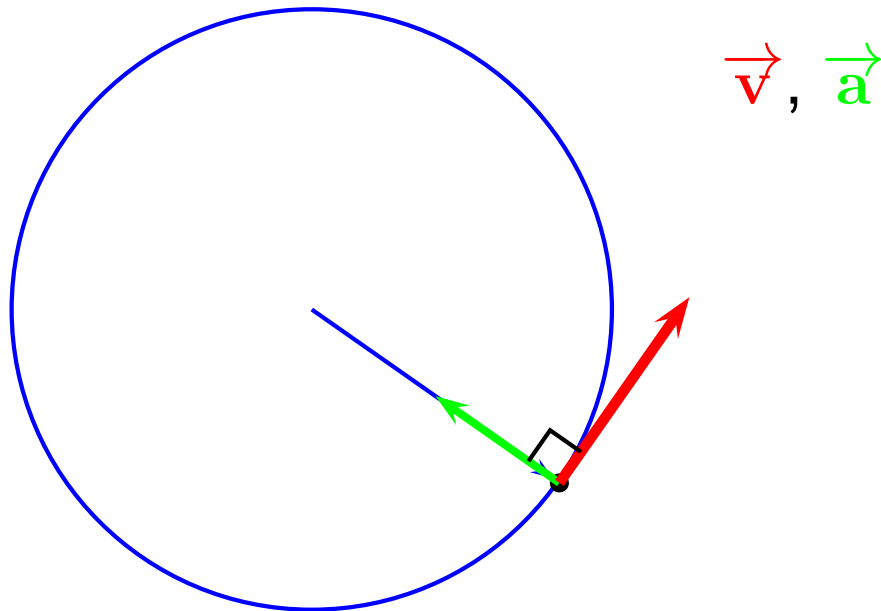
The object's acceleration must always be perpendicular to the velocity.



$\vec{v}$ ,  $\vec{a}$

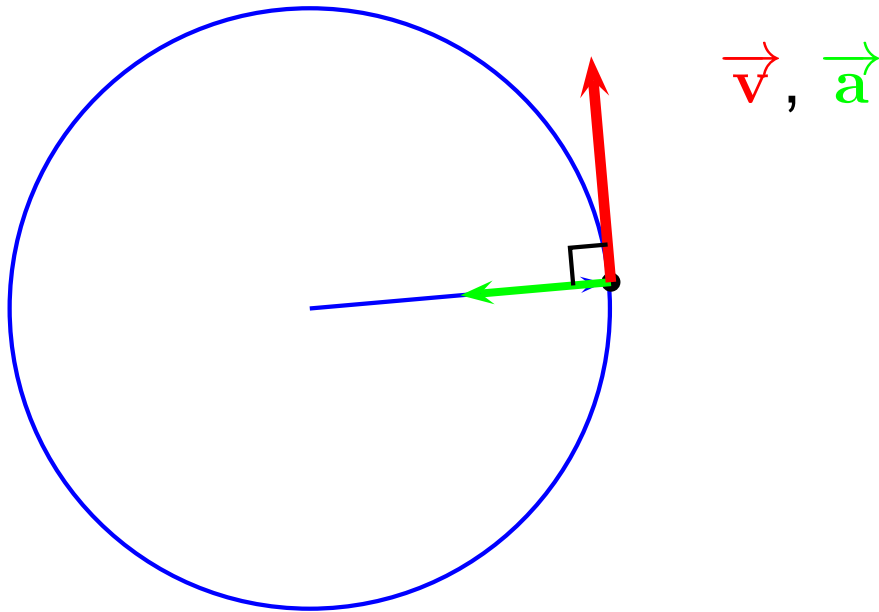
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



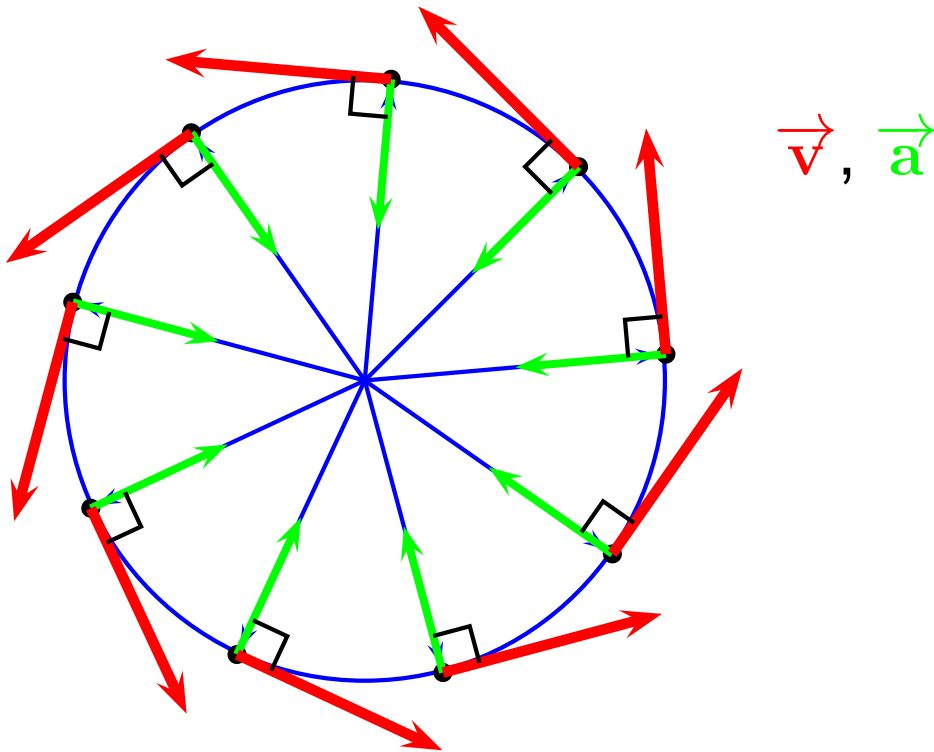
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



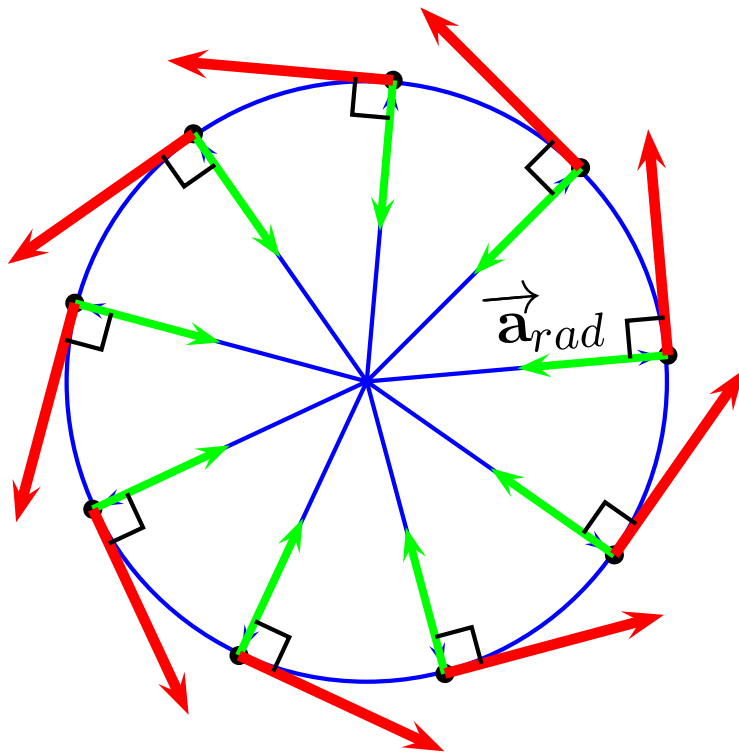
# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



# Centripetal Acceleration

The object's acceleration must always be perpendicular to the velocity.



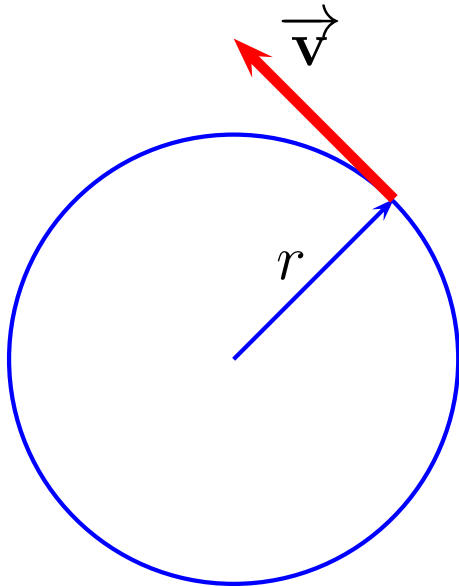
$\vec{v}$ ,  $\vec{a}$

Centripetal Acceleration -  $\vec{a}_{rad}$

The acceleration towards the center necessary for circular motion

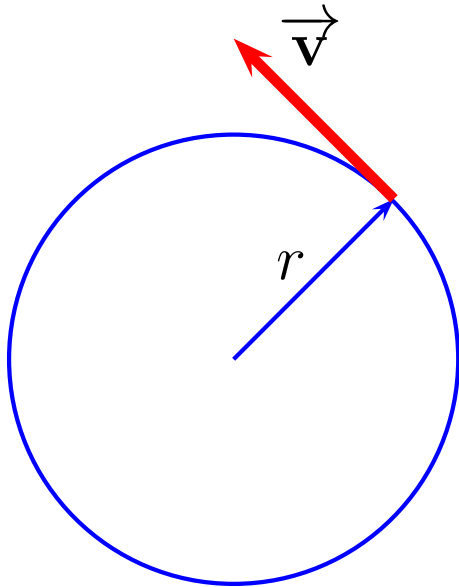


# Centripetal Acceleration II



It can be shown:  $a_{rad} = \frac{v^2}{r}$   
(See textbook for derivation)

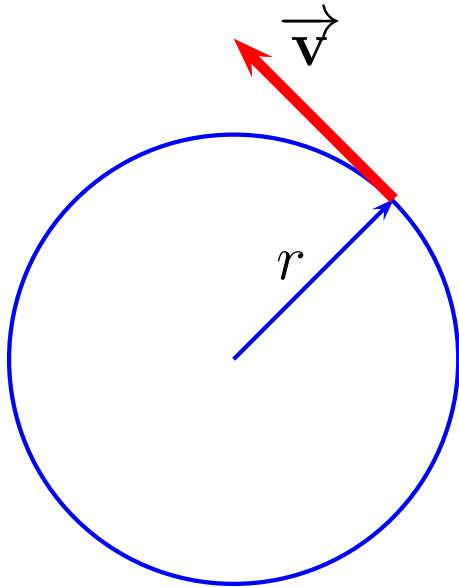
# Centripetal Acceleration II



It can be shown:  $a_{rad} = \frac{v^2}{r}$   
(See textbook for derivation)

We can also write  $a_{rad}$  in terms of the motion's period ( $T$ ).

# Centripetal Acceleration II

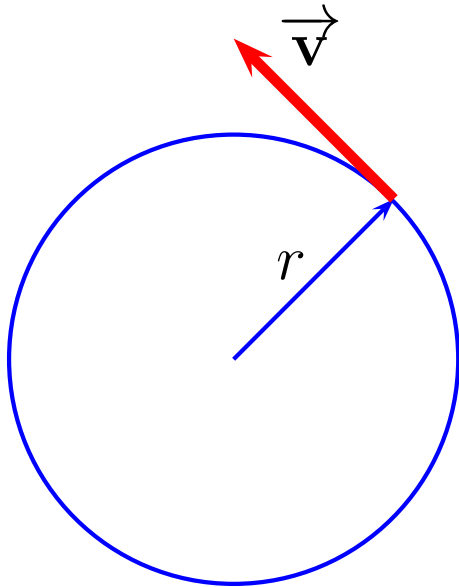


It can be shown:  $a_{rad} = \frac{v^2}{r}$   
(See textbook for derivation)

We can also write  $a_{rad}$  in terms of the motion's period ( $T$ ).

Period - The time for one revolution (once around the circle).

# Centripetal Acceleration II



It can be shown:  $a_{rad} = \frac{v^2}{r}$   
(See textbook for derivation)

We can also write  $a_{rad}$  in terms of the motion's period ( $T$ ).

Period - The time for one revolution (once around the circle).

$$T = \frac{2\pi r}{v} \Rightarrow a_{rad} = \frac{4\pi^2 r}{T^2}$$

# 2D Kinematics

In numerical problems, each component is solved for separately.

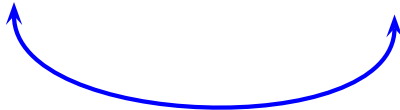
# 2D Kinematics

In numerical problems, each component is solved for separately.

$$\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} = \left( \frac{dv_x}{dt} \right) \hat{\mathbf{i}} + \left( \frac{dv_y}{dt} \right) \hat{\mathbf{j}}$$

# 2D Kinematics

In numerical problems, each component is solved for separately.


$$\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} = \left( \frac{dv_x}{dt} \right) \hat{\mathbf{i}} + \left( \frac{dv_y}{dt} \right) \hat{\mathbf{j}}$$


For Constant Acceleration:

$$v_x = v_{o,x} + a_x t$$

# 2D Kinematics

In numerical problems, each component is solved for separately.

$$\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} = \left( \frac{dv_x}{dt} \right) \hat{\mathbf{i}} + \left( \frac{dv_y}{dt} \right) \hat{\mathbf{j}}$$


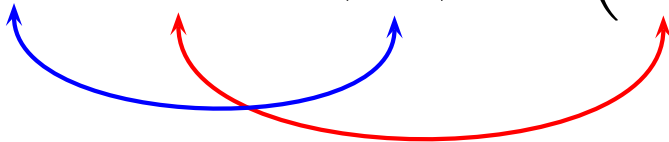
For Constant Acceleration:

$$v_y = v_{o,y} + a_y t$$



# 2D Kinematics

In numerical problems, each component is solved for separately.

$$\vec{\mathbf{a}} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}} = \left( \frac{dv_x}{dt} \right) \hat{\mathbf{i}} + \left( \frac{dv_y}{dt} \right) \hat{\mathbf{j}}$$


For Constant Acceleration:

$$v_x = v_{o,x} + a_x t$$

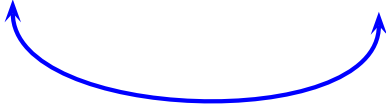
$$v_y = v_{o,y} + a_y t$$

Initial Velocity Vector:  $\vec{\mathbf{v}}_o = v_{o,x} \hat{\mathbf{i}} + v_{o,y} \hat{\mathbf{j}}$

# 2D Kinematics II

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \left( \frac{dx}{dt} \right) \hat{i} + \left( \frac{dy}{dt} \right) \hat{j}$$


# 2D Kinematics II

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \left( \frac{dx}{dt} \right) \hat{i} + \left( \frac{dy}{dt} \right) \hat{j}$$


For Constant Acceleration:

$$x = x_o + v_{o,x}t + \frac{1}{2}a_x t^2$$

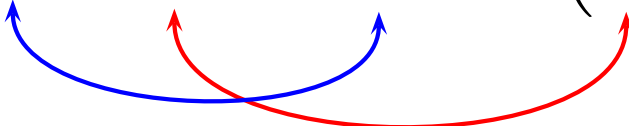
# 2D Kinematics II

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \left( \frac{dx}{dt} \right) \hat{i} + \left( \frac{dy}{dt} \right) \hat{j}$$


For Constant Acceleration:

$$y = y_o + v_{o,y}t + \frac{1}{2}a_y t^2$$

# 2D Kinematics II

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = \left( \frac{dx}{dt} \right) \hat{i} + \left( \frac{dy}{dt} \right) \hat{j}$$


For Constant Acceleration:

$$x = x_o + v_{o,x}t + \frac{1}{2}a_x t^2$$

$$y = y_o + v_{o,y}t + \frac{1}{2}a_y t^2$$

Position Vector:  $\vec{r} = x\hat{i} + y\hat{j}$

Initial Position Vector:  $\vec{r}_o = x_o\hat{i} + y_o\hat{j}$

# Example

$$x = x_o + v_{o,x}t + \frac{1}{2}a_x t^2, \quad v_x = v_{o,x} + a_x t$$

$$y = y_o + v_{o,y}t + \frac{1}{2}a_y t^2, \quad v_y = v_{o,y} + a_y t$$

Example: A car is traveling east at  $20 \text{ m/s}$  when it is hit by a truck going north. If the collision causes a constant acceleration of  $5 \text{ m/s}^2$  to the north, and the car's brakes a constant deceleration of  $3 \text{ m/s}^2$  to the west, where is the car located after 3s?

# Example

$$x = x_o + v_{o,x}t + \frac{1}{2}a_x t^2, \quad v_x = v_{o,x} + a_x t$$

$$y = y_o + v_{o,y}t + \frac{1}{2}a_y t^2, \quad v_y = v_{o,y} + a_y t$$

Example: A car is traveling east at  $20 \text{ m/s}$  when it is hit by a truck going north. If the collision causes a constant acceleration of  $5 \text{ m/s}^2$  to the north, and the car's brakes a constant deceleration of  $3 \text{ m/s}^2$  to the west, where is the car located after 3s?

- What direction is the car moving in at that point in time?