February 3, Week 3

Today: Chapter 3, Two-Dimensional Motion

Homework Assignment #3 due February 6
Mastering Physics: 3 Mastering Physics problems, 2.77, 2.85, 2.93.
Written Problem: 2.88.

Exam #1 Friday, February 10.

Practice Exam available on website.

Chapter 2 practice problems now available on Mastering Physics.

Clicker Poll

Which of the following best describes your situation?

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(a) I would like a review session at 7:30PM on Tuesday, February 7.

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(b) I would like a review session at 7:30PM on Thursday, February 9.

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(a) I would like a review session at 7:30PM on Tuesday, February 7.

(b) I would like a review session at 7:30PM on Thursday, February 9.

(c) I cannot make either of these times for a review session but would like to have one.

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Which of the following best describes your situation?

(a) I would like a review session at 7:30PM on Tuesday, February 7.

(b) I would like a review session at 7:30PM on Thursday, February 9.

(c) I cannot make either of these times for a review session but would like to have one.

(d) I have no desire for a review session but am clicking in order to get my three clicker-quiz points.

Two-Dimensional Motion

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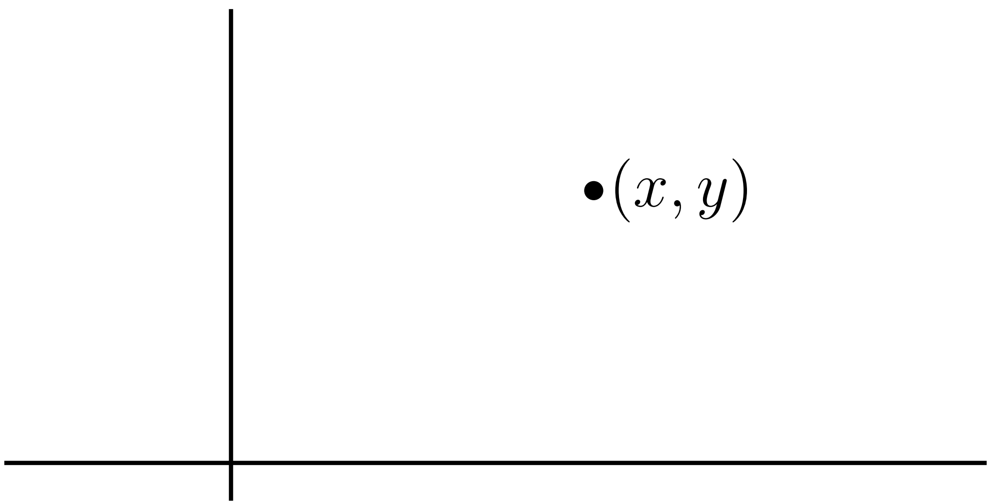
In 2D (and 3D), this means we have to know the components of the position, velocity, and acceleration vectors.

The Position Vector

In two-dimensional, cartesian coordinates, objects are located at the point (x, y).



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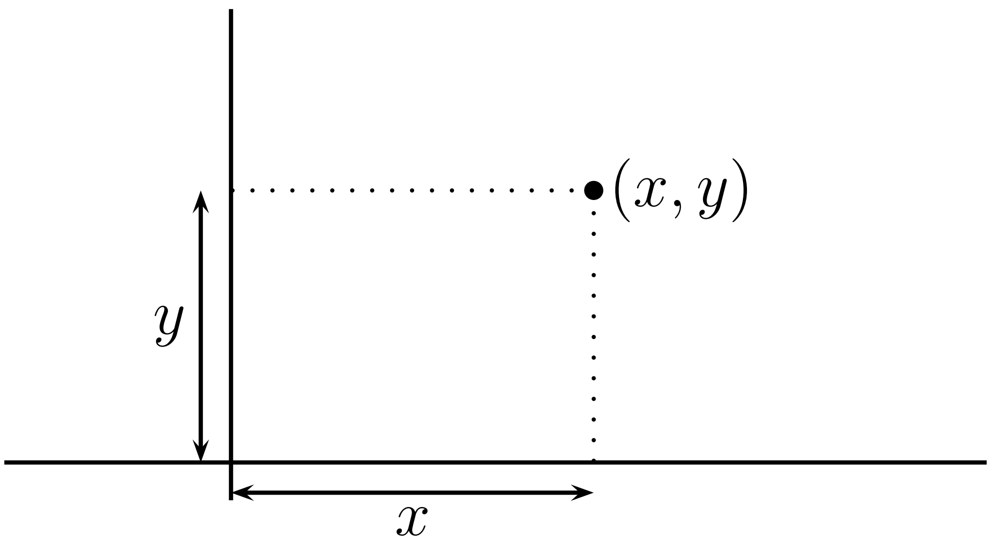

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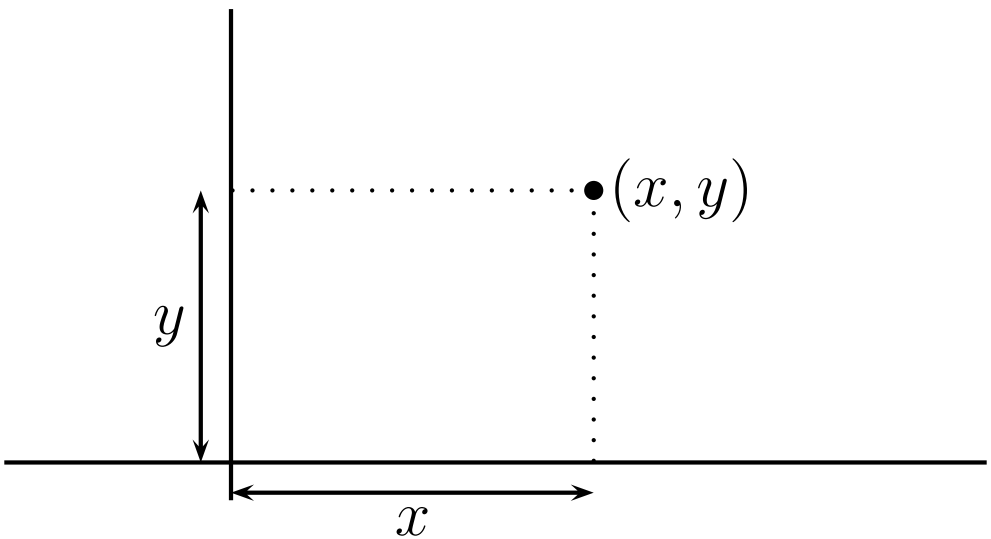

Same procedure as finding components!

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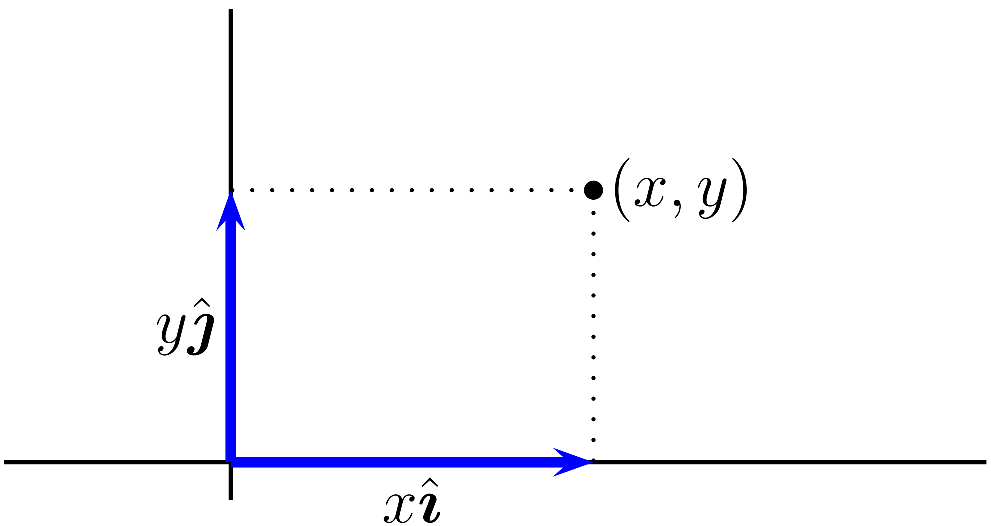

We make the cartesian coordinates the x and y components of the position vector.

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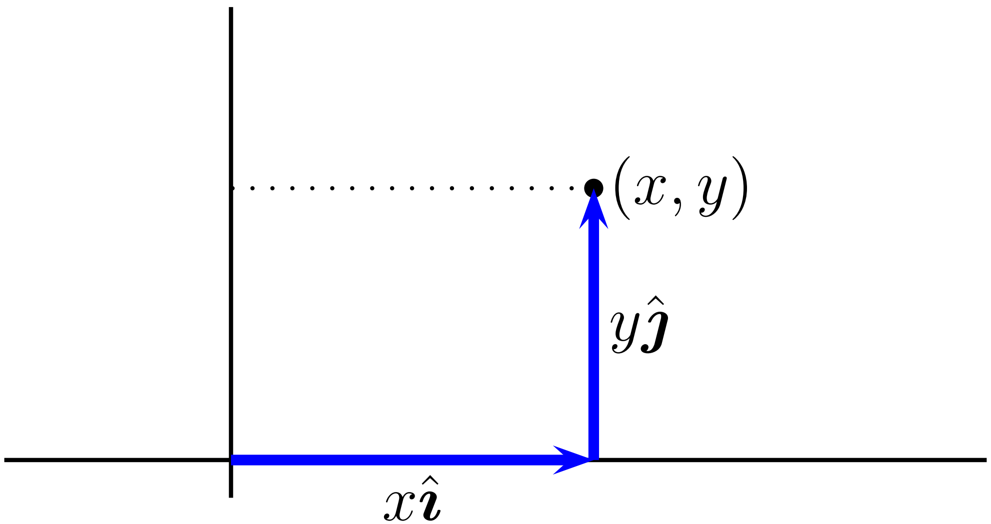

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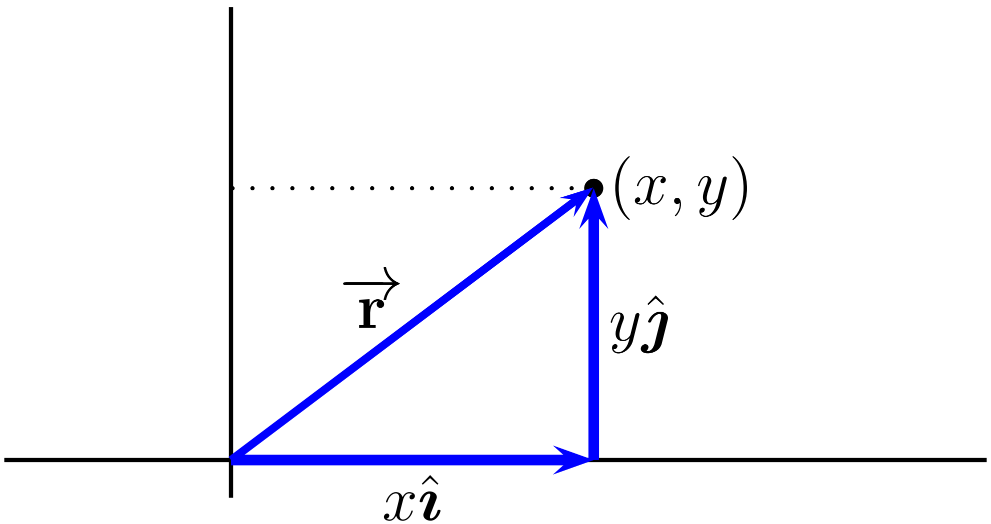

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$\Rightarrow$ the position vector goes from the origin to the object's location.

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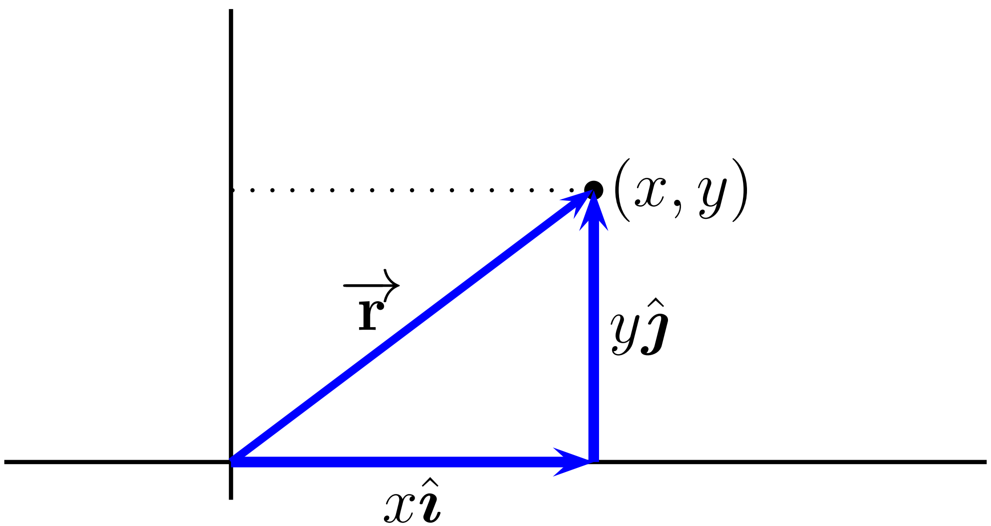

$\vec{\mathbf{r}} = x\hat{\boldsymbol{\imath}} + y\hat{\boldsymbol{\jmath}}$

$\Rightarrow$ the position vector goes from the origin to the object's location.

Distance is given by the magnitude of the position vector $\Rightarrow r = \sqrt{x^2 + y^2}$.

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Displacement has the same definition but is now a vector subtraction.

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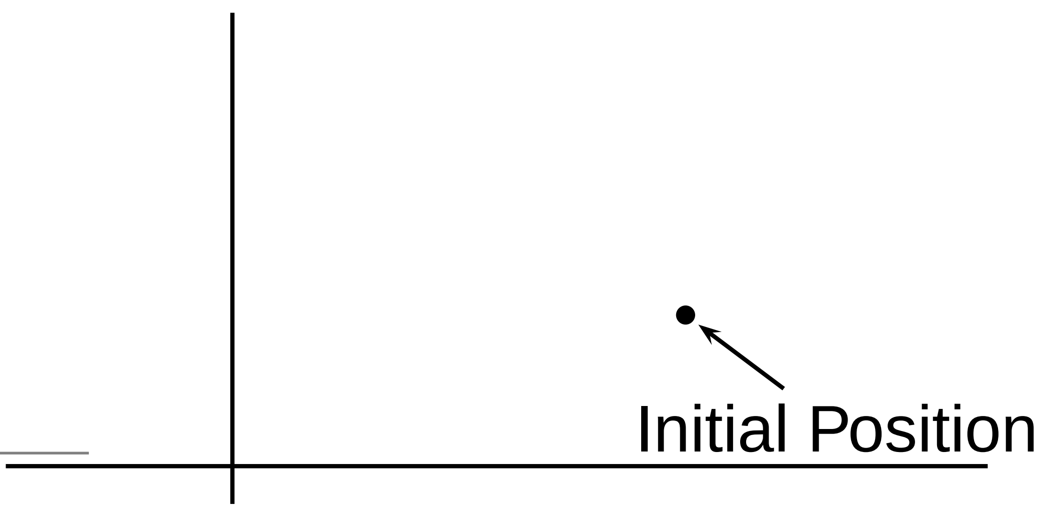

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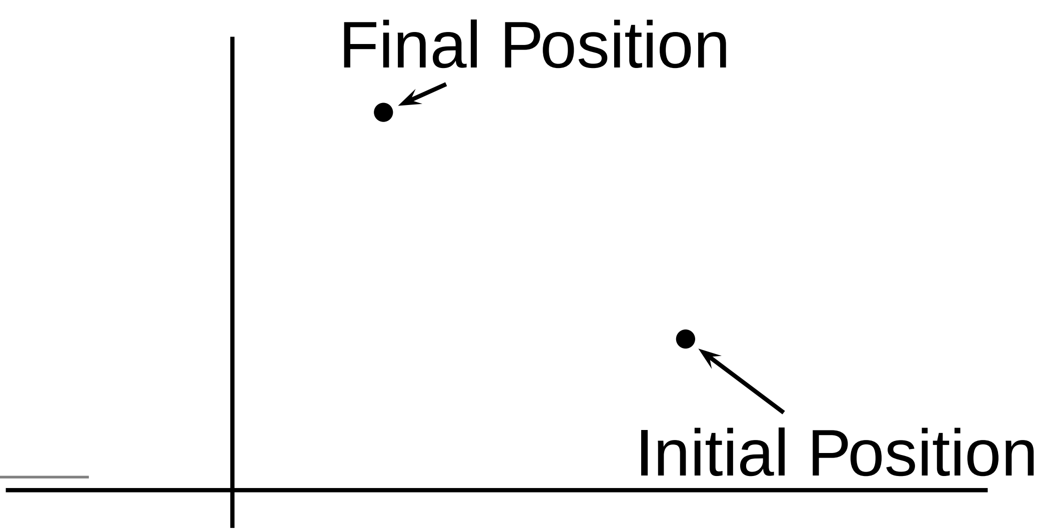

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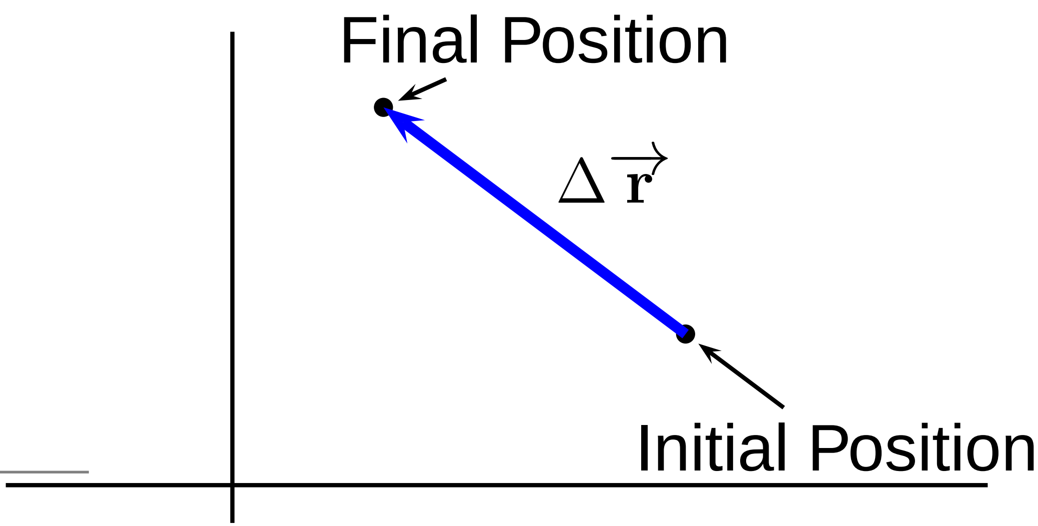

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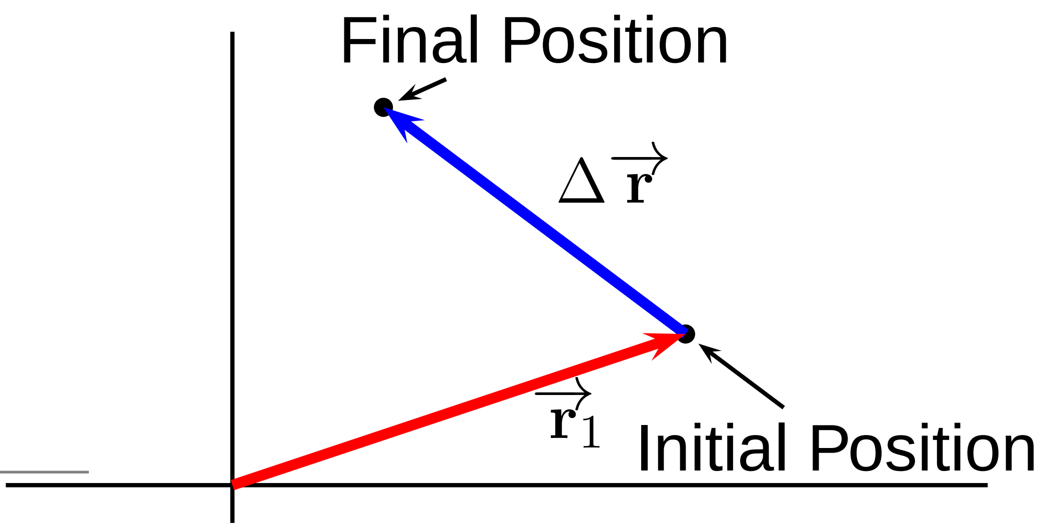

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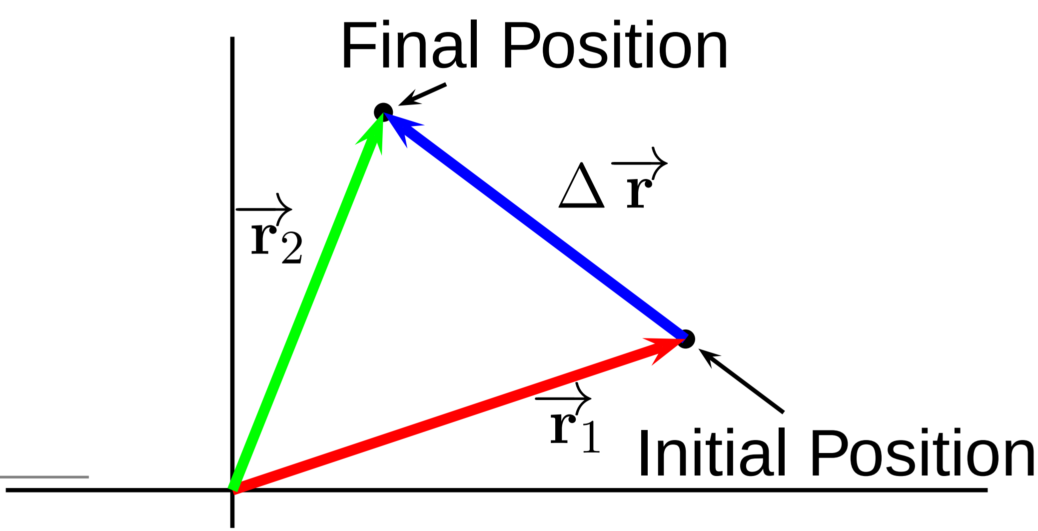

Graphical addition shows that $\overrightarrow{\mathbf{r}}_1 + \Delta \overrightarrow{\mathbf{r}} = \overrightarrow{\mathbf{r}}_2$

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In terms of components:

$$\Delta \overrightarrow{\mathbf{r}} = (x_2 - x_1)\,\hat{\boldsymbol{\imath}} + (y_2 - y_1)\,\hat{\boldsymbol{\jmath}} = \Delta x\hat{\boldsymbol{\imath}} + \Delta y\hat{\boldsymbol{\jmath}}$$

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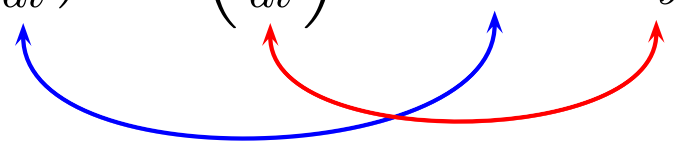
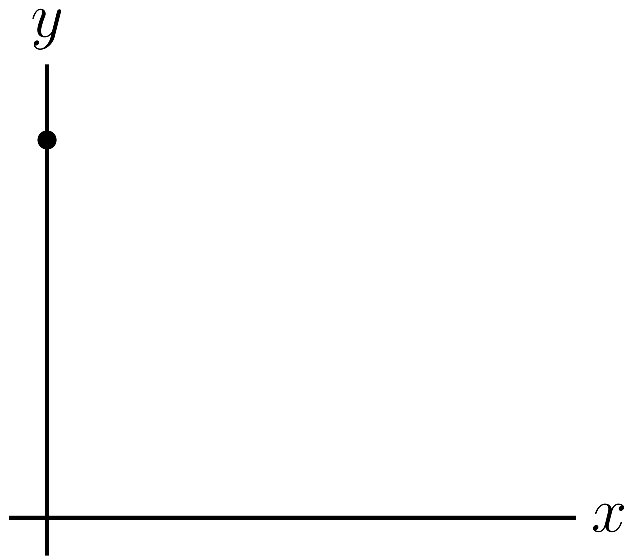


Motion in x direction.

Motion in y direction.

Velocity II



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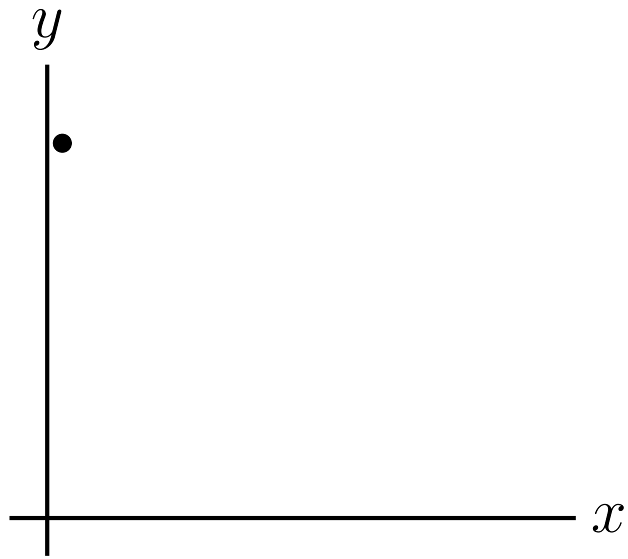

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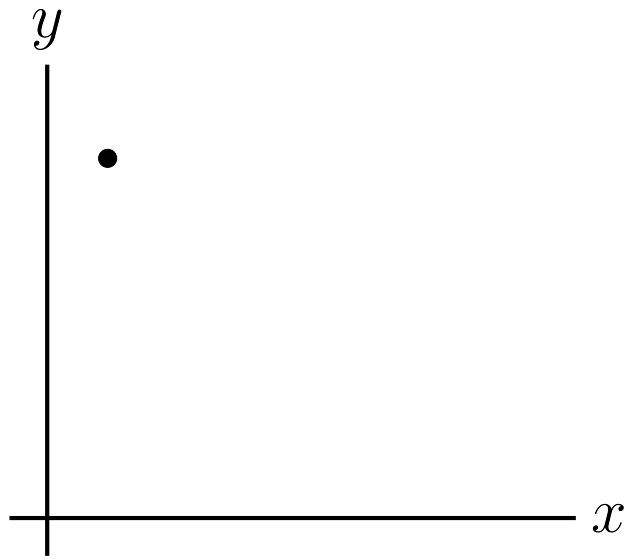

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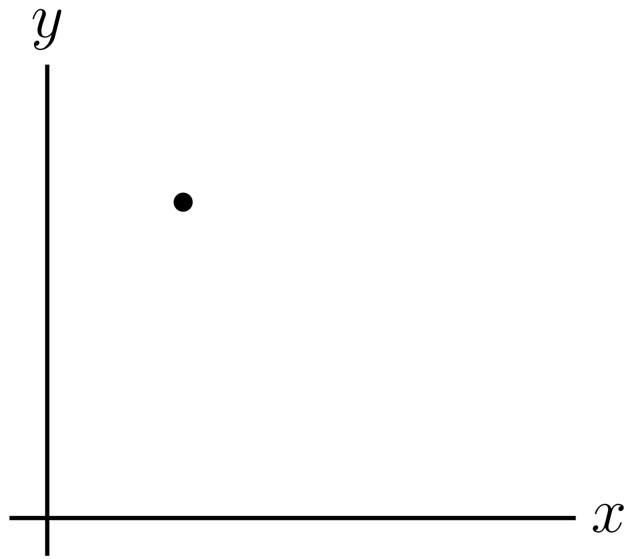

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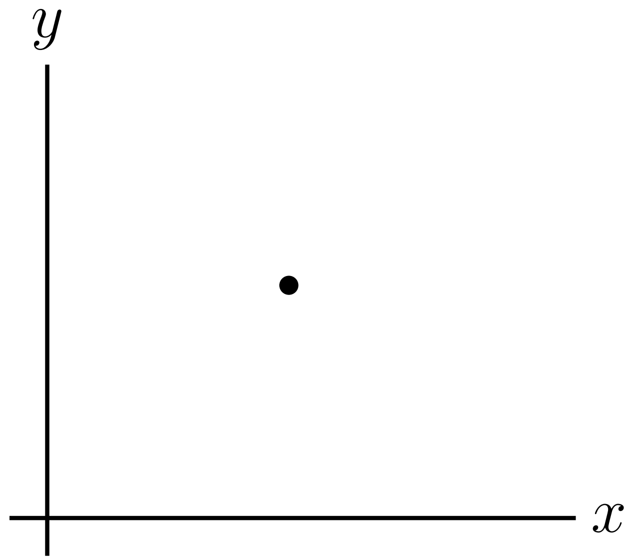

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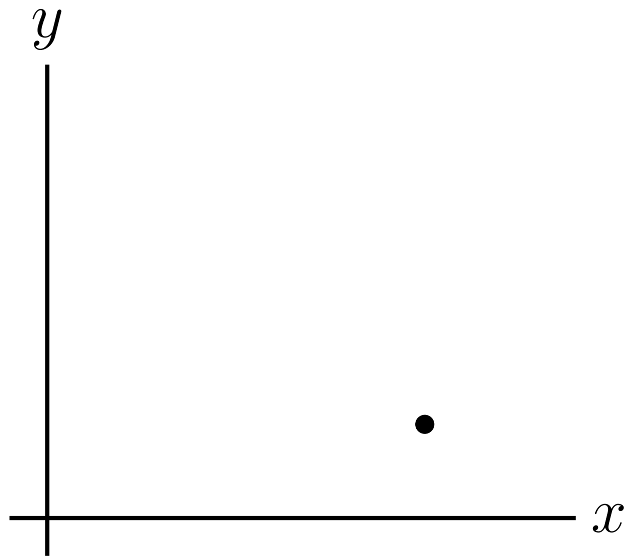

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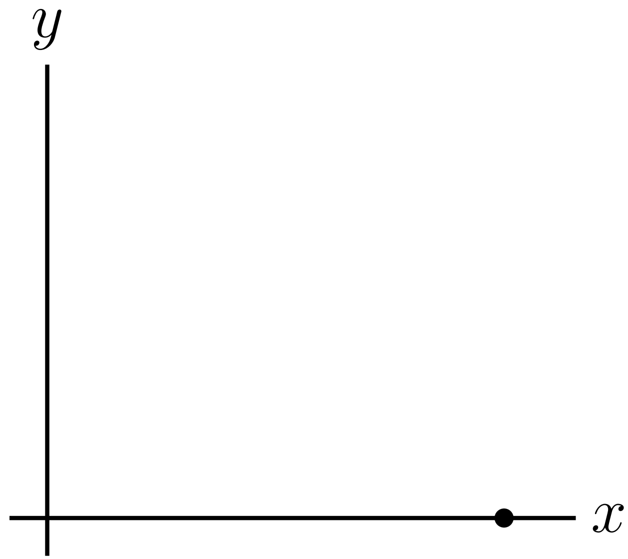

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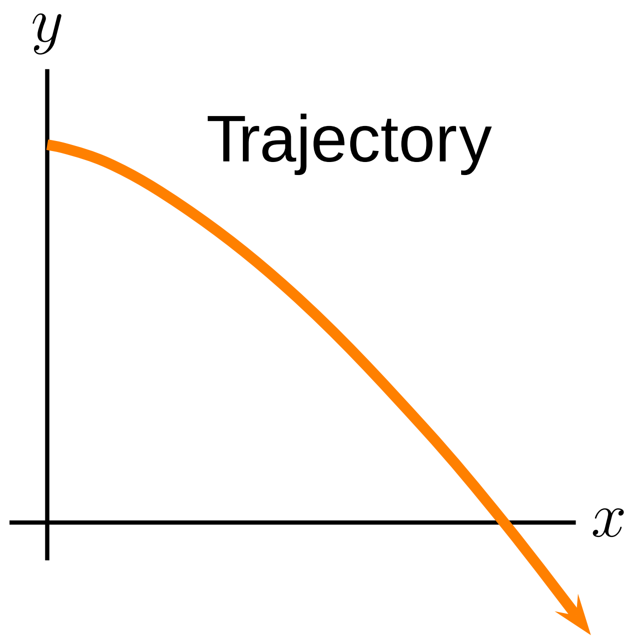

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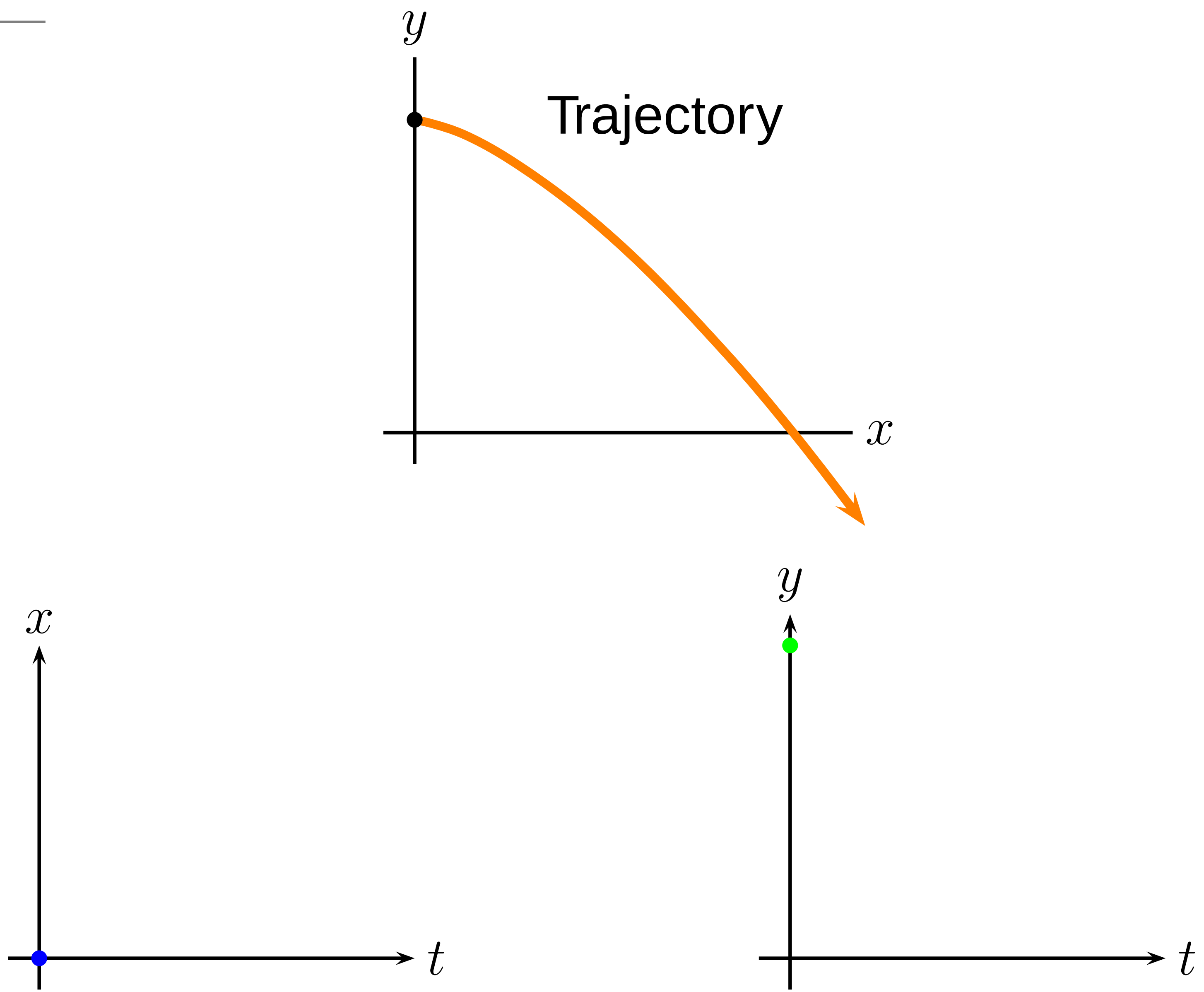

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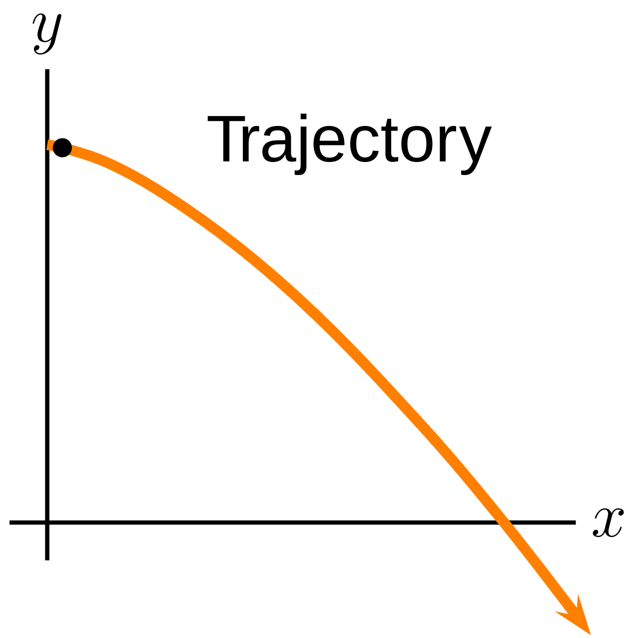

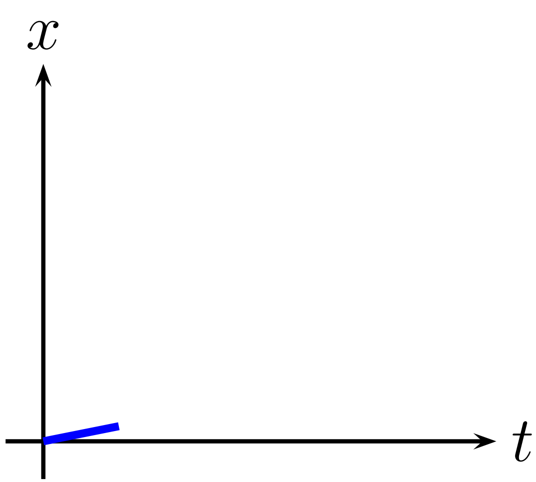

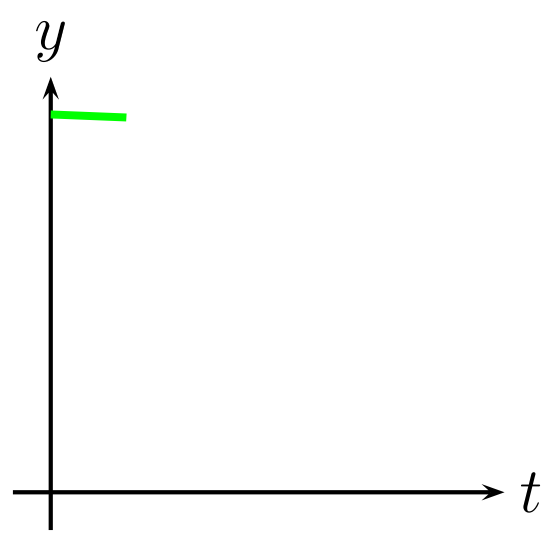

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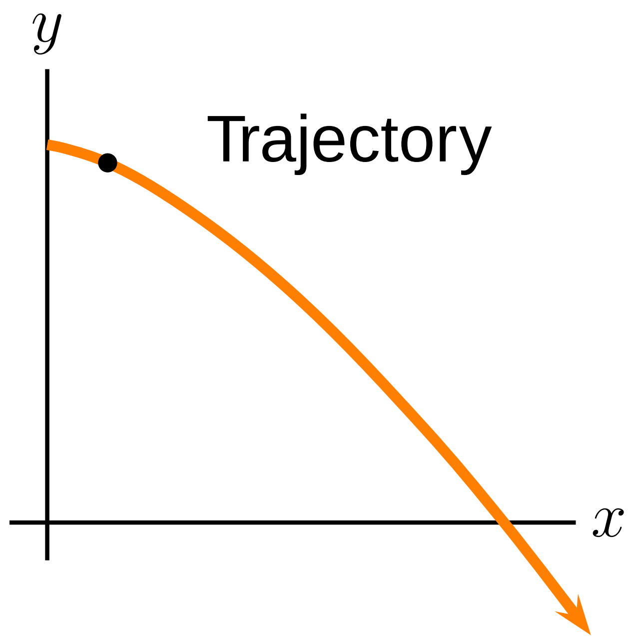

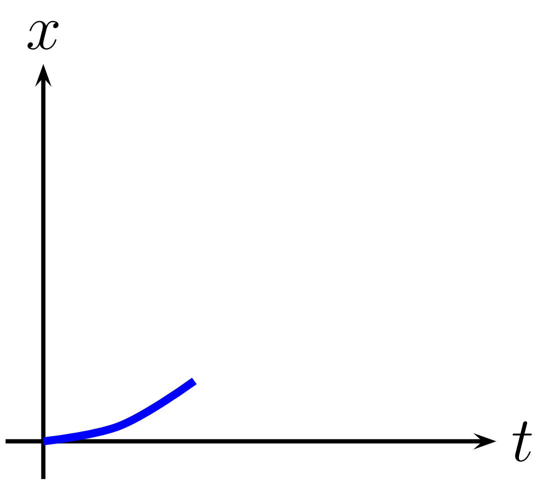

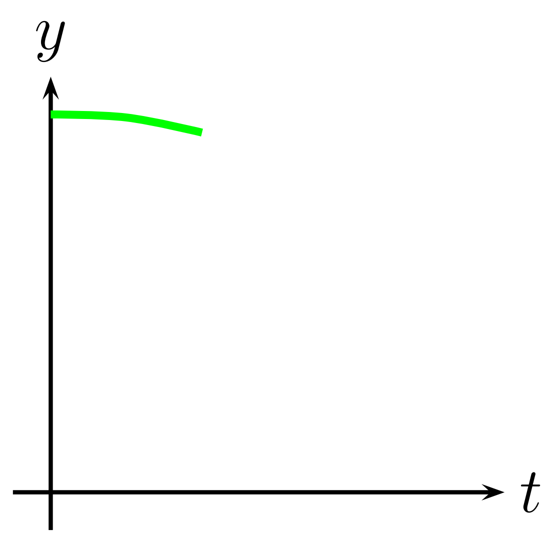

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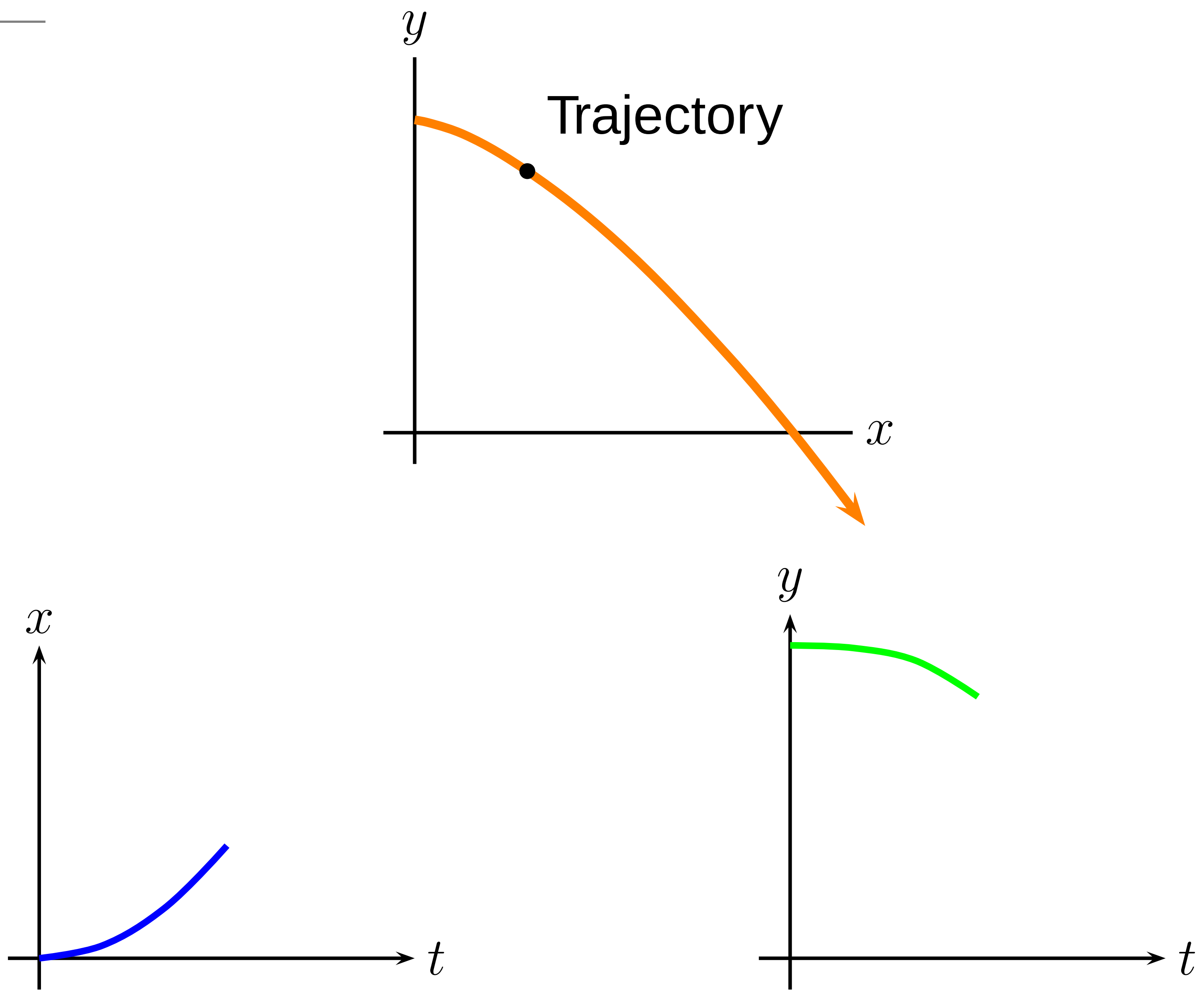

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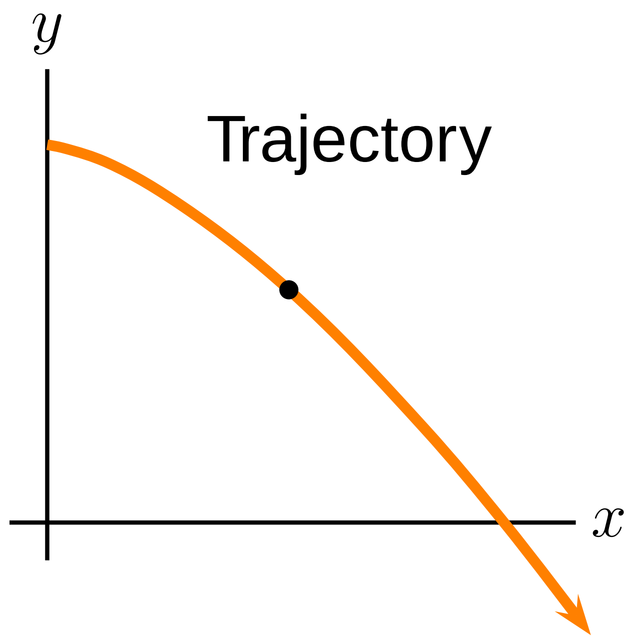

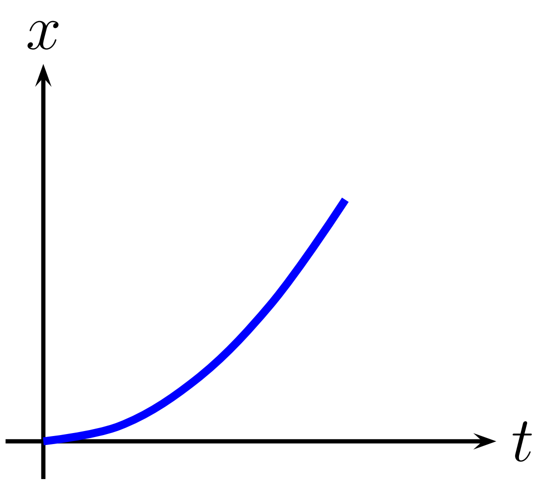

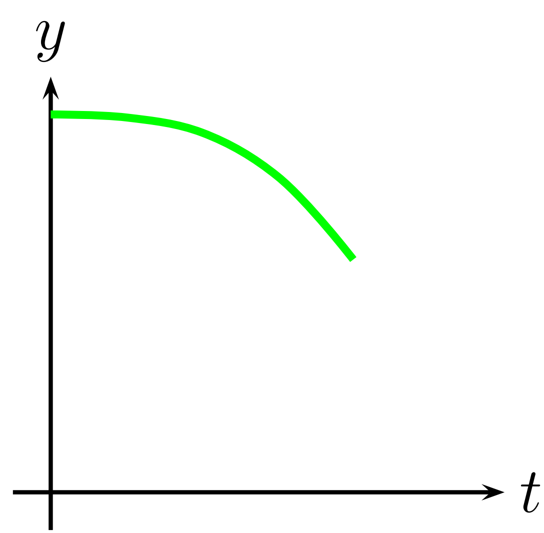

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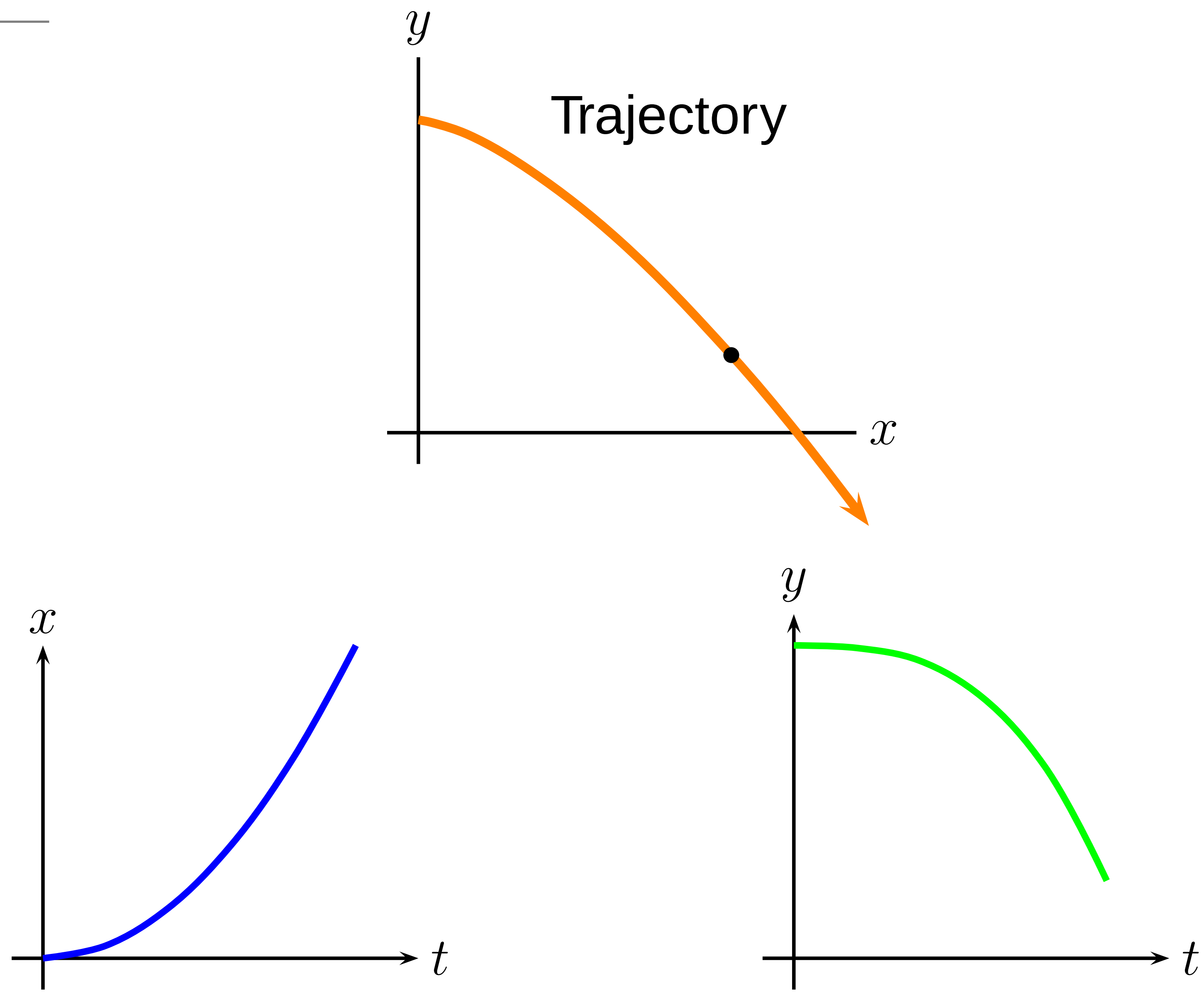

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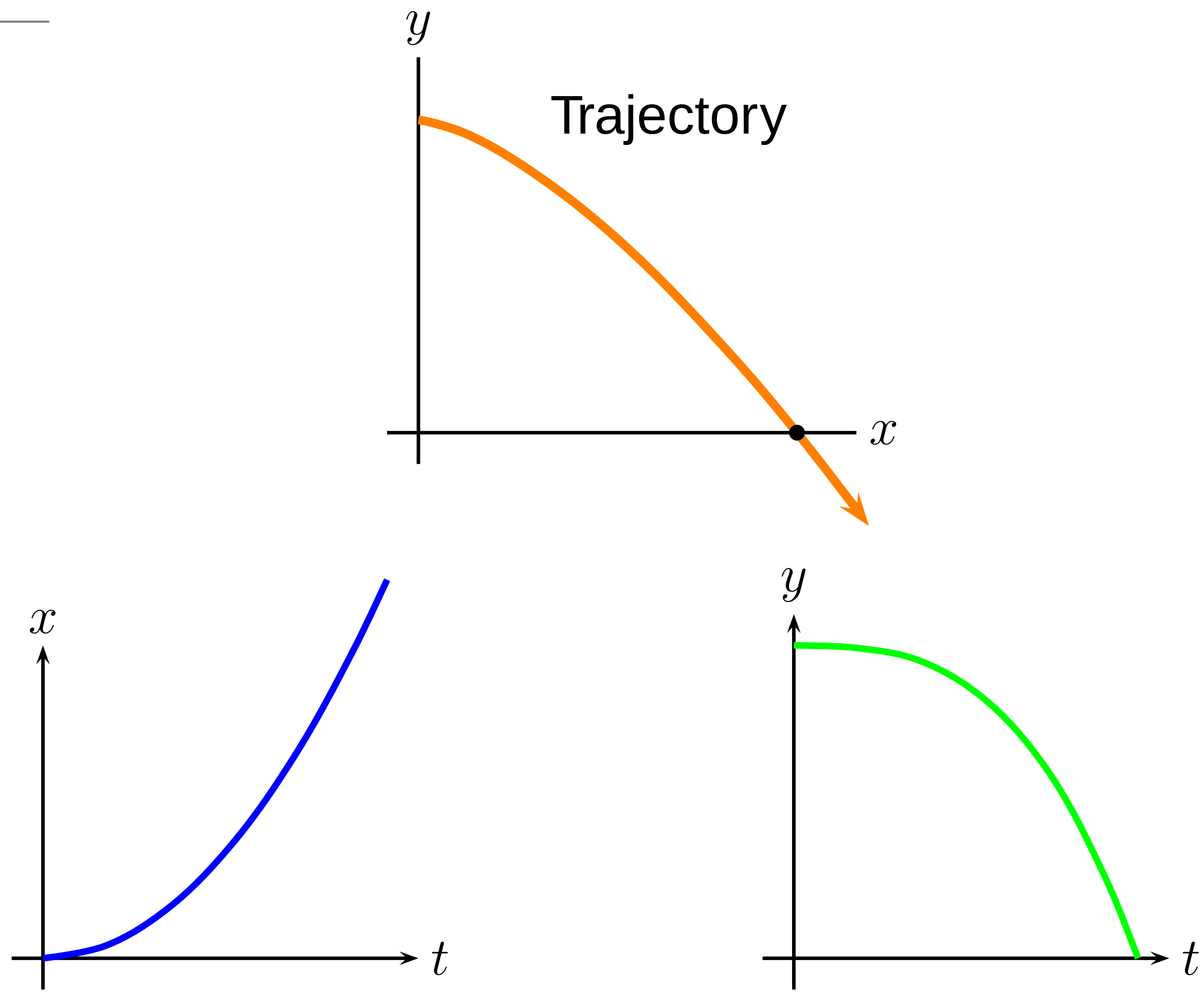

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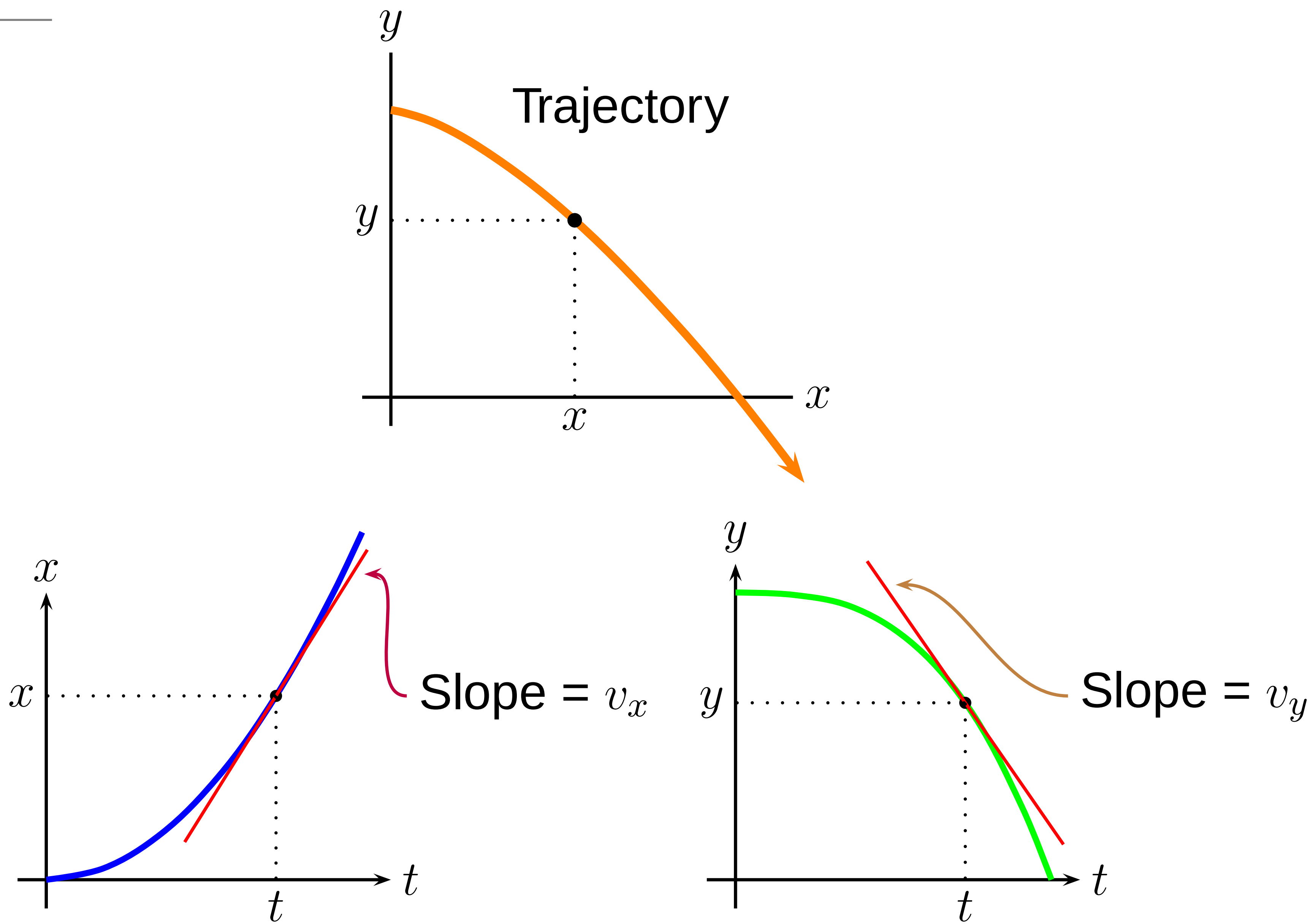

Speed

Speed is the magnitude of the velocity vector.

$$v = \sqrt{v_x^2 + v_y^2}$$

Acceleration

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Since $\vec{\mathrm{a}}$ is related to the change in $\vec{\mathrm{v}}$, **either** a change in speed or direction involves an acceleration.

While $\vec{\mathrm{a}}$ can be written in terms of components, its direction relative to $\vec{\mathrm{v}}$ is most important in describing its effect upon motion.

Direction of Acceleration

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} \Rightarrow d\vec{\mathbf{v}} \text{ in same direction as } \vec{\mathbf{a}}$$

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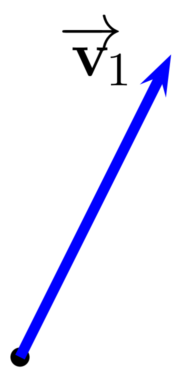

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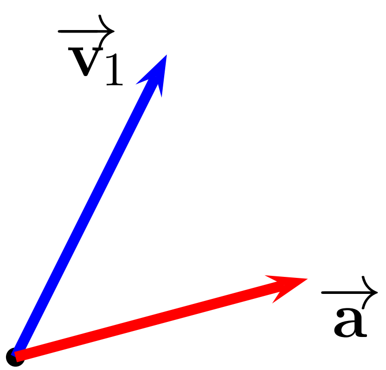

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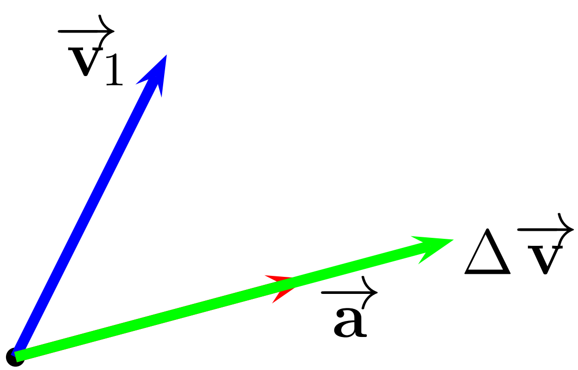

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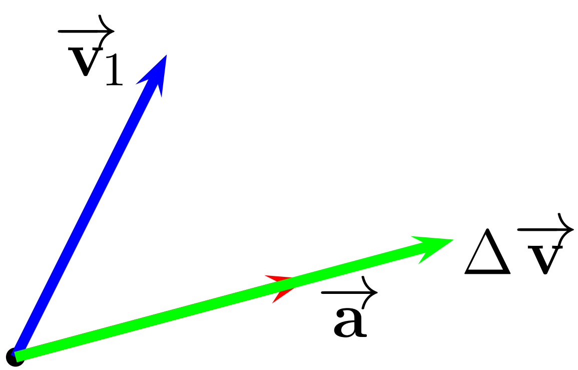
$$d\vec{\mathbf{v}} \approx \Delta\vec{\mathbf{v}} \Rightarrow \vec{\mathbf{a}} \text{ gives direction of } \Delta\vec{\mathbf{v}}$$



$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1$$

$$\Rightarrow$$

$$\vec{\mathbf{v}}_1 + \Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_2$$

Direction of Acceleration

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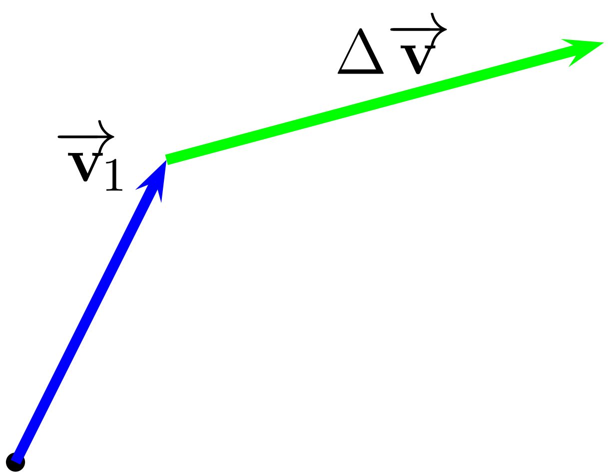

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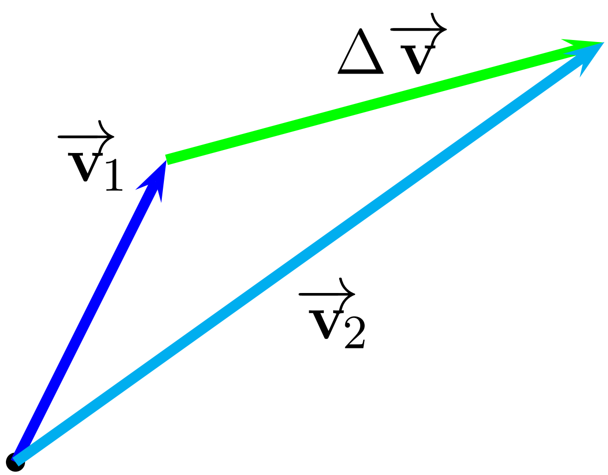

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Parallel Acceleration

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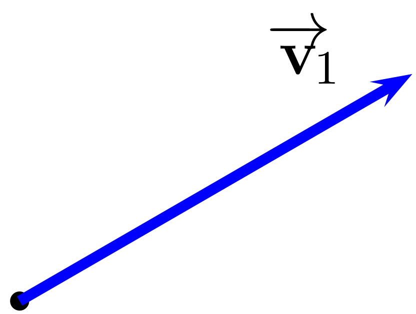

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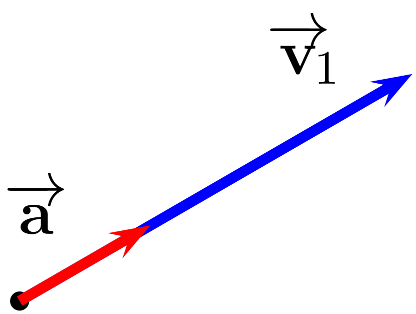

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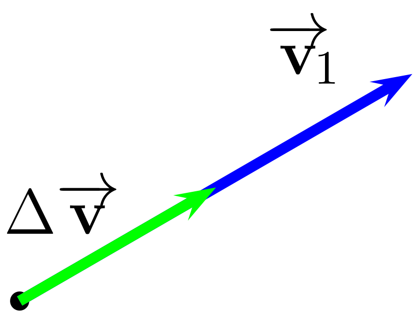

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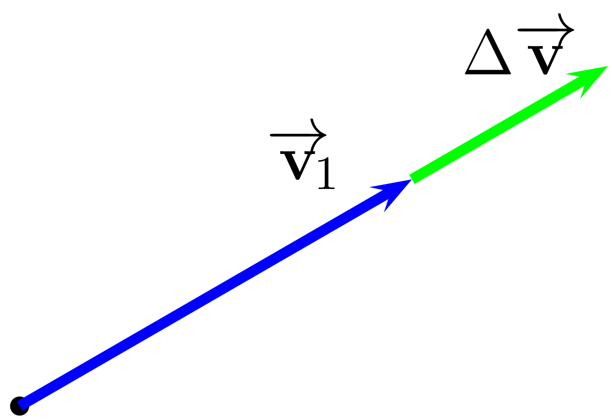

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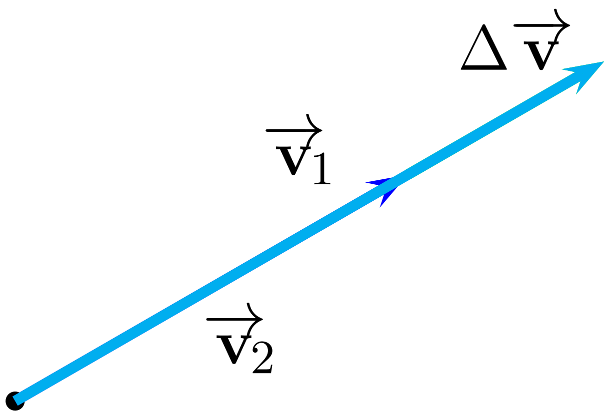

Only a change
in magniutde

Perpendicular Acceleration

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Here, we can't cheat! We have to take the limit as $\Delta\vec{\mathbf{v}} \to 0$.

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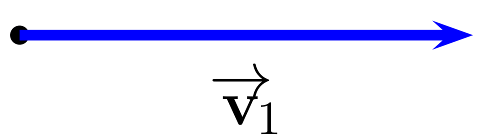

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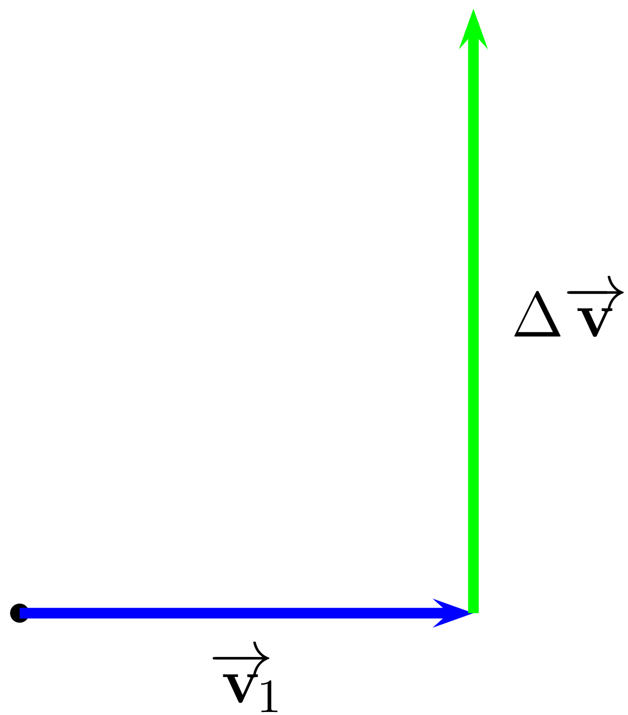

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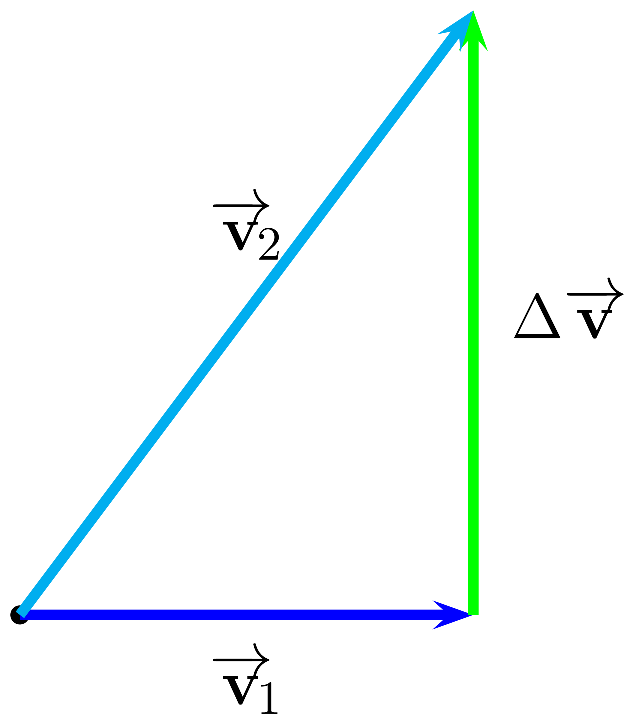

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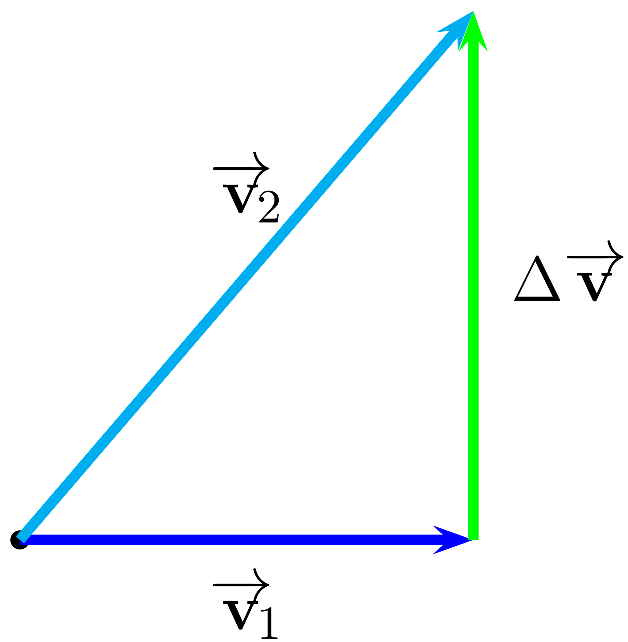

As $\Delta\vec{\mathbf{v}} \to 0$

$$v_2 \to v_1$$

but $\vec{\mathbf{v}}_2 \neq \vec{\mathbf{v}}_1$

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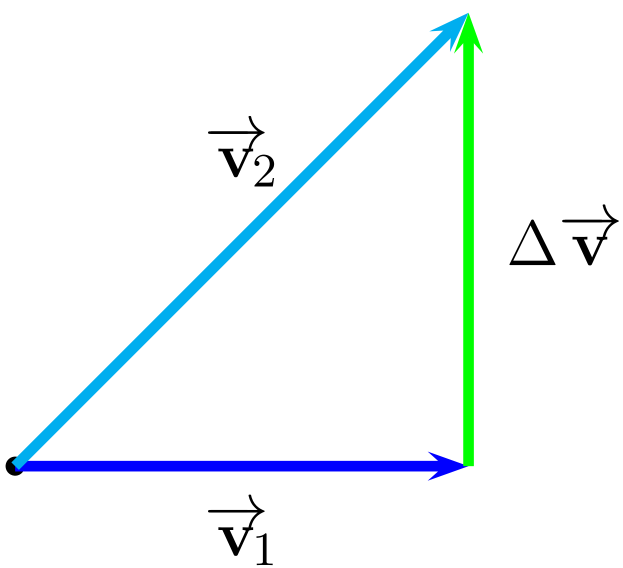

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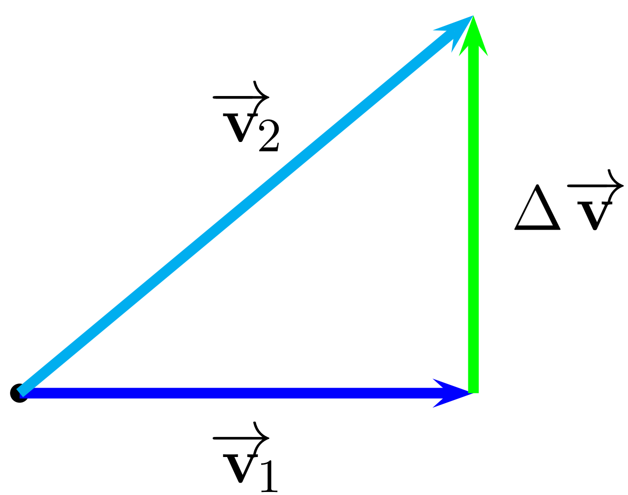

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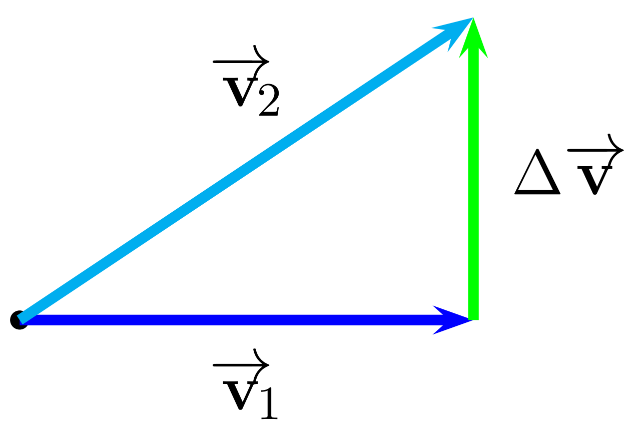

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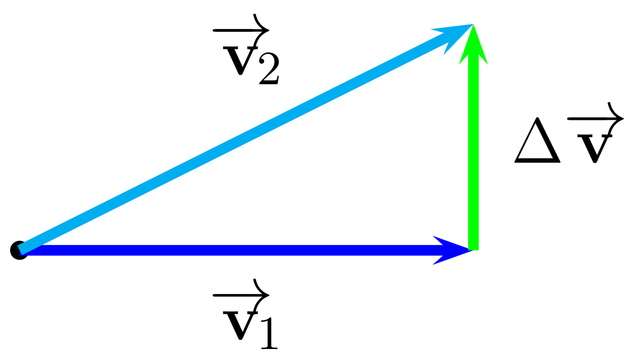

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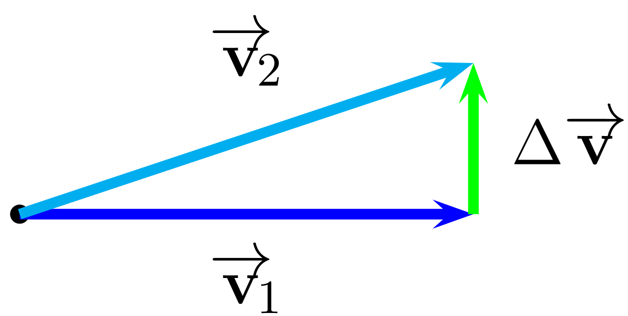

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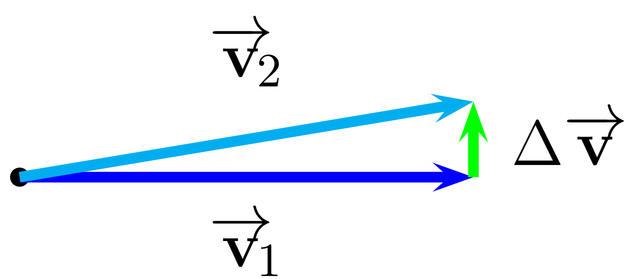

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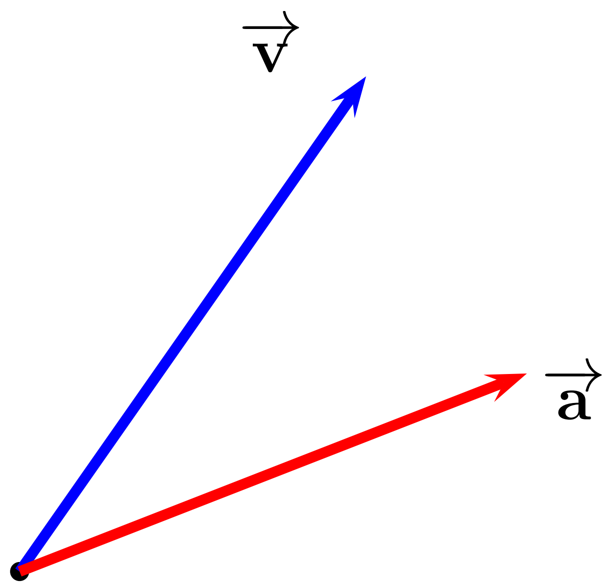
General Acceleration

An acceleration in an arbitrary direction will change both speed and direction.



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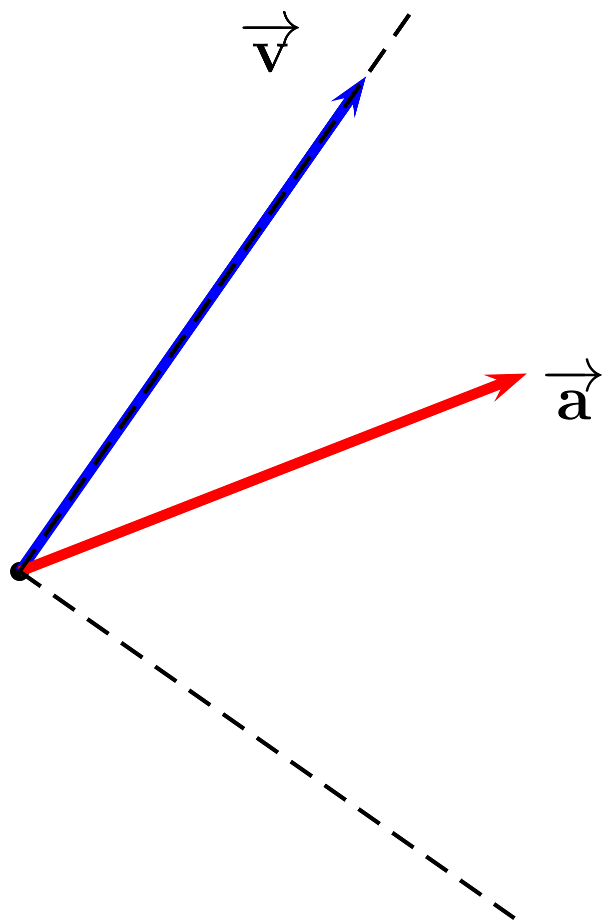

Use coordinates parallel and perpendicular to $\vec{\mathbf{v}}$

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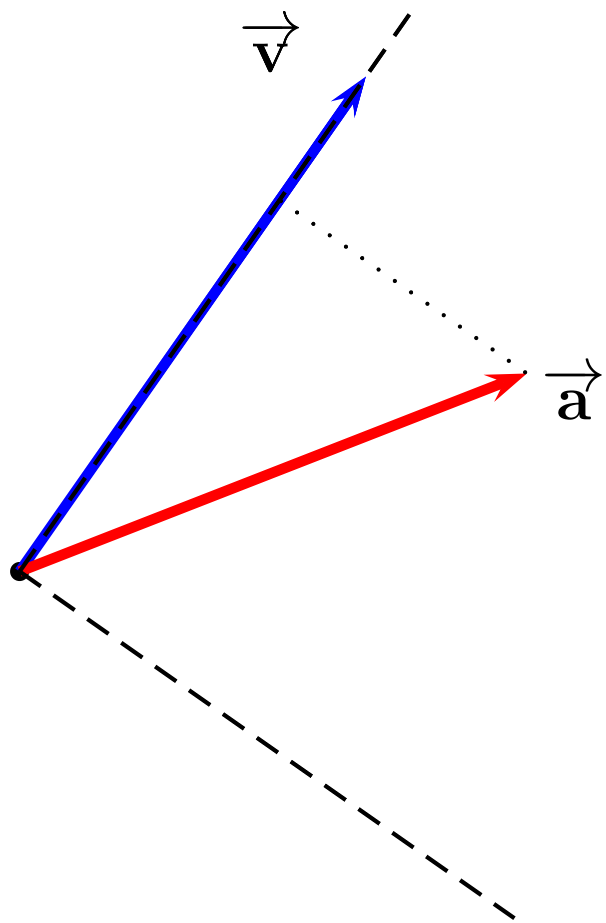

Split $\vec{\mathbf{a}}$ into a component parallel to $\vec{\mathbf{v}}$

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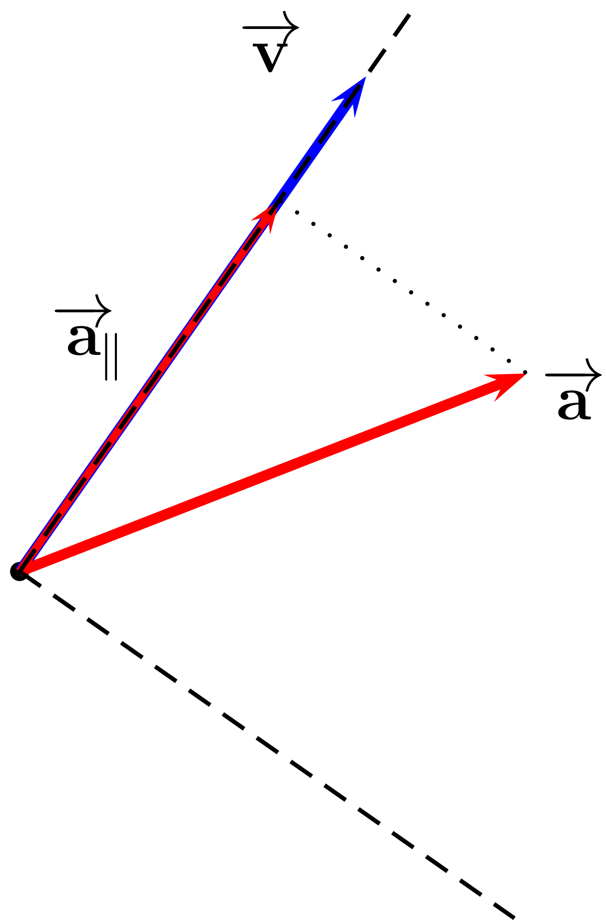

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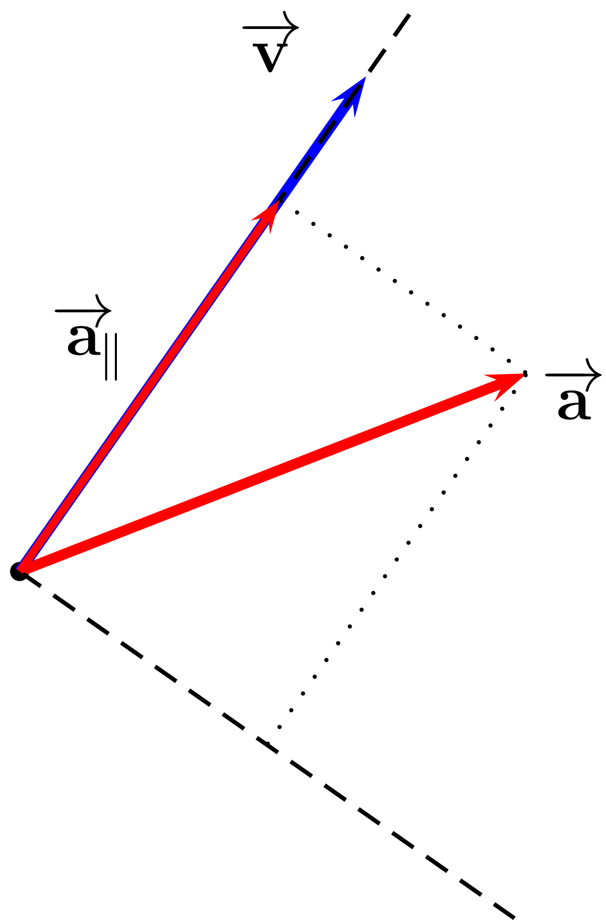

And a component perpendicular to $\vec{\mathbf{v}}$

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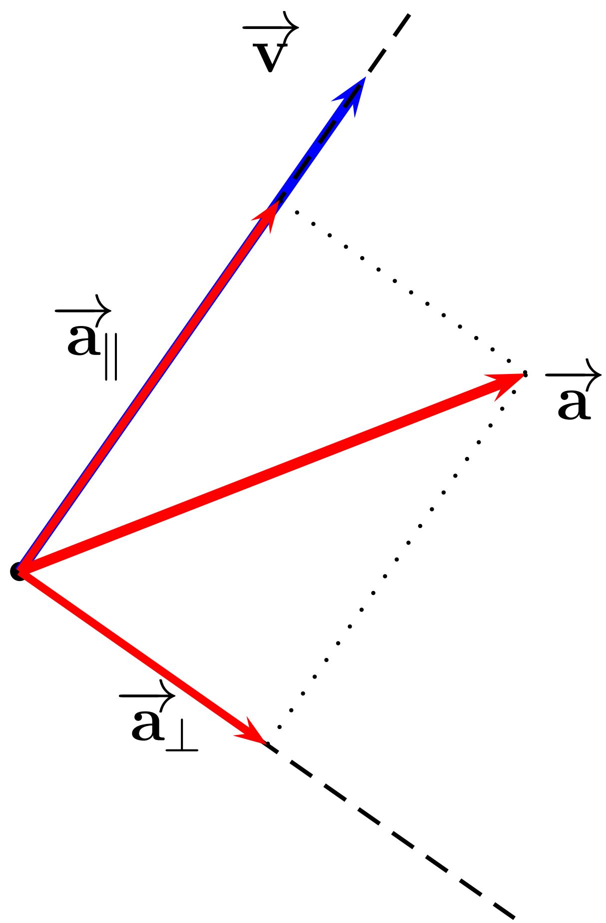

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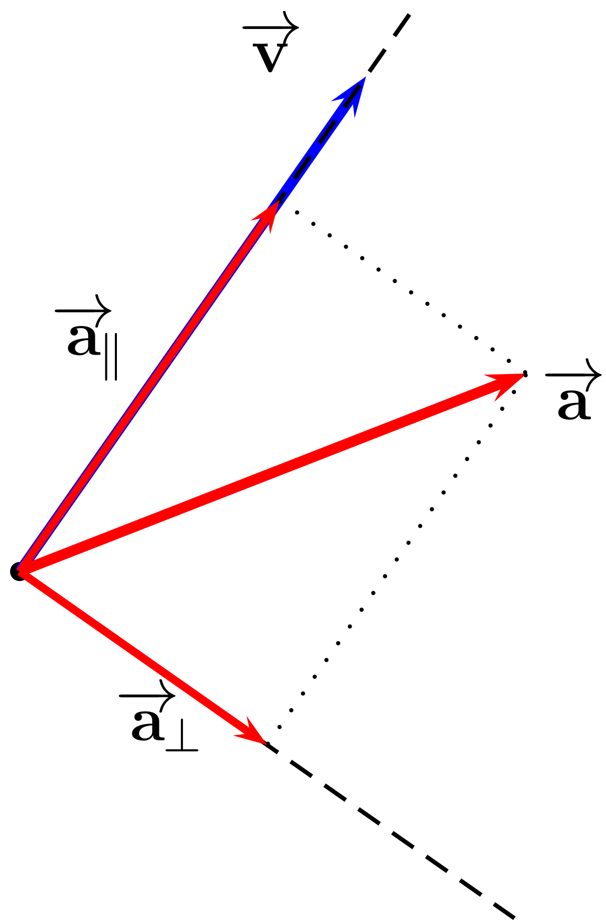

$\vec{\mathbf{a}}_{\|}$ changes speed

$\vec{\mathbf{a}}_{\perp}$ changes direction