July 15, Week 7

Today: Chapter 7, Rotational Dyanmics

Final Homework #7 now available

Newton's Second Law for Rotation

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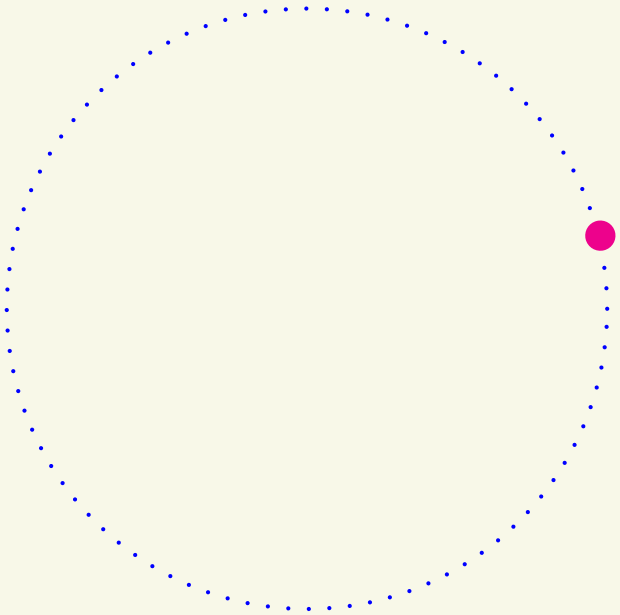

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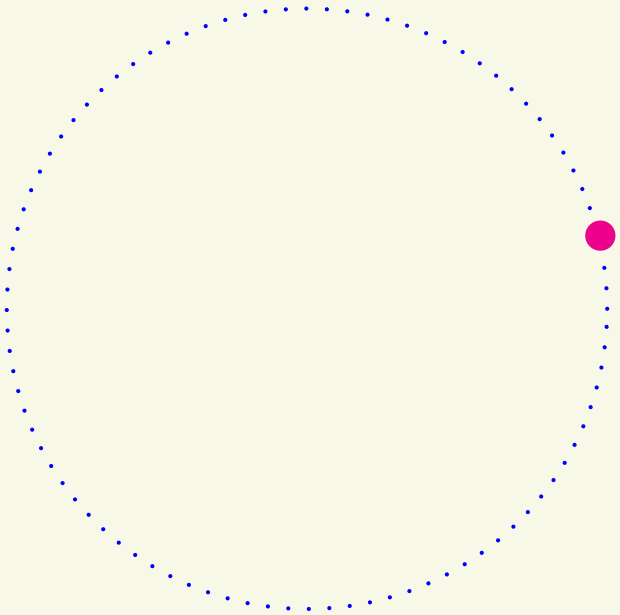

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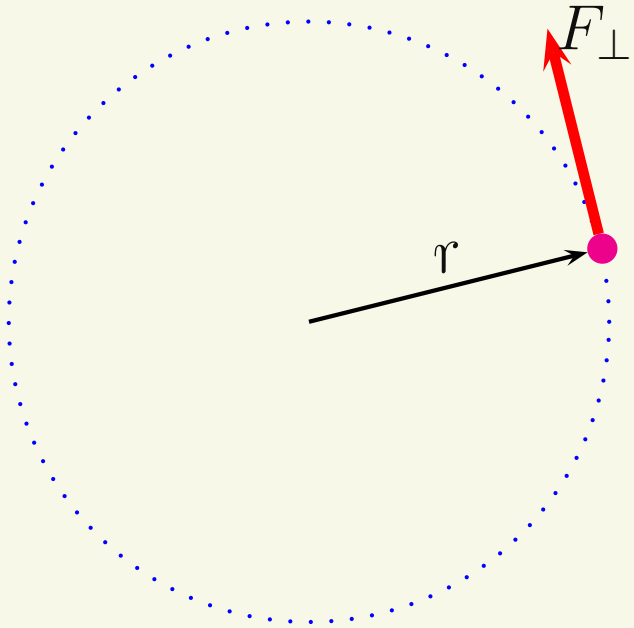

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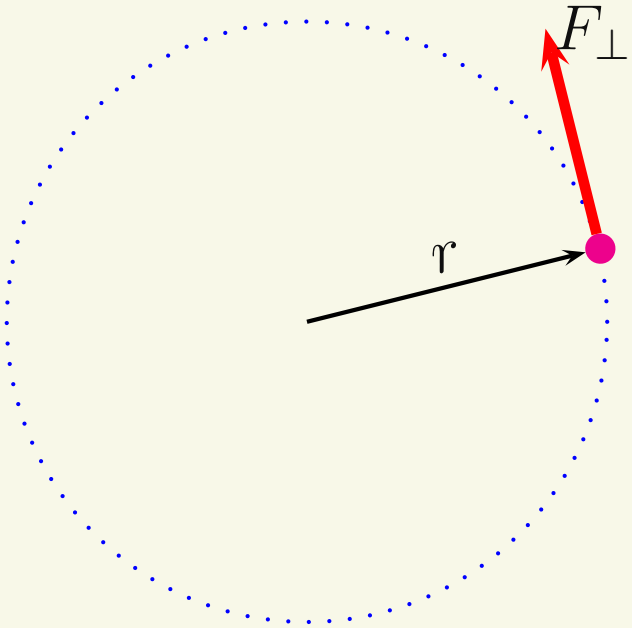

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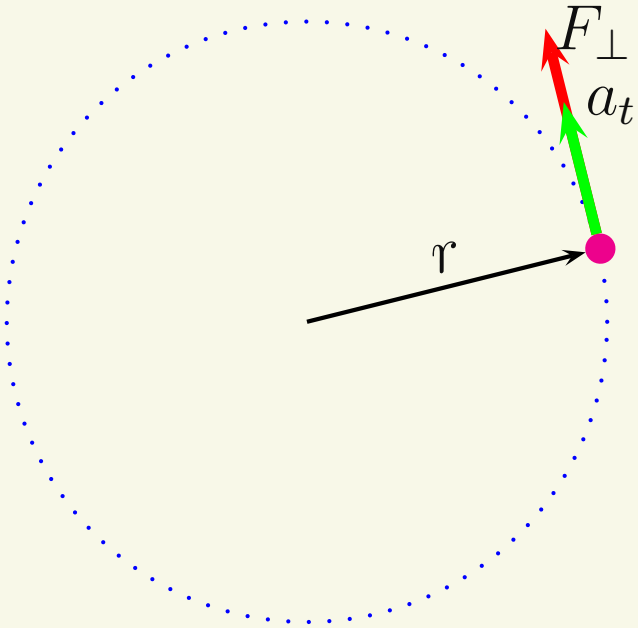

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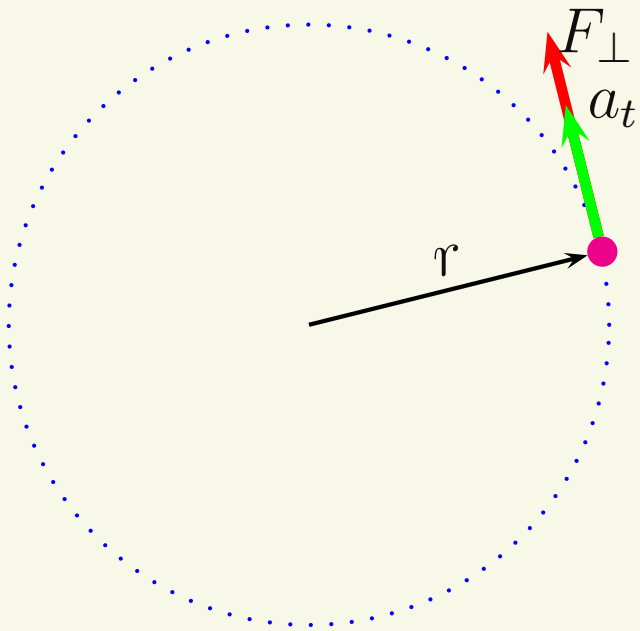

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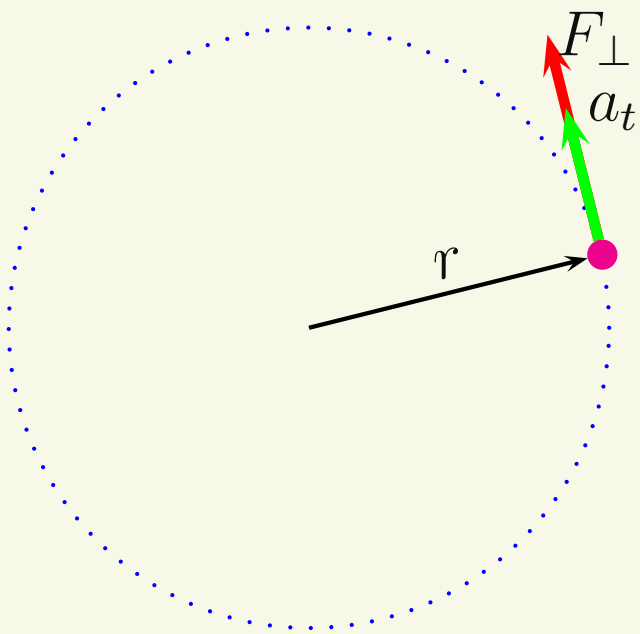

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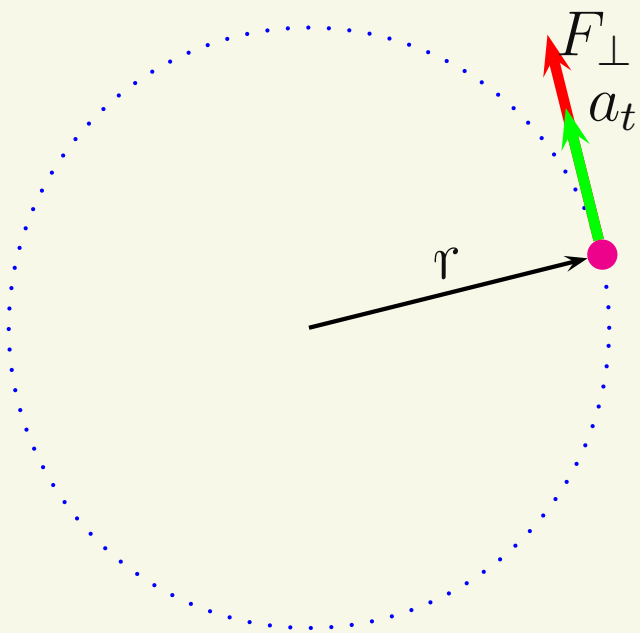

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For multiple $F_\perp$: $\boxed{\sum \tau = mr^2\alpha}$

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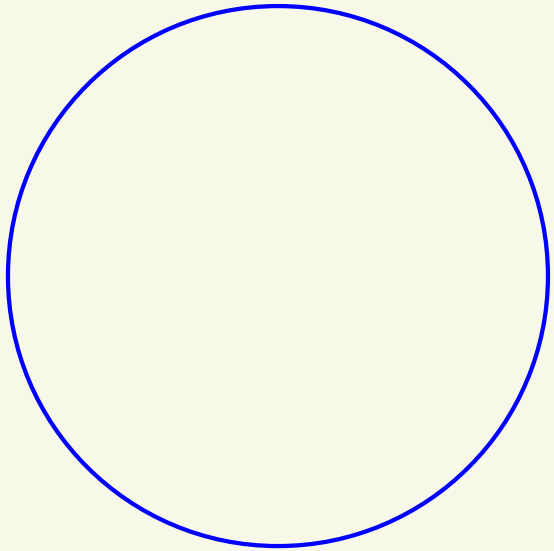

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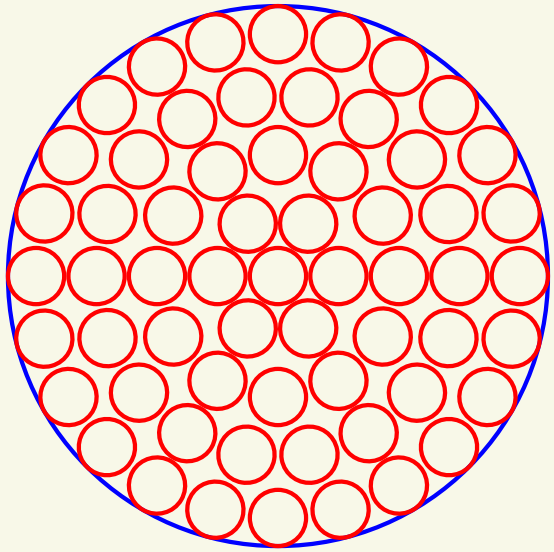

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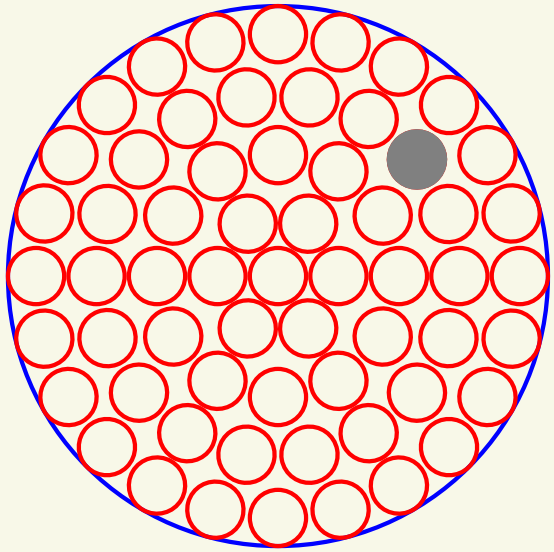

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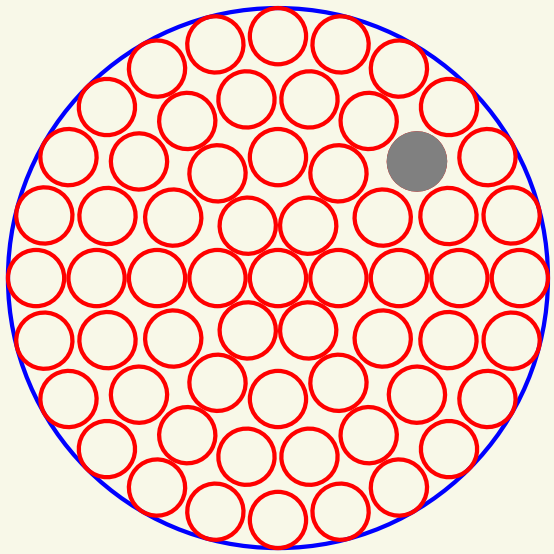

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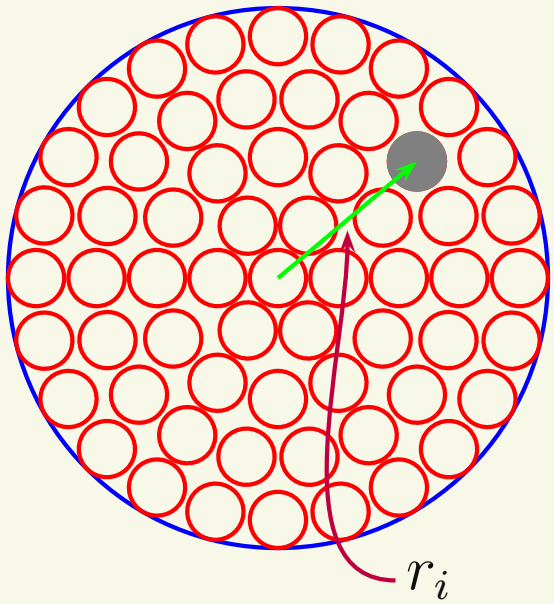

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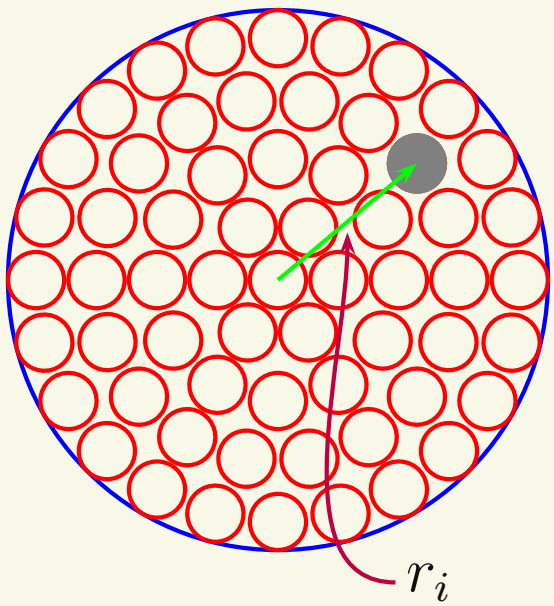

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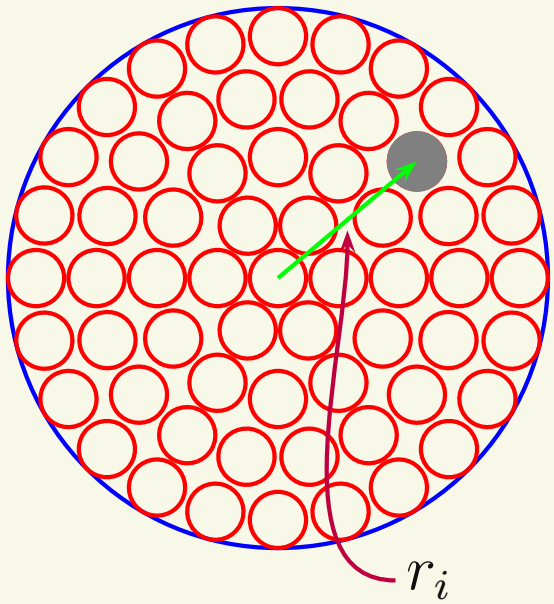

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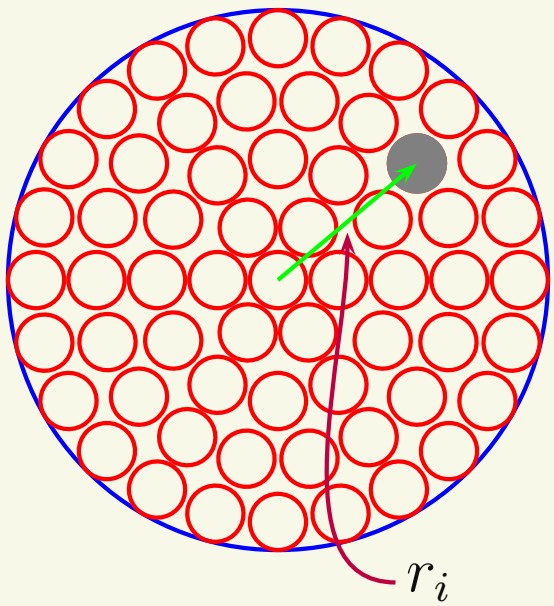

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$$\boxed{\sum \tau = I\alpha}$$

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The moment of inertia depends on:

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(b) The axis of rotation.

(c) The total mass of the object.

Standard Shapes

For standard shapes and axes, equations for moments of inertia have already been calculated. (Page 216 of our textbook.)

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Table 9.2 Moments of Inertia of Various Bodies

(a) Slender rod, axis through center

$$I = \frac{1}{12}ML^2$$

(b) Slender rod, axis through one end

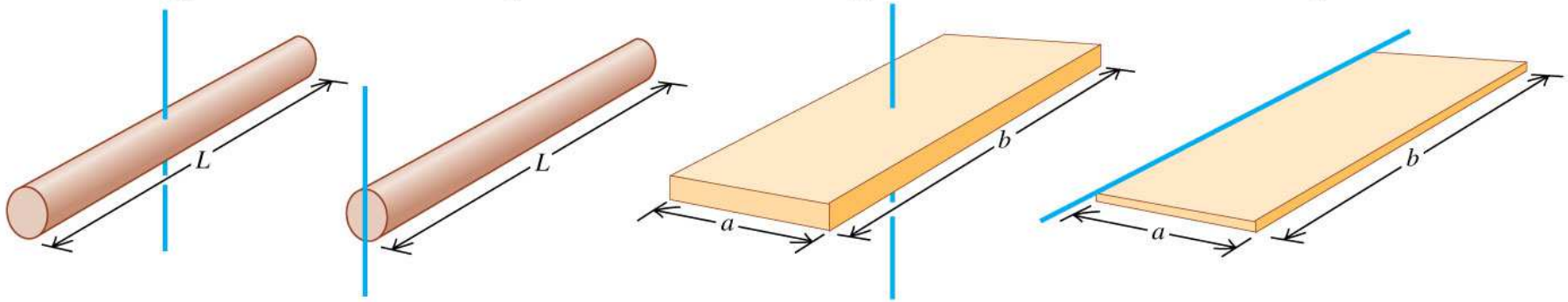
$$I = \frac{1}{3}ML^2$$

(c) Rectangular plate, axis through center

$$I = \frac{1}{12}M(a^2 + b^2)$$

(d) Thin rectangular plate, axis along edge

$$I = \frac{1}{3}Ma^2$$



(e) Hollow cylinder

$$I = \frac{1}{2}M(R_1^2 + R_2^2)$$

(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$

(g) Thin-walled hollow cylinder

$$I = MR^2$$

(h) Solid sphere

$$I = \frac{2}{5}MR^2$$

(i) Thin-walled hollow sphere

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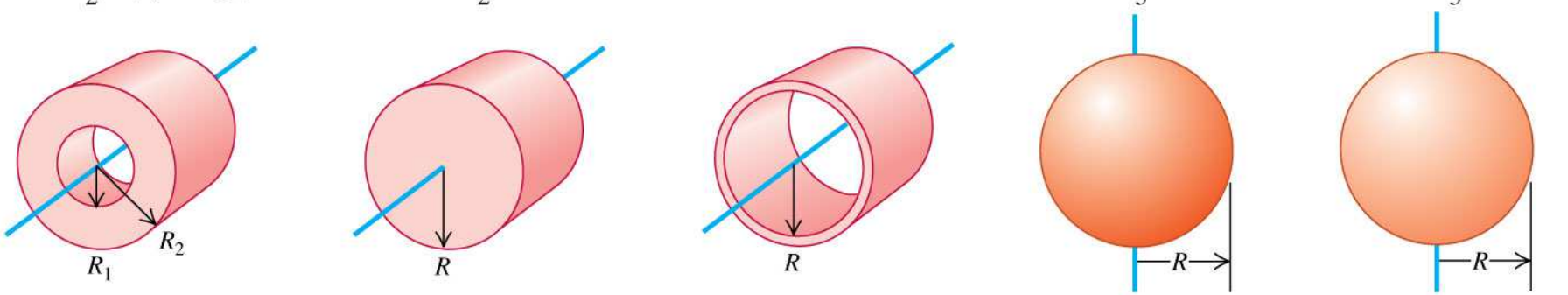

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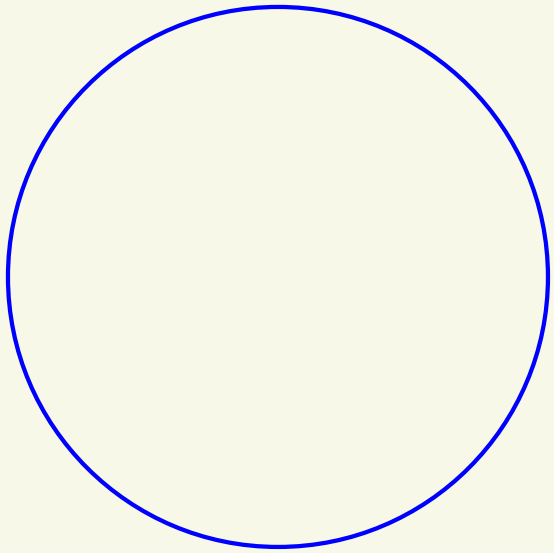

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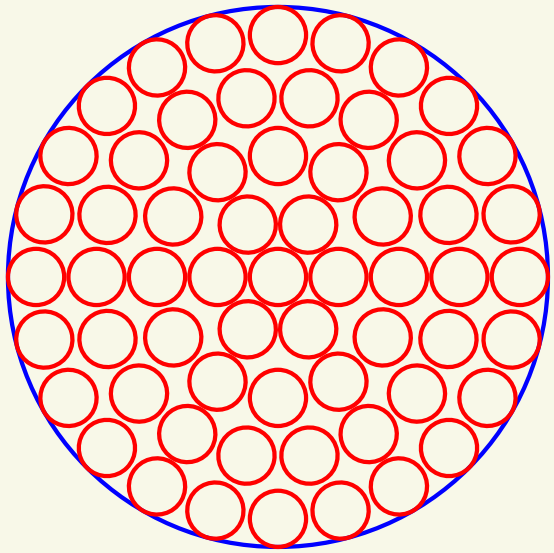

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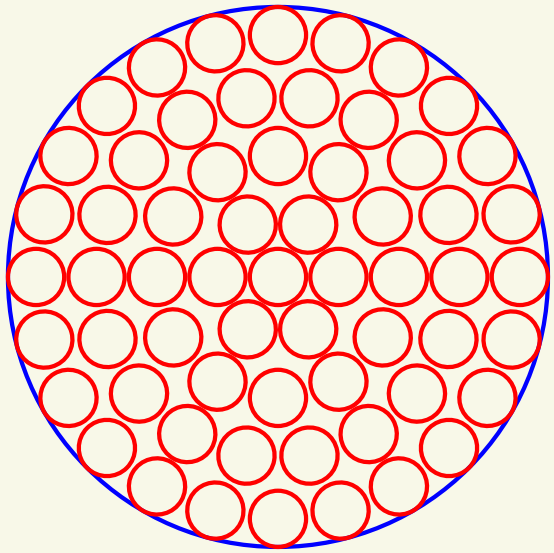

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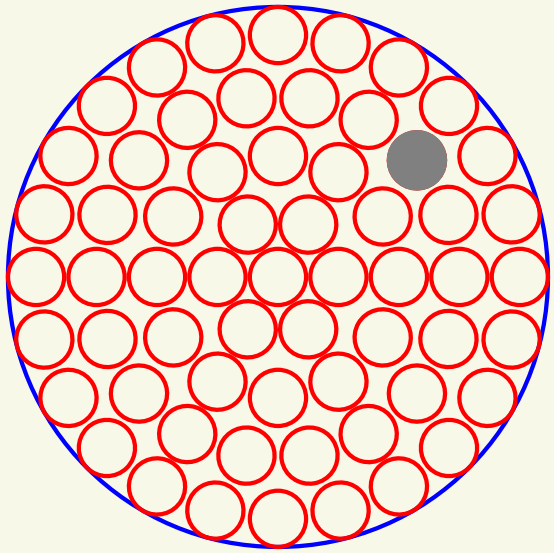

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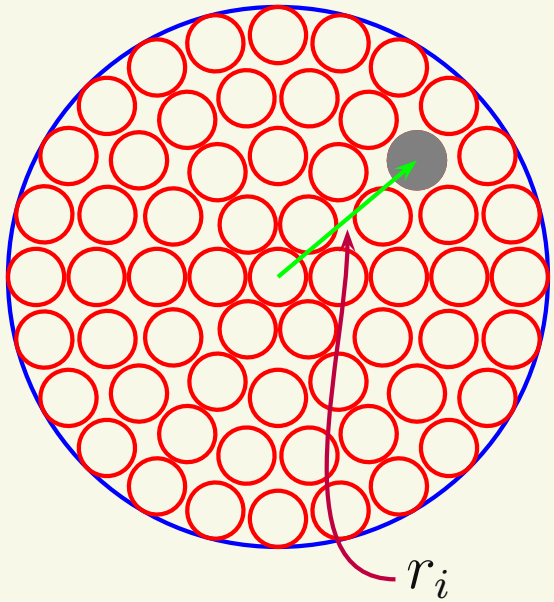

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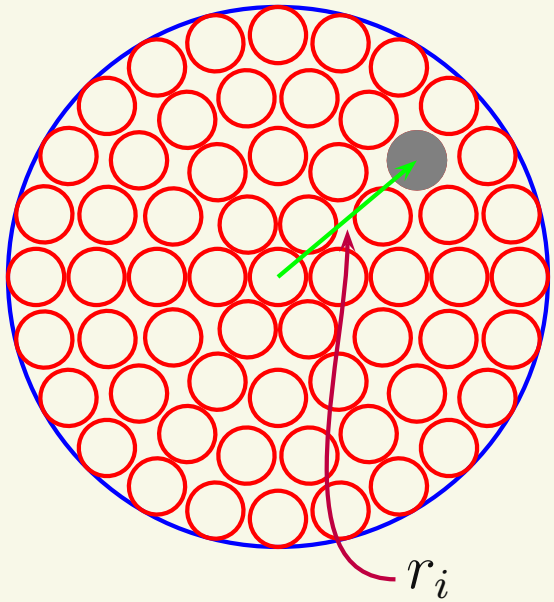

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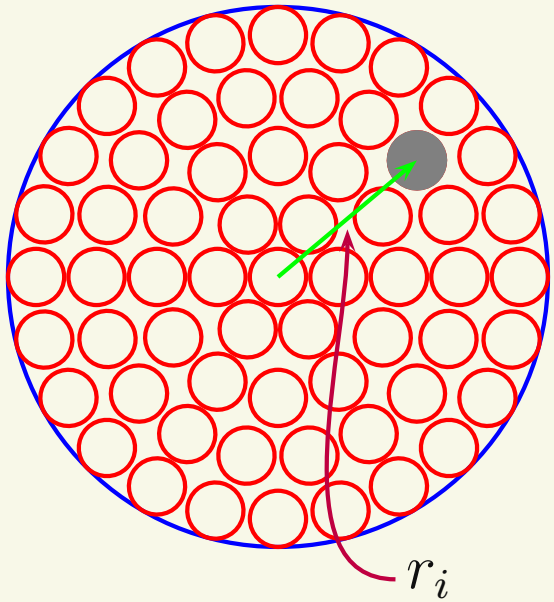

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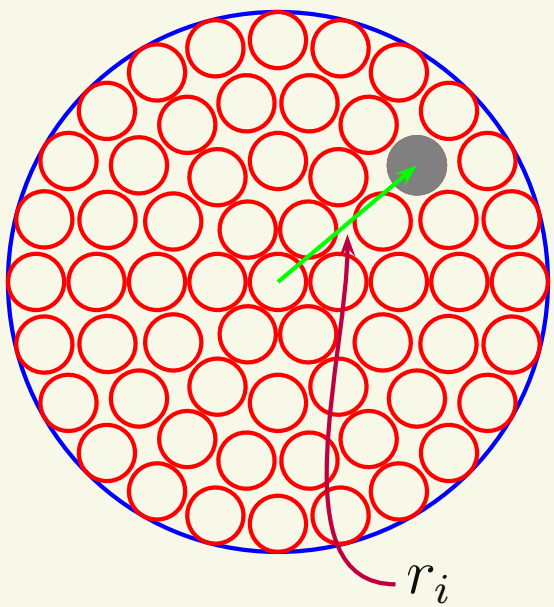

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$$K = \frac{1}{2} \left(\sum_i m_i r_i^2 \right) \omega^2 \Rightarrow \boxed{K = \frac{1}{2} I \omega^2}$$

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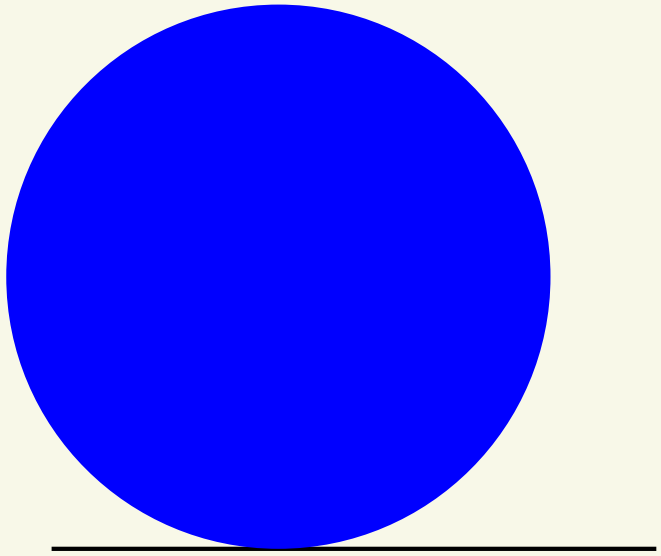

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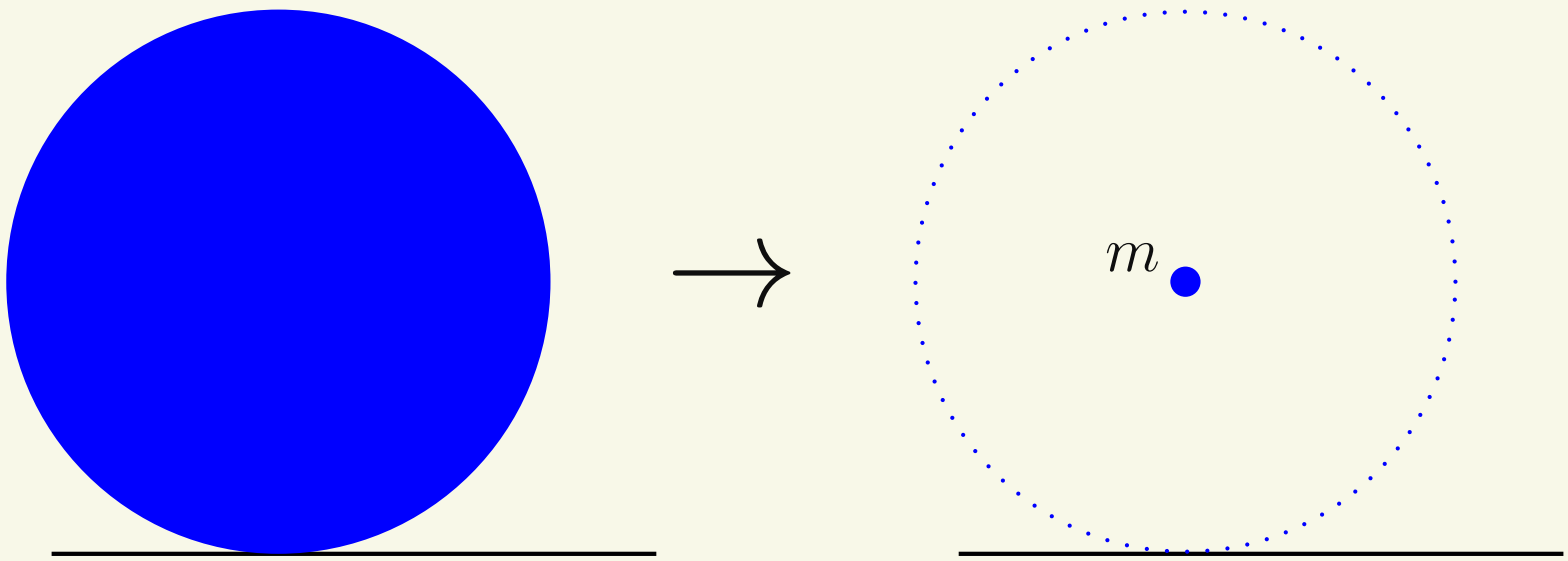

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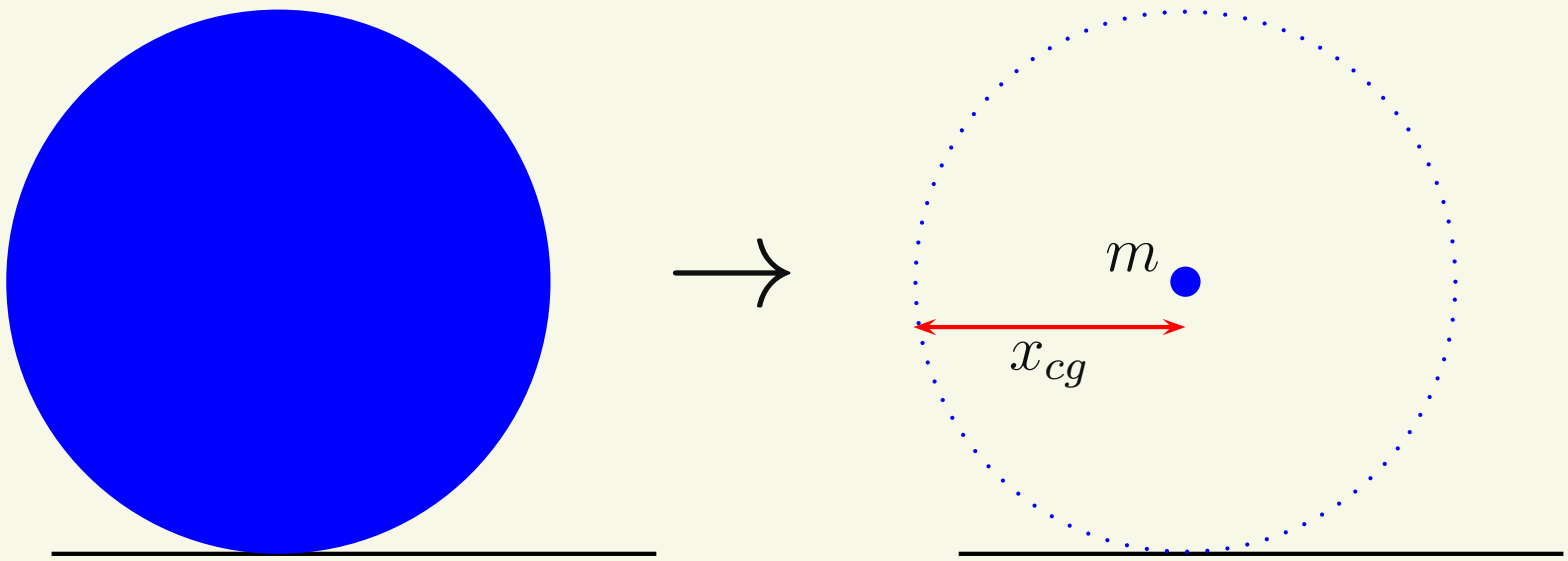

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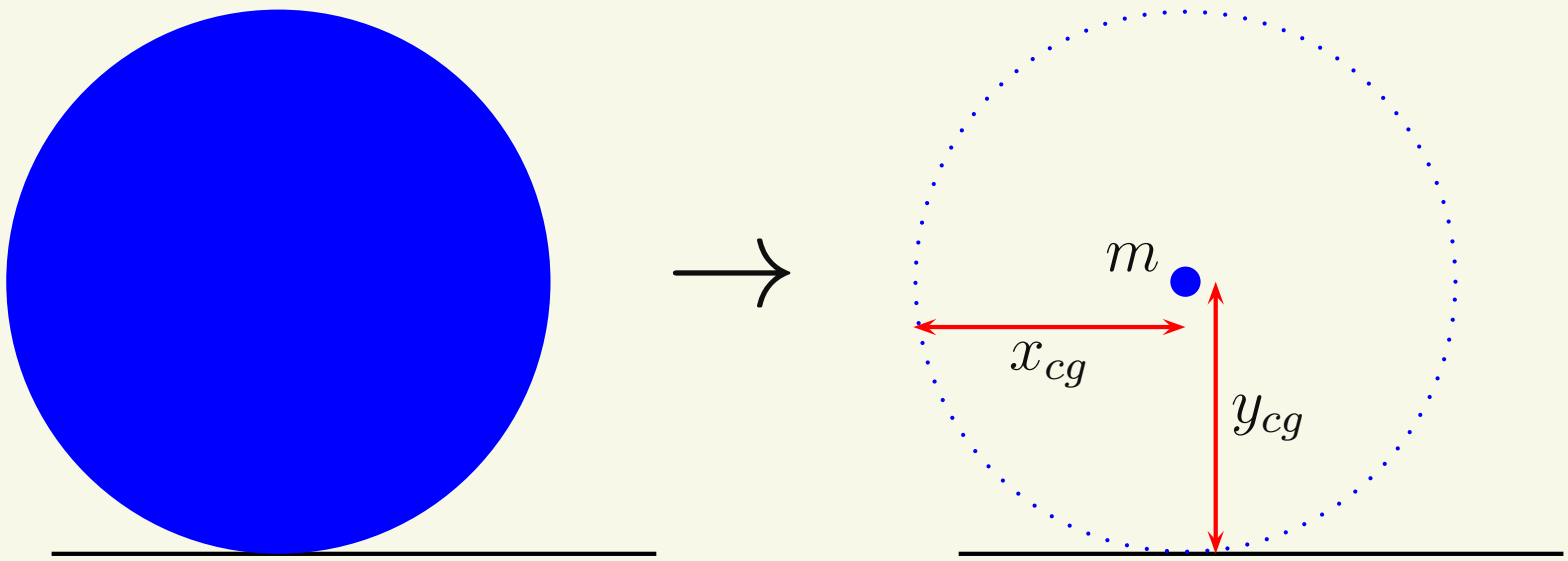

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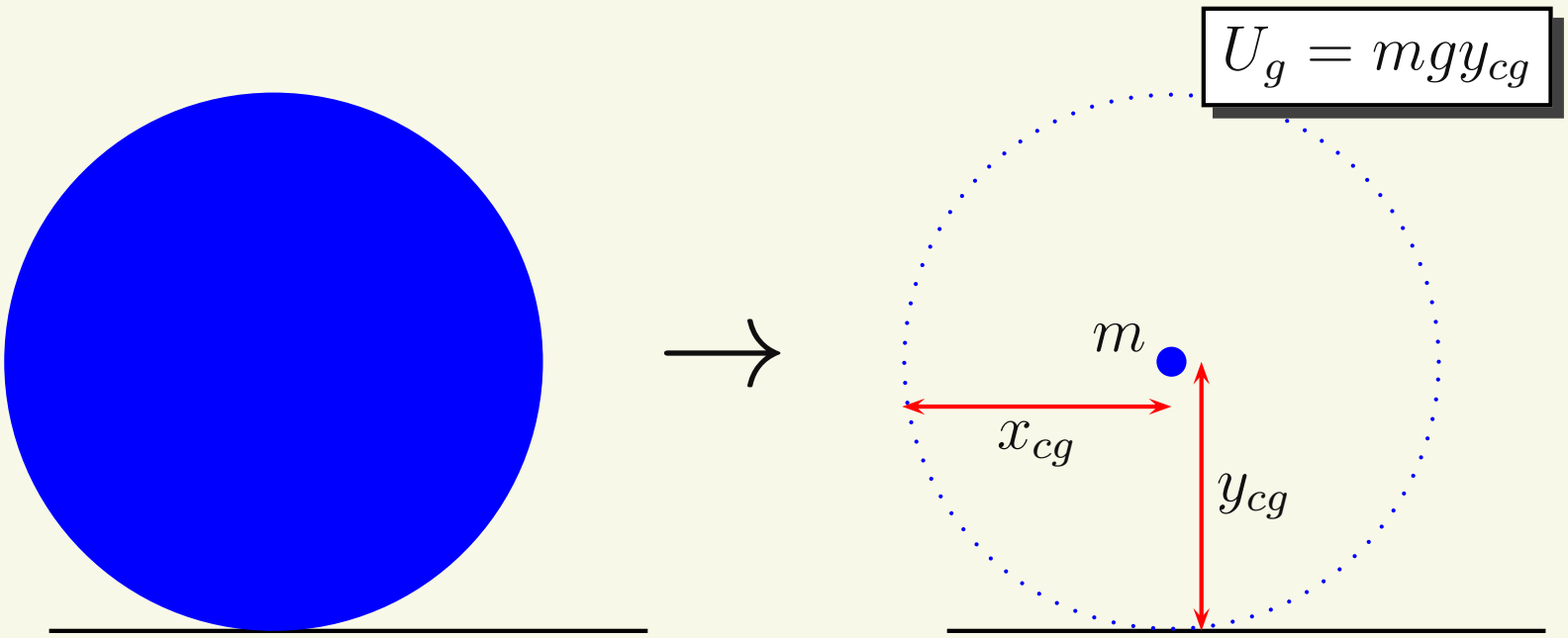

Conservation of Rotational energy

If gravity is the only force doing work on a rigid body:

$$\boxed{\frac{1}{2}I\omega_i^2 + mgy_{cg,i} = \frac{1}{2}I\omega_f^2 + mgy_{cg,f}}$$

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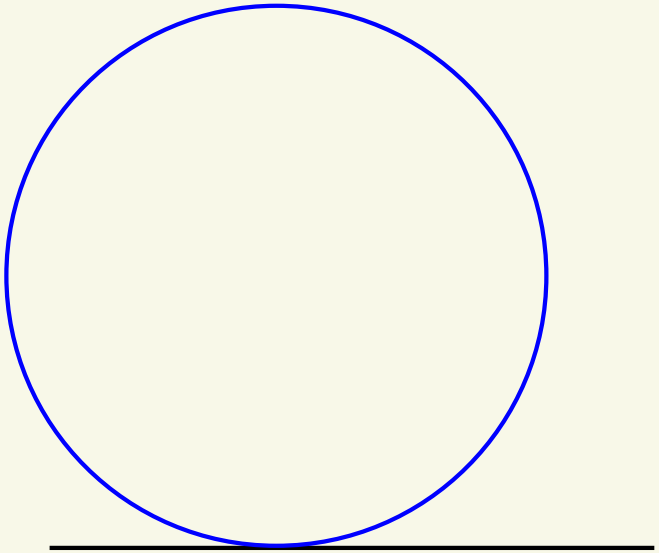
Example: A 0.15-kg meter stick is held $1\,m$ above the ground by one end. If it starts from rest, what angular speed will the stick have when it swings through the vertical?

Rolling

When an object rolls, it rotates and its center moves.



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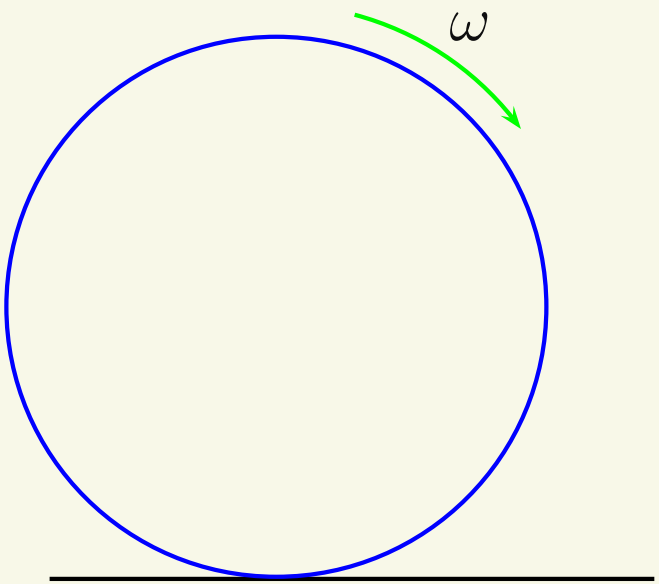

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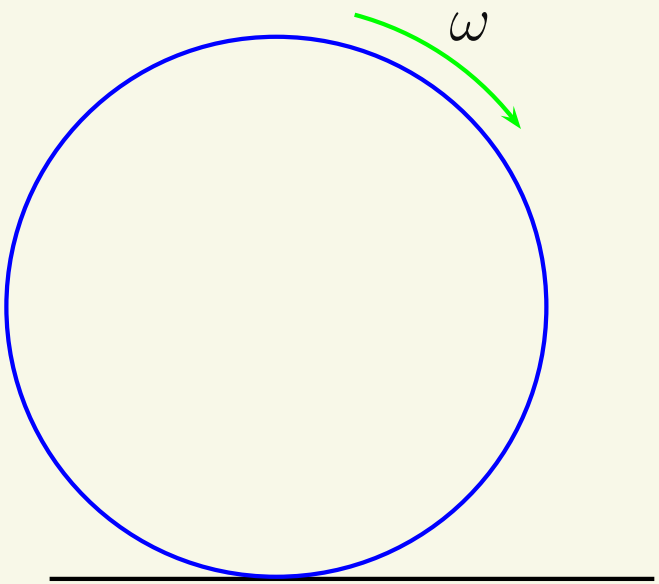

Rotational: $K_r = \frac{1}{2} I \omega^2$

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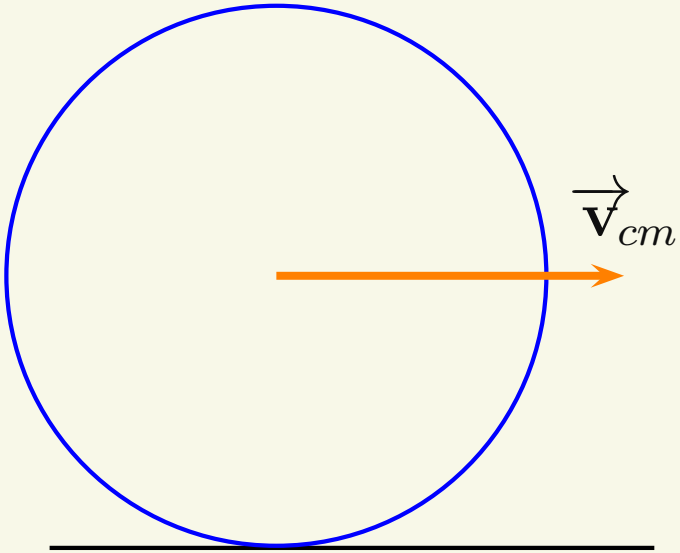

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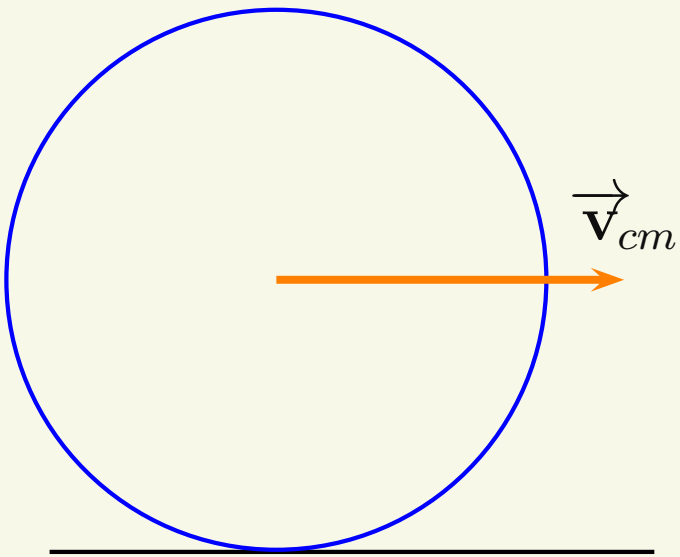

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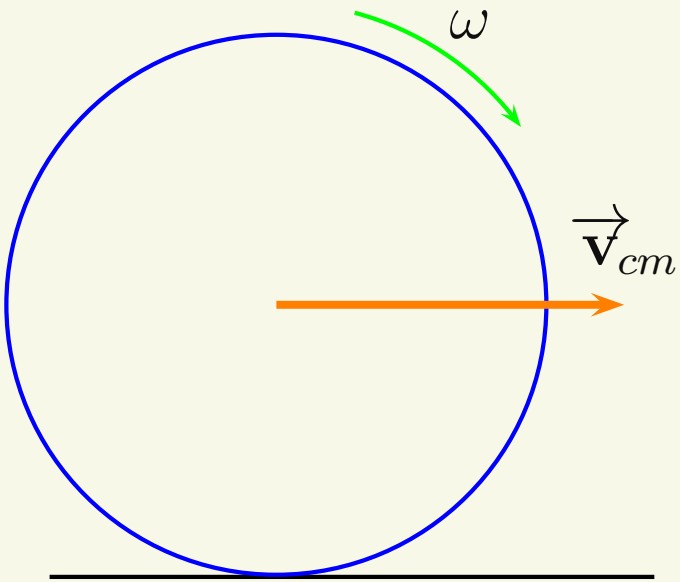

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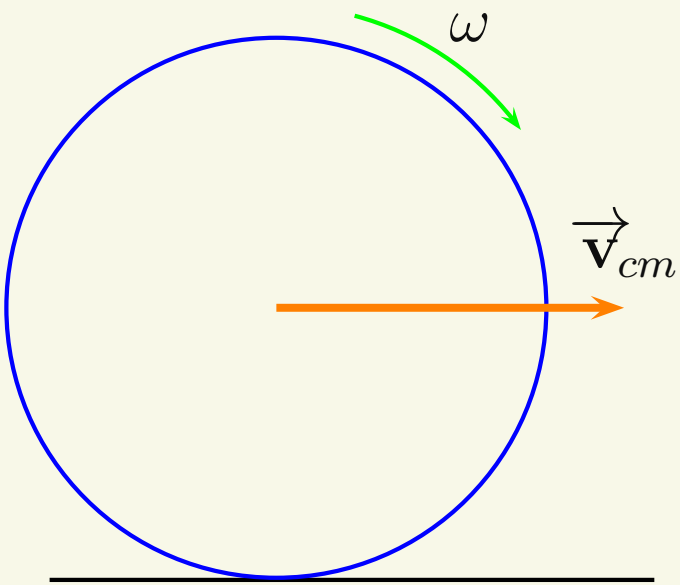

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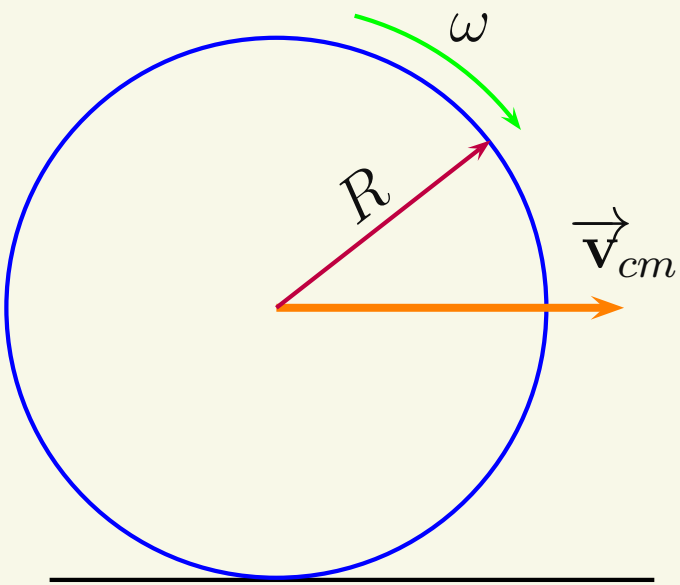

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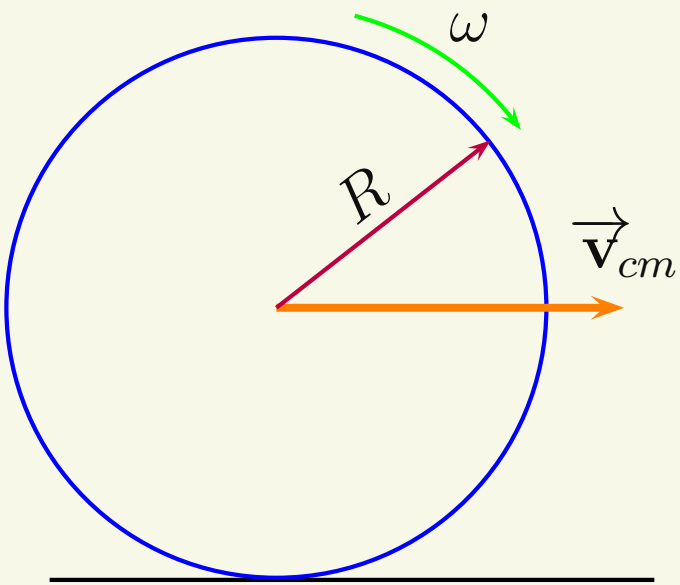

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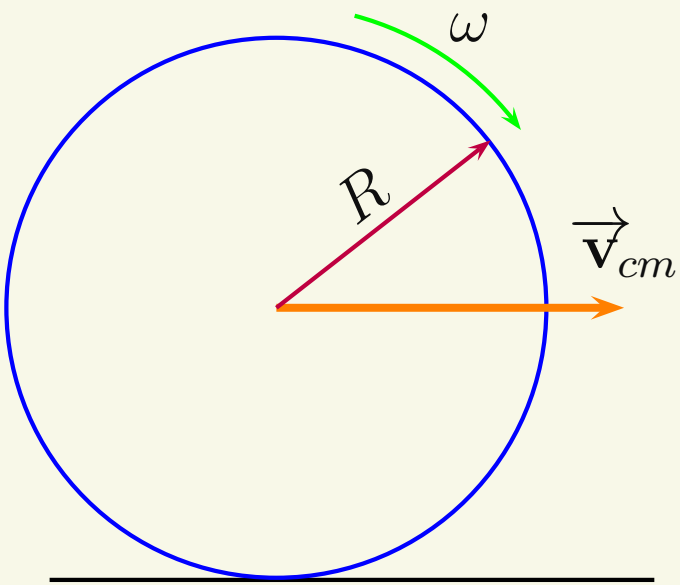

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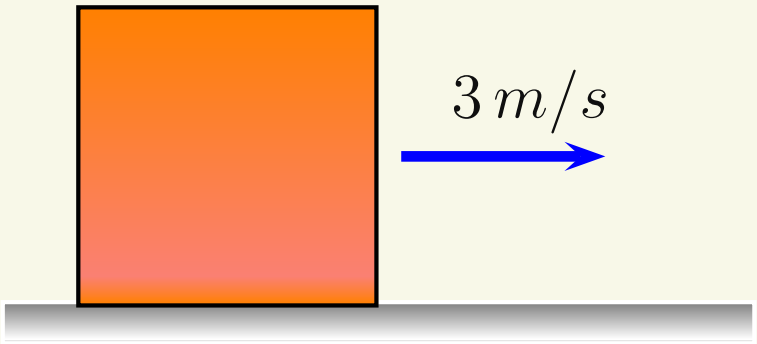
$$\boxed{K = \frac{1}{2}mv_{cm}^2\left(1 + \frac{I}{mR^2}\right)}$$

Thermal Energy Exercise

A 10-kg mass sliding to the right, initially with speed $3\,m/s$, is stopped by friction. How much thermal energy will be created by this process?



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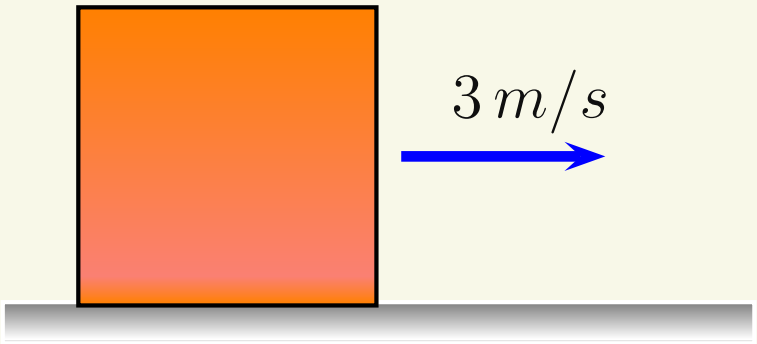

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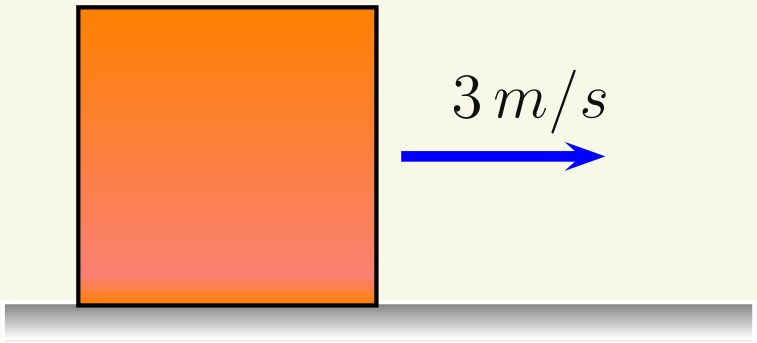

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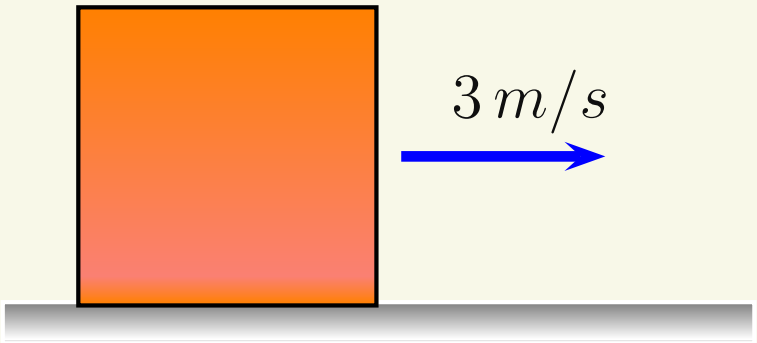

This $45\,J$ of thermal energy is absorbed by the block, the floor, and the surrounding air. Chapter 11 deals with the effect that this thermal energy has on the molecules inside each of these substances.

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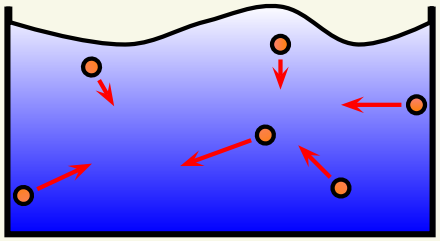

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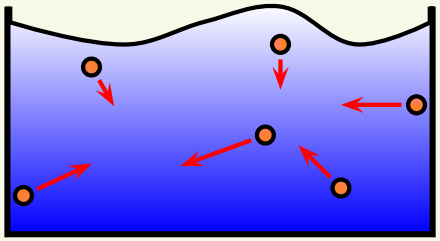
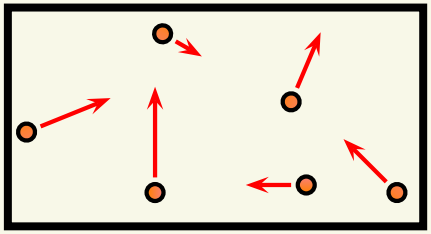


Gas - molecules can go anywhere



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