Chapter 3, Sections 3.6-3.7

3.6 - Motion in Two Dimensions: Projectile Motion

(1.) A projectile is launched horizontally at $15\,m/s$ and hits the ground $3\,s$ later. The figure shows the projectile's position every $0.5\,s$ second.

- At each dot draw a vector for the horizontal v_x and vertical v_y components of the velocity vector.
- Label each vector with a numerical value of the velocity at that point. Show your calculations below the figure.
- The length of each vector should indicate its magnitude, using the length of the $15\,m/s$ vector at $t = 0\,s$ as a reference.

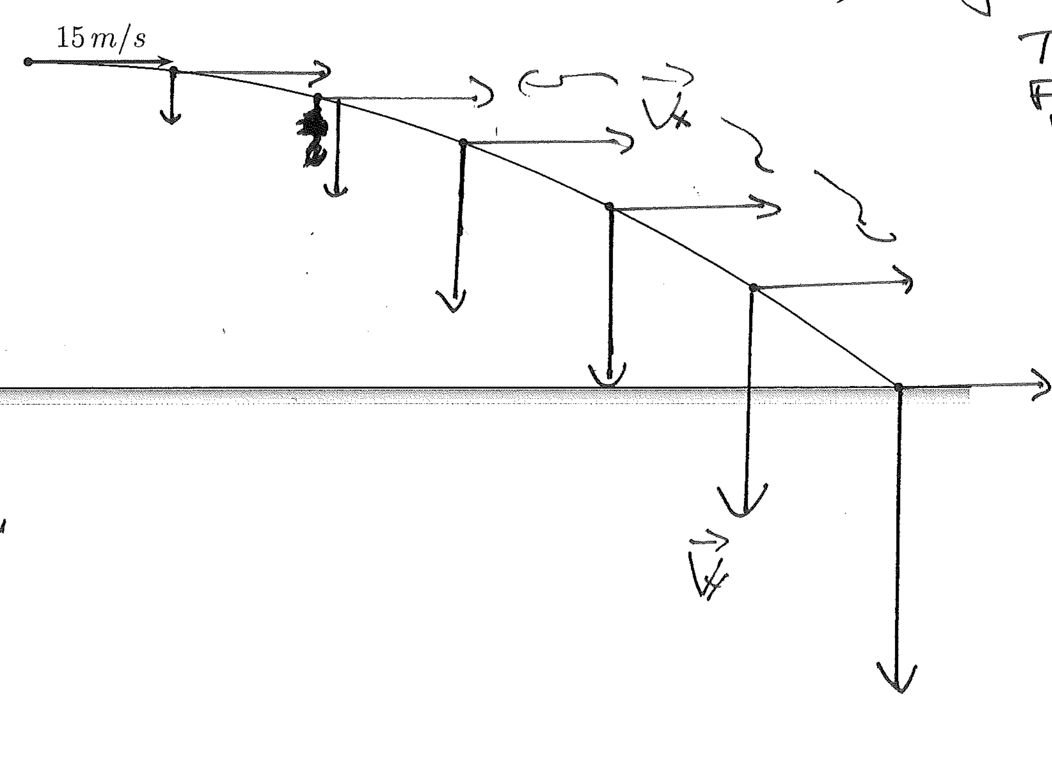
HORIZONTAL LAUNCH $\Rightarrow (V_x)_i = 15\,m/s$

$(V_y)_i = 0$

$(V_x)_f = (V_x)_i \Rightarrow V_x$ IS Constant at 15 m/s

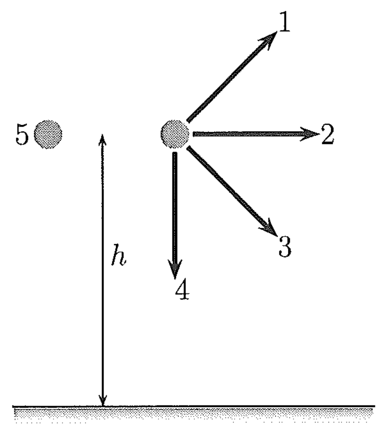
$(V_y)_f = (V_y)_i - gt \Rightarrow (V_y)_f = -gt \leftarrow$ SEE TABLE FOR VALUES

t	V_f
0	0
.5 s	-4.9 m/s ≈ -5 m/s
1 s	-9.8 m/s ≈ -10 m/s
1.5 s	-14.7 m/s ≈ -15 m/s
2 s	-19.6 m/s ≈ -20 m/s
2.5 s	-24.5 m/s ≈ -25 m/s
3 s	-29.4 m/s ≈ -30 m/s



(2.) Four balls are simultaneously launched with the same speed from the same height, h, above the ground. At the same instant, ball 5 is released from rest at the same height. Rank in order, from shortest to longest, the amount of time it takes each of these balls to hit the ground. Circle any pairs that hit the ground simultaneously.

Order: 4, 3, 2,5, 1



Explanation:

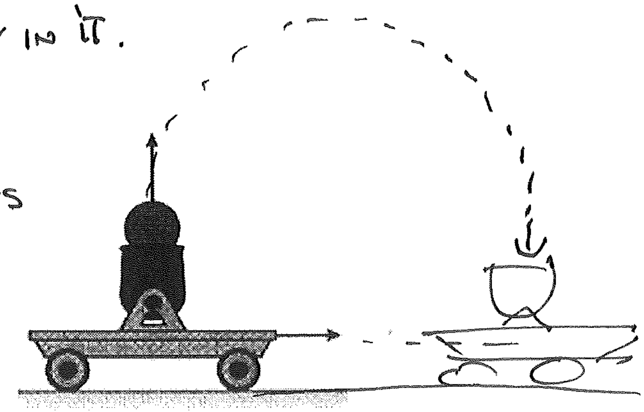
To HIT THE ground EACH BALL Must TRAVEL THE SAME VERTICAL distance h. GRAVITY CAUSES DOWNWARD ~~MOTION~~ ACCELERATION THAT IS THE SAME FOR EACH, but A DOWNWARD INITIAL VElocity will SHORTEN THE TIME while AN UPWARD INITIAL VELOCITY will Lengthen IT. THE AMOUNT OF UPWARD OR DOWNWARD INITIAL Velocity is given by $(V_y)_i \leftarrow$ THE y-component of initial velocity. Looking AT picture, we see 4 HAS NOTHING but DOWN y-component, 3 HAS SOME down, 2 HAS ZERO, 1 HAS SOME UPWARD y-component, 5 HAS ZERO AS well.

∴ THE RANKING IS AS Above.

(3.) A cart which is rolling with constant speed fires a ball straight up. When the ball comes back down, will it land in front of the launching tube, behind the launching tube, or directly in it?

DIRECTLY IN IT.

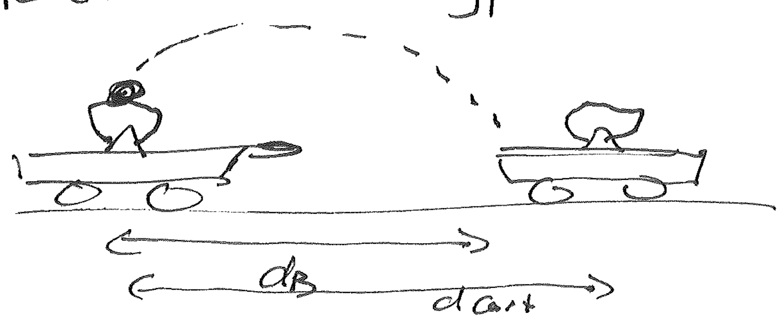
Explanation:



THE BALL IS ON THE CART WHEN IT IS LAUNCED, SO ITS INITIAL X-COMPONENT of velocity MUST BE EQUAL TO THE CART'S: $(V_x)_i = V_{cart}$. A Projectile HAS Constant V_x AND THE CART IS Rolling WITH Constant SPEED, SO IF IT TAKES A time Δt FOR BALL TO COME BACK DOWN, HORIZONTALLY THE BALL will travel A distance $d_B = (V_x)_i \Delta t$ AND THE CART, $d_{cart} = (V_{cart}) \Delta t$. Since $(V_x)_i = V_{cart}$, $d_B = d_{cart}$, SO BALL LANDS BACK IN LAUNCHER.

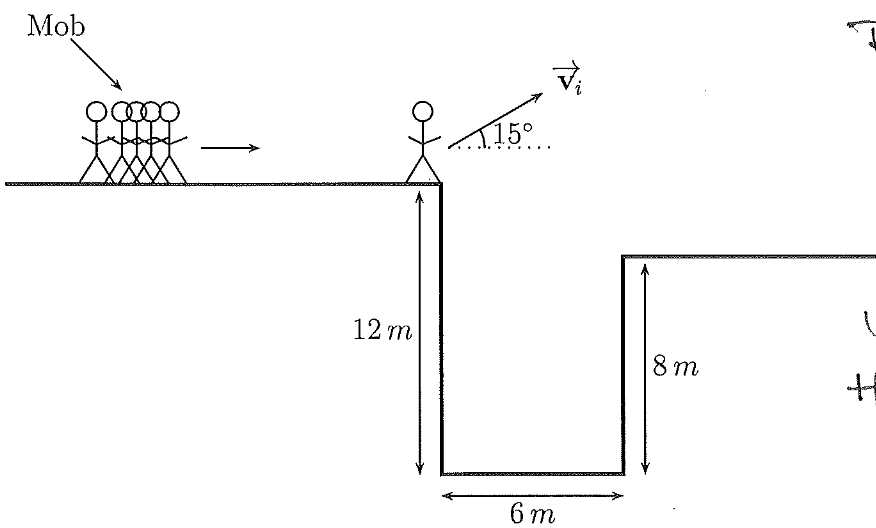
(4.) Does your answer to the previous question change if the cart is accelerating forward? Explain your answer.

NOW BALL will LAND BEHIND LAUNCHING tube. When in the AIR, THE BALL will have CONSTANT $(V_x)_i = V_{cart,i}$ ← speed of Cart when BALL WAS LAUNCED, SO BALL travels $d_B = (V_{cart,i}) \Delta t$. THE CART IS ACCELERATING, SO IT will travel FARTHER, $d_{cart} = (V_{cart,i}) \Delta t + \frac{1}{2} a \Delta t^2$



3.7 -Projectile Motion: Solving Problems

(1.) One day finds your instructor fleeing from a mob of angry physics students. As is usually the case in situations like this, he eventually finds himself caught at the edge of a 12-m high ravine. 6.0 m away is the other side of the ravine which is only 8.0 m high. (As schematically shown below.) In desperation, your instructor launches himself with speed 6.5 m/s and angle 15°. Does he make it to the other side of the ravine? Please back up your assertion with a (hopefully) correct numerical calculation.



Probably THE easiest WAY IS TO FIND THE INSTRUCTOR'S y-POSITION, $y_f = ?$ WHEN $x_f = 6m$.

With $y_i = 12m$, if $y_f < 8m$, HE DOESN'T make it.

KNOWN: $y_i = 12m$, $x_i = 0$, $x_f = 6m$

$\vec{V}_i = 6.5 m/s$ at $15° \Rightarrow (V_x)_i = 6.5 m/s \cos 15° = 6.2785 m/s$

$(V_y)_i = 6.5 m/s \sin 15° = 1.6823 m/s$

$a_x = 0, \quad a_y = -g$

UNKNOWN: $y_f = ?$, $\Delta t = ?$

STRATEGY: use $x_f = x_i + (V_x)_i \Delta t$ to find Δt

THEN $y_f = y_i + (V_y)_i \Delta t - \frac{1}{2} g \Delta t^2$ to find y_f

$X_f = X_i + (V_x)_i \Delta t \Rightarrow 6m = 0 + (6.2785 m/s)\Delta t$

$\Rightarrow \Delta t = \frac{6m}{6.2785 m/s} = 0.9556 s$

$Y_f = Y_i + (V_y)_i \Delta t - \frac{1}{2} g \Delta t^2 \Rightarrow Y_f = 12m + (1.6823 m/s)(.9556 s) - \frac{1}{2}(9.8 m/s^2)(.9556 s)^2$

$\Rightarrow Y_f = 9.13 m$ ← makes it!

You could also REVERSE THE problem. Assume $Y_f = 8m$ AND FIND X_f.

$Y_f = Y_i + (V_y)_i \Delta t - \frac{1}{2} g \Delta t^2 \Rightarrow 8m = 12m + 1.6823 m/s \Delta t - \frac{1}{2}(9.8 m/s^2)\Delta t^2$

$\Rightarrow -4m = 1.6823 m/s \Delta t - 4.9 m/s^2 \Delta t^2 \Rightarrow 4.9 m/s^2 \Delta t^2 - 1.6823 m/s \Delta t - 4m = 0$

QUADRATIC EQN.: $\Delta t = \frac{-(-1.6823 m/s) \pm \sqrt{(1.6823 m/s)^2 - 4(4.9 m/s^2)(-4m)}}{2(4.9 m/s^2)}$

$\Rightarrow \Delta t = \frac{1.6823 m/s \pm \sqrt{81.23 m^2/s^2}}{9.8 m/s^2} = 1.0913 s$ OR $\cancel{-0.748 s}$

$X_f = X_i + (V_x)_i \Delta t \Rightarrow X_f = 0 + (6.2785 m/s)(1.0913 s)$

$\Rightarrow X_f = 6.85 m$ ← LARGER THAN 6m, SO AGAIN, MAKES it.