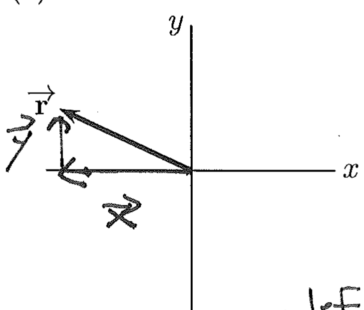
Chapter 3, Sections 3.3-3.5

3.3 - Coordinate Systems and Vector Components

(1.) For the following vectors, draw the x- and y-vector components. Label whether the scalar components are positive(+) or negative(−).

(a)

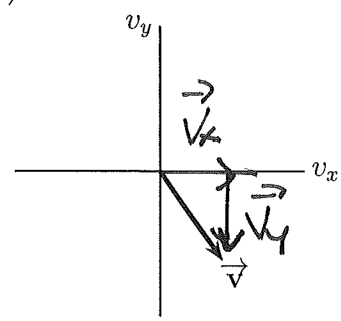


x	−
y	+

left

up

(b)

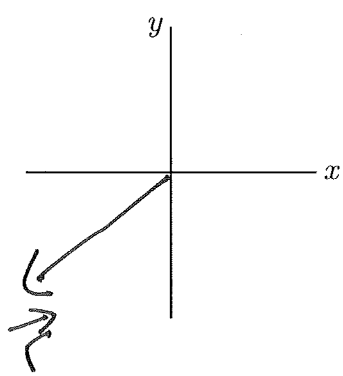


v_x	+
v_y	−

Right

DOWN

(2.) On the axes, sketch a vector whose x-component and y-component are negative.



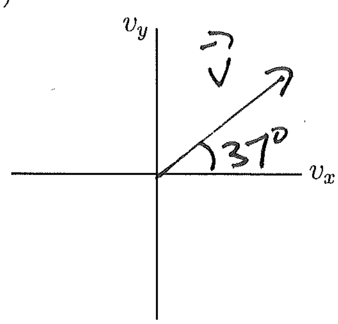
Negative x AND Negative y ⇒ 3RD QUADRANT

(3.) For the vectors given, sketch the vectors on the axes provided and find the numerical values for x- and y-scalar components. Assume the angles given are the standard angle. Show your calculations in the region below.

(a) $\vec{v} = 15\,m/s$ at 37° ← First Quadrant ⇒ positive x & positive y

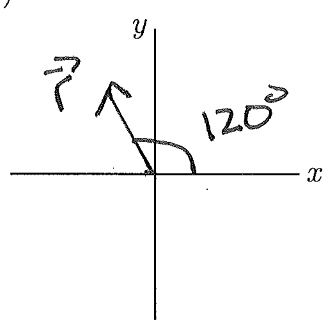
(b) $\vec{r} = 5\,m$ at 120° ← 2ND QUADRANT ⇒ Negative x & positive y

(a)



v_x	12m/s
v_y	9.0m/s

(b)



x	-2.5m
y	4.33m

Standard Angle ⇒

$V_x = V\cos\theta = 15\,m/s \cos 37°$

$\Rightarrow V_x = 11.9795\,m/s = 12\,m/s$

$V_y = V\sin\theta = 15\,m/s \sin 37°$

$\Rightarrow V_y = 9.027\,m/s = 9.0\,m/s$

$X = r\cos\theta = 5m\cos 120°$

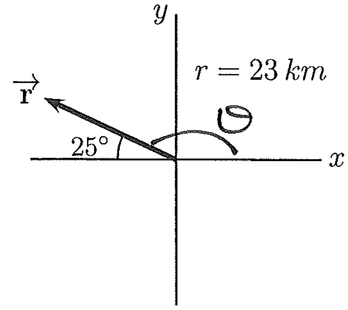
$= -2.5\,m$ ← to left

$Y = r\sin\theta = 5m\sin 120°$

$= 4.33\,m$ ← up

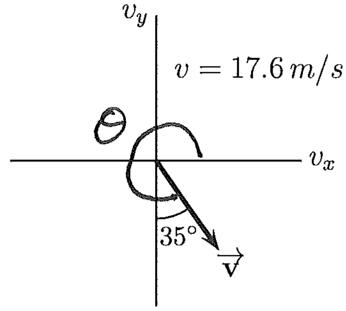
(4.) For the following vectors, find the numerical value of the x- and y-scalar components. Show your calculations in the region below.

(a)



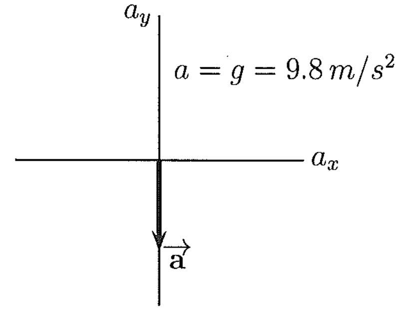
x	-20.8Km
y	9.72Km

(b)



v_x	10.1m/s
v_y	-14.4m/s

(c)



a_x	0
a_y	-9.8m/s²

EITHER USE TRIG IF you KNOW IT, OR stick WITH standard Angle.

a) $\theta = 180^\circ - 25^\circ = 155^\circ \Rightarrow x = (23\,Km)\cos 155^\circ = -20.8\,Km$

$y = (23\,Km)\sin 155^\circ = 9.72\,Km$

b) $\theta = 270^\circ + 35^\circ = 305^\circ \Rightarrow V_x = (17.6\,m/s)\cos 305^\circ = 10.1\,m/s$

$V_y = (17.6\,m/s)\sin 305^\circ = -14.4\,m/s$

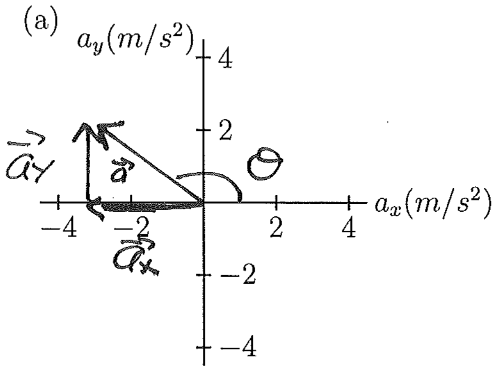
c) A Vector THAT points STRAIGHT DOWN HAS NO x-component AND Negative y-component $\Rightarrow a_x = 0,\ a_y = -9.8\,m/s^2$

(5.) For each set of components given, sketch the vector and indicate its standard angle θ on the axes provided. Then find the numerical value of the vector's magnitude and the angle θ. Show your calculations in the region below.

(a) $a_x = -3\,m/s^2$, $a_y = 2\,m/s^2$ ← 2ND QUADRANT

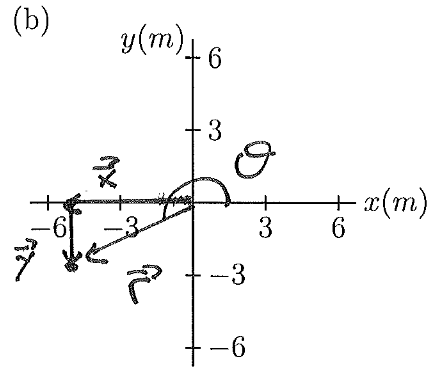
(b) $x = -5\,m$, $y = -2\,m$ ← 3RD QUADRANT

(a)



a	3.61 m/s²
θ	146.3°

(b)



r	5.39 m
θ	202°

(a) $a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-3\,m/s^2)^2 + (2\,m/s^2)^2} = \sqrt{(3\,m/s^2)^2 + (2\,m/s^2)^2} = \sqrt{9\,m^2/s^4 + 4\,m^2/s^4} = \sqrt{13\,m^2/s^4}$

$\Rightarrow a = 3.61\,m/s^2$

θ in 2ND QUADRANT $\Rightarrow$ Calculator wrong by 180° $\Rightarrow \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) + 180° = \tan^{-1}\left(-\frac{2}{3}\right) + 180°$

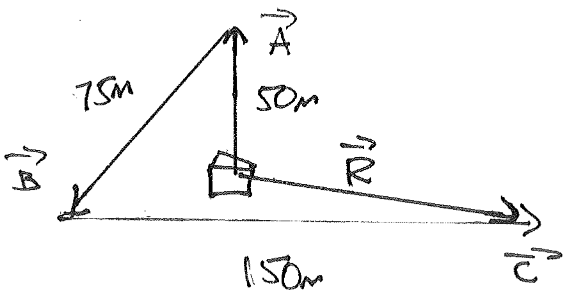
$\Rightarrow \theta = -33.69° + 180° = 146.3°$

(b) $r = \sqrt{x^2 + y^2} = \sqrt{(5\,m)^2 + (2\,m)^2} = \sqrt{25\,m^2 + 4\,m^2} = \sqrt{29\,m^2} = 5.39\,m$

3RD QUAD $\Rightarrow$ Calculator wrong by 180° $\Rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right) + 180° = \tan^{-1}\left(\frac{2}{5}\right) + 180° = 21.8° + 180°$

$= 201.8°$

(6.) A man leaves his house and walks 50 m due north. He then walks southwest 75 m. Finally, he walks 150 m east. Determine the numerical value of how far and what angle the man must walk in order to return home.



$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

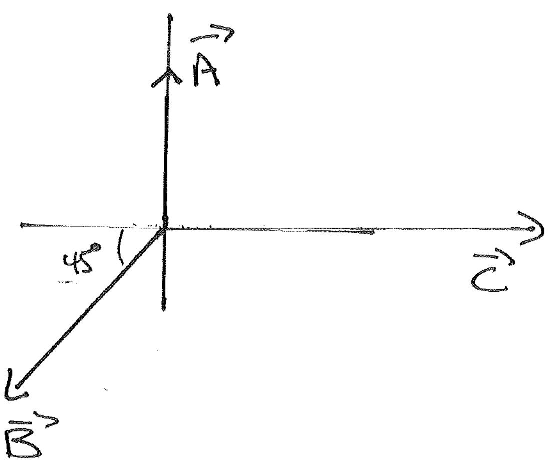
points from house to final position

So $-\vec{R}$ points back to home.

$R_x = A_x + B_x + C_x$

$R_y = A_y + B_y + C_y$

to help find components, redraw all 3 at origin



$\vec{A}$ points up ⇒ only has y-component ⇒ $A_x = 0$, $A_y = 50m$

$\vec{C}$ points right ⇒ only has x-component ⇒ $C_x = 150m$, $C_y = 0$

$\vec{B}$ has standard angle $\theta = 180° + 45° = 225°$

⇒ $B_x = 75m \cos 225° = -53.0m$

$B_y = 75m \sin 225° = -53.0m$

So $R_x = A_x + B_x + C_x = 0 - 53m + 150m = 97m$

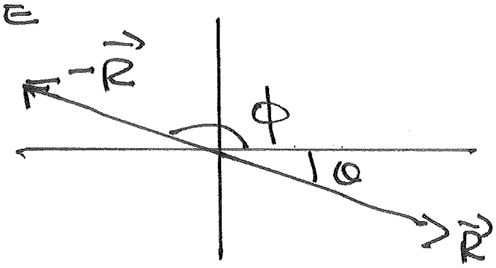
$R_y = A_y + B_y + C_y = 50m - 53m + 0 = -3m$

$\therefore R = \sqrt{R_x^2 + R_y^2} = \sqrt{(97m)^2 + (3m)^2} = \sqrt{9418m^2} = 97m$

R_x positive, R_y Negative $\Rightarrow$ 4th QUADRANT $\Rightarrow$ CAlculator OK!

$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-3}{97}\right) = -1.77°$

To walk home



$\phi = 180° - 1.77°$

$= 178°$