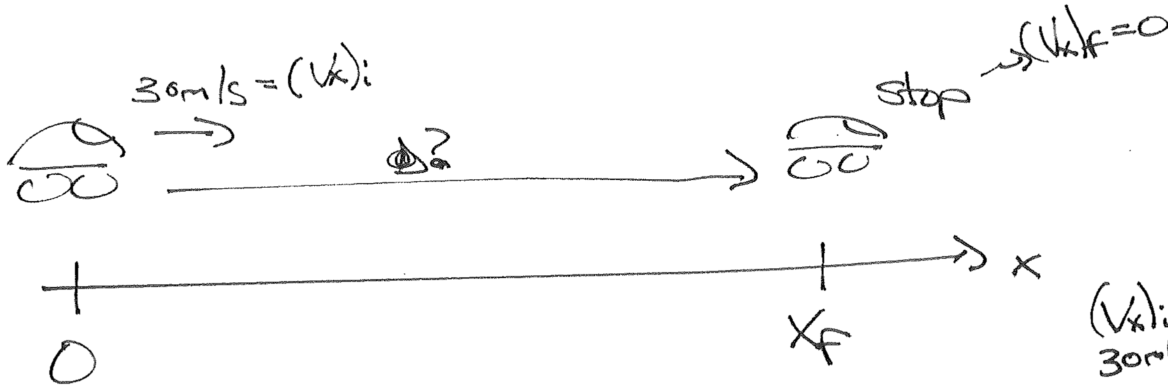
Constant Acceleration Problems

Example: A car is traveling at 30 m/s when driver hits brakes causing a constant deceleration of $2.5\,m/s^2$. How long does it take and how far does car go while stopping?

Picture should include: object at start, object at end, coordinate system, and list of variables both known and unknown.



Known

$X_i = 0$

$(V_x)_i = 30\,m/s$ $\qquad (V_x)_f = 0$

$a_x = -2.5\,m/s^2$

speed decreasing so a_x opposite to V_x

⇒ If we put $X_i = 0$ then $X_f = \Delta X = ?$

Unknown:

$X_f = ?$

$\Delta t = ?$

To find Δt: $(V_x)_f = (V_x)_i + a_x \Delta t$ will work since we know everything but Δt

$$0 = 30\,m/s - 2.5\,m/s^2\,\Delta t \Rightarrow \frac{2.5\,m/s^2\,\Delta t}{2.5\,m/s^2} = \frac{30\,m/s}{2.5\,m/s^2}$$

$$\Rightarrow \Delta t = 12\,s$$

NOW THAT WE KNOW Δt, $X_f = X_i + (V_x)_i \Delta t + \frac{1}{2} a_x \Delta t^2$

will work

$\Rightarrow X_f = 0 + (30\,m/s)(12\,s) + \frac{1}{2}(-2.5\,m/s^2)\underbrace{(12\,s)^2}_{(12s)(12s)=144s^2}$

$\Rightarrow X_f = 360\,m - \frac{1}{2}(2.5\,m/s^2)(144\,s^2)$

$X_f = 360\,m - 180\,m = 180\,m$

OF COURSE, WE COULD ALSO USE $(V_x)_f^2 = (V_x)_i^2 + 2a_x \Delta X$

$\Rightarrow 0^2 = (30\,m/s)^2 + 2(-2.5\,m/s^2)\Delta X \Rightarrow 0 = 900\,m^2/s^2 - 5\,m/s^2\,\Delta X$

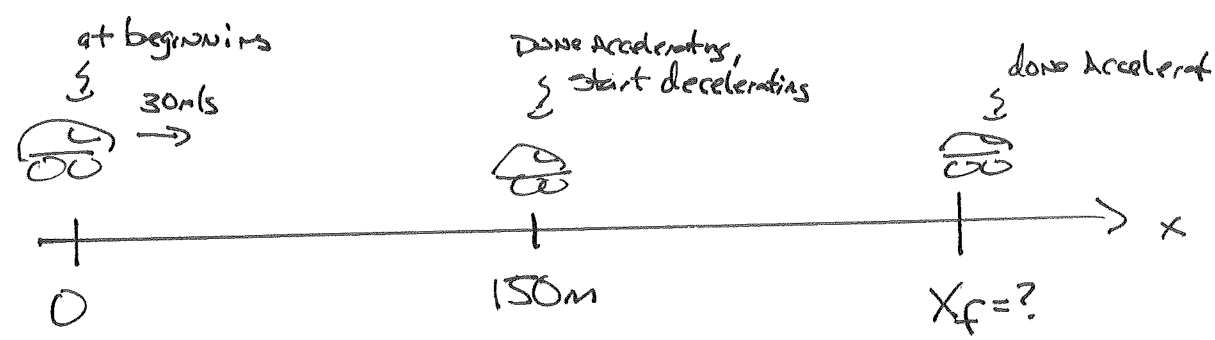
$\Rightarrow \dfrac{5\,m/s^2\,\Delta X}{5\,m/s^2} = \dfrac{900\,m^2/s^2}{5\,m/s^2} \Rightarrow \Delta X = 180\,m$

&

UNIT: $\frac{m^{\cancel{2}}}{\cancel{s^2}} \times \frac{\cancel{s^2}}{\cancel{m}} = m$

Example: Phyllis is traveling on a straight Highway with a speed of 30m/s and wishes to pass Stanley who is in the Car ahead of her. Phyllis Accelerates at $1.25\,m/s^2$ for 150m. She then decelerates back down to 30m/s in 5s. How long did it take and how far did she go in total?

Here, we have two motions. One leading directly into a second. So we draw car at 3 times.



Separate the 2 motions : Known:

1st = Accelerating	2ND = Decelerating
$X_i = 0$	$\cancel{X_{i,1} = 150m}$
$V_{i,1} = 30m/s$	$X_{i,2} = 150m$
$X_{f,1} = 150m$	$\cancel{a_{x,2} = 30m/s}$
$a_{x,1} = 1.25\,m/s^2$	$V_{f,2} = 30m/s$
	$\Delta t_2 = 5s$

We know one other fact : $V_{f,1} = V_{i,2}$ ← Final velocity of 1 is initial velocity of 2.

Unknowns:

1st	2ND
$\Delta t_1 = ?$	$X_{f,2} = ?$
$V_{f,1} = ?$	$a_{x,2} = ?$

Begin with $(V_x)_f^2 = (V_x)_i^2 + 2a\Delta x$ to find $(V_x)_f$

$(V_x)_f^2 = (30\,m/s)^2 + 2(1.25\,m/s^2)(150\,m) = 900\,m^2/s^2 + 375\,m^2/s^2 \Rightarrow$

$(V_x)_f^2 = 1275\,m^2/s^2 \Rightarrow (V_x)_f = \sqrt{1275\,m^2/s^2} = 35.7\,m/s$

Now that we have that: $(V_x)_f = (V_x)_i + a_x \Delta t$ can find Δt_1

$35.7\,m/s = 30\,m/s + 1.25\,m/s^2\,\Delta t_1 \Rightarrow 1.25\,m/s^2\,\Delta t_1 = 35.7\,m/s - 30\,m/s$

$\Rightarrow 1.25\,m/s^2\,\Delta t_1 = 5.7\,m/s \Rightarrow \Delta t_1 = \dfrac{5.7\,m/s}{1.25\,m/s^2} = 4.56\,s$

Unit: $\dfrac{\cancel{m}}{\cancel{s}} \times \dfrac{s^{\cancel{2}}}{\cancel{m}} = s$

So, now we have the following:

Known for #2	Unknown
$X_{i,2} = 150\,m$	$X_{f,2} = ?$
$V_{f,2} = 30\,m/s$	$a_{x,2} = ?$
$\Delta t_2 = 5\,s$	
$V_{i,2} = V_{f,1} = 35.7\,m/s$	

Since question asks for X_f, it's tempting to try to finish the problem right away. But all of our equations have a_x in them. We have to solve for it first.

$(V_x)_f = (V_x)_i + a_x \Delta t \Rightarrow 30\,m/s = 35.7\,m/s + a_{x,2}(5\,s) \Rightarrow a_{x,2} = \dfrac{30\,m/s - 35.7\,m/s}{5\,s}$

$\Rightarrow a_{x,2} = \dfrac{-5.7\,m/s}{5\,s} = -1.14\,m/s^2$

Now $X_f = X_i + V_{i,x}\Delta t + \frac{1}{2}a_x \Delta t^2 \Rightarrow X_f = 150\,m + 35.7\,m/s(5\,s) + \frac{1}{2}(-1.14\,m/s^2)(5\,s)^2$

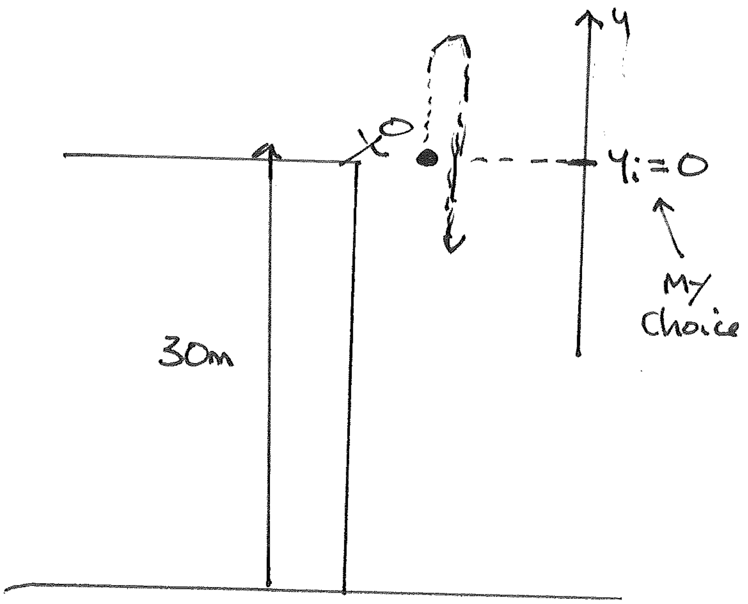
$\Rightarrow X_f = 314.25\,m = 314\,m$

To find total time, we have to take $\Delta t_1 + \Delta t_2 = \Delta t_T$

$\Delta t_T = 4.56\,s + 5\,s = 9.56\,s$

Example: A PERSON AT THE TOP OF A building 30m high throws AND egg upwards at 15m/s. If air resistance CAN be ignored.

- How fast will it be going after 3s



HARD to DRAW picture because we don't KNOW where its at.

KNOWN: $y_i = 0$

$(v_y)_i = 15\text{m/s}$ ← up is positive

$a_y = -g = -9.8\text{m/s}^2$ ← DOWN is Negative

$\Delta t = 3\text{s}$

UNKNOWN: $(v_y)_f = ?$, $y_f = ?$

$$(v_y)_f = (v_y)_i + a_y \Delta t \Rightarrow (v_y)_f = 15\text{m/s} - 9.8\text{m/s}^2(3\text{s}) \Rightarrow (v_y)_f = -14.4\text{m/s}$$

$= 14.4\text{m/s}$, DOWN

- How high, from where it was thrown, does egg go before Coming back down

upwards ⇒ positive velocity, DOWN ⇒ Negative velocity, At turnaround point at MAXimum height $v_y = 0$

SO KNOWN: $y_i = 0$

$(V_y)_i = 15\,m/s$

$a_y = -9.8\,m/s^2$

$(V_y)_f = 0$

UNKNOWN: $y_f = ?$

$\Delta t = ?$

$(V_y)_f^2 = (V_y)_i^2 + 2a\Delta y$ will give ANSWER IN one step.

$0^2 = (15\,m/s)^2 + 2(-9.8\,m/s^2)(y_f - 0) \Rightarrow 0 = 225\,m^2/s^2 - 19.6\,m/s^2\, y_f$

$\Rightarrow y_f = \dfrac{225\,m^2/s^2}{19.6\,m/s^2} = 11.479\,m = 11.5\,m$

unit: $\frac{m^2}{s^2} \times \frac{s^2}{m} = m$

How long to hit ground?

Since we put @ $y = 0$ at throwing height, ground must be at $y = -30\,m$

KNOWN: $y_i = 0$

$(V_y)_i = 15\,m/s$

$a_y = -9.8\,m/s^2$

$y_F = -30\,m$

UNKNOWN: $\Delta t = ?$

$(V_y)_F = ?$ ← Not ZERO!

$y_F = y_i + (V_y)_i \Delta t + \frac{1}{2} a_y \Delta t^2 \Rightarrow -30\,m = 0 + 15\,m/s\,\Delta t + \frac{1}{2}(-9.8\,m/s^2)\Delta t^2$

$\Rightarrow -30\,m = 15\,m/s\,\Delta t - 4.9\,m/s^2\,\Delta t^2$ ← QUADRATIC EQUATION

REARRANGE: $4.9\,m/s^2\,\Delta t^2 - 15\,m/s\,\Delta t - 30\,m = 0$

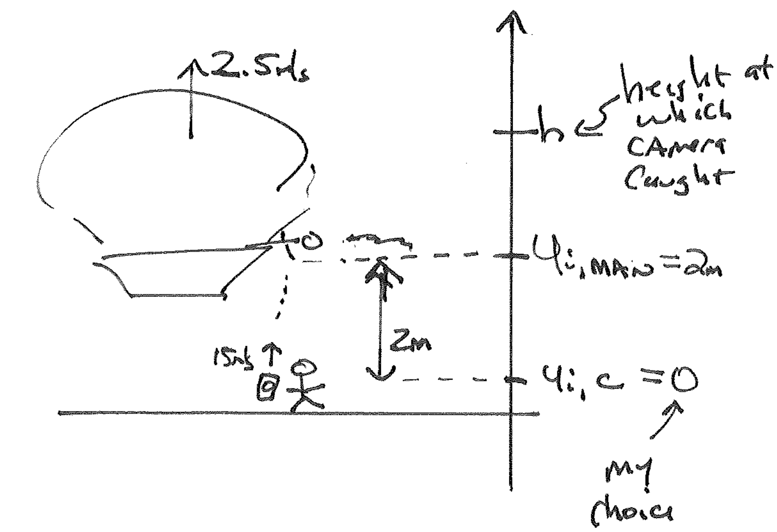
$a\Delta t^2 + b\Delta t + c = 0$

$\Rightarrow \Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta t = \dfrac{-(-15\,m/s) \pm \sqrt{(15\,m/s)^2 - 4(4.9\,m/s^2)(-30\,m)}}{2(4.9\,m/s^2)} = \dfrac{15\,m/s \pm \sqrt{813\,m^2/s^2}}{9.8\,m/s^2}$

$\Delta t = \dfrac{15\,m/s + \sqrt{813\,m^2/s^2}}{9.8\,m/s^2} = \dfrac{43.5\,m/s}{9.8\,m/s^2} = 4.44\,s$ OR $\Delta t = \dfrac{15\,m/s - \sqrt{813\,m^2/s^2}}{9.8\,m/s^2} = \dfrac{-13.5\,m/s}{9.8\,m/s^2} = -1.38\,s$

Example: A man is in a hot-air balloon which takes off and rises with constant 2.5m/s speed. Just after take off, the man notices that he forgot his camera. A friend throws the camera up to him with a speed of 15m/s. If the man is 2m above the camera when it is thrown, how high will it be when he catches his camera?



Two moving objects:

KNOWN	Man:	Camera:
	$y_{i,MAN} = 2m$	$y_{i,c} = 0$
	$V_{i,MAN} = 2.5m/s$	$V_{i,c} = 15m/s$
	$a_y = 0$	$a_y = -9.8m/s$
	$V_{f,MAN} = 2.5m/s$	

UNKNOWN:	Man:	Camera:
	$y_{f,m} = ?$	$y_{f,c} = ?$
	$\Delta t = ?$	$\Delta t = ?$

same since started at same time

MAN catches camera when $y_{f,M} = y_{f,C} = h$ ← convenient

$$y_f = y_i + (V_y)_i \Delta t + \tfrac{1}{2} a_y \Delta t^2$$

for MAN: $h = 2m + 2.5m/s\,\Delta t + 0 \Rightarrow h = 2m + 2.5m/s\,\Delta t$

Camera: $h = 0 + 15m/s\,\Delta t + \tfrac{1}{2}(-9.8m/s^2)\Delta t^2$

$$\Rightarrow h = 15m/s\,\Delta t - 4.9m/s^2\,\Delta t^2$$

$h = 2m + 2.5m/s\,\Delta t$ and $h = 15m/s\,\Delta t - 4.9m/s^2\,\Delta t^2$

$\Rightarrow 2m + 2.5m/s\,\Delta t = 15m/s\,\Delta t - 4.9m/s^2\,\Delta t^2$

$\Rightarrow 4.9m/s^2\,\Delta t^2 + 2.5m/s\,\Delta t - 15m/s\,\Delta t + 2m = 0$

$\Rightarrow 4.9m/s^2\,\Delta t^2 + (2.5m/s - 15m/s)\,\Delta t + 2m = 0$

$\Rightarrow 4.9m/s^2\,\Delta t^2 - 12.5m/s\,\Delta t + 2m = 0$ ← QUADRATIC EQUATION

$$\Delta t = \frac{-(-12.5m/s) \pm \sqrt{(12.5m/s)^2 - 4(4.9m/s^2)(2m)}}{2(4.9m/s^2)} = \frac{12.5m/s \pm \sqrt{117.05m^2/s^2}}{9.8m/s^2}$$

$$\Rightarrow \Delta t = \frac{(12.5m/s + \sqrt{117.05m^2/s^2})}{9.8m/s^2} = 2.38s$$

$$\text{OR } \Delta t = \frac{(12.5m/s - \sqrt{117.05m^2/s^2})}{9.8m/s^2} = 0.172s$$

MAN gets two tries. First at 0.172s AND AGAIN at 2.38s

For $\Delta t = 0.172s$ $(V_y)_f = (V_y)_i + a_y\,\Delta t \Rightarrow (V_y)_f = 15m/s - 9.8m/s^2(0.172s)$

$= 14.8m/s$ ← CAmera going up

$\Delta t = 2.38s \Rightarrow (V_y)_f = 15m/s - 9.8m/s^2(2.38s) = -8.32m/s$ ← CAMERA Coming back down

TO ACTUALLY FINISH Problem $h = 2m + 2.5m/s\,\Delta t \Rightarrow h = 2m + 2.5m/s\,(0.172s)$

$\Rightarrow h = 2.43m$ if MAN CATCHES CAmera ON first try