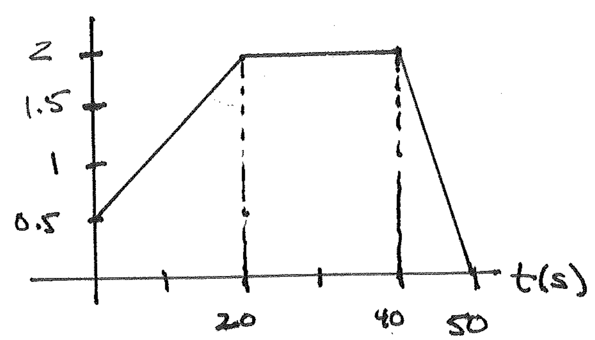
Physics 151, HW #3

6 Mastering Physics Problems
from Chapter 2

Written Problem : 2.74

WHAT Velocity vs. time graphs.. (yes, AGAIN)



NOTICE: 3 Line segments ⇒ 3 Constant Acceleration Motions

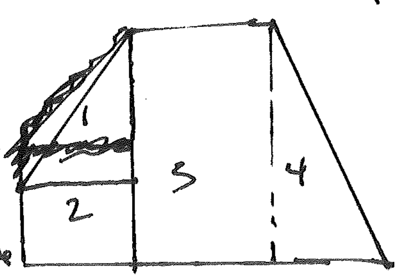
From 20 to 40s, Horizontal line ⇒ Uniform motion ⇒ $a_x = 0$

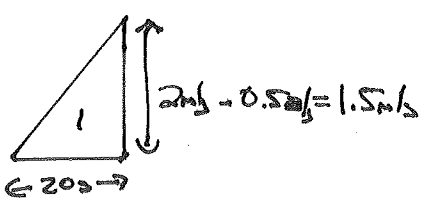
a) what is initial velocity? ⇒ initial velocity at $t=0$. Just read off of graph $V_i = 0.5\,m/s$

b) what is total distance? → For any type of motion Δx is Area under V_x vs. t.

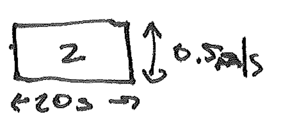
Split into 4 areas

Sorry MADE My triangle too small

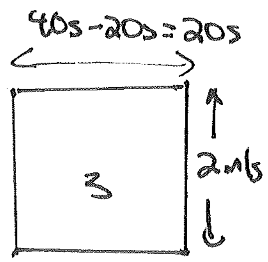




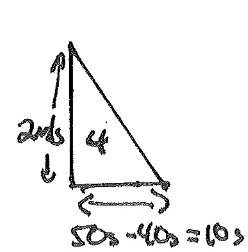
$A_1 = \frac{1}{2}(20s)(1.5\,m/s)$
$= 15\,m$



$A_2 = (20s)(0.5\,m/s) = 10\,m$



$A_3 = 20s(2\,m/s) = 40\,m$



$A_4 = \frac{1}{2}(10s)(2\,m/s)$
$= 10\,M$

$\Delta X = A_1 + A_2 + A_3 + A_4 = 15m + 10m + 40m + 10m = 75m$

c) what is a_{av} over first 20s?

$a_{av} = \frac{\Delta v}{\Delta t}$ for $t_i = 0$, $t_f = 20s$

$V_i = 0.5m/s$, $V_f = 2m/s$

$$a_{av} = \frac{V_f - V_i}{t_f - t_i} = \frac{(2m/s - 0.5m/s)}{(20s - 0)} = \frac{1.5m/s}{20s} = 0.075m/s^2$$

This question is a little silly since V_x vs t straight line ⇒ constant acceleration, so the instantaneous acceleration value for $0<t<20s$ is also $0.075m/s^2$, and both a_{av} and a_x are the slope of the line.

d) what is instantaneous acc. at $t = 45s$. → Again, straight line for 40s to 50s ⇒ instant. Acc. is constant for all times between 40s and 50s

AND $a_x = \frac{\Delta V}{\Delta t}$ = slope here use $t_i = 40s$, $t_f = 50s$ ⇒ $V_{xi} = 2.0m/s$

$V_{xf} = 0$

$$\Rightarrow a_x = \frac{(0 - 2m/s)}{(50s - 40s)} = \frac{-2m/s}{10s} = -0.2m/s^2$$

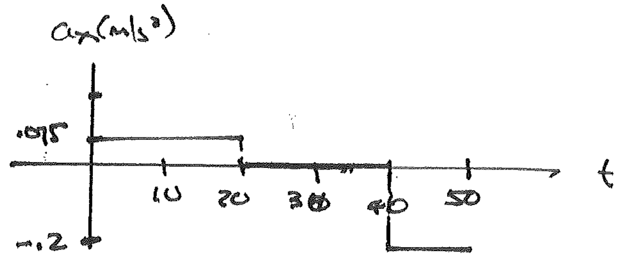
e) which is correct a_x vs t graph?

We know $a_x = 0.075\,m/s^2$ for first 20s

for 40s to 50s, $a_x = -.2\,m/s^2$. As I mentioned at very beginning

from 20s to 40s, $a_x = 0$ since ~~gra~~ V_x graph is horizontal

$\Rightarrow$ 3 constant graphs $\Rightarrow$ 3 Horizontal lines



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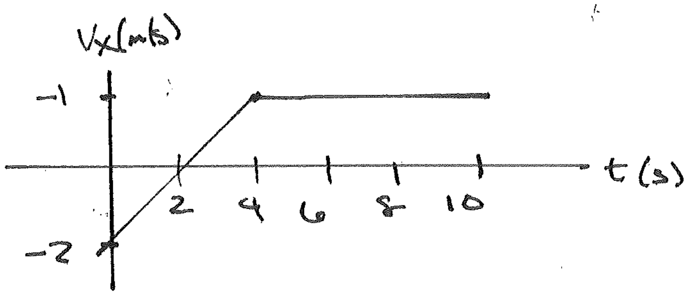


Train starts at origin

Draw position graph

Notice: for first 4s, v_x = line

$\Rightarrow$ Constant Acceleration

so x vs t is a parabola

For 4s on, v_x = horizontal line

$\Rightarrow$ Uniform motion ($a_x = 0$) so

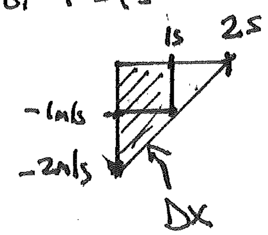
x vs t is a straight line.

To get Mastering to correctly draw a curve (like a parabola) we need to give it at least 4 points, so let's find position at

$t = 1s, 2s, 3s, 4s$ (and also at $t=0, x=0$).

Easiest to use that area under v_x vs t graph is Δx

For $t = 1s$



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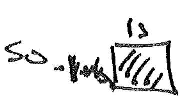
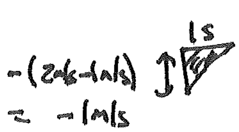


$A = -1\,m/s(1s) = -1\,m$



$A = \frac{1}{2}(1s)(-1\,m/s) = -0.5\,m$

$\Rightarrow \Delta x = -1.5\,m$

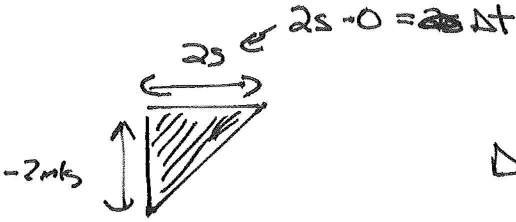
train starts from origin $\Rightarrow$ ~~x~~

$x_1 = -1.5\,m$

x at t = 1s

to find at $t=2s$, Actually easier to start from 0 Again

Since then we can just use Area of triangle.



$$\Delta X = \tfrac{1}{2}(2s)(-2m/s) = -2m$$

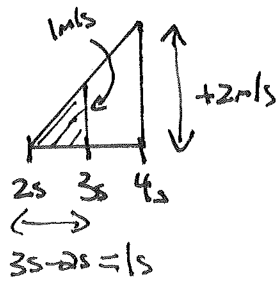
Since this from when train was at origin

$$X_2 = -2m$$

(By the way, neither of these negative values should surprise you. V_x negative ⇒ moving to left. Train starts at 0, so negative position to begin with)

$t=3s$, Here its probably easier to find area starting at $t=2s$

So now $\Delta X = X_f - X_i \Rightarrow X_f = X_i + \Delta X$. We'll need to use $X_i = X_2 = -2m$



$$\Delta X = \tfrac{1}{2}(1s)(1m/s) = 0.5m$$ ← Velocity now positive So train moving to right

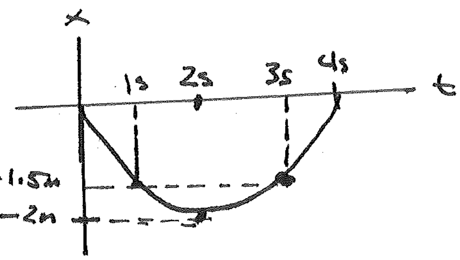
$$X_3 = X_2 + \Delta X = -2m + 0.5m = -1.5m$$

$t=4s$, Again start from 2s, picture above can be used $\Delta X = \tfrac{1}{2}(\underset{4s-2s}{2s})(2m/s) = 2m$

so $X_4 = X_2 + \Delta X = -2m + 2m = 0$

So by $t = 4s$, train is back at origin

Our parabola looks like

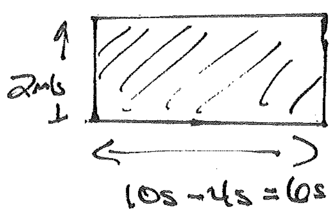


For straight line piece: Pretty simple since at $t = 4s$ $X_4 = 0$

So area under rectangle from 4s to 10s gives X_{10} since $\Delta X = X_{10} - X_4$

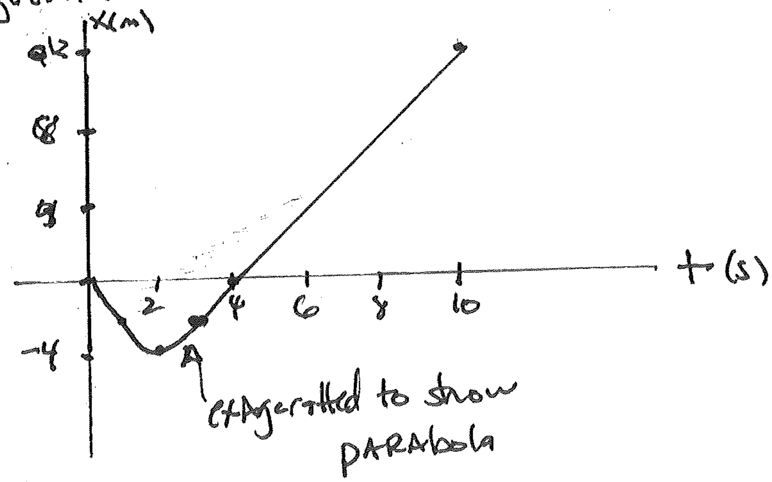
$= X_{10} - 0$

$= X_{10}$



$\Delta X = (6s)(2m/s) = 12m \Rightarrow X_{10} = 12m$

All together:



b)

Find Acceleration at $t = 3s$

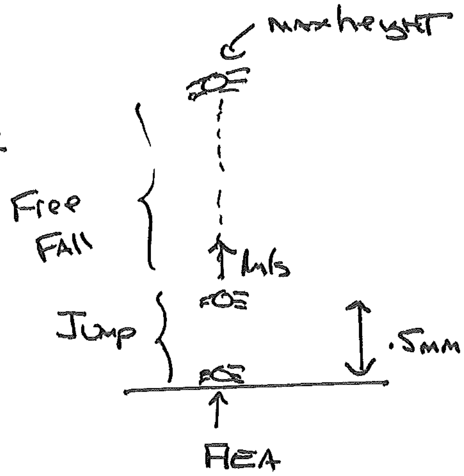
For all times between 0 and 4s, Acceleration has same value.

~~of~~ that value is slope of V_x line, so use $t_i = 0$, $t_f = 4s$

$\Rightarrow V_{xi} = -2m/s$, $V_{xf} = +2m/s$

$$a_x = \frac{\Delta V_x}{\Delta t} = \frac{V_{xf} - V_{xi}}{t_f - t_i} = \frac{+2m/s - (-2m/s)}{4s - 0} = \frac{2m/s + 2m/s}{4s} = \frac{4m/s}{4s} = 1m/s^2$$

59.



KNOWNS

For Jump: $y_i = 0$

$y_F = .5mm$

$(V_y)_i = 0$

$(V_y)_F = 1m/s$

UNKNOWNS: $a_y = ?$

a) WHAT IS FLEA'S ACCELERATION (ASSUMED Constant)

$a_y = ?$

For JUMP $V_i = 0$ (starts from rest) $V_F = 1m/s$

$y_i = 0$, $y_F = .5mm \times \frac{1m}{1000mm} = 5\times10^{-4}m$

$V_{yF}^2 = V_{yi}^2 + 2a_y(y_F - y_i) \Rightarrow (1m/s)^2 = 0^2 + 2a_y(5\times10^{-4}m - 0)$

$\Rightarrow 1m^2/s^2 = 2a_y(5\times10^{-4}m) \Rightarrow 1m^2/s^2 = a_y(1\times10^{-3}m)$

$\Rightarrow \boxed{a_y = \frac{1m^2/s^2}{1\times10^{-3}m} = 1000m/s^2}$

b) HOW LONG DOES ACCELERATION LAST?

NOW THAT WE HAVE ACCELERATION, $(V_y)_F = (V_y)_i + a_y\Delta t$ works

$\Rightarrow 1m/s = 0 + 1000m/s^2\,\Delta t \Rightarrow \boxed{\Delta t = \frac{1m/s}{1000m/s^2} = 1\times10^{-3}s = 1ms}$

Unit: $\frac{\cancel{m}}{\cancel{s}} \times \frac{s^{\cancel{2}}}{\cancel{m}} = s$

c) How High will flea go?

AFTER JUMP, FLEA IN FREE FALL $\Rightarrow a_y = -9.8 m/s^2$

Reset Problem to $V_i = 1 m/s$, $\cancel{x}_i\,y_i = .5mm = 5\times10^{-4} m$

MAXIMUM HEIGHT $\Rightarrow V_f = 0$, $\cancel{x}_f\,y_f = ?$

$V_{yf}^2 = V_{yi}^2 + 2a_y(\cancel{x}_f - \cancel{x}_i)$ will work (AGAIN)

$\Rightarrow 0 = (1 m/s)^2 + 2(-9.8 m/s^2)(\cancel{x}_f - 5\times10^{-4} m)$

$\Rightarrow 0 = 1 m^2/s^2 - 19.6 m/s^2 (\cancel{x}_f - 5\times10^{-4} m)$

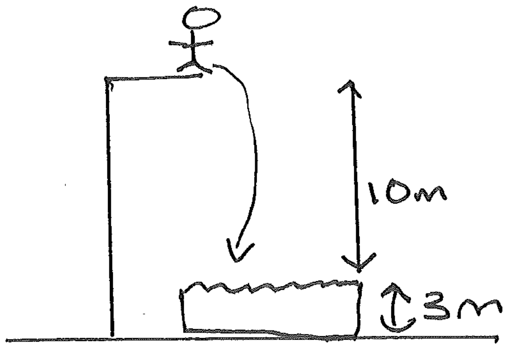
$\Rightarrow 19.6 m/s^2 (\cancel{x}_f - 5\times10^{-4} m) = 1 m^2/s^2$

$\Rightarrow \cancel{x}_f - 5\times10^{-4} m = \dfrac{1 m^2/s^2}{19.6 m/s^2} = .051 m$

$\dfrac{m^{\cancel{2}}}{\cancel{s^2}} \times \dfrac{\cancel{s^2}}{\cancel{m}} = m$

$\Rightarrow \boxed{\cancel{x_f}\,y_f = .051m + 5\times10^{-4} m = .05152m = .052m}$

61.



WHAT MINIMUM ACCELERATION NEEDED to Keep Diver From Hitting Bottom of pool?

DIVER Free FAlls for 10m then decelerates By pool.

KEY: the FINAL velocity of free fall EQUALS INITIAL VELOCITY of DECELERATION.

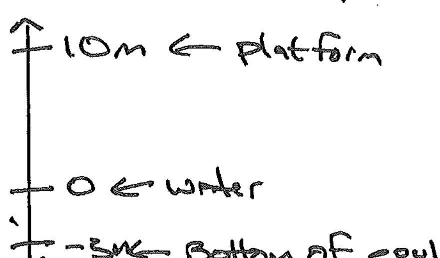
Let Free FAll Be motion #1, Deceleration be motion #2

$$V_{f_1} = V_{i_2}$$

KNOWNS

#1 : $V_{i_1} = 0$ ← Diver steps off platform, $a_1 = -9.8\,m/s^2$ ← Free fall

Coordinate system:



$y_{i_1} = 10m$, $y_{f_1} = 0$

$y_{i_2} = 0$, $y_{f_2} = -3m$

KNOWNS

#2 : $V_{f_2} = 0$ ← water stops Diver,

UNKNOWNS:
$a_2 = ?$
$V_{i_2} = ?$

For #1: $V_{f_1}^2 = V_{i_1}^2 + 2a_1(y_{f_1} - y_{i_1})$ Allows us to solve for V_{f_1}

$$V_{f_1}^2 = 0 + 2(-9.8\,m/s^2)(0-10m) = 2(-9.8\,m/s^2)(-10m)$$

$$\Rightarrow V_{f_1}^2 = 196\,m^2/s^2 \Rightarrow V_{f_1} = \pm\sqrt{196\,m^2/s^2} = -14\,m/s$$ ← choose negative because going downward

$$\Rightarrow V_{i_2} = -14\,m/s,\ V_{f_2} = 0,\ x_{i_2} = 0,\ x_{f_2} = -3m,\ a_2 = ?$$

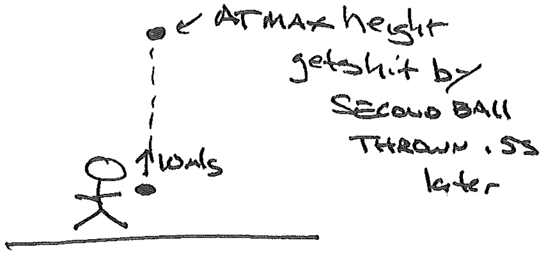
$$V_{f_2}^2 = V_i^2 + 2a_2(y_{f_2} - y_{i_2})$$ works for finding a_2

$$\Rightarrow 0 = (-14\,m/s)^2 + 2a_2(-3m - 0)$$

$$\Rightarrow 0 = 196\,m^2/s^2 - 6m\,a_2 \Rightarrow a_2 = \frac{196\,m^2/s^2}{6m}$$

$$\Rightarrow a_2 = 32.6667\,m/s^2 = \underline{\underline{32.7\,m/s^2}}$$

65.



WHAT speed NEEDED for 2ND BALL.

FOR BALL 1: $V_{i_1} = 10\,m/s$, $a = -9.8\,m/s^2$, AT MAX Height $V_{f_1} = 0$

BALL 2: $V_{i_2} = ?$, $a_2 = -9.8\,m/s^2$

IF BALL 1 takes A time Δt_1 to reACH MAX height

then $\Delta t_2 = \Delta t_1 - .5s$ ← HAS .5s less time to reACH BALL 1

SO FIND Δt_1 AND LOCATION OF MAX height, $x_{f_1} = ?$, $x_{i_1} = 0$

$$V_{f_1} = V_{i_1} + a_1 \Delta t_1 \Rightarrow 0 = 10\,m/s - 9.8\,m/s^2\,\Delta t_1 \Rightarrow \Delta t_1 = \frac{10\,m/s}{9.8\,m/s^2}$$

$$\Rightarrow \Delta t_1 = 1.02\,s$$

$$y_{f_1} = y_{i,1} + V_{i_1}\Delta t_1 + \tfrac{1}{2}a_1\Delta t_1^2 = 0 + (10\,m/s)(1.02\,s) + \tfrac{1}{2}(-9.8\,m/s^2)(1.02\,s)^2$$

$$\Rightarrow y_{f_1} = 5.1\,m$$

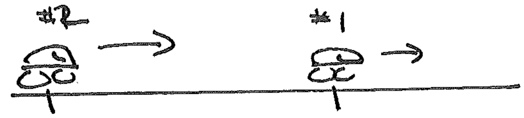
SO BALL 2: $\Delta t_2 = 1.02s - .5s = .52s$ to be at $X_{f_2} = 5.1m$ $X_{i_2} = 0$

$$y_{f_2} = y_{i_2} + V_{i_2}\Delta t_2 + \tfrac{1}{2}a_2\Delta t_2^2 \Rightarrow 5.1m = 0 + V_{i_2}(.52s) + \tfrac{1}{2}(-9.8\,m/s^2)(.52s)^2$$

$$\Rightarrow 5.1m = V_{i_2}(.52s) - 1.32m \Rightarrow V_{i_2}(.52s) = 5.1m + 1.32 = 6.42m$$

$$\Rightarrow V_{i_2} = \frac{6.42m}{.52s} = 12.356\,m/s = \underline{12.4\,m/s}$$

71.



$a_1 = 2m/s^2$ AT $t = 2s$

#2 LAUNCHED with $a_2 = 8m/s^2$

a) AT WHAT TIME DOES #2 CATCH #1?

For #1: $V_{i_1} = 0$, $a_1 = 2m/s^2$, $X_{i_1} = 0$; #2: $V_{i_2} = 0$, $a_2 = 8m/s^2$, $X_{i_2} = 0$

Let t = time at which #2 catches #1.

$\Delta t_1 = t - 0 = t$, But $\Delta t_2 = t - 2s$ ← 2s shorter.

KEY: #2 CATCHES #1 WHEN $X_{f_2} = X_{f_1}$

$$X_{f_2} = X_{i_2} + V_{i_2}\Delta t_2 + \frac{1}{2}a_2 \Delta t_2^2 \Rightarrow X_{f_2} = 0 + 0 + \frac{1}{2}(8m/s^2)(t-2s)^2$$

$$\Rightarrow X_{f_2} = 4m/s^2 (t-2s)^2$$

$$X_{f_1} = X_{i_1} + V_{i_1}\Delta t_1 + \frac{1}{2}a_1 \Delta t_1^2 = 0 + 0 + \frac{1}{2}(2m/s^2)(t)^2$$

$$\Rightarrow X_{f_1} = 1m/s^2\, t^2$$

$$X_{f_2} = X_{f_1} \Rightarrow 4m/s^2 (t-2s)^2 = 1m/s^2\, t^2 \Rightarrow (t-2s)^2 = \frac{1m/s^2}{4m/s^2}t^2$$

$$\Rightarrow \sqrt{(t-2s)^2} = \sqrt{.25t^2} \Rightarrow t - 2s = .5t$$

$$\Rightarrow t - .5t = 2s \Rightarrow t(1-.5) = 2s \Rightarrow t(.5) = 2s$$

$$\Rightarrow \boxed{t = \frac{2s}{.5} = 4s}$$

b) HOW FAR?

$$X_{f_1} = (1\,m/s^2)t^2 = (1\,m/s^2)(4s)^2 = \underline{\underline{16m}}$$

AND to CHECK $X_{f_2} = 4m/s^2\,(t-2s)^2 = 4m/s^2\,(4s-2s)^2$

$$= 4m/s^2\,(2s)^2 = \underline{\underline{16m}}$$

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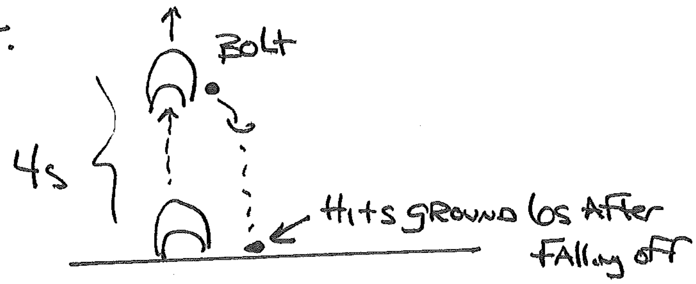

WHAT WAS ROCKET'S ACCELERATION?

KEY: THE BOLT'S INITIAL VELOCITY AND POSITION IS THE SAME AS THE ROCKET'S AT $\Delta t = 4s$

ROCKET: $V_{iR} = 0$ ← LAUNCHED FROM REST, $X_{iR} = 0$ ← LAUNCHED FROM GROUND

$\Rightarrow V_{FR} = V_{iR} + a_R \Delta t \Rightarrow V_{FR} = 0 + a_R(4s) = a_R(4s)$

$X_{FR} = X_{iR} + V_{iR}\Delta t + \frac{1}{2}a_R \Delta t^2 \Rightarrow X_{FR} = 0 + 0 + \frac{1}{2}(a_R)(4s)^2 = a_R(8s^2)$

$\therefore$ BOLT: $X_{iB} = a_R(8s^2)$, $V_{iB} = a_R(4s)$, $X_{FB} = 0$ ← HITS GROUND

$\Delta t = 6s$, $a_B = -9.8 m/s^2$ ← BOLT IN FREE FALL

$X_{FB} = X_{iB} + V_{iB}\Delta t + \frac{1}{2}a_B \Delta t^2 \Rightarrow 0 = a_R(8s^2) + a_R(4s)(6s) + \frac{1}{2}(-9.8 m/s^2)(6s)^2$

$\Rightarrow 0 = a_R(8s^2) + a_R(24s^2) - 176.4m$

$\Rightarrow a_R(8s^2 + 24s^2) = 176.4m$

$\Rightarrow a_R(32s^2) = 176.4m \Rightarrow a_R = \frac{176.4m}{32s} = 5.5125 m/s^2$

$= \underline{5.5 m/s^2}$