Solution to HW#7

March 22, 2007

19-1

a) From Figure 19-2 from text, the luminous efficiencies of the two lasers are,

\[ V(\lambda = 632.8 \text{ nm}) = 0.2 \quad V(\lambda = 441.6 \text{ nm}) = 0.02 \]

So the luminous flux ratio (equal to illuminance ratio when spot sizes are the same) is,

\[ \frac{\Phi_{\text{He-Cd}}}{\Phi_{\text{He-Ne}}} = \frac{685 \times 0.02 \times 50 \text{ mW}}{685 \times 0.2 \times 4 \text{ mW}} = 1.25 \]

The He-Cd laser appears about 1.25× brighter.

b) The luminous efficiencies are \( V(488 \text{ nm}) = 0.2 \) and \( V(543.5 \text{ nm}) = 0.95 \). We require that

\[ \Phi_{\text{argon}} = \Phi_{\text{He-Ne}} \Rightarrow P_{\text{argon}} \times 685 \times 0.2 = 0.5 \text{ mW} \times 685 \times 0.95 \]

\[ P_{\text{argon}} = 2.4 \text{ mW} \]

19-5

a) The sun subtends a solid angle of \( \Omega \) to the earth, which is

\[ \Omega = \frac{\pi (\frac{\theta}{2})^2}{r^2} = \frac{\pi \theta^2}{4} \]

The luminance of the sun is, by definition

\[ L_\nu = \frac{E_\nu}{\Omega} = \frac{10^5 \text{ lx}}{\frac{\pi}{4} \times (0.5/180 \times 3.14 \text{ rad})^2} = 1.68 \times 10^9 \text{ lx/sr} \]

b) \[ E_\nu = L_\nu \times 2\pi = 2\pi L \]
a) 

\[ M_{sys} = M_{\text{back lens surface}} \times M_{\text{translation } S_{23}} \times M_{\text{front lens surface}} \times M_{\text{translation } S_{12}} \times M_{\text{cornea surface}} = \]

\[
\begin{pmatrix}
1 & 0 \\
\frac{n_3 - n_4}{r_3 n_4} & \frac{n_3}{n_4}
\end{pmatrix}
\times
\begin{pmatrix}
1 & S_{23} \\
0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
1 & 0 \\
\frac{n_2 - n_3}{r_2 n_3} & \frac{n_2}{n_3}
\end{pmatrix}
\times
\begin{pmatrix}
1 & S_{12} \\
0 & 1
\end{pmatrix}
\times
\begin{pmatrix}
1 & 0 \\
\frac{n_1 - n_2}{r_1 n_2} & \frac{n_1}{n_2}
\end{pmatrix}
\]

\( n_1 = 1: \) air  
\( n_2 = 1.333: \) anterior chamber  
\( n_3 = 1.45: \) lens  
\( n_4 = 1.333: \) vitreous chamber  
\( r_1 = 8 \text{ mm}: \) radius of curvature of the cornea surface  
\( r_2 = 10 \text{ mm}: \) radius of curvature of the front lens surface  
\( r_3 = -6 \text{ mm}: \) radius of curvature of the back lens surface  
\( S_{12} = 3.6 \text{ mm}: \) distance from corneal vertex to front lens surface  
\( S_{23} = 3.6 \text{ mm}: \) distance from front lens surface to the back lens surface

Plug in numbers and the resulting system matrix is

\[ M_{sys} = \begin{pmatrix} 0.75846 & 5.105 \\ -0.0501 & 0.6518 \end{pmatrix} \]

b) According to Table 18-2 of the textbook, everything w.r.t the input, output planes:

front focal point to corneal vertex:

\[ F_1: \quad p = \frac{D}{C} = \frac{0.6518}{-0.0501} = -13.04 \text{ mm} \]

back focal point to back lens vertex:

\[ F_2: \quad q = -\frac{A}{C} = \frac{-0.75846}{-0.0501} = 15.18 \text{ mm} \]

front principle plane to corneal vertex:

\[ H_1: \quad r = \frac{D - n_1 / n_4}{C} = \frac{0.652 - 1/1.333}{-0.0501} = 1.963 \text{ mm} \]
back principle plane to back lens vertex:

\[ H_2 : \quad s = \frac{1 - A}{C} = \frac{1 - 0.75846}{-0.0501} = -4.82 \text{ mm} \]

Thus, from the corneal vertex (input plane)

\[ F_1 : \quad 13.04 \text{ mm} \]

\[ F_2 : \quad (15.18 + 7.2) = 22.38 \text{ mm} \]

\[ H_1 : \quad 1.964 \text{ mm} \]

\[ H_2 : \quad (7.2 - 4.82) = 2.38 \text{ mm} \]

19-13

a) The contact lens images the far point to infinity.

\[ S' = -50 \text{ cm}, \quad S = +\infty \]

\[ \frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \]

So the contact lens has \( f = -50 \text{ cm} \), \( p = 1/f = -2.00 \text{ D} \).

b) The contact lens in part a) will image the new near point to the myopic person’s near point \( S' = 15 \text{ cm} \) \((f = -50 \text{ cm})\)

\[ \frac{1}{S} + \frac{1}{S'} = \frac{1}{f} \]

\[ S = 21.43 \text{ cm} \]

c) If the corrective lens is 2 cm from the eye, in part a)

\[ S' \Rightarrow -(50 - 2) \text{ cm} = -48 \text{ cm} \]

\( S \) is still at infinity. \( f = -48 \text{ cm} \), \( P = 1/f = -2.08 \text{ D} \)

In part b),

\[ S' \Rightarrow -(15 - 2) \text{ cm} = -13 \text{ cm} \]

with \( f = -48 \text{ cm} \), \( S = 17.83 \text{ cm} \), and the new near point is \((17.83+2) = 19.83 \text{ cm}\) in front of the eye.
Assume that the distance between the corrective lens and the eye is 1.5 cm. To correct for the far point,

\[ \frac{1}{\infty} + \frac{1}{-(15 - 1.5) \text{ cm}} = \frac{1}{f_1} \Rightarrow f_1 = -13.5 \text{ cm} \]

So the front focal power is \( P_1 = 1/f_1 = -7.41 \) D. To correct for the near point, (comfortable reading distance being 40 cm)

\[ \frac{1}{(40 - 1.5) \text{ cm}} + \frac{1}{-(13 - 1.5) \text{ cm}} = \frac{1}{f_2} \Rightarrow f_2 = -16.40 \text{ cm} \]

So the back focal power is \( P_2 = 1/f_2 = -6.10 \) D.